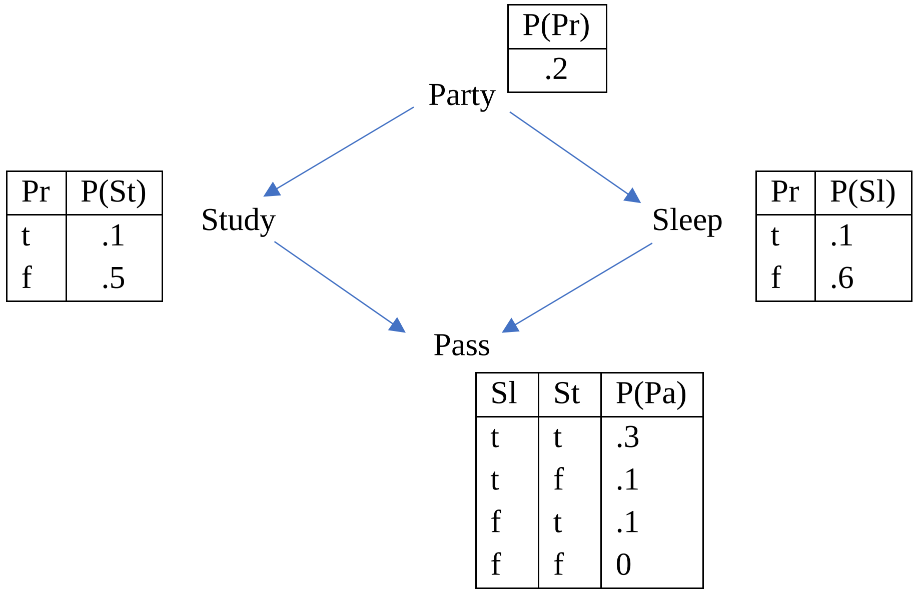
Washington State University
School of Electrical Engineering and Computer Science
Fall 2020

CptS 440/540 Artificial Intelligence
Homework 8
Due: October 29, 2020 (11:59pm pacific time)

General Instructions: Put your answers to the following problems into a PDF document and submit as an attachment under Content ➐ Homework 8 for the course CptS 440 Pullman (all sections of CptS 440 and 540 are merged under the CptS 440 Pullman section) on the Blackboard Learn system by the above deadline. Note that you may submit multiple times, but we will only grade the most recent entry submitted before the deadline.

1. Construct a Bayesian network (showing all nodes, links, and conditional probability tables) that is consistent with the below full joint probability distribution below over four Boolean random variables (Party, Sleep, Study, Pass) and consistent with the following three conditions. Round each probability in the conditional probability tables to the nearest tenth.
    - Sleep is conditionally independent of Study given Party.
    - Study is conditionally independent of Sleep given Party.
    - Pass is conditionally independent of Party given Sleep and Study.

Party	**Sleep**	**Study**	**Pass**	**Probability**
true	true	true	true	0.0216
true	true	true	false	0.0024
true	true	false	true	0.0224
true	true	false	false	0.0336
true	false	true	true	0.0216
true	false	true	false	0.0144
true	false	false	true	0.0084
true	false	false	false	0.0756
false	true	true	true	0.3024
false	true	true	false	0.0336
false	true	false	true	0.0896
false	true	false	false	0.1344
false	false	true	true	0.0864
false	false	true	false	0.0576
false	false	false	true	0.0096
false	false	false	false	0.0864



