Design and Finite Element Analysis of Varying Width Piezoelectric Cantilever Beam to Harvest Energy

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***Abstract*— The main aim of this work is to enhance the conversion of mechanical energy into electrical energy by using developed 3D finite element analysis of varying width piezoelectric cantilever energy harvester designed using block pulse functions (BPFs). The piezoelectric cantilever beam is subjected to harmonic base excitation by applying vertical acceleration of 0.2g through body load. The width of the cantilever beam is distributed at three equal lengths as discretized linear function by applying the block pulse functions. Initial study indicate that piezoelectric cantilever composed of varying width as discretized linear function using block pulse functions (BPFs) is found to generate approximately 2.5 times the voltage per unit base acceleration as generated by rectangular piezoelectric cantilever for same volume of substrate and piezoelectric material and for the same natural frequency. Further harmonic analysis is performed on the piezoelectric cantilever beam designed using BPFs to figure parameters like charge, voltage and energy produced by a unimorph varying width piezoelectric cantilever beam using Comsol Multiphysics.**

Keywords*— *Energy harvesting, Piezoelectricity, Block pulse functions.

I. INTRODUCTION

Vibration based energy harvesting has been investigated by several researchers over the last decade and cantilever beams with piezoelectric material layers are employed as piezoelectric energy harvester due to its simple configuration. Interest in piezoelectric wireless sensors with the advancement in the electronics has been increased manifold and various researches has conducted to optimize the harvested energy for the proper and efficient functioning of the wireless circuits. Although piezoelectric energy harvesting is most promising one for the wireless sensor but the major limitation is generation of sufficient amount of energy for the autonomous operation of the unit [1]- [3].

A large percentage of initial research in power harvesting with piezoelectric generators has focused on improving the efficiency of piezoelectric power harvesting systems. Initial piezoelectric energy harvester was capable of producing limited amount of power in the range of tens of microwatts to few milliwatts insufficient for the autonomous operation of the electronic circuits. In order to address this issue, several approaches have been proposed, which aim at increasing harvested energy using piezoelectric energy harvesters [4]-[8].

While most of the recent research has been conducted on rectangular, triangular and trapezoidal profiled piezoelectric cantilever beams, no analysis has been performed on other profiles due to low energy density as limited space available to place energy harvesting beams in electronic circuits [9]-[11]. Initial research by Mateu and Moll on piezoelectric cantilever type energy harvesters proved mathematically that a triangular cantilever with base and height dimensions equal to the base and length dimensions of a rectangular beam will have a higher strain and maximum deflection for a given load [12]. On similar grounds Roundy and Baker analytically proved that trapezoidal cantilever can generate more than twice the energy than a rectangular beam. But due to brittle nature of PZT any machining other than straight cut result in extensive increase in production cost [13]-[14]. As economical production of piezoelectric energy harvesters is required to implement the technology on mass level, hence to reduce the manufacturing cost and to attain good manufacturability any other profile piezoelectric cantilever beam can be designed using BPFs. In this paper a triangular piezoelectric cantilever beam is approximated as varying width cantilever beam using BPFs and analysis has done using finite element procedure. The schematic of piezoelectric beam designed using BPFs is shown in Fig. 1.

II. BLOCK PULSE FUNCTIONS

The set of block pulse functions (BPFs) is a set of orthogonal functions taken discrete valuation [15]. The (i+1)th member of

BPF, $\psi_{(\mathrm{m})}(x)$, comprised of m component functions, is defined as

$$\psi_{(m)}(x)=\begin{cases}1 & \text{for } ih\leq x\prec(i+1)h\\ 0 & Elsewhere\end{cases} \tag{1}$$

Where $h=\dfrac{T}{m}$, T is total span of function.

