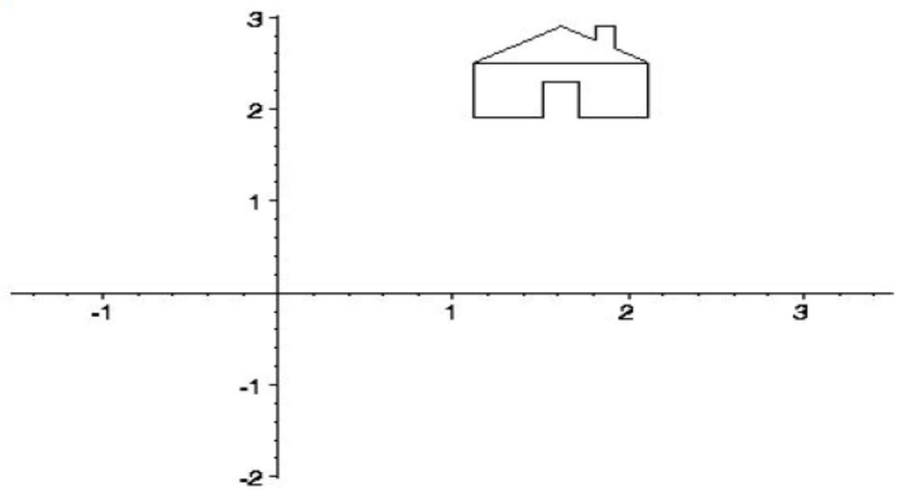
3D Geometric Transformations

TRANSFORMATION

- Transformations are a fundamental part of the computer graphics. Transformations are the movement of the object in Cartesian plane .



WHY WE USE TRANSFORMATION

- Transformation are used to position objects , to shape object , to change viewing positions , and even how something is viewed.
- In simple words transformation is used for

  1) Modeling
  2) viewing

3D TRANSFORMATION

- When the transformation takes place on a 3D plane .it is called 3D transformation.
- Generalize from 2D by including **z** coordinate

Straight forward for translation and scale, rotation more difficult

Homogeneous coordinates: 4 components

Transformation matrices: 4×4 elements

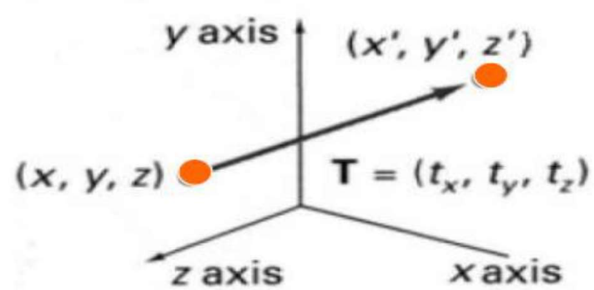
$$\begin{bmatrix} a & b & c & t_x \\ d & e & f & t_y \\ g & h & i & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3D TRANSLATION

- Moving of object is called translation.
- In 3 dimensional homogeneous coordinate representation , a point is transformed from position P = (x, y , z) to P'=(x', y', z')
- This can be written as:-

Using **P' = T . P**

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

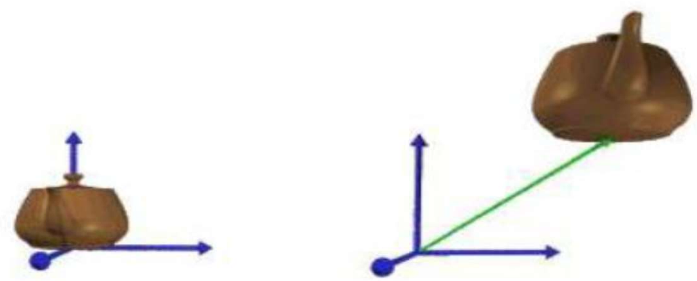


3D TRANSLATION

- The matrix representation is equivalent to the three equation.

