4-75 N beads are resting on a smooth horizontal wire which is circular at the end with radius r as shown in figure. The masses of the beads are m, $m/2$, $m/4$ ...., $m/2^{n-1}$ respectively. Find the minimum velocity which should be imparted to the first bead of mass m such that the n^{th} bead will fall in the tank shown in figure-4.148.

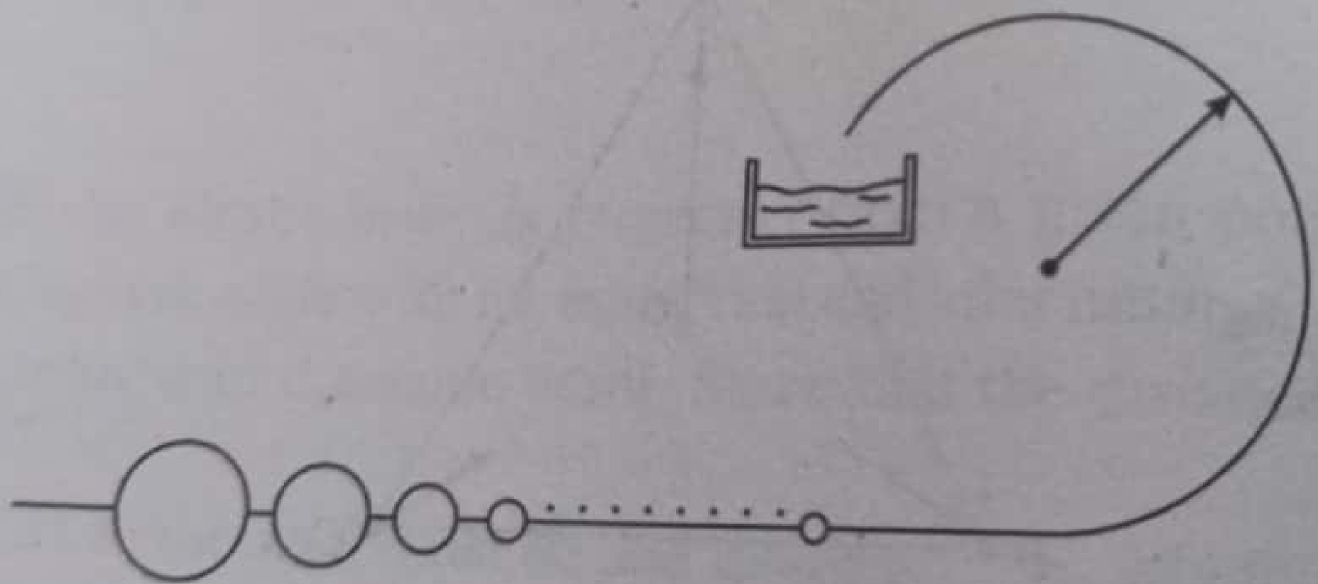

Figure 4.148

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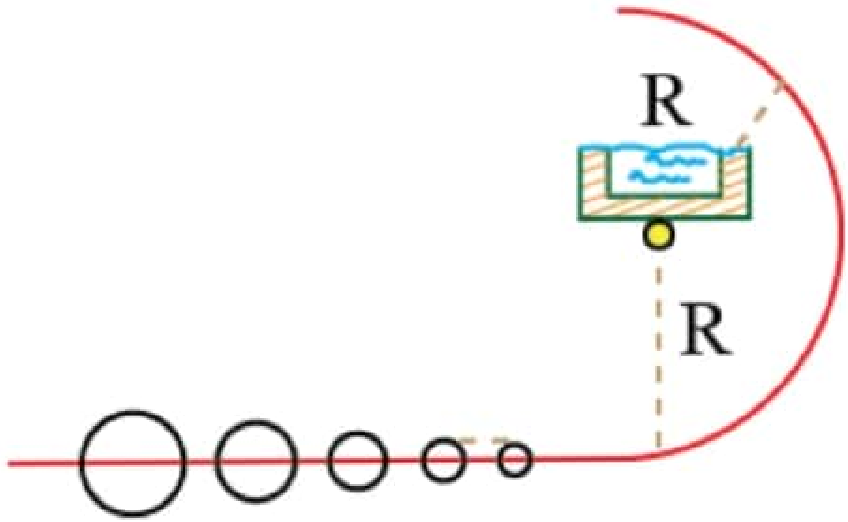
n beads are resting on a smooth horizontal wire which is circular at the end with radius R as shown. The masses of the beads are $m, \frac{m}{2}, \frac{m}{4}, \ldots, \frac{m}{2^{n-1}}$ respectively. Find the minimum velocity that should be imparted to the first bead of mass m such that the n^{th} bead will fall in the tank shown in the figure. Assume all collisions to be elastic.



