

ICMMM - 2017

# Analysis Of Perforated Piezoelectric Sandwich Smart Structure Cantilever Beam Using COMSOL

N. Sivakumar<sup>a\*</sup>, Dr. H. Kanagasabapathy<sup>b</sup>, H. P. Srikanth<sup>c</sup><sup>a</sup>Department of Mechanical Engineering, Sun College of Engineering and Technology, Nagercoil, 629902, Tamil Nadu, India<sup>b</sup>Department of Mechanical Engineering, National Engineering College, Kovilpatti, 628503, Tamil Nadu, India<sup>c</sup>Research Scholar, Visvasvaraya Technological University, Belagavi, 590018, Karnataka, India

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## Abstract

This paper addresses the analysis of smart structure to implement a cantilever beam using COMSOL. The model of the smart structure is geometrically defined and developed using finite element method with meshing in various stages. 3D static bending of a beam due to applied loading and electrostatic forces together couple so as to optimally generate electric potential distribution for a specified region which facilitates thereby for the plotting of curves to be studied in this paper. The details of the meshing reveal a definite range of voltage and applicable load to make significant displacement of the beam.

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Selection and/or Peer-review under responsibility of International Conference on Materials Manufacturing and Modelling (ICMMM - 2017).

**Keywords:** Finite element modeling; PZT-5H; perforation; sandwich piezoelectric cantilever beam; smart structure; COMSOL cantilever beam; COMSOL; static analysis; finite element tool;

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## 1. Introduction

Smart structure is the one of the fast developing area in the field of engineering because of the wide range of applications; typically these devices are used to power sensors, wireless communication systems, recent smart cars and aero elastic flight vehicle. The basic structure of MEMS design is elastic cantilever beam. Smart materials in nature have an interesting application for modern technology and materials exhibiting piezoelectric effect are no exception. The piezoelectric effect happens in some materials whenever an externally induced elastic strain leads to a variation in the electric polarization vector in the process generating a charge and an electric potential difference in the material.

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\* Corresponding author. Tel.: +91-9488323207; fax: +91-04652-281466.

E-mail address: [nsivakumar767@gmail.com](mailto:nsivakumar767@gmail.com)

Electricity from static effects from piezoelectric materials have drastically changed the way smart technologies were looked at some four decades ago, causing conversion of mechanical energy into electrical energy and vice versa in some unusual situations like charge generation from automobiles on humps to waking on pavements with piezoes underneath. Well off people to average income people now want the manufacture of sensors and actuators from hi-end environmental friendly polymeric piezos to cheap lead based lead zirconates, even suitable for astronauts for their aircrafts and space structures [1].

Jae Eun Kim et al revealed that [2] piezoelectric material has 3 absolutely separate material properties, namely he took mechanical energy and forces, electrical charge and potential, and electromechanical linearity measurements. The development resulted in piezoelectric actuators, sensors and energy harvesters useful as a prelim for static analysis and for simulation on a digital computer. A gamut of numerical examples was taken care of validating pre-conditioning to activity of piezoelectricity in these materials. The simplistic approach resulted in some of presentations by A.Y.T. Leung et al that can intrigue scientists for a while. The construction utilizing the simplistic geometry is created to analytically investigate the electromechanical characteristics of plates of piezo-ceramics [3].

In [4] novel evolutionary algorithms to fix up a splendid variety of shape control scenarios: smart composite plate structures with active element as a piezoceramic based static-electricity actuators, has been the talk of the show. Corrado Maurini et.al couldn't stop proposing a highly coupled Euler–Bernoulli differential model with mathematical operators such as 4th and higher order differential equations of laminated piezoceramic based static-electrical beams [5].