  $x' = x + t_x$, $y' = y + t_y$, $z' = z + t_z$

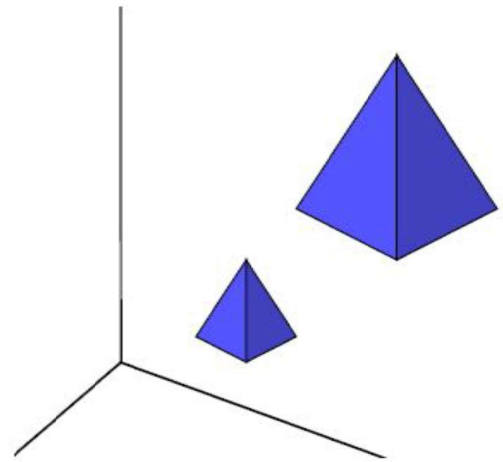
Where parameter t_{x}, t_{y}, t_{z} are specifying translation distance for the coordinate direction x , y , z are assigned any real value.



3D SCALING

- Changes the size of the object and repositions the object relative to the coordinate origin.

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



3D SCALING

- The equations for scaling

$$x' = x \cdot sx$$

$$S_{sx,sy,sz} \; \square \quad y' = y \cdot sy$$

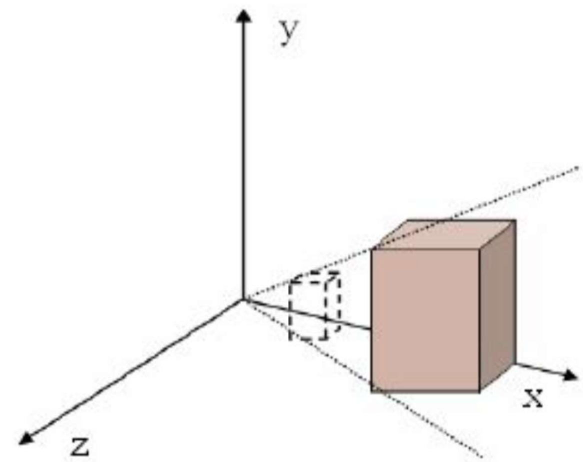
$$z' = z \cdot sz$$

Classwork

Derive the transformation matrix for scaling a 3D object keeping the point (Xa, Ya, Za) fixed

Fixed Point Scaling

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & (1-s_x)x_f \\ 0 & s_y & 0 & (1-s_y)y_f \\ 0 & 0 & s_z & (1-s_z)z_f \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

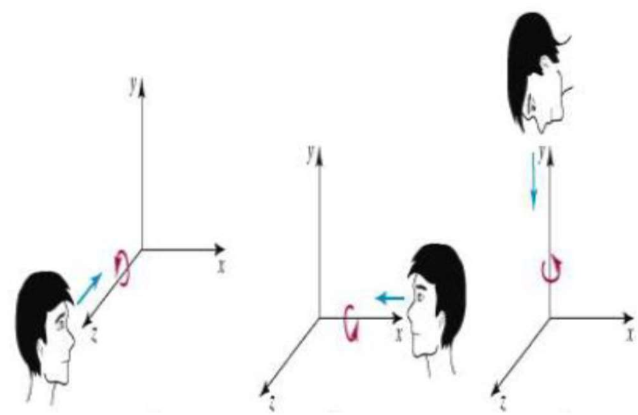


3D ROTATION

Where an object is to be rotated about an axis that is parallel to one of the coordinate axis, we can obtain the desired rotation with the following transformation sequence.

