Washington State University
School of Electrical Engineering and Computer Science
Fall 2020

CptS 440/540 Artificial Intelligence
Homework 7
Due: October 22, 2020 (11:59pm pacific time)

General Instructions: Put your answers to the following problems into a PDF document and submit as an attachment under Content ❼ Homework 7 for the course CptS 440 Pullman (all sections of CptS 440 and 540 are merged under the CptS 440 Pullman section) on the Blackboard Learn system by the above deadline. Note that you may submit multiple times, but we will only grade the most recent entry submitted before the deadline.

1. Suppose you are given the following full joint probability distribution over three random variables: *Grade*, *Study*, *Sleep*. *Grade* has domain {A, B, C}, *Study* has domain {yes, no}, and *Sleep* has domain {2,4,6}. Compute the probabilities below. Show your work.

	Grade:	A		B		C	
	Study:	yes	no	yes	no	yes	no
Sleep:	6	0.10	0.05	0.08	0.06	0.03	0.08
	4	0.06	0.03	0.06	0.04	0.04	0.10
	2	0.02	0.01	0.04	0.02	0.06	0.12

   a. P(Grade=B, Study=yes, Sleep=4).
      P = **0.06**
   b. P(Study=yes, Sleep=4).
      P(A, yes, 4) = 0.06, P(B, yes, 4)=0.06, P(C, yes, 4) = 0.04
      P = P(A) + P(B) + P(C) = **0.16**
   c. P(Sleep=4).
      P = 0.06 + 0.03 + 0.06 + 0.04 + 0.04 + 0.10 = **0.23**
   d. P((Grade=C) $\vee$ (Sleep=6)).
      P = P(Grade=C) + P(Sleep=6) – P(Grade=C, Sleep=6)
      P = (0.03 + 0.04 + 0.06 + 0.08 + 0.10 + 0.12) + (0.10 + 0.05 + 0.08 + 0.06 + 0.03 + 0.08) – (0.03+ 0.08) = **0.72**
   e. P(Grade=A | Study=no, Sleep=2).
      P = 0.01/(0.01 + 0.02 + 0.12) = **0.067**
   f. P(Grade=A | Study=yes).
      P = (0.10 + 0.06+0.02)/((0.10 + 0.06 + 0.02) + (0.08 + 0.06 + 0.04) + (0.03 +0.04 + 0.06)) = 0.18 / (0.18 + 0.18 + 0.13) = 0.18 / 0.49 = **0.367**

2. Currently, it is estimated that the probability of someone having Coronavirus is 5%. Current tests have a false positive rate of 5% and a false negative rate of 15%. Suppose you take the

Coronavirus test and the result is positive. Compute the probability a person has the virus given that they test positive. Show your work.
P(virus=True) = 0.05
P(virus=False) = 0.95
P(test=False | virus=True) =0.15, P(test=True | virus=True) = 0.85
P(test=True | virus=False) = 0.05, P(test=False | virus=False) = 0.95

P(virus=True | test=True) = P(virus ∧ test=True) / P(test=True)
=**P(test=True | virus=True)P(virus=True) / P (test=True)**

P(test=True) = P(test=True ∧ virus=True) + P(test=True ∧ virus=False)
= P(test=True | virus=True)P(virus=True) + P(test=True | virus=False)P(virus=False)
= (0.85)(0.05) + (0.05)(0.95)
=0.09
P(virus=True | test=True) = **P(test=True | virus=True)P(virus=True) / P (test=True)**
=(0.85)(0.05)/0.09
=0.472

3. Suppose we have the 3x3 Wumpus world shown to the right. Your agent has visited locations (1,1) and (2,1). The agent observes a breeze in (2,1), but no breeze in (1,1). Given this information, we want to compute the probability of a pit in (3,1). You may use $p_{x,y}$ and $\neg p_{x,y}$ as shorthand notation for $Pit_{x,y}$=true and $Pit_{x,y}$=false, respectively. Similarly, you may use $b_{x,y}$ and $\neg b_{x,y}$ as shorthand notation for $Breeze_{x,y}$=true and $Breeze_{x,y}$=false, respectively.

