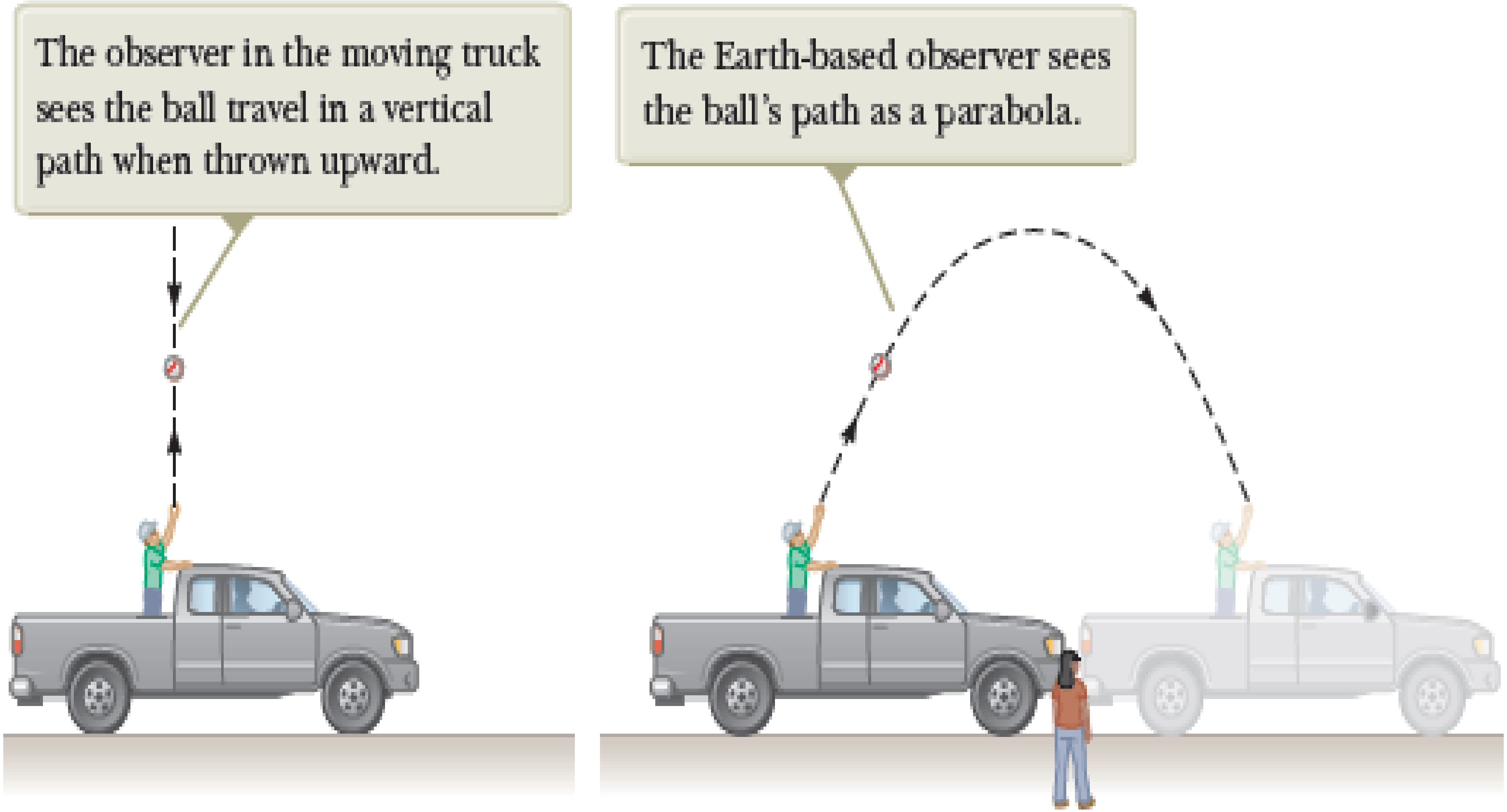
RELATIVITY

Mechanics of bodies moving at speed comparable to the speed of light

If $v \ll c$ — **Classical Mechanics/Newtonian Mechanics**

If $v \to c$ — **Relativistic Mechanics**

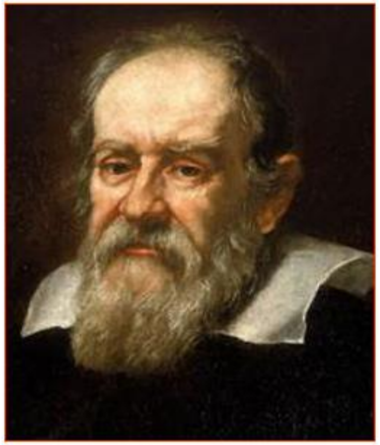
In the theory of relativity, laws of Newtonian mechanics are found incomplete. But Results and laws of Relativistic Mechanics lead to laws of Newtonian Mechanics in the limit $v \ll c$, whereas converse is not true



Relativity in Newtonian Mechanics

Galilean Transformations

Equations which relate the measurements of stationary frame of reference with the moving frame moving with $v << c$