Finite element modelling (FEM) of multi-layered laminated sandwiched structures with scattered piezoceramic based static-electric sensor and electromechanical actuator layers and a system control system is notably the highlight of the paper in [6]. Shapes such as beam, discs and spherical elements have been created inculcated with the stiffness, weight and electromechanical full or tight coupling effects of the piezoceramic based laminates. The influence of temperature on the electrical and mechanical characteristics and the coupling amongst them are rather brought out exquisitely through FEM.

The idea of an inertial vibrating mode energy harvester system to excite a sensor inside a rubber tire is also experimented with in some literature. This harvester is customized to the tire's vibration spectra and the interfered with acceleration impulses [7].

A paper [8], took shear deformation aspects regarding beam theory and a non-Newtonian form of motion: Hamilton's principle, and obtained an efficient and precision tuned finite element (FEM) model in order to be applicable to shape control of laminated sandwich type of composite beams with dispersed piezoelectric actuators everywhere on it. This shape control and later damage detection of a structured breakdown threshold evaluation for a cantilever composite beam are studied by utilizing the demo pieces and genetic optimization algorithms. The study revealed that at one of the optimal actuator placement and the optimal values of voltages can be determined for a particular point under investigation of a certain scenario. Universally, for maximum effectiveness, a great deal numbers of actuators are required to be implemented besides the beam and more so in skewed strain regions. It was got from the deductions that a few numbers of actuators with this optimal placement and optimal voltage reading could render shape control of the type of cantilever beam quite sufficiently. The static characteristics and its control for multilayered laminated sandwich discs with impregnated and/or polishing with bondable piezoelectric layers capable of undergoing mechanical loading and/or electric load/potential is the subject of study in this paper [9]. This investigation envisages the static practical response of a cantilever beam mechanically actuated utilizing piezoceramic patches. The goal of experiment here was to investigate the response of an electrostatically deflected, piezo-actuated rigid cantilever aluminium beam. Static deflection practical responses are acutely rare, and often intriguing solutions came from out these experiments and their differentiation with finite element (FE) computational solutions [10].

To prove the effectiveness of bending due to electrostatic forces and electric potential distribution simulation is done using COMSOL under the structural analysis module with stationary study. This paper is divided as follows; The Model Definition is described in Section 2. COMSOL Implementation is given in section 3; Results are discussed in section 4 and conclusion is given in section 5.

## 2. Model Definition

Ceramic based piezoelectric materials are structurally anisotropic materials. Their orientation in different directions of a incision crystal revealing Euclidian angle has a direct effect on the parameters altering in the piezoelectric constitutive relationships and the ultimate result of a wide spectrum of the wave propagating out of it [11]. In this paper a 3 layer sandwiched piezoelectric cantilever beam as shown in fig. 1 is considered for a detailed investigation.

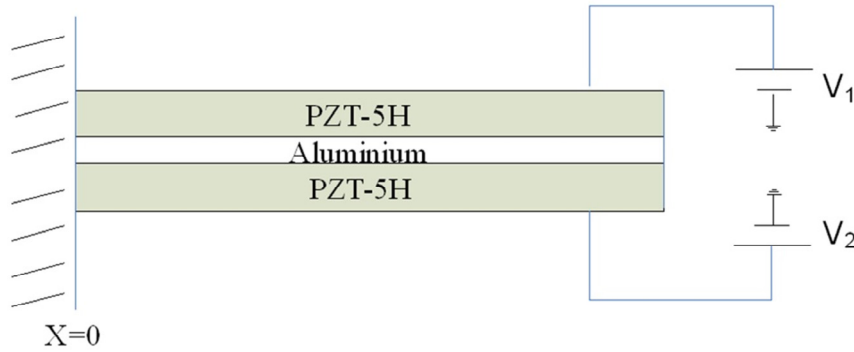


Fig. 1. General three layer piezoelectric sandwich cantilever beam.

