GATE ASSIGNMENT-1

AI25BTECH11004-B.JASWANTH

General Aptitude (GA)

Q.1. "I have not yet decided what I will do this evening; I __________ visit a friend." (GATE ST 2023)

(A) mite

(B) would

(C) might

(D) didn't

Q.2. Eject : Insert :: Advance : __________
(By word meaning) (GATE ST 2023)

(A) Advent

(B) Progress

(C) Retreat

(D) Loan

Q.3. In the given figure, PQRSTV is a regular hexagon with each side of length 5 cm. A circle is drawn with its centre at V such that it passes through P. What is the area (in cm^2) of the shaded region? (The diagram is representative) (GATE ST 2023)

images/Q.3image.png

(A) $\frac{25\pi}{3}$

(B) $\frac{20\pi}{3}$

(C) 6π

(D) 7π

Q.4. A duck named Donald Duck says "All ducks always lie."
Based only on the information above, which one of the following statements can be logically inferred with certainty? (GATE ST 2023)

(A) Donald Duck always lies.

(B) Donald Duck always tells the truth.

(C) Donald Duck's statement is true.

(D) Donald Duck's statement is false.

Q.5. A line of symmetry is defined as a line that divides a figure into two parts in a way such that each part is a mirror image of the other part about that line.

The figure below consists of 20 unit squares arranged as shown. In addition to the given black squares, upto 5 more may be coloured black. Which one among the following options depicts the minimum number of boxes that must be coloured black to achieve two lines of symmetry? (The figure is representative) (GATE ST 2023)

images/Q.5image.png

(A) d

(B) c, d, i

(C) c, i

(D) c, d, i, f, g

Q.6. Based only on the truth of the statement 'Some humans are intelligent', which one of the following options can be logically inferred with certainty? (GATE ST 2023)

(A) No human is intelligent.

(B) All humans are intelligent.

(C) Some non-humans are intelligent.

(D) Some intelligent beings are humans.

Q.7. Which one of the options can be inferred about the mean, median, and mode for the given probability distribution (i.e., probability mass function), $P(x)$, of a variable x? (GATE ST 2023)

images/Q.7image.png

(A) mean = median $\neq$ mode

(B) mean = median = mode

(C) mean $\neq$ median = mode

(D) mean $\neq$ mode = median

Q.8. The James Webb telescope, recently launched in space, is giving humankind unprecedented access to the depths of time by imaging very old stars formed almost 13 billion years ago. Astrophysicists and cosmologists believe that this odyssey in space may even shed light on the existence of dark matter. Dark matter is supposed to interact only via the gravitational interaction and not through the electromagnetic-, the weak- or the strong-interaction. This may justify the epithet “dark” in dark matter.

Based on the above paragraph, which one of the following statements is FALSE? (GATE ST 2023)

(A) No other telescope has captured images of stars older than those captured by the James Webb telescope.

(B) People other than astrophysicists and cosmologists may also believe in the existence of dark matter.

(C) The James Webb telescope could be of use in the research on dark matter.

(D) If dark matter was known to interact via the strong-interaction, then the epithet “dark” would be justified.

Q.9. Let $a = 30!$, $b = 50!$, and $c = 100!$. Consider the following numbers: $\log_a c$, $\log_c a$, $\log_b a$, $\log_a b$.
Which one of the following inequalities is CORRECT? (GATE ST 2023)

(A) $\log_c a < \log_b a < \log_a b < \log_a c$

(B) $\log_c a < \log_a b < \log_b a < \log_b c$

(C) $\log_c a < \log_b a < \log_a c < \log_a b$

(D) $\log_b a < \log_c a < \log_a b < \log_a c$

Q.10. A square of side length 4 cm is given. The boundary of the shaded region is defined by one semi-circle on the top and two circular arcs at the bottom, each of radius 2 cm, as shown.
The area of the shaded region is ________________ cm^2. (GATE ST 2023)

images/Q.10image.png

(A) 8

(B) 4

(C) 12

(D) 10

Q.11. The area of the region bounded by the parabola $x = -y^2$ and the line $y = x + 2$ equals (GATE ST 2023)

(A) $\frac{3}{2}$

(B) $\frac{7}{2}$

(C) $\frac{9}{2}$

(D) 9

Q.12. Let A be a 3×3 real matrix having eigenvalues $1, 0, -1$. If $B = A^2 + 2A + I_3$, where I_3 is the 3×3 identity matrix, then which one of the following statements is true? (GATE ST 2023)

(A) $B^3 - 5B^2 + 4B = 0$

(B) $B^3 - 5B^2 - 4B = 0$

(C) $B^3 + 5B^2 - 4B = 0$

(D) $B^3 + 5B^2 + 4B = 0$

Q.13. Consider the following statements:

(I) Let A and B be two $n \times n$ real matrices. If B is invertible, then $\text{rank}(BA) = \text{rank}(A)$.

(II) Let A be an $n \times n$ real matrix. If $A^2\mathbf{x} = \mathbf{b}$ has a solution for every $\mathbf{b} \in \mathbb{R}^n$, then $A\mathbf{x} = \mathbf{b}$ also has a solution for every $\mathbf{b} \in \mathbb{R}^n$.