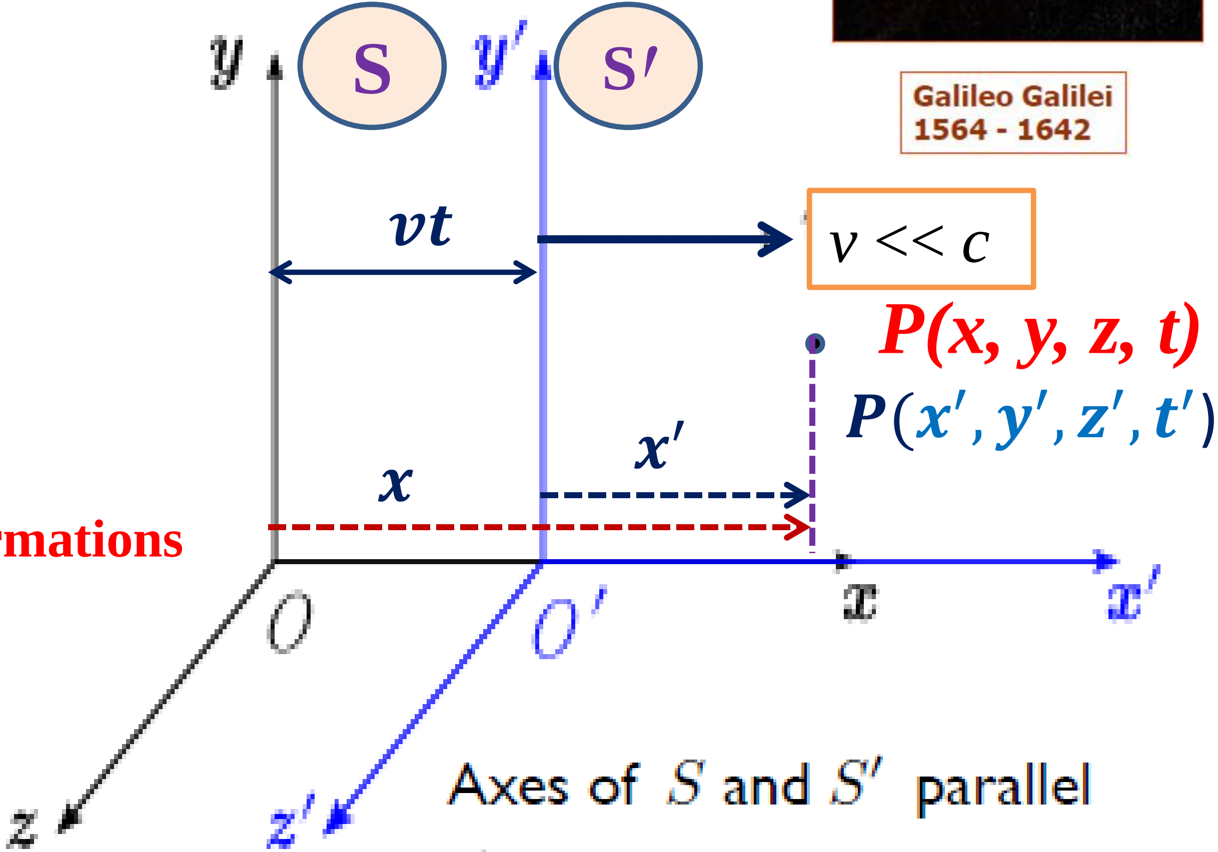


Galileo Galilei
1564 - 1642

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

Inverse Galilean Transformations

$$x = x' + vt'$$
$$y = y'$$
$$z = z'$$
$$t = t'$$



Axes of S and S' parallel

$\vec{v}$ parallel to x-direction

Origins coincide at $t = t' = 0$

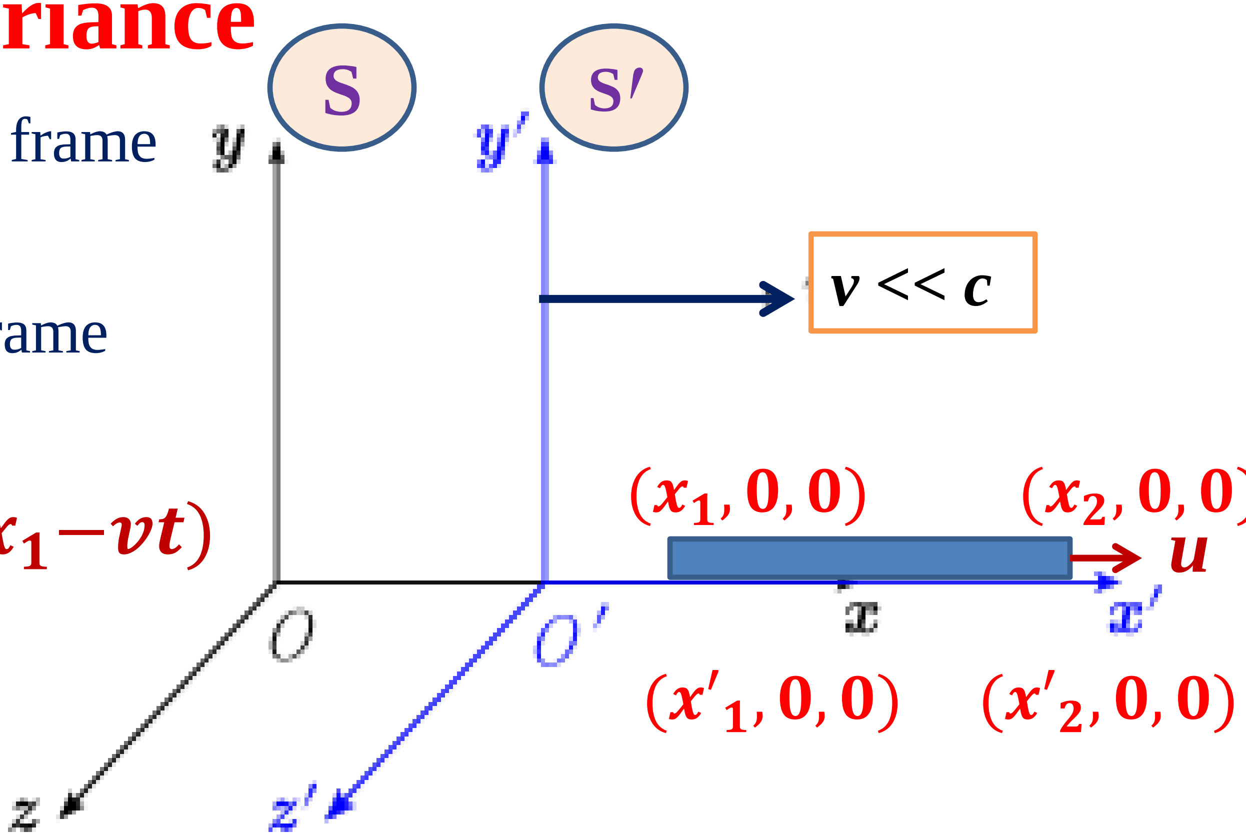
Galilean Invariance

Length as per stationary frame

$$L = x_2 - x_1$$

Length as per Moving frame

$$L' = x'_2 - x'_1$$
$$= (x_2 - vt) - (x_1 - vt)$$
$$= x_2 - x_1$$
$$L' = L$$



If bar moves with speed ***u*** parallel to its length and ***P*** is any point on bar with coordinates measured by two frames as ***x*** and ***x'*** such that

$$x' = x - vt$$
$$\frac{dx'}{dt} = \frac{dx}{dt} - v$$
$$u' = u - v$$

These results are compatible with Newtonian Mechanics but not consistent with the theory of ***RELATIVITY***