The host beam comprises of PZT-5H sandwich material within which an aluminium layer under lies and its specifications are described in Table 1. Here the trials undertaken are described with modification in geometry and materials. A beam of length 100 mm will be sandwiched with a thickness of 2 mm of aluminium and a total thickness of 18 mm upto a width of 30 mm of a PZT-5H will be undergoing body deformation as to lead to a defined displacement, due to a voltage applied into it. To supplement the converse piezo electric effect of stain generated charge distribution. A linear force is applied at the tip of the cantilever beam on one side while the other end is clamped.

A voltage of -10 V on the top surface is applied while the lower piezo electric layer is subjected to varying electric potential stating from 5 V. All the possible combinations of voltages at the bottom surface together with a fixed edge loading (seen in fig. 2) are studied in a pursuit to control the deformation of the beam lest there be damage beyond notice on simulation screen. In direct piezo electric effect we expect the deformation to increase with applied voltage and on the other hand the converse piezo electric effect will lead us to anticipate a voltage generation due to edge loading from the forces applied (see fig. 4). The forces here are not laterally applied but a rather oblique direction is advised and the resolutions into component forces are given in Table 2.

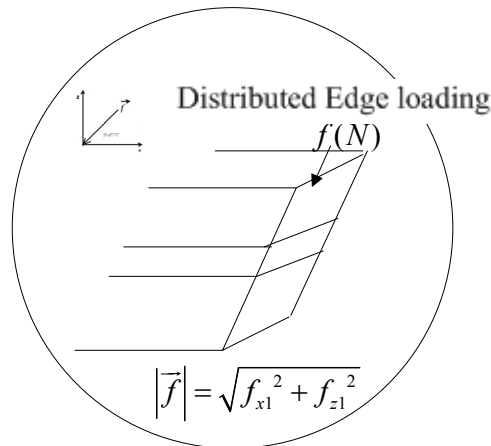


Fig. 2. Force resolution of distributed edge loading

Piezoelectric finite element modeling is done based on the V. Piefort thesis [15] and it is given below.

$$\{T\} = [c^E]\{S\} - [e]^T\{E\} \quad (1)$$

$$\{D\} = [e]\{S\} + [\varepsilon^S]\{E\} \quad (2)$$

Large values of the stress vector  $\{T\}$  are known to influence the electric flux density vector  $\{D\}$  and the strain

Table 1: Material properties of three layer piezoelectric sandwich cantilever beam

Material	Density	Young's modulus	Possion ratio
PZT-5H	7500kg/m <sup>3</sup>		
Al-aluminium	2700 kg/m <sup>3</sup>	70 GPa	0.35
3ply laminate	730 Kg/m <sup>3</sup>	10.89 GPa	0.36

Table 2: Force resolution of edge loading

$f_{x1}$ (N)	$f_{z1}$ (N)	$\sqrt{f_{x1}^2 + f_{z1}^2}$ (N)
10	20	22.36
40	60	72.11
70	80	106.30
100	100	141.42

Vector $\{S\}$ . Sometime in this contemporarily the same parameters are inclined to exert a sphere of influence to the electric field vector $\left\{\begin{bmatrix} \rightarrow \\ E \end{bmatrix}\right\}$ , which find their definitions only on the basis of the elastic matrix $\left[c^E\right]$ , the piezoelectric matrix designated  $[e]$  and the dielectric vector $[\varepsilon^S]$ . The anisotropic symmetric matrix applies to both the electric as well as mechanical effects as the material is the same that undergoes inverse and direct piezoelectric effect:

$$[c] = \begin{bmatrix} c_{aa} & c_{ab} & c_{ac} & c_{ad} & c_{ae} & c_{af} \\ & c_{bb} & c_{bc} & c_{bd} & c_{be} & c_{bf} \\ & & c_{cc} & c_{cd} & c_{ce} & c_{cf} \\ & & & c_{dd} & c_{de} & c_{df} \\ & & & & c_{ee} & c_{ef} \\ 6 \times 6 \text{ symmetry} & & & & & c_{ff} \end{bmatrix} \quad (3)$$