Which of the above statements is/are true? (GATE ST 2023)

(A) Only (I)

(B) Only (II)

(C) Both (I) and (II)

(D) Neither (I) nor (II)

Q.14. Consider the probability space $(\Omega, \mathcal{G}, P)$, where $\Omega = [0,2]$ and $\mathcal{G} = \{\varnothing, \Omega, [0,1], (1,2]\}$. Let X and Y be two functions on Ω defined as

and

$X(\omega) = \begin{cases} 1 & \omega \in [0,1] \\ 2 & \omega \in (1,2] \end{cases}$

$Y(\omega) = \begin{cases} 2 & \omega \in [0,1.5] \\ 3 & \omega \in (1.5,2] \end{cases}$ Then which one of the following statements is true? (GATE ST 2023)

(A) X is a random variable with respect to $\mathcal{G}$, but Y is not a random variable with respect to $\mathcal{G}$

(B) Y is a random variable with respect to $\mathcal{G}$, but X is not a random variable with respect to $\mathcal{G}$

(C) Neither X nor Y is a random variable with respect to $\mathcal{G}$

(D) Both X and Y are random variables with respect to $\mathcal{G}$

Q.15. Let $\Phi(\cdot)$ denote the cumulative distributive function of a standard normal random variable. If the random variable X has the cumulative distribution function

$$F(x) = \begin{cases} \Phi(x) & x < -1 \\ \Phi(x+1) & x \geq -1 \end{cases}$$

then which one of the following statements is true? (GATE ST 2023)

(A) $P(X \leq -1) = \frac{1}{2}$

(B) $P(X = -1) = \frac{1}{2}$

(C) $P(X < -1) = \frac{1}{2}$

(D) $P(X \leq 0) = \frac{1}{2}$

Q.16. Let X be a random variable with probability density function

$$f(x) = \begin{cases} \alpha\lambda x^{\alpha-1} e^{-\lambda x^{\alpha}}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

where $\alpha > 0$and $\lambda > 0$. If the median of X is 1 and the third quantile is 2, then (α, λ) equals (GATE ST 2023)

(A) $(1, \log_e 2)$

(B) $(1, 1)$

(C) $(2, \log_e 2)$image

(D) $(1, \log_e 3)$

Q.17. Let X be a random variable having poission distribution with mean $\lambda > 0$. Then $E\left(\frac{1}{X+1} \middle| X > 0\right)$ equals (GATE ST 2023)

(A) $\frac{1-e^{-\lambda}-\lambda e^{-\lambda}}{\lambda(1-e^{-\lambda})}$

(B) $\frac{1-e^{-\lambda}}{\lambda}$

(C) $\frac{1-e^{-\lambda}-\lambda e^{-\lambda}}{\lambda}$

(D) $\frac{1-e^{-\lambda}}{\lambda+1}$

Q.18. Suppose that X has the probability density function

$$f(x) = \begin{cases} \frac{\lambda^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

where $\alpha > 0$ and $\lambda > 0$. Which one of the following statements is NOT true? (GATE ST 2023)

(A) $E(X)$ exists for all $\alpha > 0$ and $\lambda > 0$

(B) Variance of X exists for all $\alpha > 0$ and $\lambda > 0$

(C) $E(1/X)$ exists for all $\alpha > 0$ and $\lambda > 0$

(D) $E(\log_e(1 + X))$ exists for all $\alpha > 0$ and $\lambda > 0$

Q.19. Let (X, Y) have joint probability density function

$$f(x, y) = \begin{cases} 8xy, & 0 < x < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

If $E(X \mid Y = y_0) = \frac{1}{2}$, then y_0 equals (GATE ST 2023)

(A) $\frac{3}{4}$

(B) $\frac{1}{2}$

(C) $\frac{1}{3}$

(D) $\frac{2}{3}$

Q.20. Suppose there are 5 boxes, each containing 3 blue pens, 1 red pen, and 2 black pens. One pen is drawn at random from each of these 5 boxes.If the random variable X_1 denotes the total number of blue pens drawn and the random variable X_2 denotes the total number of red pens drawn, Then $P(X_1 = 2, X_2 = 1)$ equals (GATE ST 2023)

(A) $\frac{5}{36}$

(B) $\frac{5}{18}$

(C) $\frac{5}{12}$

(D) $\frac{5}{9}$

Q.21. Let $\{X_n\}_{n\geq1}$ and $\{Y_n\}_{n\geq1}$ be two sequences of random variables and X and Y be two random variables, all of them defined on the same probability space. Which one of the following statements is true? (GATE ST 2023)

(A) If $\{X_n\}_{n\geq1}$ converges in distribution to a real constant c, then $\{X_n\}_{n\geq1}$ converges in probability to c.

(B) If $\{X_n\}_{n\geq1}$ converges in probability to X, then $\{X_n\}_{n\geq1}$ converges in 3rd mean to X.