Ether/Search for an Absolute frame of Reference

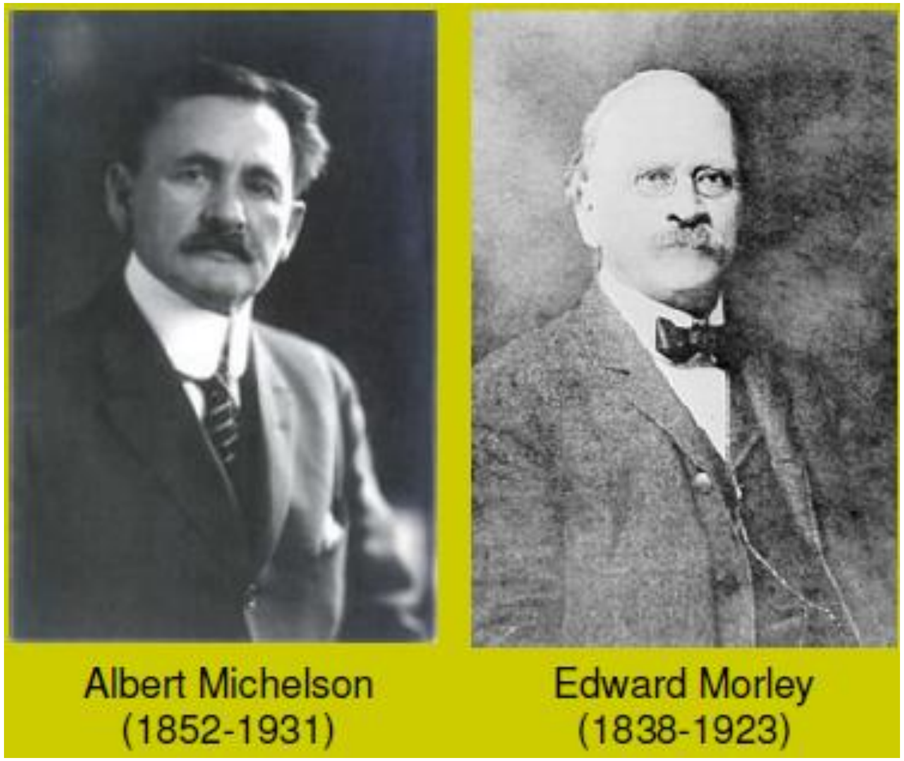
The wave nature of light suggested that there existed a propagation medium called the **ether**

Ether was proposed as an **absolute reference** in which the speed of light was constant and from which other measurements could be made

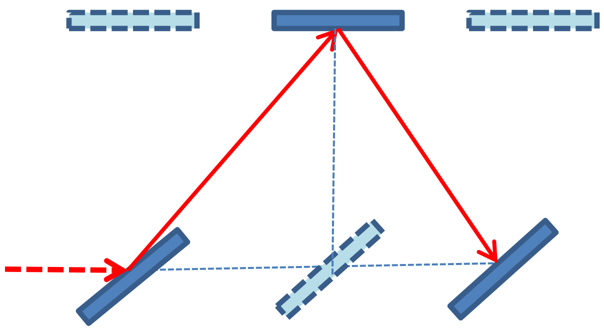
The Ether;
- Was transparent
- Had zero density
- Was everywhere

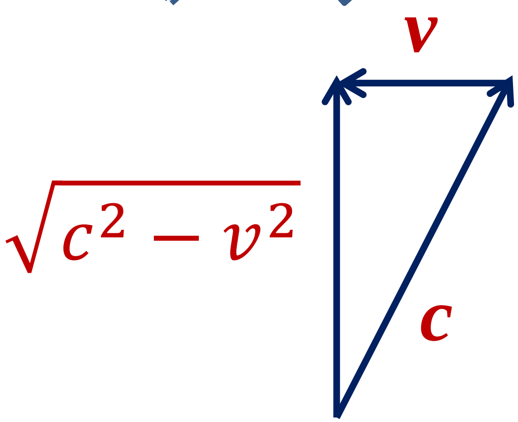
The **Michelson-Morley experiment** was an attempt to show the existence of ether