$i=0,1,2,\ldots(m-1)$

A square integrable function $f(x)$ can be expanded in to m terms using BPFs in $x=\left[0,x\right)$ as

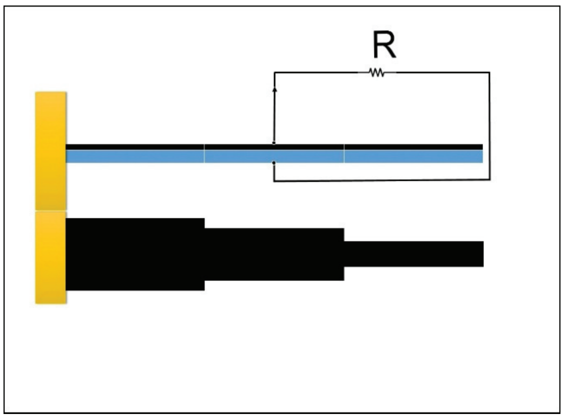

Fig. 1 SCHEMATIC OF PIEZOELECTRIC CANTILEVER BEAM

$$f(x)\approx\left[C_0 \quad C_1 \quad C_2 \quad \ldots C_{(m-1)}\right]\begin{Bmatrix}\psi_0\\ \psi_1\\ \psi_2\\ \vdots\\ \psi_{(m-1)}\end{Bmatrix}$$

$$\cong C^T\psi_{(m)}(x) \tag{2}$$

$$\text{Where}\quad C_i=\frac{1}{h}\int_{h}^{(i+1)h} f(x)\psi_i dx \tag{3}$$

III. DESIGN OF VARYING WIDTH BEAM

A linear function profile with length l is used to construct the varying width cantilever beam using BPFs. Three step discretization is done by using m = 3. In the discretization the piezoelectric generator is divided in to three equal lengths and width is calculated using BPFs as per (2).

For three step discretization, m=3

Profile function $f(x)=W\times\left(\dfrac{-x}{l}+1\right)$

Where W = Width of fixed end of the beam.

l = length of beam.

$$f(x)=\left[C_0 \quad C_1 \quad C_2\right]\begin{Bmatrix}\psi_0\\ \psi_1\\ \psi_2\end{Bmatrix}$$

Where

$$C_0=\frac{3}{l}\int_0^{\frac{l}{3}} W\times\left(\frac{-x}{l}+1\right)dx\cong 0.83W$$

$$C_1=\frac{3}{l}\int_{\frac{l}{3}}^{\frac{2l}{3}} W\times\left(\frac{-x}{l}+1\right)dx\cong 0.50W$$

$$C_2=\frac{3}{l}\int_{\frac{2l}{3}}^{l} W\times\left(\frac{-x}{l}+1\right)dx\cong 0.17W$$

Therefore,

Width of the piezoelectric beam at fixed end, at l/3 and at 2l/3 are given as

$$\left[W_x\right]=\left[0.83W \quad 0.50W \quad 0.17W\right]$$

Where $[W_x]$ is width of piezoelectic beam.

Put 0.83W= w, we get

$$\left[W_x\right]=\left[w \quad 0.60w \quad 0.20w\right]$$

$$\left[W_x\right]=\begin{cases}w & 0\leq length\leq\dfrac{l}{3}\\ 0.60w & \dfrac{l}{3}\prec length\leq\dfrac{2l}{3}\\ 0.20w & \dfrac{2l}{3}\prec length\leq l\end{cases} \tag{4}$$

Comparison is drawn between the varying width and rectangular piezoelectric beams. Length, volume of piezoelectric and substrate material and natural frequency of both the beams kept same to draw unbiased comparison between the traditional rectangular piezoelectric beam and beam designed using BPFs as shown in Fig. 2.

IV. Case Study

This section presents the case study using finite element analysis in Comsol Multiphysics to compare the traditional rectangular piezoelectric cantilever beam and beam designed using BPFs. The finite element method is first verified against the analytical results drawn by Erturk and Inman [16]. After validating the finite element method used, power generated by the varying width piezoelectric beam designed using BPFs is studied. The main aim of this work is to increase the power generation in a cost effective manner so that energy harvesting system can be manufactured for mass use. The properties of substrate material and piezoelectric material for both type of beams has taken from reference data used for the validation of the finite element method and shown in TABLE I. and TABLE II. To make the fair comparison between rectangular and varying width piezoelectric cantilever energy harvester both beams analysed with same volume of substrate and piezoelectric materials and same natural frequency. Hence possess same material and electromechanical parameters but different geometric parameters to match the natural frequency.

