PROBABILITY AND STATISTICS

LAB ASSIGNMENT - 4
(Mathematical Expectation, Moments and Functions of Random Variables)

Name: Jasween Kaur Brar
Roll No: 102017187
Sub-group: 3CS8

1. The probability distribution of X, the number of imperfections per 10 meters of a synthetic fabric in continuous rolls of uniform width, is given as

x	0	1	2	3	4
$p(x)$	0.41	0.37	0.16	0.05	0.01

Find the average number of imperfections per 10 meters of this fabric.

(Try functions **sum( )**, **weighted.mean( )**, c(a **%*%** b) to find expected value/mean.

```
# Q1
# sum
x<-c(0,1,2,3,4)
px<-c(0.41,0.37,0.16,0.05,0.01)
sm<-sum(x*px)
sm

# weighted mean
wm<-weighted.mean(x,px)
wm

# c(a%*%b)
exp_val<-c(x%*%px)
exp_val
```

Output :

```
> # Q1
> # sum
> x<-c(0,1,2,3,4)
> px<-c(0.41,0.37,0.16,0.05,0.01)
> sm<-sum(x*px)
> sm
[1] 0.88
>
> # weighted mean
> wm<-weighted.mean(x,px)
> wm
[1] 0.88
>
> # c(a%*%b)
> exp_val<-c(x%*%px)
> exp_val
[1] 0.88
```

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2. The time T, in days, required for the completion of a contracted project is a random variable with probability density function $f(t) = 0.1\ e^{(-0.1t)}$ for $t > 0$ and 0 otherwise. Find the expected value of T.
Use function **integrate( )** to find the expected value of continuous random variable T.

```
# Q2
fun<-function(t)
{
  return(t*0.1*(exp(-0.1*t)))
}

fun(1)

exp_value<-integrate(fun,lower=0,upper=Inf)
exp_value
print(exp_value$value)
```

Output :

```
> fun<-function(t)
+ {
+   return(t*0.1*(exp(-0.1*t)))
+ }
>
> fun(1)
[1] 0.09048374
>
> exp_value<-integrate(fun,lower=0,upper=Inf)
> exp_value
10 with absolute error < 6.7e-05
> print(exp_value$value)
[1] 10
```

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3. A bookstore purchases three copies of a book at $6.00 each and sells them for $12.00 each. Unsold copies are returned for $2.00 each. Let X = {number of copies sold} and Y = {net revenue}. If the probability mass function of X is

x	0	1	2	3
$p(x)$	0.1	0.2	0.2	0.5

Find the expected value of Y.

```
#Q3
func=function(x)
{
  x=12*x+2*(3-x)-18
}
x<-c(0, 1, 2, 3)
pX<-c(0.1, 0.2, 0.2, 0.5)
eX<-(sum(x*pX))
print(func(eX))
```

Output :

```
> #Q3
> func=function(x)
+ {
+   x=12*x+2*(3-x)-18
+ }
> x<-c(0, 1, 2, 3)
> pX<-c(0.1, 0.2, 0.2, 0.5)
> eX<-(sum(x*pX))
> print(func(eX))
[1] 9
```

---

4. Find the first and second moments about the origin of the random variable X with probability density function $f(x) = 0.5e^{-|x|}$, $1 < x < 10$ and 0 otherwise. Further use the results to find Mean and Variance.
(kth moment = $E(X^k)$, Mean = first moment and Variance = second moment – Mean^2.

```
#Q4
first=function(x)
{
  x=x*0.5*exp(-abs(x))
}
second=function(x)
{
  x=x*first(x)
}
firstmoment=integrate(first, lower=1, upper=10)
secondmoment=integrate(second, lower=1, upper=10)

print(firstmoment$value)

print(secondmoment$value)

#Mean
print(firstmoment$value)

#Variance
print(secondmoment$value-firstmoment$value*firstmoment$value)
```

Output :

```
>
> print(firstmoment$value)
[1] 0.3676297
>
> print(secondmoment$value)
[1] 0.9169292
>
> #Mean
> print(firstmoment$value)
[1] 0.3676297
>
> #Variance
> print(secondmoment$value-firstmoment$value*firstmoment$value)
[1] 0.7817776
```

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5. Let X be a geometric random variable with probability distribution

$$f(x) = \frac{3}{4}\left(\frac{1}{4}\right)^{x-1}, x = 1,2,3, \ldots$$

Write a function to find the probability distribution of the random variable $Y = X^2$ and find probability of Y for X = 3. Further, use it to find the expected value and variance of Y for X = 1,2,3,4,5.

```
#Q5
fun3=function(x)
{
  return(3/4 * (1/4)^(x-1))
}
func4<-function(y)
{
  return(3/4 * (1/4)^(sqrt(y)-1))
}
x=3
y=x^2
func4(y)
x=c(1, 2, 3, 4, 5)
h<-x^2
prob<-func4(h)
expect<-sum(h*prob)
print(expect)
expect2<-sum((h^2)*prob)
fin<-expect2-expect^2
print(fin)
```

Output :

```
> func4(y)
[1] 0.046875
> x=c(1, 2, 3, 4, 5)
> h<-x^2
> prob<-func4(h)
> expect<-sum(h*prob)
> print(expect)
[1] 2.182617
> expect2<-sum((h^2)*prob)
> fin<-expect2-expect^2
> print(fin)
[1] 7.614112
```