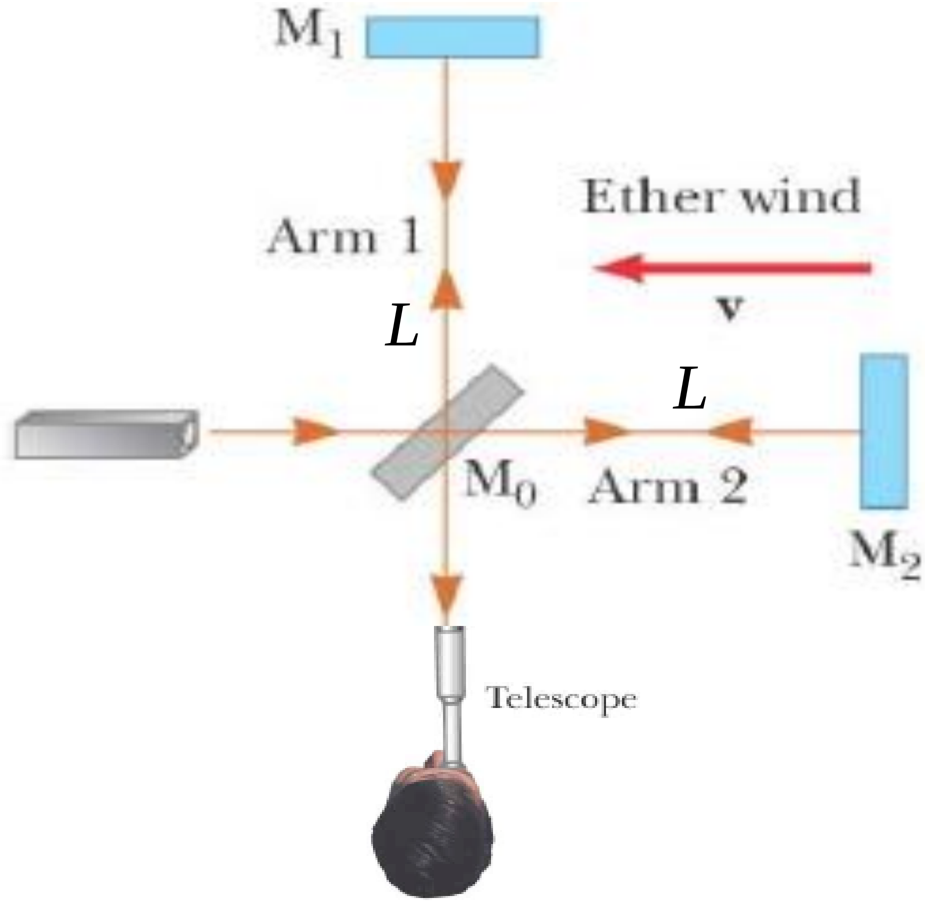
Michelson-Morley Experiment



Michelson and Morley performed an experiment to detect the presence of **Ether**. Experimental setup was similar to Michelson’s interferometer.





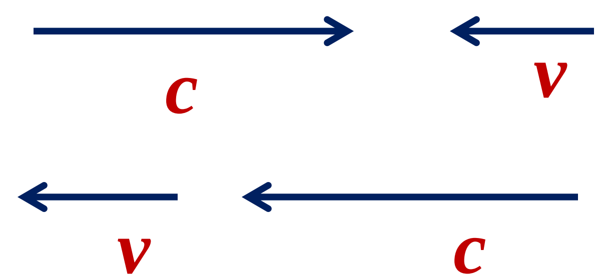


Time for Arm 1

$$t_1 = \frac{2L}{c'} = \frac{2L}{\sqrt{c^2 - v^2}}$$

Time for Arm 2

$$t_2 = \frac{L}{c-v} + \frac{L}{c+v} = \frac{2Lc}{c^2 - v^2}$$



Optical Path difference between two rays

$$\Delta x = c(t_1 - t_1) = c\left[\frac{2Lc}{c^2 - v^2} - \frac{2L}{\sqrt{c^2 - v^2}}\right]$$

$$= c\left[\frac{2L}{c\left(1 - \frac{v^2}{c^2}\right)} - \frac{2L}{c\sqrt{1 - \frac{v^2}{c^2}}}\right]$$

$$= 2L\left[\left(1 - \frac{v^2}{c^2}\right)^{-1} - \left(1 - \frac{v^2}{c^2}\right)^{-1/2}\right]$$

$$\cong 2L\left[\left(1 + \frac{v^2}{c^2}\right) - \left(1 + \frac{v^2}{2c^2}\right)\right]$$

$$\Delta x = L\left[\frac{v^2}{c^2}\right]$$

If setup is rotated through 90º then new path difference is

$$\Delta x' = -L\left[\frac{v^2}{c^2}\right]$$

Change in Path difference

$$\Delta = 2L\frac{v^2}{c^2}$$

If n number of fringe shifts occur then

$$n\lambda = 2L\frac{v^2}{c^2}$$

For the experiment, expected result was

$$\boxed{n = 0.404}$$

Experiment was sensitive enough to detect a shift of 0.01

Experiment was carried out at different places and at different times round the year

But no such fringe shift was ever detected

The result was termed as

NEGATIVE or NULL RESULT

Conclusions drawn from Null Results:

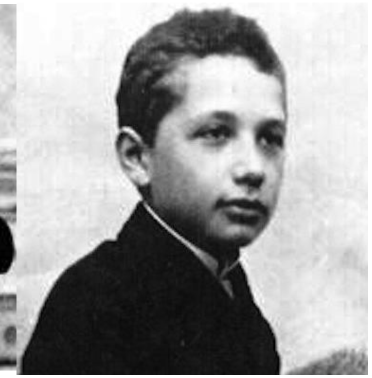
1. **There *no ether like medium* permeating the entire universe**

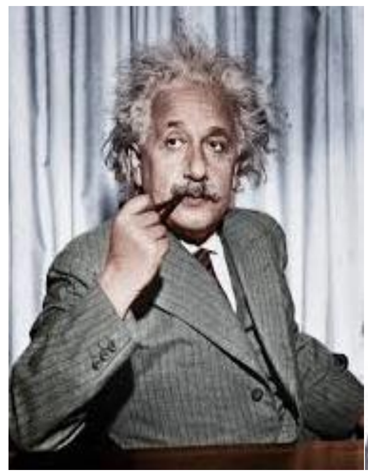
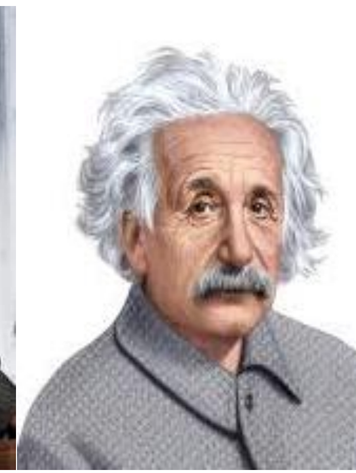
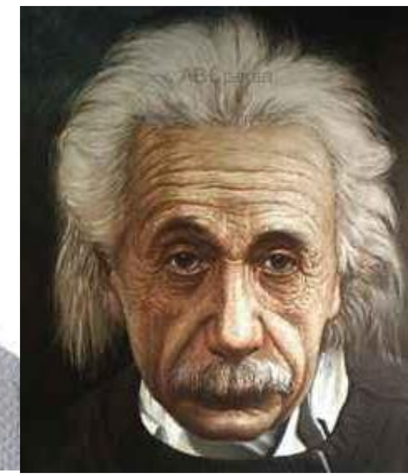
2. **Speed of light in free space is *constant* in all the direction of propagation and is not affected due to motion of observer or source ($c = 2.998 \times 10^8\ m/s$)**

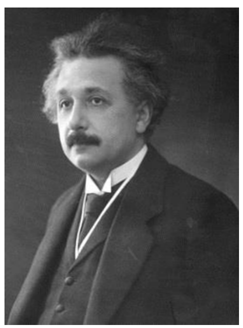
In 1907, Michelson wins the Nobel Prize in Physics, mostly for this famous “failed” experiment.