Coordinate axis rotation

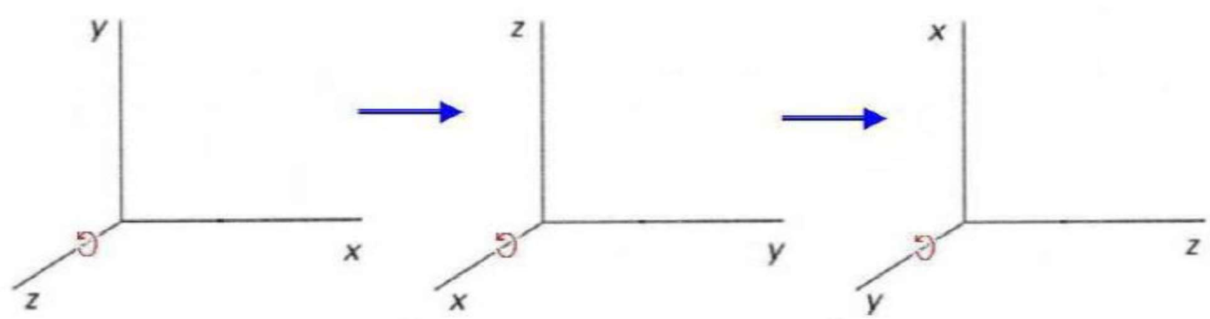
Z- axis Rotation(Roll)
Y-axis Rotation(Yaw)
X-axis Rotation(Pitch)



COORDINATE AXIS ROTATION

- Obtain rotations around other axes through cyclic permutation of coordinate parameters:

$$x \rightarrow y \rightarrow z \rightarrow x$$



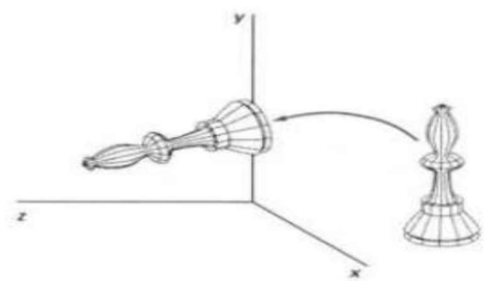
X-AXIS ROTATION

The equation for X-axis rotation

$x' = x$

$y' = y \cos\theta - z \sin\theta$

$z' = y \sin\theta + z \cos\theta$



$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Y-AXIS ROTATION

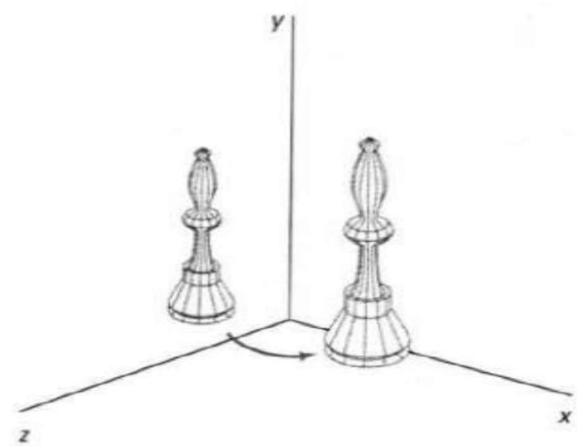
The equation for Y-axis rotaion

$\mathbf{x' = x\cos\theta + z\sin\theta}$

$\mathbf{y' = y}$

$\mathbf{z' = \quad z\cos\theta - x\sin\theta}$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