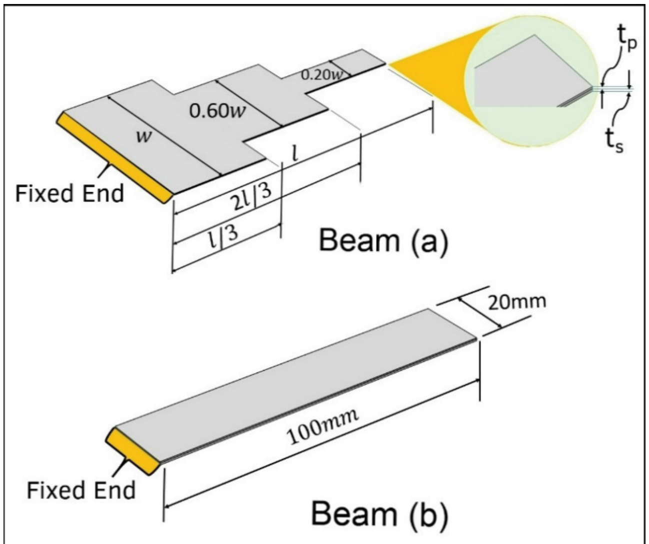

Fig. 2 Beam(a)-An unimorph piezoelectric energy harvester designed using BPFs.

Beam(b)-An unimorph rectangular piezoelectric cantilever beam from [16].

TABLE I. GEOMETRIC PARAMETERS OF SAMPLE HARVESTERS.

Parameter	**Beam (a)**	**Beam (b) [16]**
Length of the Beam, l (mm)	100	100
Width of the beam's fixed end, w (mm)	60	20
Thickness of the substrate, t_s (mm)	0.276	0.5
Thickness of the PZT-5A, t_p (mm)	0.222	0.4

TABLE II. MATERIAL AND ELECTROMECHANICAL PARAMETERS OF BOTH THE HARVESTERS.

PZT-5A		**Substrate Material**	
Mass Density (Kg/m^3)	7800	Young's modulus (GPa)	100
Permittivity (nF/m)	15.94	Mass Density (Kg/m^3)	7165
$C_{11}^E; C_{22}^E$ (GPa)	120.3	Damping Constant	0.2
C_{12}^E (GPa)	75.2		
$C_{13}^E; C_{23}^E$ (GPa)	75.1		
C_{33}^E (GPa)	110.9		
C_{66}^E (GPa)	22.7		
$e_{31}^E; e_{32}^E$ (C/m^2)	-5.2		
e_{33}^E (C/m^2)	15.9		

V. Validation

The finite element analysis results are validated against the analytical results drawn by Erturk and Inman [16]. The first fundamental frequency of the beam (b) found to be 48.915333 as shown in Fig. 3 using finite element simulations and results match with the results of the [16] where first natural frequency come out to be 48.8Hz at R=$10^6\Omega$. At this high resistance value the voltage obtained can be approximated as open circuit voltage. Further the rectangular cantilever beam subjected to frequency domain analysis under 0.2g acceleration. Maximum displacement and open circuit voltage per unit acceleration is plotted in fig. 4. The results are in good agreement with the analytically obtained results in [16] The difference in the result due to presence of errors in using finite element method and to approximation of open circuit voltage at R=$10^5\Omega$. Further as the generated voltage in the simulations using finite element analysis is lower side of the reference [16] results so the procedure can be used to approximate voltage generation from another piezoelectric cantilever beam.