Electric flux being constant becomes  $c^D$ , this can be the feed also :

$$c^D = [c^E + \left(\frac{e^2}{s^S}\right)] \quad (4)$$

The stereoscopic view of the electric field vector  $\{E\}$  is even more chopped and portrayed out using the piezoelectric stress matrix  $[e]$  spread along in the x, y and z Cartesian coordinates as to the stress vector appears as distributed in x, y, z, xy, yz, and xz:

$$[e]_{6 \times 3} = \begin{bmatrix} e_{aa} & e_{ab} & e_{ac} \\ e_{ba} & e_{bb} & e_{bc} \\ e_{ca} & e_{cb} & e_{cc} \\ e_{da} & e_{db} & e_{dc} \\ e_{ea} & e_{eb} & e_{ec} \\ e_{fa} & e_{fb} & e_{fc} \end{bmatrix}_{6 \times 3} \quad (5)$$

The piezoelectric matrix  $[e]$  has again a form having the piezoelectric strain matrix adopted in integrity in the matrix of  $[d]$ . This  $[d]$  transformed to the piezoelectric stress matrix  $[e]$  having with it the elasticity matrix  $[c]$  looks like:

$$[e]_{6 \times 3} = ([c]_{6 \times 6} \cdot [d]_{6 \times 3}) \quad (6)$$

The kinetics and kinematics of the piezoelectric elements are understood using the field of stress induced displacement field  $\{U\}$  and the unknown electric potential  $\{\Phi\}$  observed upon a mesh of minuscule area of the structure under investigation. These are indispensably corresponding to the node values of displacement  $\{U_i\}$  and electric potential  $\{\Phi_i\}$  due to the mean of the shape functions of displacement  $[N_U]$  and electric potential  $[N_\Phi]$ .

$$\{U\} = [N_U]\{U_i\} \quad (7)$$

$$[\Phi] = [N_\Phi]\{\Phi_i\} \quad (8)$$

The derivatives of the displacement shape function  $[B_U]$  and electric field shape function  $[B_\Phi]$  also have a role in this scenario, where the nodal surfaces and edges have a particular affinity for strain field  $\{S\}$  and the electric field  $\{E\}$  as defined by

$$\{S\} = [D][N_U]\{U_i\} = [B_U]\{U\} \quad (9)$$

$$\{E\} = -\nabla[N_\Phi]\{\Phi_i\} = -[B_\Phi]\{\Phi_i\} \quad (10)$$

where  $\nabla$  is the gradient vector operator and  $[D]$  is the derivative operator (replaceable by the  $\partial$  in 3-D) defined such as

$$\{S\} = [D]\{U\} \quad (11)$$

These are the elements of piezoelectric stress: strain relationship with electric potential and electric displacement as far as the matrices and numerical techniques are concerned. There are again formulations based on Kirchhoff's element. Mindlin element are utilized for the mesh calculation in the various geometries of the selected study and all the values so generated are utilized for computation of the same select set of piezoelectric matrices and these are done with other software packages that are having compatibility with COMSOL [12].

### 3. COMSOL Implementation

The algebraic components in the piezoelectric material (PZT-5H) attributes ascribe a Cartesian base unit, wherein the poling direction happens to be the z direction. Due to the poling alignment of the piezoelectric layer in this model being directed along the x-axis, there necessitates utility of an interior coordinate system in the property's settings of PZT-5H in order to rotate the piezoelectric based sandwich cantilever beam. By observing the stroboscopic effect in sandwich beam a defined internal coordinate system is reoriented at about 90 degrees across the global y-axis. With the help of this coordinate system in the piezoelectric material (PZT-5H) with modified settings it will allow the rotation of the structure so as to align the polarization orientation along the x-axis. All the complex modeling has been carried out in direct stationary solvers with solver type SPOOLES. More information about the implementation procedure for any structure shall be available through the help guide in COMSOL Package [13]. The COMSOL package is particularly user friendly due largely to the predesigned physics interfaces, employing the governing equations from a vast variety of physics boundary value problem and initial value problem for many homogeneous and in-homogeneous differential equations into a model. Here piezoelectric sensor design and development are introduced as a utility to detect the train on rail track beholding the measurement point, a rendezvous where sensor meets the track. At a critical threshold, the output response can be thrown to mechanically actuate the boundary territories and other alarm equipment [14].