(C) If $\{X_n\}_{n\geq 1}$ converges in distribution to X and $\{Y_n\}_{n\geq 1}$ converges in distribution to Y, then $\{X_n+Y_n\}_{n\geq 1}$ converges in distribution to $X+Y$.

(D) If $\{E(X_n)\}_{n\geq 1}$ converges to $E(X)$, then $\{X_n\}_{n\geq 1}$ converges in 1st mean to X.

Q.22. Let X be a random variable with probability density function

$$f(x;\lambda)=\begin{cases}\frac{1}{\lambda}e^{-x/\lambda}, & x>0\\ 0, & \text{otherwise}\end{cases}$$

where $\lambda>0$ is an unknown parameter. Let $Y_1,\ldots,Y_n$ be a random sample of size n from a population having the same distribution as X^2.
If $\overline{Y}=\frac{1}{n}\sum_{i=1}^{n}Y_i$, then which one of the following statements is true? (GATE ST 2023)

(A) $\sqrt{\overline{Y}/2}$ is a method of moments estimator of λ

(B) $\sqrt{\overline{Y}}$ is a method of moments estimator of λ

(C) $\frac{1}{2}\sqrt{\overline{Y}}$ is a method of moments estimator of λ

(D) $2\sqrt{\overline{Y}}$ is a method of moments estimator of λ

Q.23. Let $X_1,\ldots,X_n$ be a random sample of size n(≥ 2) from a population having probability density function

$$f(x;\theta)=\begin{cases}\frac{2}{\theta x(-\ln x)}e^{-\frac{(\ln x)^2}{\theta}}, & 0<x<1\\ 0, & \text{otherwise}\end{cases}$$

where $\theta>0$ is an unknown parameter.Then which one of the following statements is true? (GATE ST 2023)

(A) $\frac{1}{n}\sum_{i=1}^{n}(\ln X_i)^2$ is the maximum likelihood estimator of θ

(B) $\frac{1}{n-1}\sum_{i=1}^{n}(\ln X_i)^2$ is the maximum likelihood estimator of θ

(C) $\frac{1}{n}\sum_{i=1}^{n}\ln X_i$ is the maximum likelihood estimator of θ

(D) $\frac{1}{n-1}\sum_{i=1}^{n}\ln X_i$ is the maximum likelihood estimator of θ

Q.24. Let X_1,X_2,.....,X_n be a random Sample of size n from a population having uniform distribution over the interval $(1/3,\theta)$,where $\theta>1/3$ is an unknown parameter.If Y= max$\{X_1,X_2,....X_n\}$,then Which one of the following statements is true? (GATE ST 2023)

(A) $\left(\frac{n+1}{n}\right)(Y-\frac{1}{3})+\frac{1}{3}$ is an unbiased estimator 0f θ

(B) $\left(\frac{n}{n+1}\right)(Y-\frac{1}{3})+\frac{1}{3}$ is an unbiased estimator 0f θ

(C) $\left(\frac{n+1}{n}\right)(Y+\frac{1}{3})-\frac{1}{3}$ is an unbiased estimator 0f θ

(D) Y is an unbiased estimator 0f θ

Q.25. Suppose that $X_1,X_2,\ldots,X_n,Y_1,Y_2,\ldots,Y_n$ are independent and identically distributed random vectors each having $N_p(\mu,\Sigma)$ distribution, where Σ is non-singular, $p>1$ and $n>1$. If

$$\overline{X}=\frac{1}{n}\sum_{i=1}^{n}X_i,\quad \overline{Y}=\frac{1}{n}\sum_{i=1}^{n}Y_i,$$

then which one of the following statements is true? (GATE ST 2023)

(A) There exists $c>0$ such that $c\,(\overline{X}-\mu)^{\top}\Sigma^{-1}(\overline{X}-\mu)$ has χ^2 distribution with p degrees of freedom

(B) There exists $c > 0$ such that $c\,(\overline{X} - \overline{Y})^\top \Sigma^{-1} (\overline{X} - \overline{Y})$ has χ^2 distribution with $(p-1)$ degrees of freedom

(C) There exists $c > 0$ such that $c\sum_{i=1}^{n} (X_i - \overline{X})^\top \Sigma^{-1} (X_i - \overline{X})$ has χ^2 distribution with p degrees of freedom

(D) There exists $c > 0$ such that $c\sum_{i=1}^{n} (X_i - Y_i - \overline{X} + \overline{Y})^\top \Sigma^{-1} (X_i - Y_i - \overline{X} + \overline{Y})$ has χ^2 distribution with p degrees of freedom

Q.26. Consider the following regression model

$$y_k = \alpha_0 + \alpha_1 \log_e k + \epsilon_k, \quad k = 1, 2, \ldots, n,$$

where ϵ_k's are independent and identically distributed random variables each having probability density function

$$f(x) = \frac{1}{2} e^{-|x|}, \quad x \in \mathbb{R}.$$

Then which one of the following statements is true? (GATE ST 2023)