This question has single correct option

A. $\left(\frac{3}{4}\right)^{n-1} \sqrt{5gR}$

B. $\left(\frac{4}{3}\right)^{n-1} \sqrt{5gR}$

C. $\left(\frac{3}{4}\right)^{n-1} 2\sqrt{gR}$

D. $\left(\frac{4}{3}\right)^{n-1} 2\sqrt{gR}$

Answer:

C. $\left(\frac{3}{4}\right)^{n-1} 2\sqrt{gR}$

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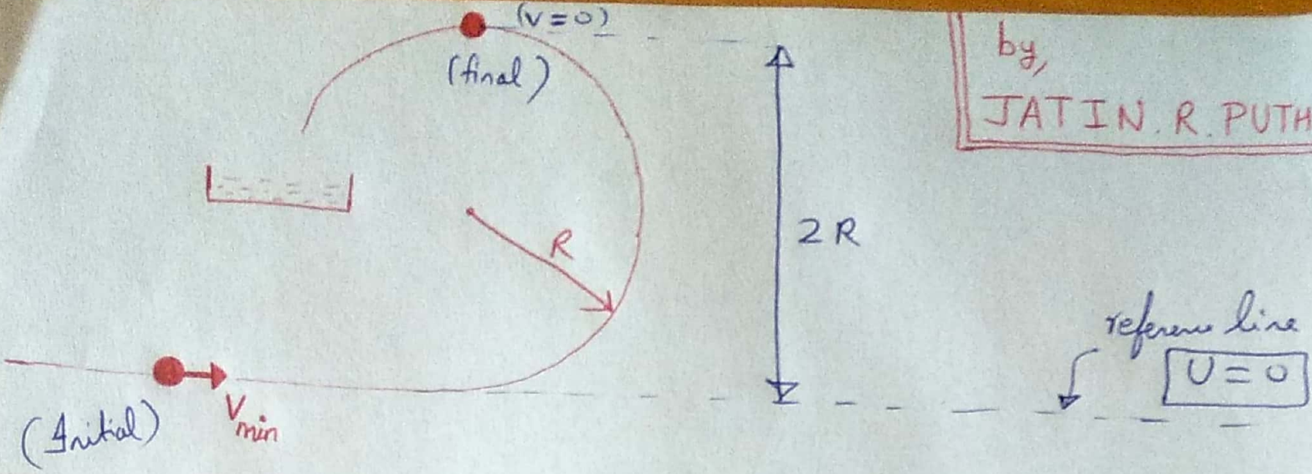

by,
JATIN. R. PUTHRAN

For a particle/bead to fall in the tank the minimum velocity required should be sufficient enough to be able to take the particle to the uppermost point.

By Energy Conservation, $\boxed{E_i = E_f}$

$$U_i + K_i = U_f + K_f$$

$$0 + \frac{1}{2} M V_{min}^2 = Mg(2R) + 0$$

$$\therefore \boxed{V_{min} = \sqrt{4gR}}$$

So this is the velocity with which the smallest bead/ n^{th} bead/ bead with mass $\frac{m}{2^{n-1}}$ must be launched with is given as,

$$\therefore \boxed{V_n = V_{min} = \sqrt{4gR}} \quad (1)$$

(Velocity of n^{th} bead / bead with mass $\frac{m}{2^{n-1}}$.)

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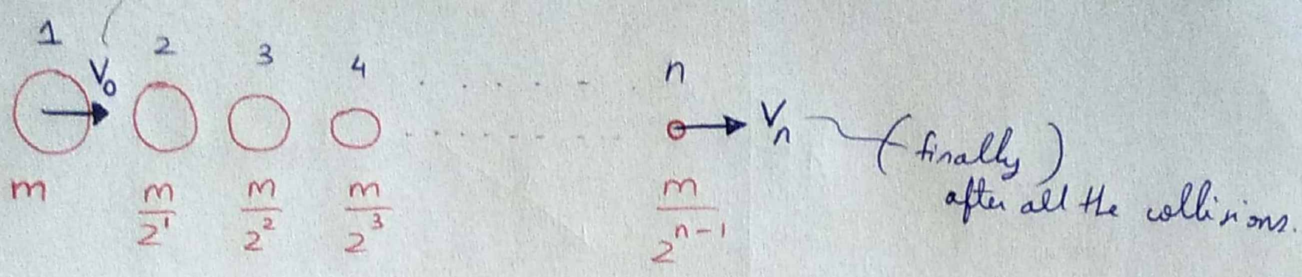
Let initial velocity imparted to '1' be "V_0"



Now, consider perfectly elastic collision of '1' & '2'.