### 4. Results and discussion

Piezoelectricity is that phenomenon wherein the electric charge accumulates in certain solid materials in the crystalline spaces called "voids" viz ceramics, counteracting to applied mechanical stresses. The piezoelectric effect is known so far as the linear electromechanical interplay amongst the mechanical and the electrical activities in crystalline materials (absolutely solid) with no inversion of crystal symmetry [15] allowable.

A 3D surface plot of all the three sandwich cantilever beam has been carried out using COMSOL by constructing the required model within the COMSOL CAD. This CAD tool facilitates the CAD modeling simultaneously with material selection. The present case involves all the three models with a basic hetero layered PZT-5H and Aluminium defined in the material's library in COMSOL. Other than this the changes in the second and third type of model is the presence of perforation in the lateral sides passing through the materials. Later the last among them will be taken for rigorous investigation by filling the perforation with 3 ply laminate type of ply wood so as to avoid the void in the previous case [16].

All the trials will consist of stress and surface deformation and in order to keep this restricted in the lateral displacement, PZT-5H was chosen with a  $-x$  and  $+z$  orientation of poling. These materials will constitute a family of similar structured sandwiching piezo devices as to constitute a smart structural monitoring device. All the cases under study are depicted in figure 4 below. These cases exhibited saturation in the deformable quantity with applied potential greater than 40 V. Hence a region of interest in the case of direct piezo electric effect under 40 V is evident. This holds the objective of observing deformation with possibilities to overlook applied electric field (DC with stationary study). This is extended with simultaneous application of edge loading. The loading considered was such that until a point of time the materials breaks down the load shall not cease to act upon the piezo electric cantilever beam at the free end of the rigid body. Thus this force is considered to be in the  $x$ - $z$  plane acted upon vectorially and whose resultant shall be known by the law of parallelogram of forces.

The meshing of all the three models were carried out with the cylindrical shape of perforation and a tetrahedron type of hexagonal meshes were generated. During this meshing all the degrees of freedom were allowed for the combination of forces and voltages (100 N in 10 different combinations and 40 V in 8 combinations with a total of 800 different possibilities). Where the need arises for restricting a select range of values for any of these parameters, it is done so as with example- voltage with displacement curves being considered under no load.

In the Von-Mises stresses, the deformation is observed in the surface total displacement wherein the stress is maximum at the location near the fixed end and also a little away from the tip of the free end of the cantilever, there being distributed. While in the perforated beam, the stress is at the fixed end and at the tip. These cases are even skewed in the plywood filled cantilever wherein the stress is most likely to occur at the fixed end only. All the combinations of electric potential and forces are considered.

A graph of all the select combination of  $V_{dc}$ ,  $fx1$  and  $fz1$  under consideration are being plotted for different parameters under the study of stationary type of descriptions. Here the solvers such as SPOOLES will undertake all the Finite Element Analysis to an extent of pivot threshold of 0.1. All of the forces and displacements are part of the structural mechanics module wherein solid mechanics and piezo electric effect are brought into one group. The structural behavior of the piezoelectric material is defined with the boundary condition as shown in the Fig. 3 below.