(A) The maximum likelihood estimator of α_0 does not exist

(B) The maximum likelihood estimator of α_1 does not exist

(C) The least squares estimator of α_0 exists and is unique

(D) The least squares estimator of α_1 exists, but it is not unique
S

Q.27. uppose that $X_1, X_2, \ldots, X_n$ are independent and identically distributed random variables each having probability density function $f(\cdot)$ and median θ. We want to test

$$H_0 : \theta = \theta_0 \quad \text{against} \quad H_1 : \theta > \theta_0.$$

Consider a test that rejects H_0 if $S > c$ for some c depending on the size of the test, where S is the cardinality of the set $\{i : X_i > \theta_0,\ 1 \le i \le n\}$. Then which one of the following statements is true? (GATE ST 2023)

(A) Under H_0, the distribution of S depends on $f(\cdot)$

(B) Under H_1, the distribution of S does not depend on $f(\cdot)$

(C) The power function depends on θ

(D) The power function does not depend on θ

Q.28. Suppose that x is an observed sample of size 1 from a population with probability density function $f(\cdot)$. Based on x, consider testing

$$H_0 : f(y) = \frac{1}{\sqrt{2\pi}} e^{-y^2/2}, \quad y \in \mathbb{R} \quad \text{against} \quad H_1 : f(y) = \frac{1}{2} e^{-|y|}, \quad y \in \mathbb{R}.$$

Then which one of the following statements is true? (GATE ST 2023)

(A) The most powerful test rejects H_0 if $|x| > c$ for some $c > 0$

(B) The most powerful test rejects H_0 if $|x| < c$ for some $c > 0$

(C) The most powerful test rejects H_0 if $\big||x| - 1\big| > c$ for some $c > 0$

(D) The most powerful test rejects H_0 if $\big||x| - 1\big| < c$ for some $c > 0$

Q.29. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x, y) = xy$. Then the maximum value (rounded off to two decimal places) of f on the ellipse

$$x^2 + 2y^2 = 1$$

equals ____________. (GATE ST 2023)

Q.30. Let A be 2×2 real matrix such that $AB = BA$ for all 2×2 real B. If trace(A) equals 5, then det(A) (rounded off to two decimal places) equals ________________ (GATE ST 2023)

Q.31. Two defective bulbs are present in a set of five bulbs. To remove the two defective bulbs, the bulbs are chosen randomly one by one and tested. If X denotes the minimum number of bulbs that must be tested to find out the two defective bulbs, then $P(X = 3)$ (rounded off to two decimal places) equals ________________ (GATE ST 2023)

Q.32. Let $\{X_n\}_{n\geq 1}$ be a sequence of independent and identically distributed random variables each having mean 4 and variance 9. If

$$Y_n = \frac{1}{n}\sum_{i=1}^{n} X_i, \quad n \geq 1,$$

then $\lim_{n\to\infty} E\left[\left(\frac{Y_n - 4}{\sqrt{n}}\right)^2\right]$ (in integer) equals ________________ (GATE ST 2023)

Q.33. Let $\{W_t\}_{t\geq 0}$ be a standard Brownian motion. Then $E(W_4^2 \mid W_2 = 2)$ (in integer) equals ________________ GATE ST 2023

Q.34. Let $\{X_n\}_{n\geq 1}$ be a Markov chain with state space $\{1, 2, 3\}$ and transition probability matrix

$$\begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/3 & 1/3 & 1/3 \\ 0 & 1/2 & 1/2 \end{bmatrix}.$$

Then $P(X_2 = 1 \mid X_1 = 1, X_3 = 2)$ (rounded off to two decimal places) equals ________________ (GATE ST 2023)

Q.35. Suppose $(X_1, X_2, X_3) has N_3(\mu, \Sigma)$ distribution with $\mu = (0, 0, 0)^\top$ and

$$\Sigma = \begin{bmatrix} 2 & 2 & 1 \\ 2 & 5 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

Given that $\Phi(-0.5) = 0.3085$, where $\Phi(\cdot)$ denotes the cumulative distributive function of a standard normal random variable, $P\left((X_1 - 2X_2 + 2X_3)^2 < \frac{7}{2}\right)$ (rounded off to two decimal places) equals ________________ (GATE ST 2023)

Q.36. Let A be an $n \times n$ real matrix. Consider the following statements:

(I) If A is symmetric, then there exists $c \geq 0$ such that $A + cI_n$ is symmetric and positive definite,where I_n is the $n \times n$ identity matrix.

(II) If A is symmetric and positive definite, then there exists a symmetric and positive definite B such that $A = B^2$.

Which of the above statements is/are true? (GATE ST 2023)

(A) Only (I)

(B) Only (II)

(C) Both (I) and (II)

(D) Neither (I) nor (II)

Q.37. Let X be a random variable with probability density function

$$f(x) = \begin{cases} \frac{1}{x^2}, & \text{if } x \geq 1, \\ 0, & \text{otherwise.} \end{cases}$$

If $Y = \log_e X$, then $P(Y < 1 \,|\, Y < 2)$ equals (GATE ST 2023)

(A) $\dfrac{e}{1+e}$

(B) $\dfrac{e^{-1}}{e+1}$

(C) $\dfrac{1}{1+e}$

(D) $\dfrac{1}{e-1}$

Q.38. Let $\{N(t)\}_{t\geq 0}$ be a Poisson process with rate 1.Consider the following statements.