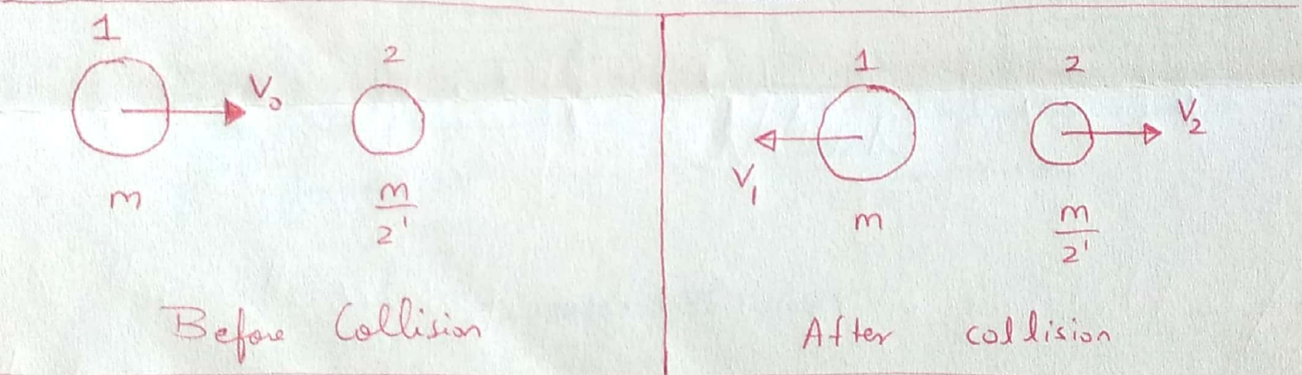
We know,

$$e = \frac{V_{sep}}{V_{app}} = \frac{V_2 - V_1}{u_1 - u_2}$$

$$1 = \frac{V_2 - V_1}{V_0 - 0}$$

$$\Rightarrow \quad V_0 = V_2 - V_1$$

$$\Rightarrow \quad \boxed{V_1 = V_2 - V_0} \quad \text{--- (2)}$$



By conservation of Linear Momentum, $\boxed{P_i = P_f}$

$$mV_0 + 0 = mV_1 + \frac{m}{2}V_2$$

$$\boxed{V_0 = V_1 + \frac{V_2}{2}} \quad \text{--- (3)}$$

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Putting value of 'V_1' from ② in ③,

We get,

$$V_0 = (V_2 - V_0) + \frac{V_2}{2}$$

$$V_0 + V_0 = V_2 + \frac{V_2}{2}$$

$$2V_0 = \frac{3V_2}{2}$$

$$V_0 = \frac{3}{4} V_2 \quad (4)$$

This means that if we impart a velocity of 'V_0' to '1' then it's value is equal to $\left(\frac{3}{4}\right)^{th}$ of the velocity with which '2' leaves after collision.

So, by same way if we proceed we will always get the same relation for every two bodies.

i.e $V_2 = \frac{3}{4} V_3$, $V_3 = \frac{3}{4} V_4$, $V_4 = \frac{3}{4} V_5$ & so on.

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And atlast,
$$V_{n-1} = \frac{3}{4} V_n$$

$\therefore$ Back substituting we will obtain,

$$V_0 = \underbrace{\left(\frac{3}{4}\right) \times \left(\frac{3}{4}\right) \times \left(\frac{3}{4}\right) \times \cdots\cdots}_{(n-1)\text{ times}} \times V_n$$

$$\boxed{V_0 = \left(\frac{3}{4}\right)^{n-1} V_n} = \left(\frac{3}{4}\right)^{n-1} V_{min}$$

From (1),
$$V_0 = \left(\frac{3}{4}\right)^{n-1} \times \sqrt{4gR}$$

$$\therefore \boxed{V_0 = \left(\frac{3}{4}\right)^{n-1} \sqrt{4gR}}$$ Ans.

The velocity with which '1' i.e. bead of mass 'm' (minimum) must be imparted.

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Please subscribe to my YouTube channel if you liked my content.
The channel name goes by my name :
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(^~^)
The video explanation of the solution is uploaded on my YouTube channel :)

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