4-59 A wedge of mass M rests on an absolutely smooth, horizontal surface. A block of mass m is placed on the wedge, inclined at an angle α to the horizontal. All the surfaces are frictionless. Assuming that the system was at rest at the initial moment, find the velocity of the wedge when the block slides down the plane through a vertical height h.

Ans. [$\sqrt{\dfrac{2m^2 gh\cos^2\alpha}{(M+m)(M+m\sin^2\alpha)}}$]


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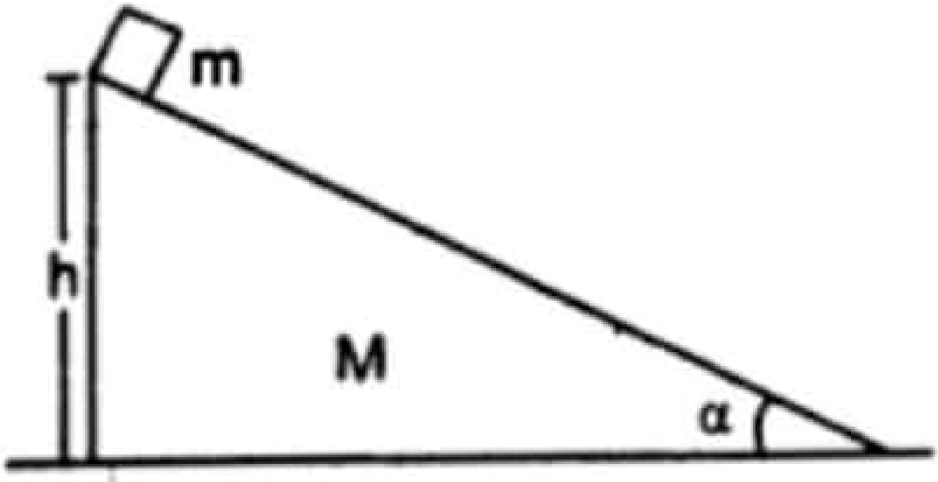
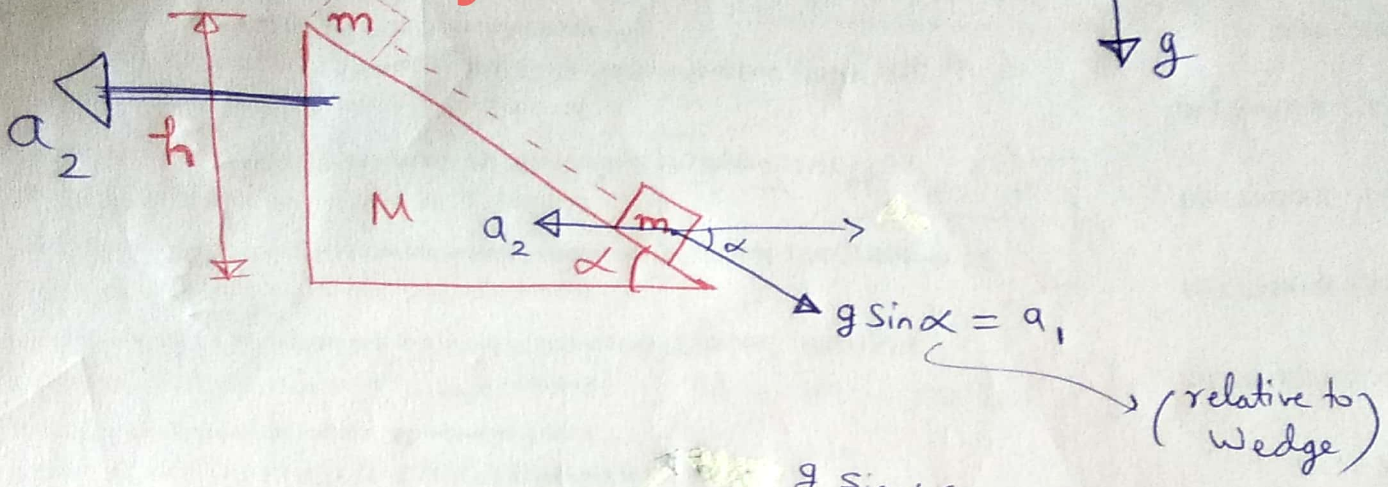
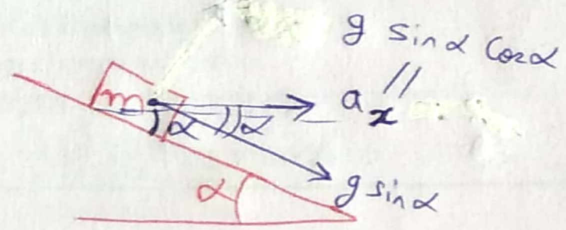
A block of mass m is placed on a triangular block of mass M, which in turn is placed on horizontal surface as shown in figure Assuming frictionless surfaces find the velocity of the triangular block when the smaller block reaches the bottom end.