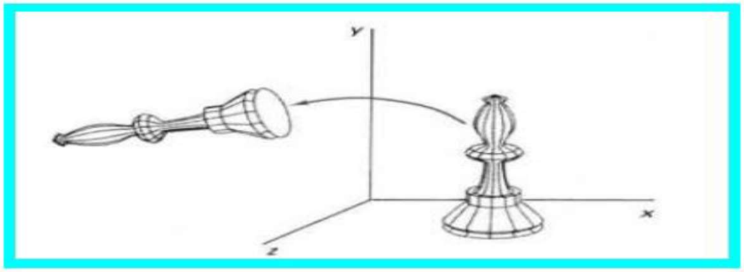
Z-AXIS ROTATION

The equation for Y-axis rotaion

$x' = x \cos\theta - y \sin\theta$

$y' = x \sin\theta + y \cos\theta$

$z' = z$



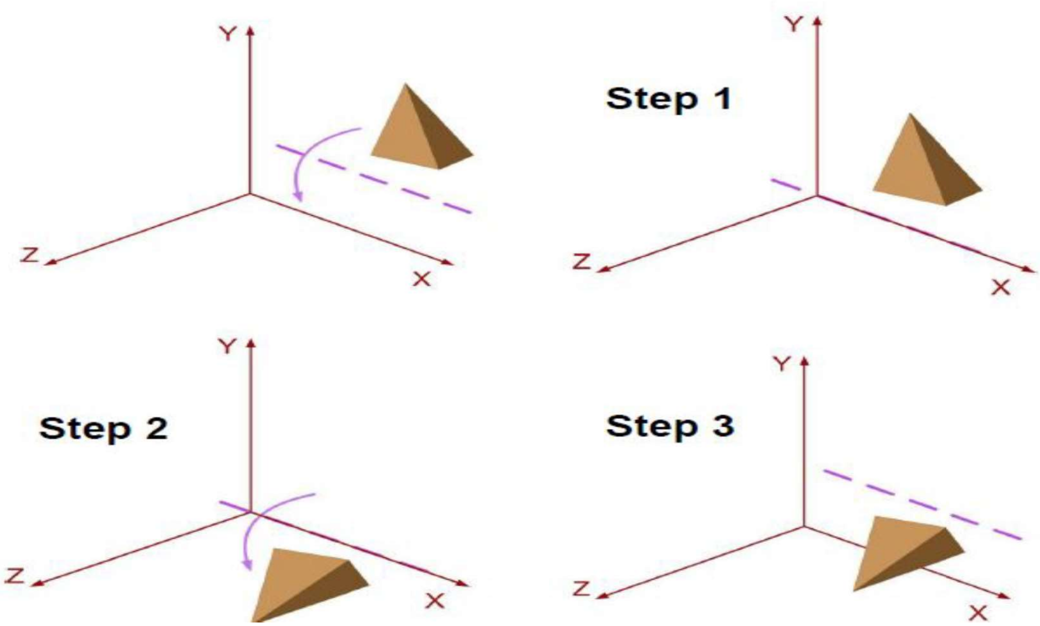
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

A homogenous co-ordinate point $P[3, 2, 1,]$ is translated in x, y and z-direction by -2, -2 and -2 unit respectively followed by successive rotation $60°$ about X-axis. Find the final position of homogenous co-ordinate.