(I) $P(N(3) = 3 \mid N(5) = 5) = \binom{5}{3}(3/5)^3(2/5)^2$.

(II) If S_5 denotes the time of the occurence of the 5th event for the above poission process, then $E(S_5 \mid N(5) = 3) = 7$.

Which of the above statements is/are true? (GATE ST 2023)

(A) Only (I)

(B) Only (II)

(C) Both (I) and (II)

(D) Neither (I) nor (II)

Q.39. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from a population having probability density function

$$f(x;\mu) = \begin{cases} e^{-(x-\mu)}, & \text{if } \mu \leq x < \infty, \\ 0, & \text{otherwise}, \end{cases}$$

where $\mu \in \mathbb{R}$ is an unknown parameter. If $\hat{M}$ is the maximum likelihood estimator of the median of X_1, then which one of the following statements is true? (GATE ST 2023)

(A) $P(\hat{M} \leq 2) = 1 - e^{-n(1-\log_e 2)}$ if $\mu = 1$

(B) $P(\hat{M} \leq 1) = 1 - e^{-n\log_e 2}$ if $\mu = 1$

(C) $P(\hat{M} \leq 3) = 1 - e^{-n(1-\log_e 2)}$ if $\mu = 1$

(D) $P(\hat{M} \leq 4) = 1 - e^{-n(2\log_e 2-1)}$ if $\mu = 1$

Q.40. Let X_1,X_2,......,$X_1$0 be a random sample of size 10 from a population having $N(0, \theta^2)$ distribution, where $\theta > 0$ is an unknown parameter.
Let $T = \frac{1}{10}\sum_{i=1}^{10} X_i^2$. If the mean square error of $cT(c > 0)$,as an estimator of θ^2),is minimized at $c = c_0$,then the value of c_0 equals (GATE ST 2023)

(A) $\frac{5}{6}$

(B) $\frac{2}{3}$

(C) $\frac{3}{5}$

(D) $\frac{1}{2}$

Q.41. Suppose that $\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_{10}$ are independent and identically distributed random vectors each having $N_2(\boldsymbol{\mu}, \Sigma)$ distribution, where Σ is non-singular. If

$$U = \frac{1}{1 + (\bar{\mathbf{X}} - \boldsymbol{\mu})^T \Sigma^{-1} (\bar{\mathbf{X}} - \boldsymbol{\mu})},$$

where

$$\bar{\mathbf{X}} = \frac{1}{10}\sum_{i=1}^{10}\mathbf{X}_i,$$

then the value of $\log_e P\left(U \leq \frac{1}{2}\right)$ equals (GATE ST 2023)

(A) -5

(B) -10

(C) -2

(D) -1

Q.42. Suppose that (X, Y) has joint probability mass function

$$P(X = 0, Y = 0) = P(X = 1, Y = 1) = \theta, \quad P(X = 1, Y = 0) = P(X = 0, Y = 1) = \frac{1}{2} - \theta,$$

where $0 \leq \theta \leq \frac{1}{2}$ is an unknown parameter.
Consider testing

$$H_0 : \theta = \frac{1}{4} \quad \text{against} \quad H_1 : \theta = \frac{1}{3},$$

based on a random sample $\{(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)\}$ from the above probability mass function.
Let M be the cardinality of the set $\{i : X_i = Y_i,\ 1 \leq i \leq n\}$. If m is the observed value of M, then which one of the following statements is true? (GATE ST 2023)

(A) The likelihood ratio test rejects H_0 if $m > c$ for some c

(B) The likelihood ratio test rejects H_0 if $m < c$ for some c

(C) The likelihood ratio test rejects H_0 if $c_1 < m < c_2$ for some c_1 and c_2

(D) The likelihood ratio test rejects H_0 if $m < c_1$ or $m > c_2$ for some c_1 and c_2

Q.43. Let $g(x) = f(x) + f(2 - x)$ for all $x \in [0, 2]$, where $f : [0, 2] \to \mathbb{R}$ is continuous on $[0, 2]$ and twice differentiable on $(0, 2)$. If g' denotes the derivative of g and f'' denotes the second derivative of f, then which one of the following statements is **NOT** true? (GATE ST 2023)

(A) There exists $c \in (0, 2)$ such that $g'(c) = 0$

(B) If $f'' > 0$ on $(0, 2)$, then g is strictly decreasing on $(0, 1)$

(C) If $f'' < 0$ on $(0, 2)$, then g is strictly increasing on $(1, 2)$

(D) If $f'' = 0$ on $(0, 2)$, then g is a constant function

Q.44. For subsets $\mathcal{T}, \mathcal{S} \subset \mathbb{R}^n$, let $L(U)$ denote their span. Which is NOT true? (GATE ST 2023)

(A) If $\mathcal{T}$ is proper subset of $\mathcal{S}$ then $L(\mathcal{T}) is a proper subset of L(\mathcal{S})$