Horizontal Comp. of 'a_1' will be, $a_x = g\sin\alpha\cos\alpha$,

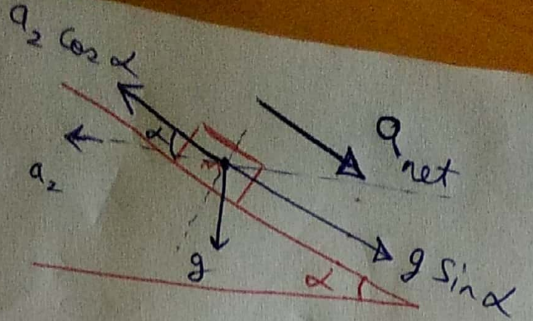
for which ~~wo~~ wedge 'M' will accelarate towards left

i.e. $\boxed{a_x = a_2}$.

By C.O.M (in horizontal direction, $F_{ext} = 0$)

$$ma_x = (M+m)a_2$$

$$a_2 = \frac{ma_x}{(M+m)} = \frac{mg\sin\alpha\cos\alpha}{(M+m)}$$



Absolute Resultant accelaration of 'm' on wedge 'M' along direction of incline / motion will be,

$$a_{net} = a = g \sin\alpha - a_2 \cos\alpha$$

$$a_{net} = g \sin\alpha - \left(\frac{mg \sin\alpha \cos\alpha}{(m+M)}\right) \cos\alpha$$

