

Consider the following Quadratic function

$$f(x) = \frac{1}{2}X^T Q X - b^T X.$$

Check if Q is a positive definite matrix or not and implement the conjugate gradient descent algorithm to find the minimum of the quadratic function.

```

1 import numpy as np

1 #function
2 def function(Q, b, x):
3     return 0.5 * np.dot(np.dot(x.T, Q), x) - np.dot(b.T, x)
4
5 #Check if Q is positive deffinite or not
6 def isposdef(Q):
7     if np.all(np.linalg.eigvals(Q) > 0):
8         return True
9     else:
10        return False

1 import numpy as np
2
3 def conjugate_gradient_descent(Q, b, x0, max_iter=1000, tol=1e-6):
4     if not isposdef(Q):
5         raise ValueError("Q is not a positive definite matrix.")
6     #step 1
7     x = x0
8     g = Q @ x - b
9     d = -g
10    #step - 2
11    alpha = g.T @ g / (d.T @ Q @ d)
12    num_iter = 0
13
14    #step - 3
15    while np.linalg.norm(g) > tol and num_iter < max_iter:
16        x = x + alpha * d
17        g_prev = g
18        g = Q @ x - b
19        beta = (g.T @ g) / (g_prev.T @ g_prev)
20        d = -g + beta * d
21        alpha = g.T @ g / (d.T @ Q @ d)
22        num_iter += 1
23    return x, num_iter
24

1 # Test the implementation
2 Q = np.array([[6, -4], [-4, 4]])
3 b = np.array([-4,0])
4 x0 = np.array([0,0])
5 print("Q is positive definite matrix:", isposdef(Q))

    Q is positive definite matrix: True

1 x ,num_iter = conjugate_gradient_descent(Q, b, x0)
2 print("Q is positive definite matrix:", isposdef(Q))
3 print("Minimum value of the Quadratic function:", function(Q, b, x))
4 print("Minimum point of the Quadratic function:", x)
5 print("Total number of iteration :", num_iter)

    Q is positive definite matrix: True
    Minimum value of the Quadratic function: -4.0
    Minimum point of the Quadratic function: [-2. -2.]
    Total number of iteration : 2
<ipython-input-17-1cc4068c35e2>:21: RuntimeWarning: invalid value encountered in double_scalars
    alpha = g.T @ g / (d.T @ Q @ d)

```

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