SPECIAL THEORY OF RELATIVITY

In 1905, Albert Einstein changed our perception of the world forever & developed a new theory of relativity at the age of 26, that abolishes the concept of absolute space & time



Postulates:

1 **All laws of physics (mechanics and electricity & magnetism) are the same in all inertial frames of reference.**

2 **The speed of light in vacuum is constant and independent of the motion of the source or the motion of the observer.**

On the basis of these postulates, Einstein concluded the following:

1. **No material particle can travel with speed equal to or greater than the speed of light**

2. **All clocks in spaceship run slow by a factor of** $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

3. **Length of spaceship decrease by a factor of** $\sqrt{1-\frac{v^2}{c^2}}$

4. **Mass of spaceship increase by a factor of** $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

5. **Mass and Energy are inter-convertible and follow the law** $E = mc^2$

6. **Velocity addition follow the law** $\frac{v_1 \pm v_2}{1 \pm \frac{v_1 v_2}{c^2}}$

7. **Relativistic Doppler effect is given by:**

$$\boxed{\nu = \sqrt{\frac{1-\mathrm{v}/c}{1+\mathrm{v}/c}}\,\nu_0}$$

Lorentz Transformations

Equations which relate the measurements of stationary frame of reference with the moving frame moving with $v \rightarrow c$

Galilean Transformations were

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

These transformations were not consistent with the theory of ***RELATIVITY,*** and hence new transformations were developed

Let transformations be:

$$x' = \beta(x - vt)$$
$$y' = y$$
$$z' = z$$
$$t' \neq t$$

Inverse Transformations

$$x = \beta(x' + vt')$$
$$y = y'$$
$$z = z'$$
$$t \neq t'$$

HERE

$$\beta = \beta(v) = \beta(-v)$$

and

$$\beta \rightarrow 1 \quad \text{if} \quad v \ll c$$

$$x = \beta(x' + vt')$$

$$x' = \beta(x - vt)$$

$$x = \beta[\beta(x - vt) + vt']$$

$$(1 - \beta^2)x = -\beta^2 vt + \beta vt'$$

$$t' = \beta t + \frac{(1 - \beta^2)}{\beta v}x$$

$\boxed{Clearly \quad t' \neq t}$

Now, suppose a source of light emits a light flash such that the distance travelled by a stationary and moving frame be:

$$x = ct$$

And $x' = ct'$

$$\beta(x - vt) = c\left[\beta t + \frac{(1 - \beta^2)}{\beta v}x\right]$$

Separate the value of x from here to get $x = ct$

We get

$$x = ct\left[\frac{\left(1+\frac{v}{c}\right)}{1-\frac{c}{v}\left(\frac{1-\beta^2}{\beta^2}\right)}\right]$$

Transformation will be valid if and only if

$$\left[\frac{\left(1+\frac{v}{c}\right)}{1-\frac{c}{v}\left(\frac{1-\beta^2}{\beta^2}\right)}\right] = 1 \quad \Longrightarrow \quad \boxed{\beta = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}}$$

Lorentz transformations are:

$$x' = \frac{x - vt}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1-\frac{v^2}{c^2}}}$$

Inverse Lorentz transformations

$$x = \frac{x' + vt'}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$y = y'$$

$$z = z'$$

$$t = \frac{t' + \frac{v}{c^2}x'}{\sqrt{1-\frac{v^2}{c^2}}}$$

Time Dilation/Relativity of Time

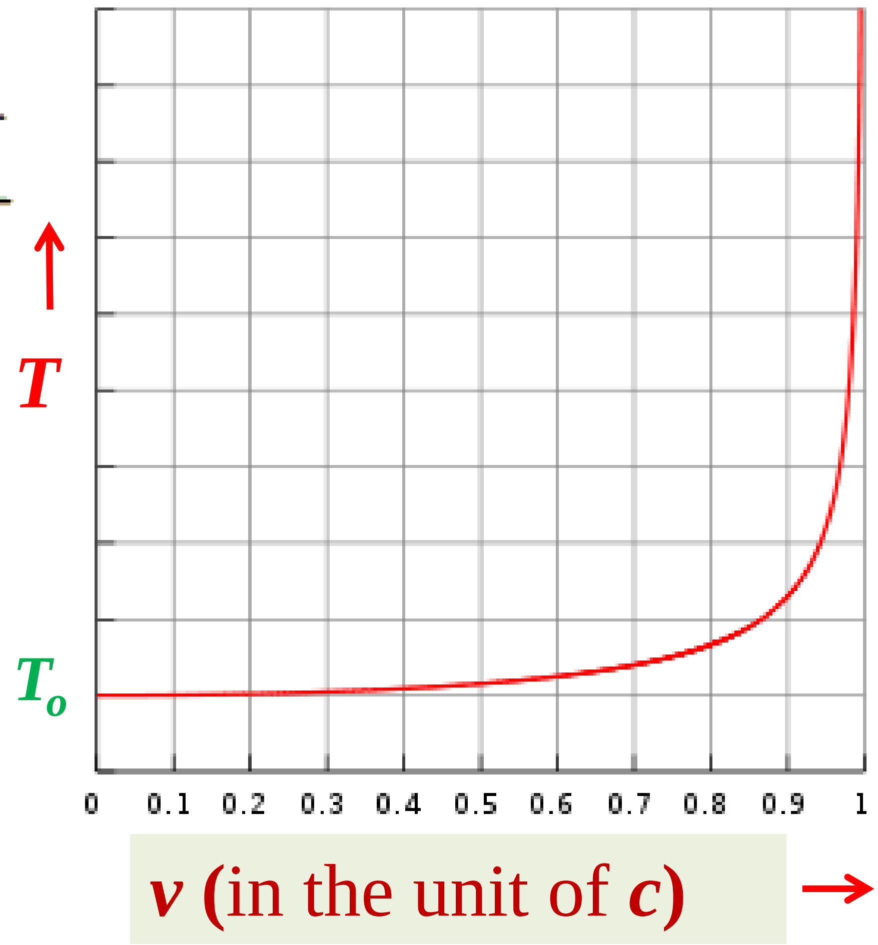
If the time interval $T_0 = t'_2 - t'_1$ is measured in the moving reference frame,

then $T = t_2 - t_1$ can be calculated using the Lorentz transformation.