$$= g \sin\alpha \left[1 - \frac{m \cos^2\alpha}{m+M}\right]$$

$$= \frac{g \sin\alpha}{(m+M)} \left[M + m - m \cos^2\alpha\right]$$

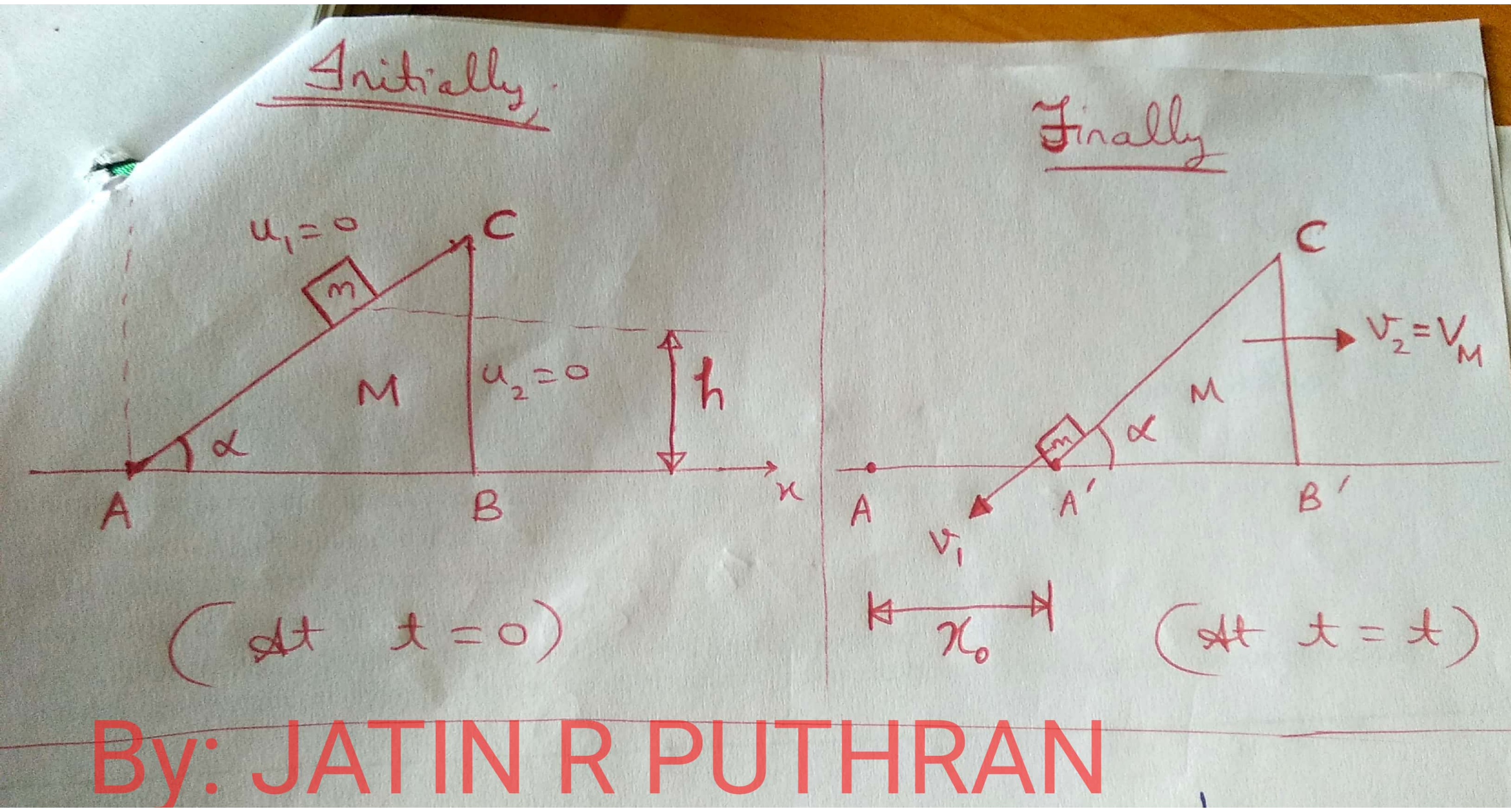
$$\boxed{a_{net} = \frac{g \sin\alpha}{(m+M)} \times \left[M + m \sin^2\alpha\right]}$$

Now, $\because\ s = ut + \frac{1}{2} a t^2$ (apply along incline / motion)

$$\frac{h}{\sin\alpha} = 0 + \frac{1}{2} \times a_{net} \times t^2$$

Time to reach bottom,

$$\boxed{t = \sqrt{\frac{2h}{a_{net} \sin\alpha}}}$$



Initially
$u_1 = 0$
C
m
M
$u_2 = 0$
h
α
A
B
x
(At $t = 0$)
Finally
C
M
$V_2 = V_M$
m
α
A
A'
B'
V_1
x_0
(At $t = t$)
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So, velocity of [redacted] Wedge ('M') after time 't' will be,

$$V_M = u + a_2 t \qquad (\because u = 0)$$

$$V_M = a_2 t = \frac{mg\sin\alpha\cos\alpha}{(M+m)}\sqrt{\frac{2h}{a_{net}\sin\alpha}}$$

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$$V_M = \sqrt{\frac{2m^2g^2\sin^{\cancel{2}}\alpha\cos^2\alpha\, h}{(M+m)^2\times\cancel{\sin\alpha}\times a_{net}}}$$

$$V_M = \sqrt{\frac{2m^2g^{\cancel{2}}\cancel{\sin\alpha}\cos^2\alpha\, h}{(M+m)^{\cancel{2}}}\times\frac{\cancel{(m+M)}}{\cancel{g\sin\alpha}\,[M+m\sin^2\alpha]}}$$

$$\boxed{V_M = \sqrt{\frac{2m^2gh\cos^2\alpha}{(M+m)[M+m\sin^2\alpha]}}}$$

Ans

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(^~^)
The video explanation of the solution is uploaded on my YouTube channel :)


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