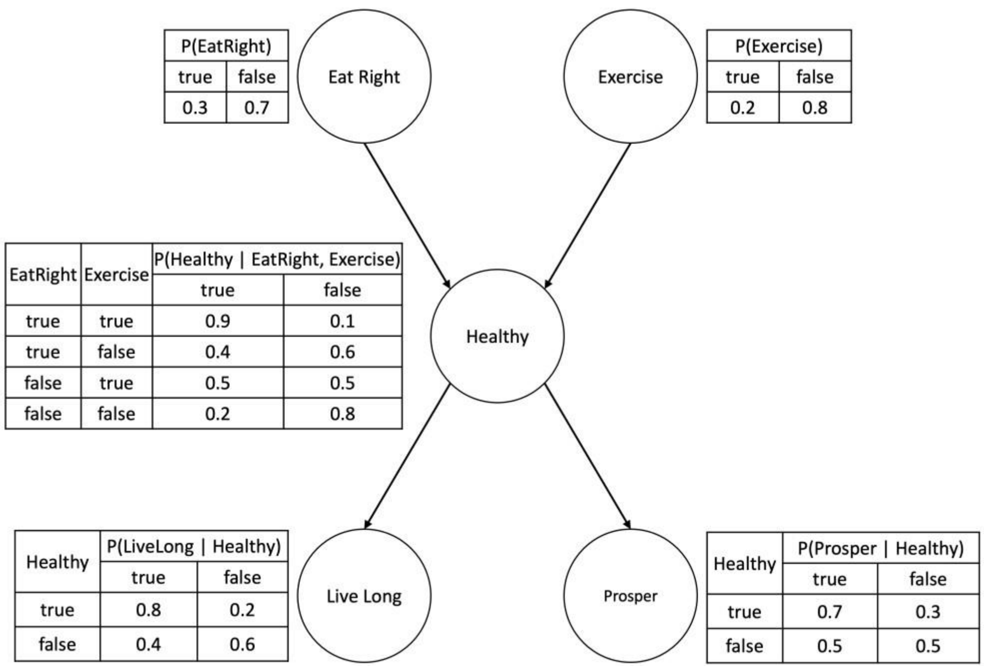
P(Pr)
.2

Pr	P(St)
t	.1
f	.5

Pr	P(Sl)
t	.1
f	.6

Sl	St	P(Pa)
t	t	.3
t	f	.1
f	t	.1
f	f	0

2. Compute the probabilities below based on the following Bayesian network. Show your work.



P(EatRight)	
true	false
0.3	0.7

P(Exercise)	
true	false
0.2	0.8

EatRight	Exercise	P(Healthy | EatRight, Exercise)	
		true	false
true	true	0.9	0.1
true	false	0.4	0.6
false	true	0.5	0.5
false	false	0.2	0.8

Healthy	P(LiveLong | Healthy)	
	true	false
true	0.8	0.2
false	0.4	0.6

Healthy	P(Prosper | Healthy)	
	true	false
true	0.7	0.3
false	0.5	0.5

a. P(EatRight=true, Exercise=true, Healthy=true, LiveLong=true, Prosper=true)?
   0.3 * 0.2 * 0.9 * 0.8 * 0.7 = **0.03024**
b. P(Healthy=true | Exercise=false)?
   P(H,E) = (0.4+0.2)/2 = 0.3
   P(H,E)/P(E) = 0.3/0.8 = **0.375**
c. P(LiveLong=true | EatRight=true, Exercise=true)?
   (0.3* 0.2 * .9 *.8) + (0.3 * 0.2 *.1 * .4) = 0.0432 + 0.0024 = **0.0456**
d. P(EatRight=true | LiveLong=true, Prosper=true)?
   =α**P**(ER, LL, P)

$$= \alpha \sum_{EX} \sum_{H} P(ER, LL, P, H, EX) \mathrm{P(LL|H)P(P|H)}$$

$$= \alpha \sum_{EX} \sum_{H} P(ER)P(EX)P(H|ER, EX)\text{P(LL|H)}\ P(P|H)$$

$$= \alpha\text{P(ER)} \sum_{EX} \text{P(EX)} \sum_{H} P(H|ER, EX)\text{P(LL|H)}\ P(P|H)$$

$$= \alpha[\text{P(ER)}\{\text{P(EX)}[\text{P(H|ER, EX)P(LL|H)}P(P|H) + \text{P}(\neg\text{H|ER, EX})\text{P(LL|}\neg\text{H)}P(P|\neg H)] + P(\neg EX)[P(\text{H|ER}, \neg\text{EX})\text{P(LL|H)}P(P|H)] + \text{P}(\neg\text{H|ER}, \neg\text{EX})\text{P(LL|}\neg\text{H)}P(P|\neg H)]\}$$

=0.3(0.2(0.9*0.8*0.7 + 0.1*0.4*0.3)+0.8(0.4*0.8*0.7 + 0.6*0.4*0.5))= α 0.11352

=0.11352/((1.2/2) *(1.2/2) **)=** 0.11352/0.36 = **0.315**

e. **P**(Prosper | EatRight=false, Exercise=false)?

=α**P**(P, ¬ER, ¬EX)

=α<P(P, ¬ER, ¬EX), P(¬P, ¬ER, ¬EX)>

α < 0.7*0.8*0.2*0.7 + 0.7*0.8*0.8*0.5, 0.7*0.8*0.2*0.3 + 0.7*0.8*0.8*0.5>

= α <0.3024, 0.2576>

=<0.54, 0.46>

3. What would be the most likely sample from applying direct sampling to the Bayesian network in Problem 2? What is this sample's probability?

   P(EatRight=False, Exercise=False, healthy=False, LiveLong=False, prosper=False)

   0.7 * 0.8 * 0.8 * 0.6 * 0.5 = **0.1344**

   P(EatRight=False, Exercise=False, healthy=False, LiveLong=False, prosper=True)

   0.7 * 0.8 * 0.8 * 0.6 * 0.5 = **0.1344**

4. *CptS 540 Students Only*. Consider the Bayesian network below, where each of the five random variables have a domain of 4 values. What is the minimum number of probabilities needed to completely describe the full joint probability distribution for this scenario? Justify your answer.