(B) $L(L(\mathcal{S})) = L(\mathcal{S})$

(C) $L(\mathcal{T} \cup \mathcal{S}) = \{u + v : u \in L(\mathcal{T}), v \in L(\mathcal{S})\}$

(D) If $\alpha, \beta and \gamma$ are three vectors in R^n such that $\alpha + 2\beta + 3\gamma = 0$, then $L(\{\alpha, \beta\}) = L(\{\beta, \gamma\})$

Q.45. Let f be a continuous function from $[0, 1]$ to the set of all real numbers.Then which one of the following statements is

NOT true? (GATE ST 2023)

(A) For any $\{x_n\}_{n\geq 1}$ in [0,1], $\sum_{n=1}^{\infty} \frac{f(x_n)}{n^2}$ is absolutely convergent

(B) If $|f(x)| = 1$ for all $x \in [0,1]$, then $|\int_0^1 f(x)\,dx| = 1$

(C) If $\{x_n\}_{n\geq 1}$ is a sequence in [0,1] such that $\{f(x_n)\}_{n\geq 1}$ is convergent then $\{x_n\}_{n\geq 1}$ is convergent

(D) If f is also monotonically increasing,then the image of f is given by $[f(0), f(1)]$

Q.46. Let X be a random variable with cumulative distribution function

$$F(x) = \begin{cases} 0, & x < -1, \\ \frac{1}{4}(x+1), & -1 \leq x < 0, \\ \frac{1}{4}(x+3), & 0 \leq x < 1, \\ 1, & x \geq 1. \end{cases}$$

Which one of the following statements is true? (GATE ST 2023)

(A) $\lim_{n\to\infty} P\left(-\frac{1}{2} + \frac{1}{n} < X < -\frac{1}{n}\right) = \frac{5}{8}$

(B) $\lim_{n\to\infty} P\left(-\frac{1}{2} - \frac{1}{n} < X < \frac{1}{n}\right) = \frac{5}{8}$

(C) $\lim_{n\to\infty} P\left(X = \frac{1}{n}\right) = \frac{1}{2}$

(D) $P(X = 0) = \frac{1}{3}$

Q.47. pmf: $p(x, y) = \frac{c}{2^{x+y+2}}$ for $x = 0, 1, \ldots; y = 0, 1, \ldots; x \neq y$. Which is true? (GATE ST 2023)

(A) $c = \frac{1}{2}$

(B) $c = \frac{1}{4}$

(C) $c > 1$

(D) X and Y independent

Q.48. Let $X_1, X_2, \ldots, X_{10}$ be a random sample of size 10 from a $N_3(\mu, \Sigma)$ distribution, where μ and non-singular Σ are unknown parameters. If

$$\overline{X}_1 = \frac{1}{5}\sum_{i=1}^{5} X_i, \quad \overline{X}_2 = \frac{1}{5}\sum_{i=6}^{10} X_i,$$

$$S_1 = \frac{1}{4}\sum_{i=1}^{5}(X_i - \overline{X}_1)(X_i - \overline{X}_1)^\top, \quad S_2 = \frac{1}{4}\sum_{i=6}^{10}(X_i - \overline{X}_2)(X_i - \overline{X}_2)^\top,$$

then which one of the following statements is NOT true? (GATE ST 2023)

(A)

$$\frac{5}{6}(\bar{X}_1 - \mu)^T S_1^{-1}(\bar{X}_1 - \mu) \text{ follows an } F\text{-distribution with 3 and 2 degrees of freedom.}$$

(B)

$$\frac{6}{5}(\bar{X}_1 - \mu)^T S_1^{-1}(\bar{X}_1 - \mu) \text{ follows an } F\text{-distribution with 2 and 3 degrees of freedom.}$$

(C)

$$4\,(S_1 + S_2) \text{ follows a Wishart distribution of order 3 with 8 degrees of freedom.}$$

(D)

$$5\,(S_1 + S_2) \text{ follows a Wishart distribution of order 3 with 10 degrees of freedom.}$$

Q.49. Which of the following sets is/are countable? (GATE ST 2023)

(A) The set of all functions from $\{1, 2, 3, \ldots, 10\}$ to the set of all rational numbers

(B) The set of all functions from the set of all natural numbers to $\{0, 1\}$

(C) The set of all integer-valued sequences with only finitely many nonzero terms

(D) The set of all integer-valued sequences converging to 1

Q.50. For a given real number a, let $a^+ = \max\{a, 0\}$ and $a^- = \max\{-a, 0\}$. If $\{x_n\}_{n\geq1}$ is a sequence of real numbers, then which of the following statements is/are true? (GATE ST 2023)

(A) If $\{x_n\}_{n\geq1}$ converges, then both $\{x_n^+\}_{n\geq1}$ and $\{x_n^-\}_{n\geq1}$ converge

(B) If $\{x_n\}_{n\geq1}$ converges to 0, then both $\{x_n^+\}_{n\geq1}$ and $\{x_n^-\}_{n\geq1}$ converge to 0