¬B, OK	B, OK A→	P?

Specifically,

a. Define the sets: *breeze*, *known*, *frontier* and *other*.
Breeze = $\{\neg b_{1,1}, b_{1,2},\}$
Known = $\{\neg p_{1,1}, \neg p_{2,1}\}$
Frontier = $\{Pit_{1,2}, Pit_{2,2}\}$
Other = {(1,3), (2,3), (3,3), (3,2)}

b. Following the method in the textbook and lecture, compute the probability *distribution* $P(Pit_{3,1} \mid breeze, known)$. Show your work.

$P(Pit_{xy})=0.2$
$P(p_{3,1} \mid breeze, known) = P(p_{3,1}, breeze, known)/ P(breeze, known)$
$= \alpha P(p_{1,3}, breeze, known)$
$=\alpha \sum_{frontier,other} P\big(p_{3,1}, breeze, known, frontier, other\big)$
$=\alpha \sum_{F,O} P\big(breeze \mid p_{3,1}, known, F, O\big)\, P(p_{3,1}, known, F, O)$
$=\alpha \sum_{F} P\big(breeze \mid p_{3,1}, known, F\big) \sum_{O} P(p_{3,1}, known, F, O)$
$=\alpha \sum_{F} P\big(breeze \mid p_{3,1}, known, F\big) \sum_{O} P(p_{3,1}), P(known), P(F), P(O)$

$=\alpha \sum_F P\big(breeze \mid p_{3,1}, known, F\big)\, P(p_{3,1}), P(known), P(F) \sum_O P(O)$

$=\alpha \sum_F P\big(breeze \mid p_{3,1}, known, F\big)\, P(p_{3,1}) P(known) P(F)(1)$

$=\alpha P(p_{3,1}) P(known) \sum_F P\big(breeze \mid p_{3,1}, known, F\big)\, P(F)$

$\alpha' = \alpha P(known)$

$=\alpha' P(p_{3,1}) \sum_F P\big(breeze \mid p_{3,1}, known, F\big)\, P(F)$

F = {($pit_{1,2}$, $pit_{2,2}$), (¬$pit_{1,2}$, $pit_{2,2}$), ($pit_{1,2}$, ¬$pit_{2,2}$), (¬$pit_{1,2}$, ¬$pit_{2,2}$)}

$=\alpha' P(p_{3,1}) \sum_F P\big(breeze \mid p_{3,1}, known, F\big)\, P(F)$

$=\alpha' < (pit_{3,1} = true, pit_{3,1} = false >$

$=\alpha' < P\big(p_{3,1}\big)[P\big(breeze \mid p_{3,1}, known, pit_{1,2}, pit_{2,2}\big) P\big(pit_{1,2}\big), P\big(pit_{2,2}\big) >$

$+ \cdots (\neg \text{pit1,2}, \text{pit2,2}) + \cdots (\text{pit1,2}, \neg \text{pit2,2}) + \cdots (\neg \text{pit1,2}, \neg \text{pit2,2})]$

$=\alpha' < (0.2)[(0)(0.2)(0.2) + (1)(0.8)(0.2) + (0)(0.2)(0.8) + (1)(0.8)(0.8)] >$

$, (0.8)[(0)(0.2)(0.2) + (1)(0.8)(0.2) + (0)(0.2)(0.8) + (0)(0.8)(0.8)] >$

$=\alpha' < 0.2(0.16 + 0.64), 0.8(0.16) >$

$=\alpha' < 0.2(0.16 + 0.64), 0.8(0.16) >$

$=\alpha' < 0.16, 0.128 >$

=<0.56, 0.44>

P(Pit$_{3,1}$ | *breeze, known*)=0.56

4. *CptS 540 Students Only*. Suppose you want to reduce the false positive rate of the test in question 2 so that you are at least 50% certain that a person has the Coronavirus if they test positive. What false positive rate would achieve this goal? Show your work. 2