$$T = t_2 - t_1 = \frac{t'_2 + \frac{vx'_2}{c^2} - t'_1 - \frac{vx'_1}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The time measurements made in the moving frame are made at the same location, so the expression reduces to:

$$T = \frac{T_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$



Length Contraction/Relativity of Length

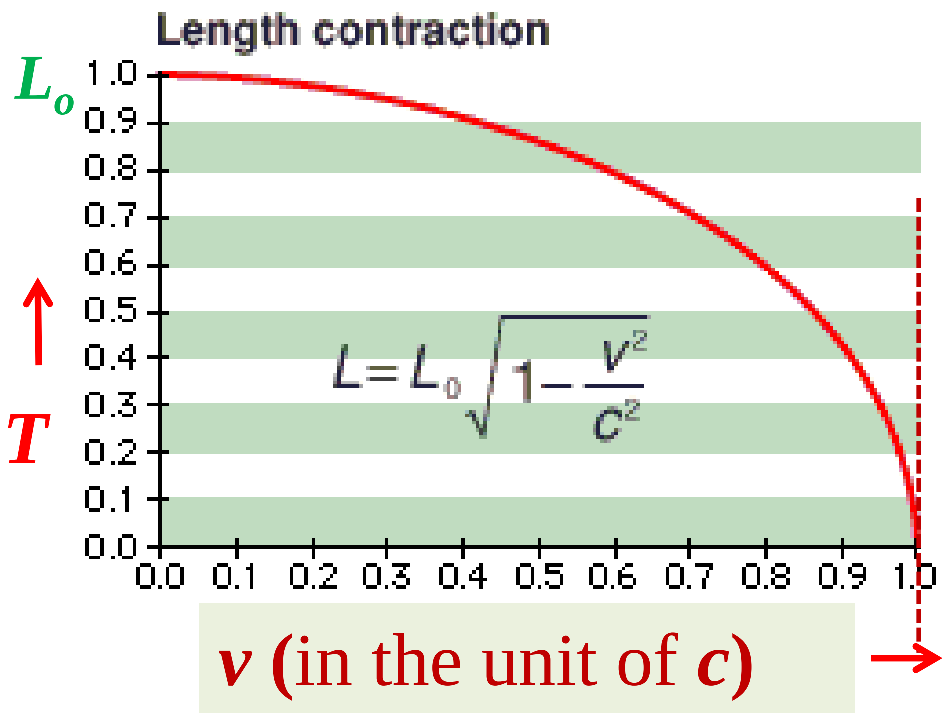
If the length $L_0 = x_2' - x_1'$ is measured in the moving reference frame,

then $L = x_2 - x_1$ can be calculated using the Lorentz transformation.

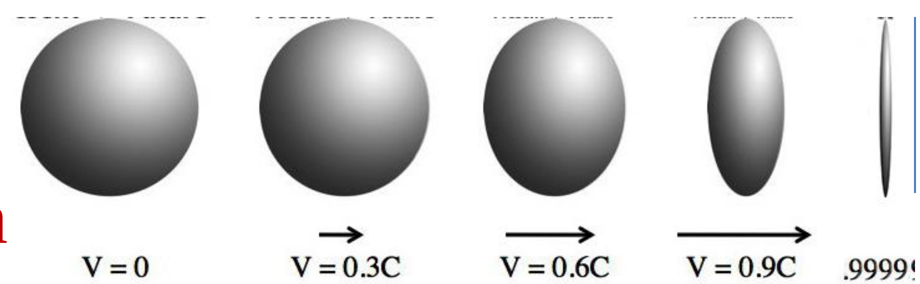
$$L_0 = x_2' - x_1' = \frac{x_2 - vt_2 - x_1 + vt_1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

But since the two measurements made in the fixed frame are made simultaneously in that frame, $t_2 = t_1$, and