(C) If both $\{x_n^+\}_{n\geq1}$ and $\{x_n^-\}_{n\geq1}$ converge, then $\{x_n\}_{n\geq1}$ converges

(D) If $\{x_n^2\}_{n\geq1}$ converges, then both $\{x_n^+\}_{n\geq1}$ and $\{x_n^-\}_{n\geq1}$ converge

Q.51. Let A be a 3×3 real matrix such that

$$A\begin{bmatrix}0\\1\\1\end{bmatrix} = \begin{bmatrix}4\\0\\0\end{bmatrix}, \quad A\begin{bmatrix}1\\0\\1\end{bmatrix} = \begin{bmatrix}0\\4\\0\end{bmatrix} and \quad A\begin{bmatrix}1\\1\\0\end{bmatrix} = \begin{bmatrix}0\\0\\4\end{bmatrix}.$$

Which of the following statements is/are true? (GATE ST 2023)

(A) $A\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}2\\2\\-2\end{bmatrix}$

(B) $A\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}2\\-2\\2\end{bmatrix}$

(C) $A\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}2\\0\\2\end{bmatrix}$

(D) $A\begin{bmatrix}1\\2\\3\end{bmatrix} = \begin{bmatrix}8\\4\\0\end{bmatrix}$

Q.52. Let X be a positive valued continuous random variable with finite mean. If $Y = \lfloor X \rfloor$, the largest integer less than or equal to X,then which of the following statements is/are true? (GATE ST 2023)

(A) $P(Y \leq u) \leq P(X \leq u)$ for all $u \geq 0$

(B) $P(Y \geq u) \leq P(X \geq u)$ for all $u \geq 0$

(C) $E(X) < E(Y)$

(D) $E(X) > E(Y)$

Q.53. Let X be a random variable with probability density function

$$f(x) = \begin{cases} e^{-x}, & \text{if } x \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

For $a < b$, if $U(a, b)$ denotes the uniform distribution over the interval (a, b), then which of the following statements is/are true? (GATE ST 2023)

(A) e^{-X} follows $U(-1, 0)$ distribution

(B) $1 - e^{-X}$ follows $U(0, 2)$ distribution

(C) $2e^{-X} - 1$ follows $U(-1, 1)$ distribution

(D) The probability mass function of $Y = [X]$ is

$$P(Y = k) = (1 - e^{-1})\, e^{-k}, \quad k = 0, 1, 2, \ldots,$$

where $[x]$ denotes the largest integer not exceeding x

Q.54. Suppose that X is a disccrete random variable with the following probability mass function

$P(X = 0) = \frac{1}{2}(1 + e^{-1})$

$P(X = k) = \frac{e^{-1}}{2\,k!}$, $k = 1, 2, 3, \ldots.$

Which of the following is/are true? (GATE ST 2023)

(A) $E(X) = 1$

(B) $E(X) < 1$

(C) $E(X \mid X > 0) < \frac{1}{2}$

(D) $E(X \mid X > 0) > \frac{1}{2}$

Q.55. Suppose that U and V are two independent and identically distributed random variables each having probability density function

$$f(x) = \begin{cases} \lambda^2 x e^{-\lambda x}, & if\, x > 0 \\ 0, & otherwise \end{cases}$$

where $\lambda > 0$. Which of the following statements is/are true? (GATE ST 2023)

(A) The distribution of $U - V$ is symmetric about 0

(B) The distribution of UV does not depend on λ

(C) The distribution of U/V does not depend on λ

(D) The distribution of U/V is symmetric about 1

Q.56. (X, Y) have joint probability mass function

$$p(x, y) = \begin{cases} \dfrac{e^{-2}}{x!(y - x)!}, & if \text{ x=0,1,\ldots,y; y=0,1,2,\ldots,} \\ 0, & \text{otherwise.} \end{cases}$$

Then which of the following statements is/are true? (GATE ST 2023)

(A) $E(X \mid Y = 4) = 2$

(B) The moment generating function of Y is $e^{2(e^v-1)}$ for all $v \in \mathbb{R}$

(C) $E(X) = 2$

(D) The joint moment generating function of (X, Y) is $e^{-2+(1+e^u)e^v}$ for all $(u, v) \in \mathbb{R}^2$

Q.57. Let $\{X_n\}_{n\geq 1}$ be a sequence of independent and identically distributed random variables with mean 0 and variance 1, all of them defined on the same probability space. For $n = 1, 2, 3, \ldots,$ let

$$Y_n = \frac{1}{n}\left(X_1X_2 + X_3X_4 + \cdots + X_{2n-1}X_{2n}\right).$$

Then which of the following statements is/are true? (GATE ST 2023)

(A) $\{\sqrt{n}\, Y_n\}_{n\geq 1}$ converges in distribution to a standard normal random variable