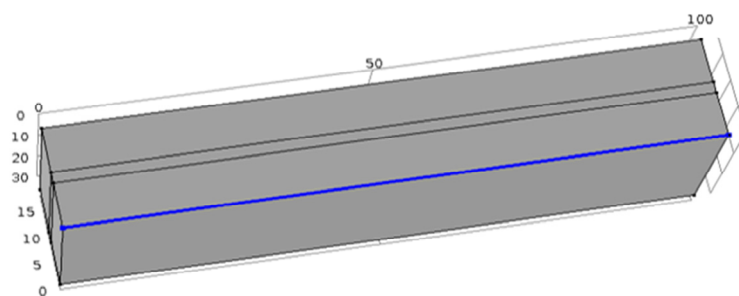


Fig. 3: The selection of edge for the all study undertaken in COMSOL

In some of the plots of one dimension there were distinguishing similarities for all the three cases of example 1, 2 and 3. In the force versus displacement graph there is a minimum value of displacement even at no load. Later all the displacements do not form a continuum. For discrete values of forces there exists instantaneous displacement for all the three examples considered.

Their pattern is somewhat wavy and varies without any abrupt change in the applied edge loading. The edge loading being applied at oblique angle should be depending on the rigidity of the cantilever beam. However all the three examples at the magnitude considered fixed, displays equal displacements for equally applied edge loading.

In the case of potential generated throughout the material, the anomalous voltage for the zero volts is observed in the displacement. These displacements are increasing randomly with the applied voltage. Hence their utility in allowing the specified displacement is voltage dependent. All the values conformed to example 1, 2 and 3 which is observed in Table 3.

Table 3: Voltage and displacement observation in example 1, 2 and 3

Voltage (Vdc)	Displacement (mm)
0	0.2
5	0.2
10	0.05
15	0.2
20	0.4
25	0.6
30	0.8
35	1.0
40	1.15

From Fig. 4, the stored energy densities versus displacement of the bottom piezo layer it is clear that in all the above cases whether be it without any crevices or with crevices and filled with filler material as in our case of 3 ply laminate, it is undisputed justification from the similarities that law of conservation of charge is preserved. In the example 1 of purely sandwiched type laminated piezoelectric cantilever, the energy density diagram is nearly smooth and falling to zero value rapidly and there is ebb and tide to indicate nonlinearity being initiated. In the example 2 of perforated cantilever beam there is a clear indication of folding of the graph while displacement increases. This is true even in the case of example 3, wherein there is a similar fall in the stored energy density analogous to the previous two examples.

It is observed from Von-Mises stress in the surface deformation that an applied edge load at the non-perforated sandwich cantilever beam undergoes minimal displacement, the perforated being the greatest and the 3 ply laminated one being the least.

However observation and intuition leads us to analyze with the help of second law of Newton while material density fluctuates. In the figures of the Von-Mises stresses, there arises a symbolic identity for every structure due to its displacement. Quantitatively, there is quite a lot of displacement happening in the perforated structure. Nevertheless, there is a great tendency for this perforation to be exclusive as far as stress and surface plot profiles describe. It is so since these are not visibly extruded in either the complete sandwiching or the plywood filled perforated structures. This is notably an isolated case where in the perforation can be filled to adapt to a situation of continuous uniformity of material selection and thereby regain physical properties akin to that of the laminated sandwiching of piezoelectric elements in the cantilever beam. This is depicted in Fig. 5 by comparing all the three types of cantilever structures so constructed.

While this is true with the shape phenomenon, the converse is not reciprocated with the electric potential of the material under study. These materials are in such a combination as to exhibit direct and inverse piezo electric effect. These phenomena are now understood with the help of Fig. 6 wherein the surface plots of electric potential for the perforated cantilever beam don't quite behave as expected but rather the sandwich piezo electric continuous beam or three ply laminate filled cantilever beam are displaying excellent electric potential contours across the profile. This is even more evident when we consider how naïve the perforated cantilever beam under study really happens to be from the figures. We see  $fx1$  is 100N while  $Vdc$  is zero or 40 V and  $fz1$  is 0 N shows significant variation and in the case of  $fx1$  being zero or both  $fx1$  and  $fz1$  are 100 N while  $Vdc$  is zero or 40 V has no signature in electric potential surface plots.