$$\boxed{L = L_0\sqrt{1 - \frac{v^2}{c^2}}}$$



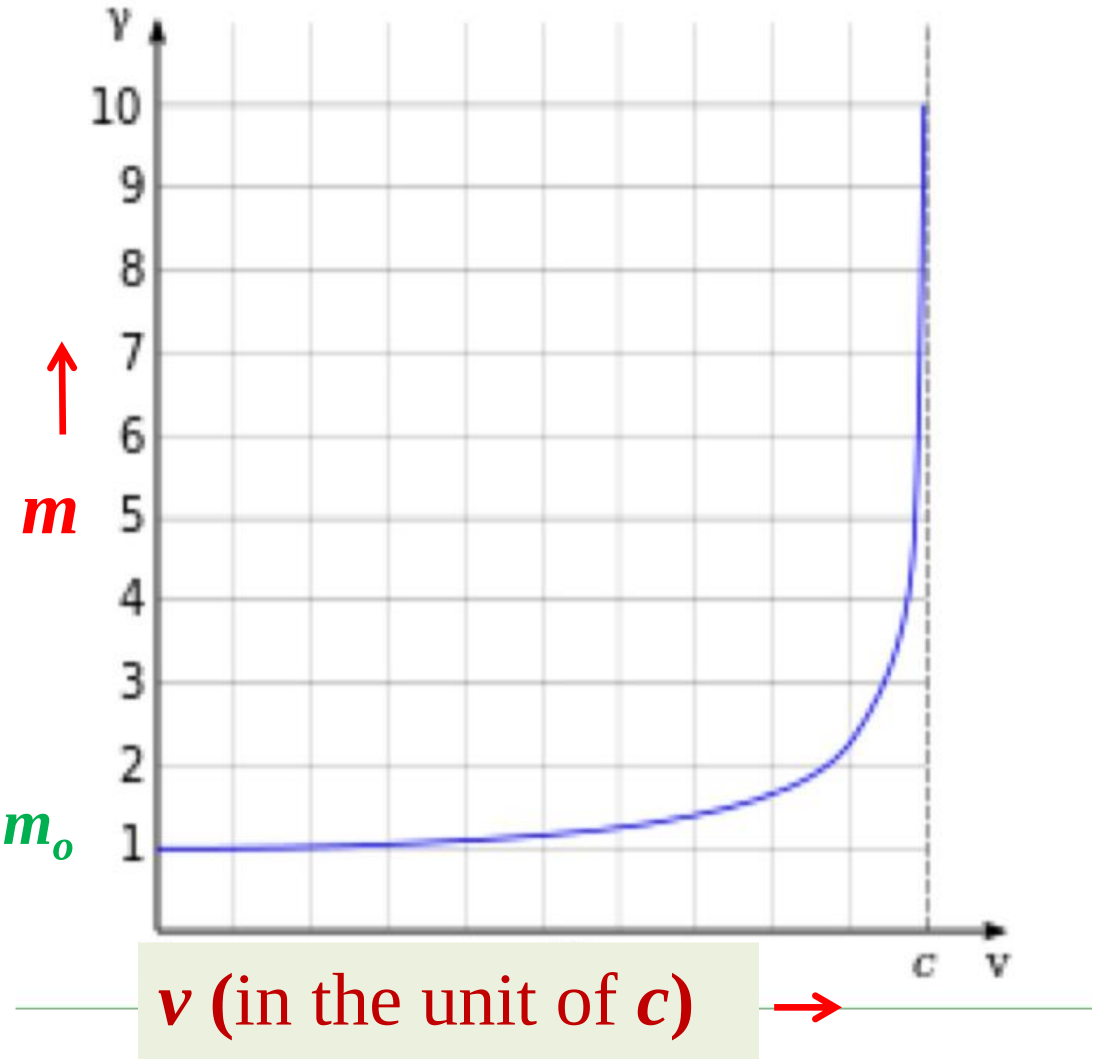
Lorentz-Fitzgerald Contraction



Relativity of Mass

Rest mass m_o & relativistic mass m of a body are related as:

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$



Einstein's Mass- Energy Relation

Let a body is being accelerated by an external force such that small increase in its KE when small work is done on it is:

$$dE_k = dw = F.dx = \frac{d(mv)}{dt}.dx = d(mv).v$$

$$= (m.dv + v.dm).v = mv.dv + v^2.dm$$

$$\boxed{dE_k = mv.dv + v^2.dm}$$

Also $\quad \boldsymbol{m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}} \Rightarrow m^2 = \frac{{m_0}^2c^2}{\boldsymbol{c^2 - v^2}}$

$$m^2(\boldsymbol{c^2 - v^2}) = {m_0}^2c^2$$

Differentiating both sides

$$2m.dm(\boldsymbol{c^2 - v^2}) + m^2(0 - 2v.dv) = 0$$

$$\boxed{dm.\boldsymbol{c^2} = mv.dv + v^2.dm}$$

$$dE_k = dm.c^2$$

Integrating both sides with proper limits

$$\int_o^{E_k} dE_k = c^2 \int_{m_o}^{m} dm$$

$$E_k = c^2(m - m_o)$$

$$\underbrace{E_k + m_o c^2}_{\text{Total energy}} = mc^2$$

$$\boxed{E = mc^2}$$

$$\boxed{E_k = c^2(m - m_o)}$$

Relativistic Kinetic energy

Deduction of classical Results

$$E_k = c^2(m - m_o) = c^2\left(\frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} - m_o\right)$$

$$= m_o c^2\left[\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} - 1\right]$$

$$E_k = m_o c^2\left[\left(1 + \frac{v^2}{2c^2} + \cdots\right) - 1\right]$$

In classical limit $v \ll c$

$$E_k = m_o c^2\left[\frac{v^2}{2c^2}\right] = \frac{1}{2} m_o v^2$$

and In classical limit $m = m_o$

$$\boxed{E_k = \frac{1}{2} m v^2}$$

Moment- Energy Relation

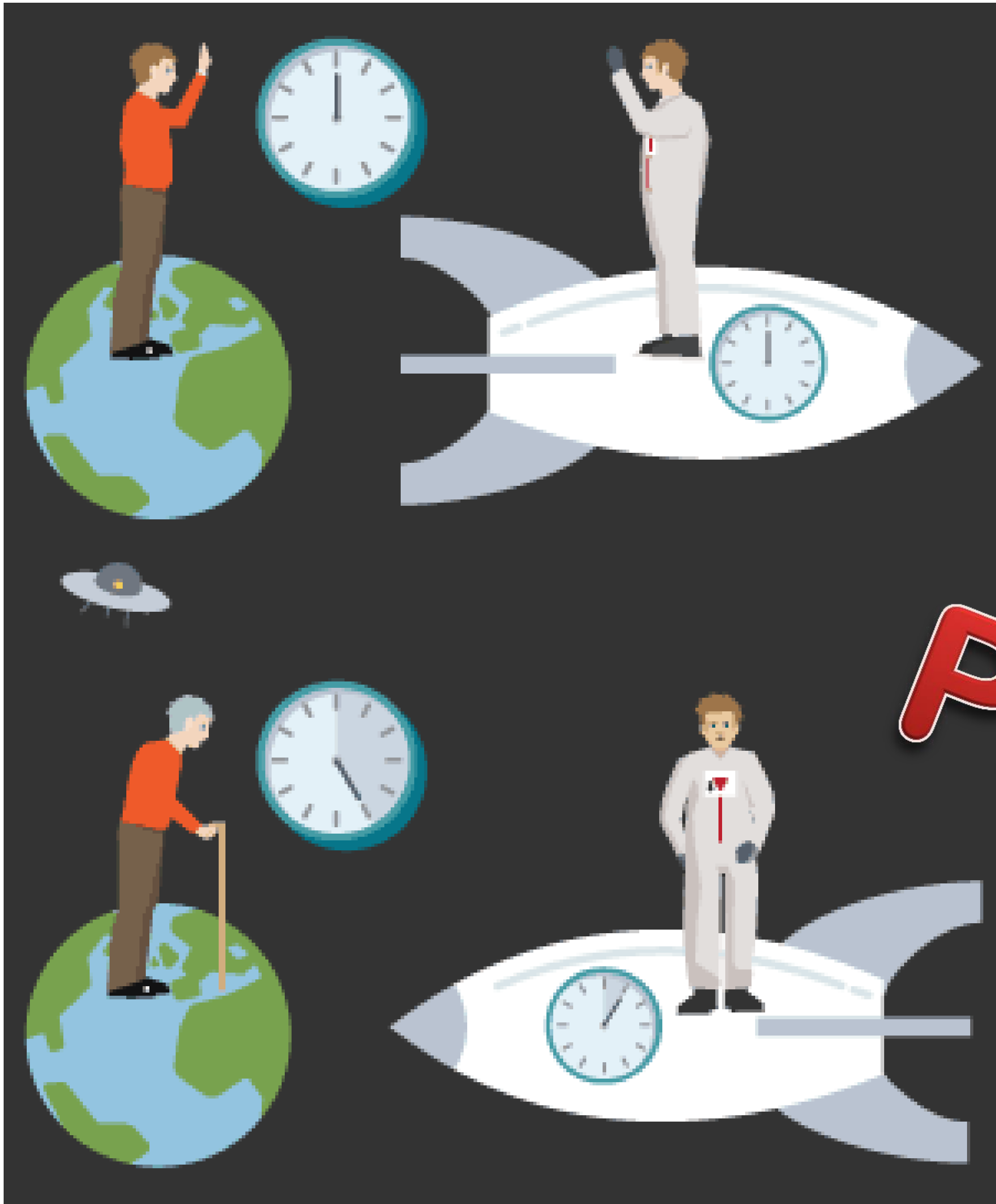
$$E = mc^2 = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} c^2 \Rightarrow E^2 = \frac{{m_o}^2 c^4}{1 - \frac{v^2}{c^2}}$$