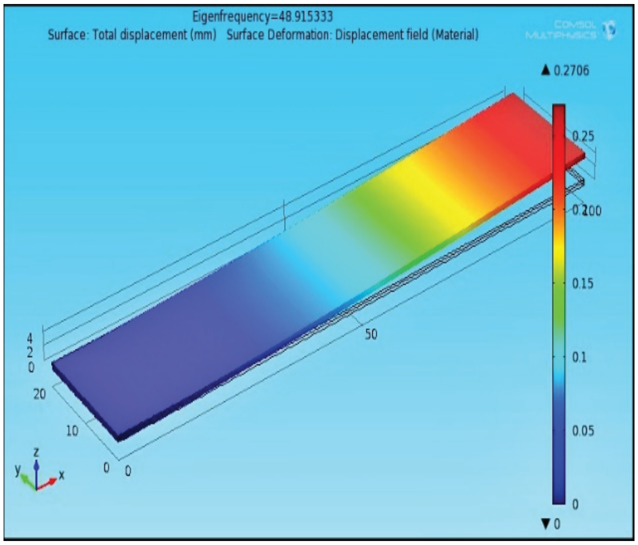

Fig. 3 MODEL OF Beam (b) FOR VALIDATION

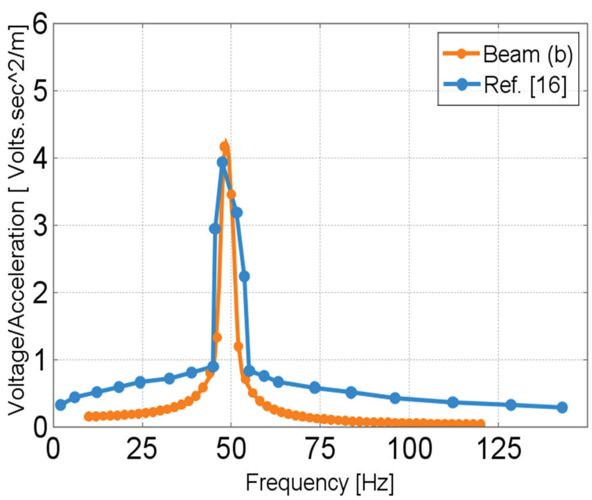

Fig. 4 FREQUENCY-VOLTAGE RESPONSE OF MODELLED Beam (b) AND [16] RESULTS.

VI. ANALYSIS

A unimorph cantilever beam (a) as shown in Fig. 5 is used to harvest energy using PZT-5A piezoelectric material layer positioned on the top of the supporting layer. PZT-5A layer is poled along the thickness direction result in transverse (d_{31}) operation mode. Such configuration is chosen since it enables the condition of low frequency to be fulfilled. The design of the piezoelectric cantilever is selected by taking in to consideration it low acceleration (< 1g) and low excitation frequencies (20-200Hz) [17]. These conditions are correspond to common environmental vibrations.

A validated finite element model is used to analyse the energy harvesters in Comsol Multiphysics with values of parameters listed in the TABLE I and TABLE II.

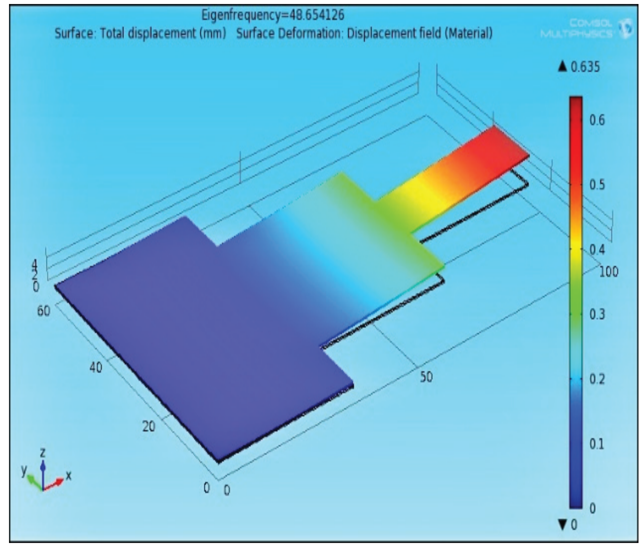

Fig. 5 MODEL OF Beam (a) FOR NATURAL FREQUENCY OF 48.65 HZ.

Piezoelectric layer got electrode at top and bottom faces and due to low thickness of electrodes the mechanical behavior of the electrodes can be neglected.