$$P' \begin{bmatrix} 1 & 0.864 & -0.5 & 1 \end{bmatrix}$$

Rotation about an axis parallel to coordinate axis

- E.g. x-axis



STEPS

1. Translate the object so as to coincide rotation axis to parallel co-ordinate axis

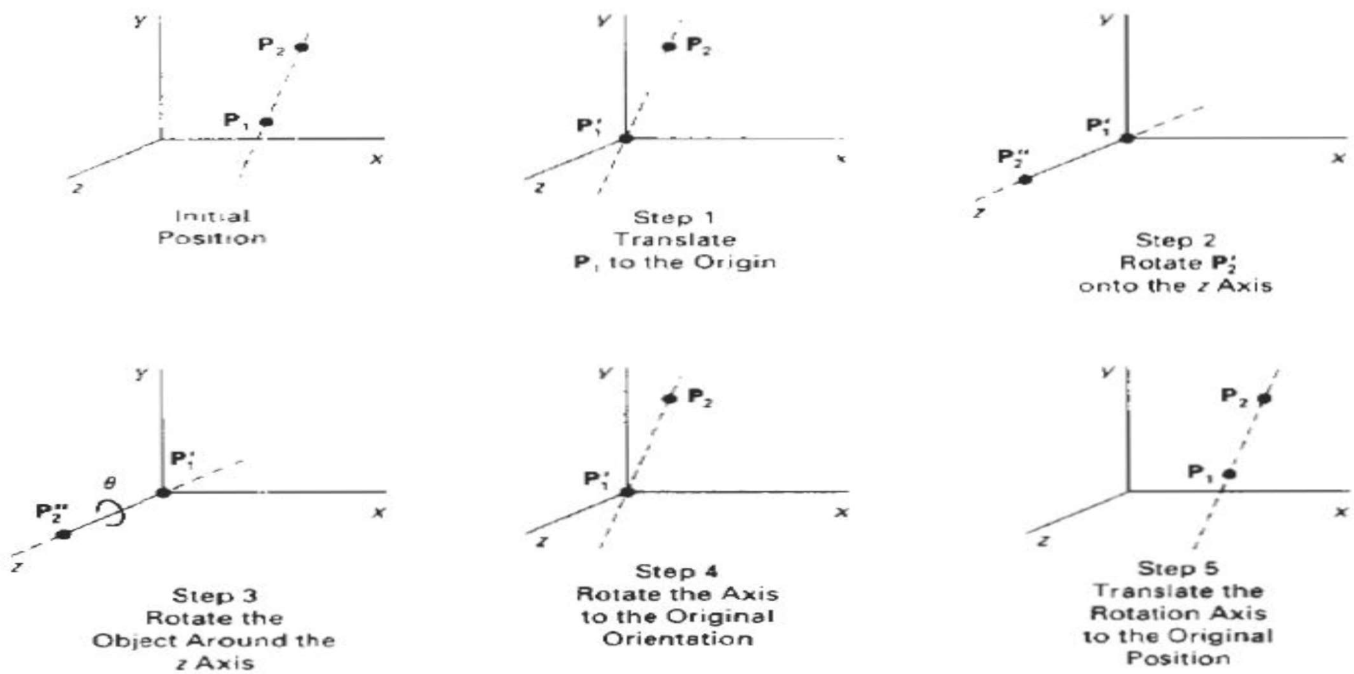
2. Perform the rotation about x-axis

3. Translate back the object so as to move rotation axis to original position

Problem

Rotate a point (5,4,2) about an axis defined by two points (10,10,15) (10,10, 25) by 45 degree. Show the final transformation matrix and rotated point coordinate.

General 3D Rotation



General 3D Rotation

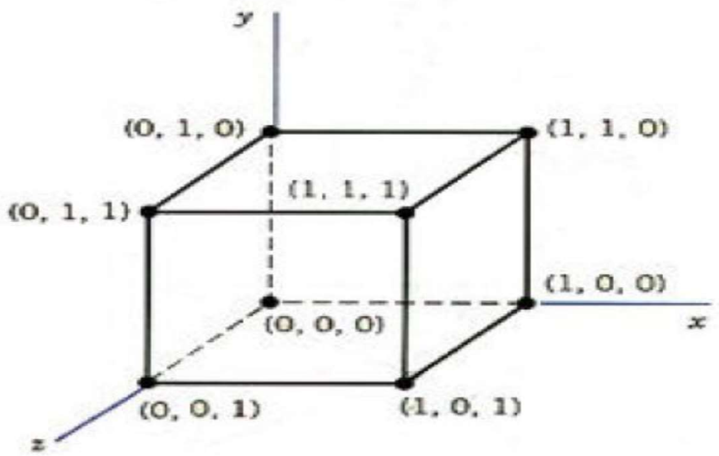
Rotation about an Arbitrary Axis

Basic Idea

1. Translate the object so that the rotation axis passes through the coordinate origin
2. Rotate the object so that the axis of rotation coincides with one of the coordinate axes
3. Perform the specified rotation about the coordinate axis
4. Apply inverse rotations to bring the rotation axis back to its original orientation
5. Apply the inverse translation to bring the rotation axis back to its original position.

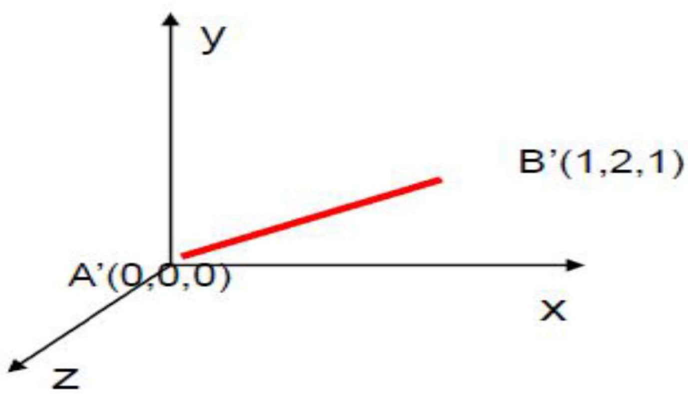
Problem Arbitrary axis Rotation

Find the new coordinates of a unit cube 90°-rotated about an axis defined by its endpoints A(2,1,0) and B(3,3,1).