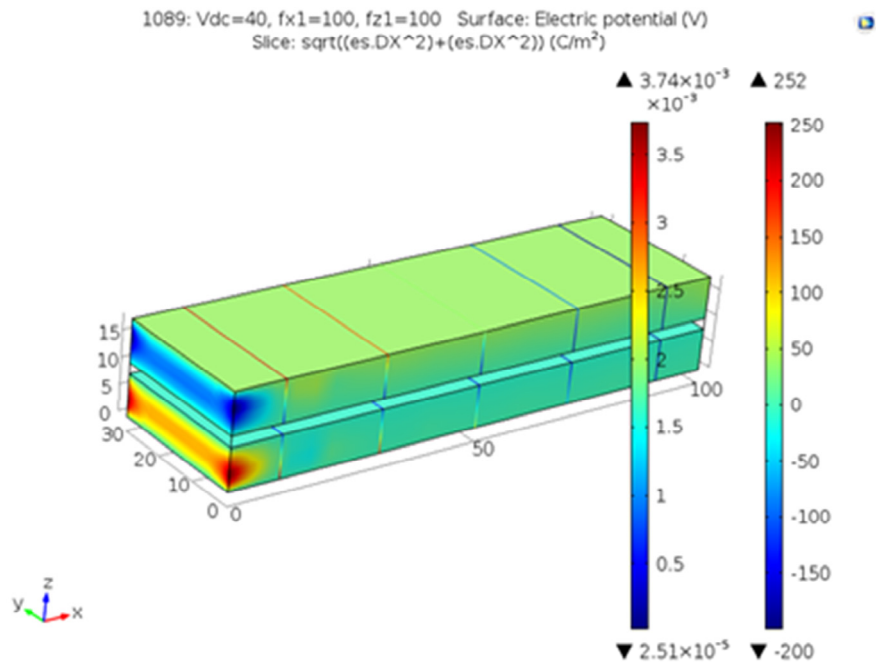


Fig. 4. Comparison of stress, voltage distribution and 3D plot of the three types of piezoelectric sandwich cantilever beam.

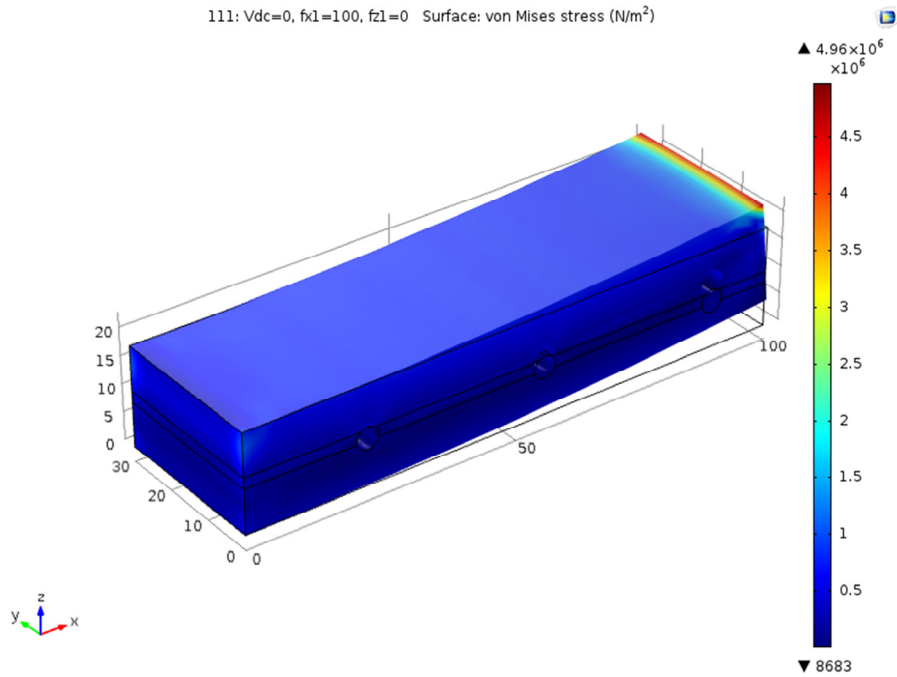


Fig. 5: A Von-Mises stress surface displacement plots for various output parameters