Electrostatic boundary conditions are provided by putting top face on floating potential, bottom face is grounded and on all other faces condition of zero charge is applied. The model is meshed with physical controlled mesh and element size fine.

The varying width cantilever beam is subjected to finite element analysis using Comsol Multiphysics that revealed the fundamental frequency of beam (a) is 48.654126Hz, Fig. (5). Minor mismatch in the natural frequency of beam (a) and beam (b) can be neglected.

For the dynamic analysis the model beams are subjected to harmonic base excitation by applying vertical acceleration through body load equal to F_Z=aρ, where a=Ng (N= 0.2÷1 and g = 9.81m/s^2) and ρ is density of the corresponding material. The two piezoelectric cantilever beams, Beam (a) - Piezoelectric varying width cantilever beam designed using BPFs and Beam (b) - Piezoelectric cantilever beam from [16], are compared using Comsol Multiphysics.

The generated open circuit voltage during harmonic base excitation of beam (a) and (b) per unit acceleration magnitude of a= 0.2g and near the resonance frequency are summarized in Fig. 6. It was found that magnitude of the generated peak open circuit voltage of beam (a) is considerably higher, approx. 2.5 times, when compared with the generated open circuit voltage of beam (b), Fig. 6.

Higher generation of open circuit voltage is due to better distribution of strains over length of the beam (a) under same environmental conditions as shown in Fig. 7 and 8. Hence beam (a) designed using BPFs can give higher and better distributed strains to harvest more energy from vibrations and can be inducted in energy harvesting devices. Further harmonic analysis is performed on the piezoelectric cantilever Beam (a). Voltage, current and power generated by the beam (a) near the resonance frequency are shown in Fig 9.

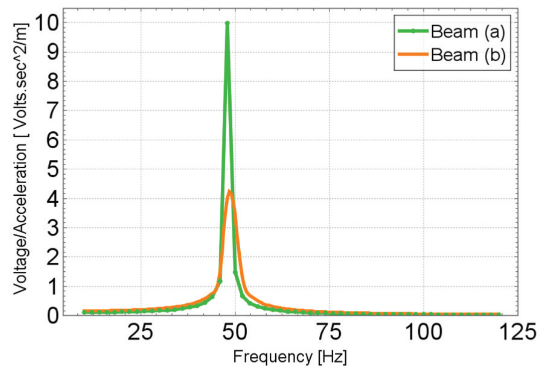

Fig. 6 FREQUENCY-DOMAIN ANALYSIS OF Beam (a) AND Beam (b)

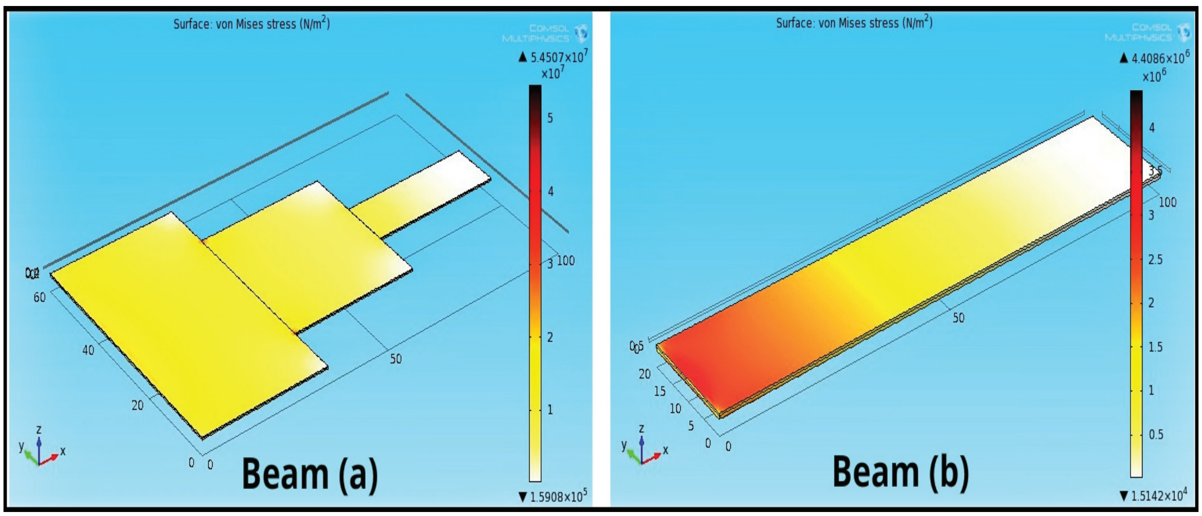

Fig. 7 VON MISES STRESS PLOT FOR Beam (a) AND Beam (b)