A Unit Cube

- Step1. Translate point A to the origin

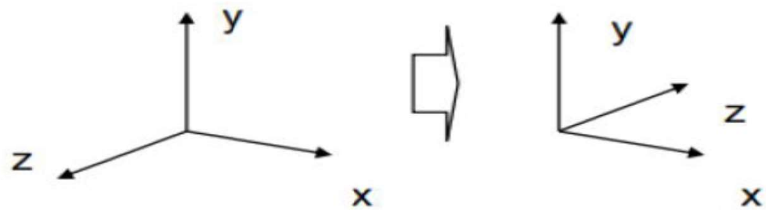


$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Reflections

- A three-dimensional reflection can be performed relative to a selected reflection axis or with respect to a selected refection plane.
- Reflections relative to a given axis are equivalent to $18o^0$ rotations about that axis.
- Reflections with respect to a plane are equivalent to 180^0 rotations in four-dimensional space.
- Reflection plane can be a coordinate plane (either xy, xz, or yz)

- Reflection in xy plane:
- This transformation changes the sign of the z coordinates, Leaving the x and y-coordinate values unchanged.

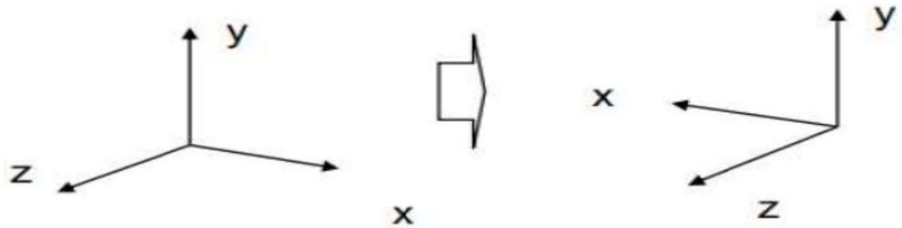


  - x' = x
  - y' = y
  - z' = -z

- The matrix representation for this reflection of points relative to the x-y plane is:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- Reflection in yz plane:
  - This transformation changes the sign of the x coordinates, Leaving the z and y-coordinate values unchanged.

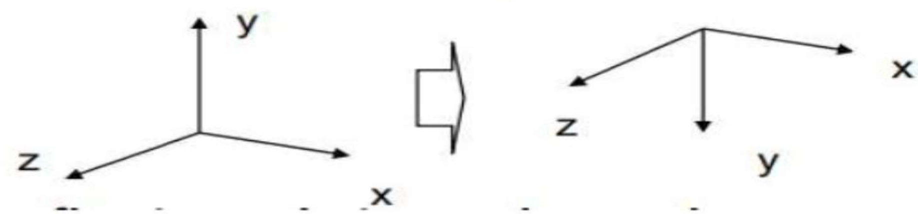


- x' = -x
  - y'=y
  - z'=z
  - The matrix representation for this reflection of points relative to the x-y plane is:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Reflection in XZ plane:

- This transformation changes the sign of the y coordinates, Leaving the z and x-coordinate values unchanged.



- y'= -y
- x'=x
- z'=z
- The matrix representation for this reflection of points relative to the x-y plane is:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Shears

- Shearing transformations produce distortions in the shapes of objects.
- The transformation equations for a Z-axis shear is :
  - $x' = x + a.z$
  - $y' = y + b.z$
  - $z' = z$

The matrix representation for a z-axis shear is:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & a & 0 \\ 0 & 1 & b & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

END of 3D Transformations