- Sigmoid Function

$$f(x) = \frac{1}{1+e^{-x}}$$

- $0 < f(x) < 1$
- Can give value $[0,1]$, given a line

- $$f(x) = \frac{1}{1+e^{-x}}$$

$$= \frac{1}{1+e^{-(mx+c)}}$$

$$= \frac{1}{1+e^{-y}}$$

- If line is given to you, predict object is line this side / that side.

$$p(x) = \frac{1}{1+e^{-(mx+c)}}$$

$$= \frac{e^{(mx+c)}}{1+e^{mx+c}}$$

- $h(\theta) = mx+c$

$$p(x) = \frac{e^{h(\theta)}}{1+e^{h(\theta)}} \quad \text{--- (I)}$$

$$\Rightarrow 1-p(x) = 1 - \frac{e^{h(\theta)}}{1+e^{h(\theta)}} \quad \text{--- (II)}$$

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$$\frac{(I)}{(II)} \quad \frac{p(x)}{1-p(x)} = \frac{e^{h(\theta)}}{1+e^{h(\theta)}} \Bigg/ 1 - \frac{e^{h(\theta)}}{1+e^{h(\theta)}}$$

$$\Rightarrow \frac{p(x)}{1-p(x)} = \frac{e^{h(\theta)}}{\cancel{1+e^{h(\theta)}}} \Bigg/ \frac{1+\cancel{e^{h(\theta)}} - \cancel{e^{h(\theta)}}}{\cancel{1+e^{h(\theta)}}}$$

$$\Rightarrow \frac{p(x)}{1-p(x)} = e^{h(\theta)}$$

$$\Rightarrow \log\left(\frac{p(x)}{1-p(x)}\right) = \log\left(e^{mx+c}\right)$$

$$\Rightarrow \boxed{\log\left(\frac{p(x)}{1-p(x)}\right) = mx+c}$$

Logistic Regression Formula

$$\Rightarrow \frac{p(x)}{1-p(x)} = e^{mx+c}$$

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- Cost Function

$$J(\theta) = -\frac{1}{m} \sum_{i=1}^{m} \left[ y^{i} \log(\hat{p}^{(i)}) + (1-y^{(i)}) \log(1-\hat{p}^{(i)}) \right]$$

for single instance,

$$\text{Cost}(p(x), y) = \begin{cases} -\log(p(x)), & \text{if } y=1 \\ -\log(1-p(x)), & \text{if } y=0 \end{cases}$$

- minimize $J(\theta)$ ; cost ↓, Error ↓

  - we choose this as line or separator

- Multiple Logistic Function

$$p(x) = \frac{1}{1+e^{-x}}$$

$$= \frac{1}{1+e^{-(m_1x_1 + m_2x_2 + \cdots + c)}}$$