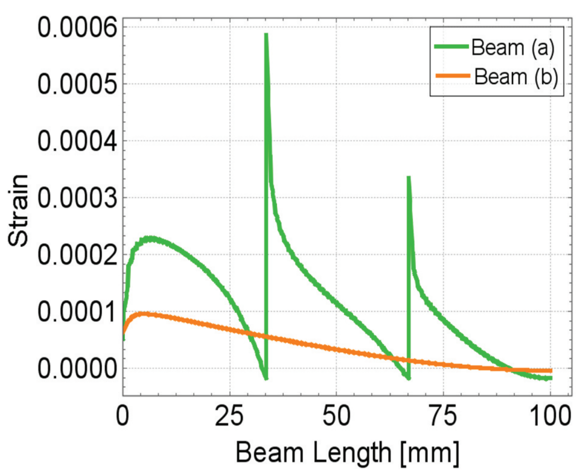

Fig. 8 STRAIN DISTRIBUTION ALONG LENGTH OF Beam (a) AND Beam (b)

VII. CONCLUSION

The main goal of this case study was to analyse the energy harvested by the varying width beam constructed using BPFs in comparison with the other constant width piezoelectric cantilever beam for same volume of substrate and piezoelectric material and natural frequency. In the study, two piezoelectric cantilever type harvesters were investigated using Comsol Multiphysics and open circuit voltage generated across piezoelectric layers are compared. It is found that the beam configuration with piezoelectric element layered as distributed linear function in three steps using BPFs is more efficient and voltage generation is approximately 2.5 times higher than the voltage produced by constant width piezoelectric cantilever. Further research can be initiated to optimize the dimensions of the piezoelectric layer to minimize the cost and maximize the generated power.

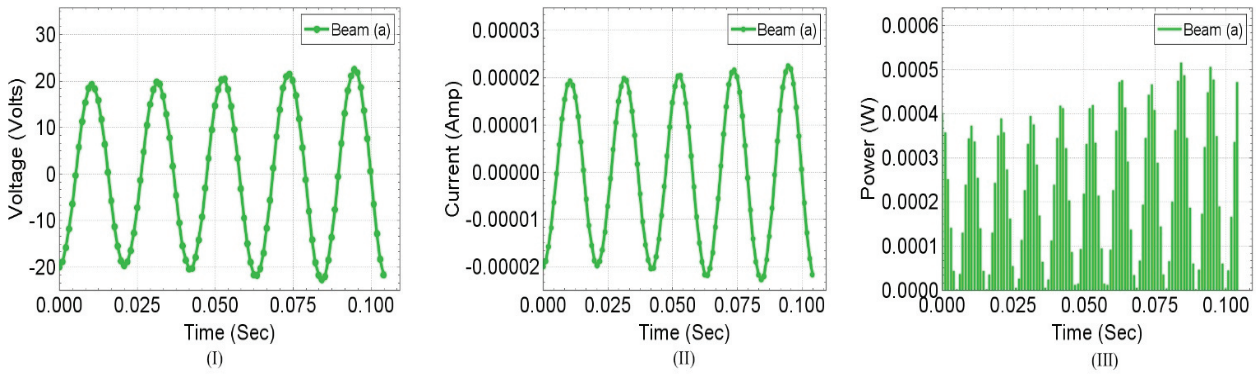

Fig. 9 (I)-VOLTAGE GENERATION WITH TIME FOR Beam (a). (II)-CURRENT GENERATION WITH TIME FOR Beam (a). (III)-POWER GENERATION WITH TIME FOR Beam (a)

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