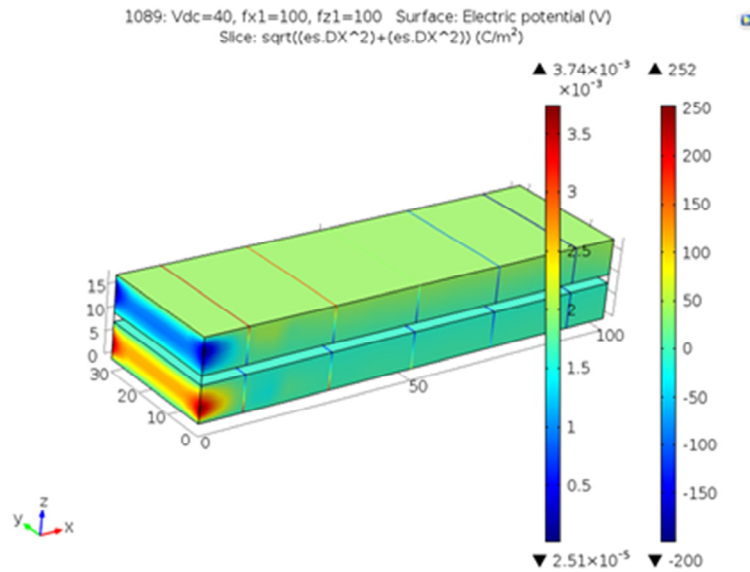


Fig. 6: Electric potential in surface plots for all the combinations of the input parameters

## 5. Conclusion

The present paper discusses all the stationary analysis of a cantilever beam in the three geometries discussed. These have been analyzed for their voltages and displacements while sweeping the parameters with a finite element analysis tool-COMSOL. This study has revealed some intriguing aspects like the anomalous behavior of displacements under no load and under no applied voltage. This also made some observation into the displacement and electric potential surface plots generated within the material with perforation and filler -3 ply laminate. Perforation is making the cantilever structure into very good bending device and ply wood inside the perforation has enabled it to restrict the bending and reconcile with its un-perforated structure and shows similar electric potential distribution across the material. The beauty of the study is the great amount of rigidity the persists in the use of 3-ply laminate as filler for the perforation as is evident in the figures for Von-Mises stress surface displacement plots.

This work has expanded our understanding of the piezoelectric cantilever beam with PZT-5H. The same analysis holds for other piezos with known poling direction within the piezoelectric orientation. The simulation study shall be in the defined room temperature but however, its study underwater, high temperature and high pressure will be very interesting. This apart, the application of this beam in structural health monitoring as a strain gauge or as sensor in force/pressure detection is to be realized. It also can be fitted with the tip of strong conductors and used as a cantilever tip for probing forces like it is done in Atomic Force Microscopy (AFM). The material can be varied in the beam as with Aluminium and substituted for better conductivity studies etc. This apart, perforations introduced were in the lateral position but even the top through bottom is possible. All the forces from multiple directions are to be considered and torsional forces also need inspection. The study was limited to stationary analysis because of frugal computational capabilities but the same structures can undergo time dependent forces so as to reveal a more realistic point of introspection. Hence a complete gamut of possibilities has opened with this selected study and more is to be understood from a simulation point of view in order to better appreciate the capabilities of finite element modeling of mechanical structures.

## Acknowledgements

The authors greatly acknowledged the guidance and support given by **Dr. N. Amuthan**, Professor and Head, Department of Electrical and Electronics Engineering, AMC Engineering College, Bengaluru.

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