(B) $\{Y_n\}_{n\geq 1}$ converges in 2nd mean to 0

(C) $\left\{Y_n + \frac{1}{n}\right\}_{n\geq 1}$ converges in probability to 0

(D) $\{X_n\}_{n\geq 1}$ converges almost surely to 0

Q.58. Consider the following regression model

$y_t = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + \epsilon_t$, t=1,2,.......,100,

where α_0,$\alpha_1$and α_2 are unknown parameters and ϵ_t 's are independent and identically distributed random variables each having N(μ,1) distribution with $\mu \in \mathbb{R}$ unknown.Then which of the following statements is/are true? (GATE ST 2023)

(A) There exists an unbiased estimator of α_1

(B) There exists an unbiased estimator of α_2

(C) There exists an unbiased estimator of α_0

(D) There exists an unbiased estimator of μ

Q.59. Consider the orthonormal set

$$v_1 = \left\{ \begin{bmatrix} \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix}, \quad v_2 = \begin{bmatrix} \frac{1}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \end{bmatrix}, \quad v_3 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{bmatrix}.\right.$$

with respect to the standard innner product on R^3. If $u = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ is the vector such that inner products of u with v_1,v_2 and v_3 ,respectively,then $a^2 + b^2 + c^2$ (integer) equals ________________ (GATE ST 2023)

Q.60. Consider the probability space $(\Omega, \mathcal{G}, P)$, $\Omega = \{1, 2, 3, 4\}$, $\mathcal{G} = \{\varnothing, \Omega, \{1\}, \{4\}, \{2, 3\}, \{1, 4\}, \{1, 2, 3\}, \{2, 3, 4\}\}$,and $P(\{1\}) = \frac{1}{4}$. $X(1) = 1$, $X(2) = X(3) = 2$and $X(4) = 3$. If $P(X \leq 2) = \frac{3}{4}$, then $P(\{1, 4\})$ (rounded off to two decimal places) equals ________________ (GATE ST 2023)

Q.61. Let $\{X_n\}_{n\geq 1}$ be a sequence of independent and identically distributed random variables each having probability density function

$$f(x) = \begin{cases} e^{-x}, & \text{if } x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

For $n \geq 1$, let $Y_n = |X_{2n} - X_{2n-1}|$.If$\bar{Y}_n = \frac{1}{n}\sum_{i=1}^{n} Y_i, \quad n \geq 1$, and $\{\sqrt{n}\,(e^{-\bar{Y}_n} - e^{-1})\}_{n\geq 1}$ converges in distribution to a normal random variable with mean 0 and variance σ^2, then σ^2 (rounded off to two decimal places) equals ___________.
GATE ST 2023

Q.62. Consider a birth-death process states $\{0, 1, 2, 3\}$.The birth rates are given by $\lambda_0 = 1$, $\lambda_1 = 1$, $\lambda_2 = 2$ and $\lambda_3 = 0$.The death rates are given by $\mu_0 = 0$, $\mu_1 = 1$, $\mu_2 = 1$ and $\mu_3 = 1$. If $[\pi_0, \pi_1, \pi_2, \pi_3]$ is the unique stationary distribution,then $\pi_0 + 2\pi_1 + 3\pi_2 + 4\pi_3$ (rounded off the two decimal places) equals _________________ (GATE ST 2023)

Q.63. Let $\{-1, -\frac{1}{2}, 1, \frac{5}{2}, 3\}$ be a realization of a random sample of size 5 from a population having $N\left(\frac{1}{2}, \sigma^2\right)$ distribution, where $\sigma > 0$ is an unknown parameter. Let T be an unbiased estimator of σ^2 whose variance attains the Cramer-Rao lower bound. Then based on the above data, the realized value of T (rounded off to two decimal places) equals ___________.
GATE ST 2023

Q.64. Let X be a random sample of size 1 from a population with cumulative distribution function

$$F(x) = \begin{cases} 0, & \text{if } x < 0, \\ 1 - (1-x)^{\theta}, & \text{if } 0 \leq x < 1, \\ 1, & \text{if } x \geq 1, \end{cases}$$

where $\theta > 0$ is an unknown parameter. To test $H_0 : \theta = 1$ against $H_1 : \theta = 2$, consider using the critical region

$$\{x \in \mathbb{R} : x < 0.5\}.$$

If α and β denote the level and power of the test, respectively, then $\alpha + \beta$ (rounded off to two decimal places) equals ___________. (GATE ST 2023)

Q.65. Let $\{0.13,\ 0.12,\ 0.78,\ 0.51\}$ be a realization of a random sample of size 4 from a population with cumulative distribution function $F(\cdot)$. Consider testing

$$H_0 : F = F_0 \quad \text{against} \quad H_1 : F \neq F_0,$$

where

$$F_0(x) = \begin{cases} 0, & \text{if } x < 0, \\ x, & \text{if } 0 \leq x < 1, \\ 1, & \text{if } x \geq 1. \end{cases}$$

Let D denote the Kolmogorov-Smirnov test statistic. If $P(D > 0.669) = 0.01$ under H_0 and

$$\psi = \begin{cases} 1, & \text{if } H_0 \text{ is accepted at level } 0.01, \\ 0, & \text{otherwise,} \end{cases}$$

then, based on the given data, the observed value of $D + \psi$ (rounded off to two decimal places) equals ___________.
GATE ST 2023