$$p = mv = \frac{m_o v}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow p^2 c^2 = \frac{{m_o}^2 c^2 v^2}{1 - \frac{v^2}{c^2}}$$

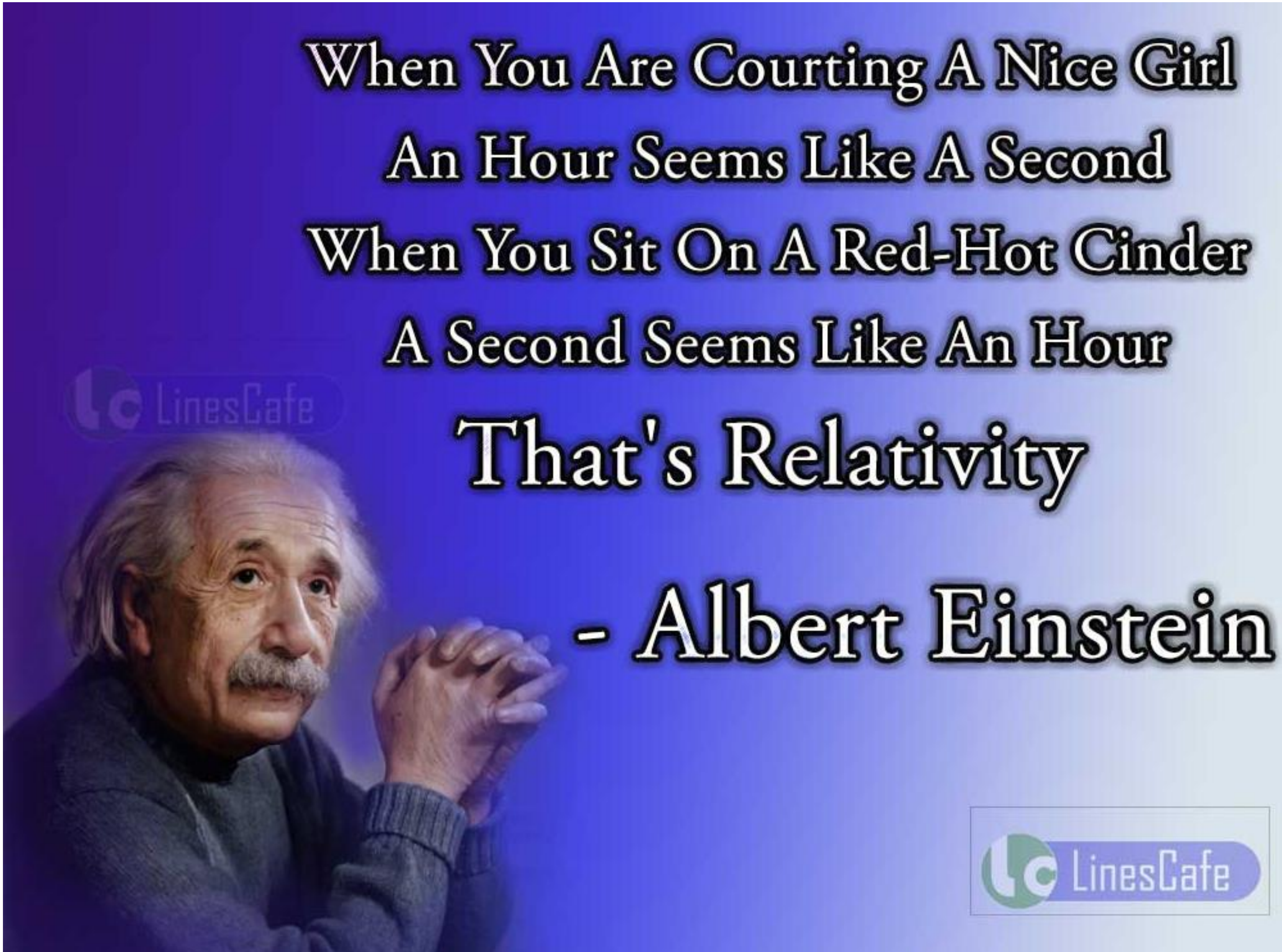
$$E^2 - p^2 c^2 = \frac{{m_o}^2 c^4}{1 - \frac{v^2}{c^2}} \left(1 - \frac{v^2}{c^2}\right) \Rightarrow \boxed{E = \sqrt{p^2 c^2 + {m_o}^2 c^4}}$$

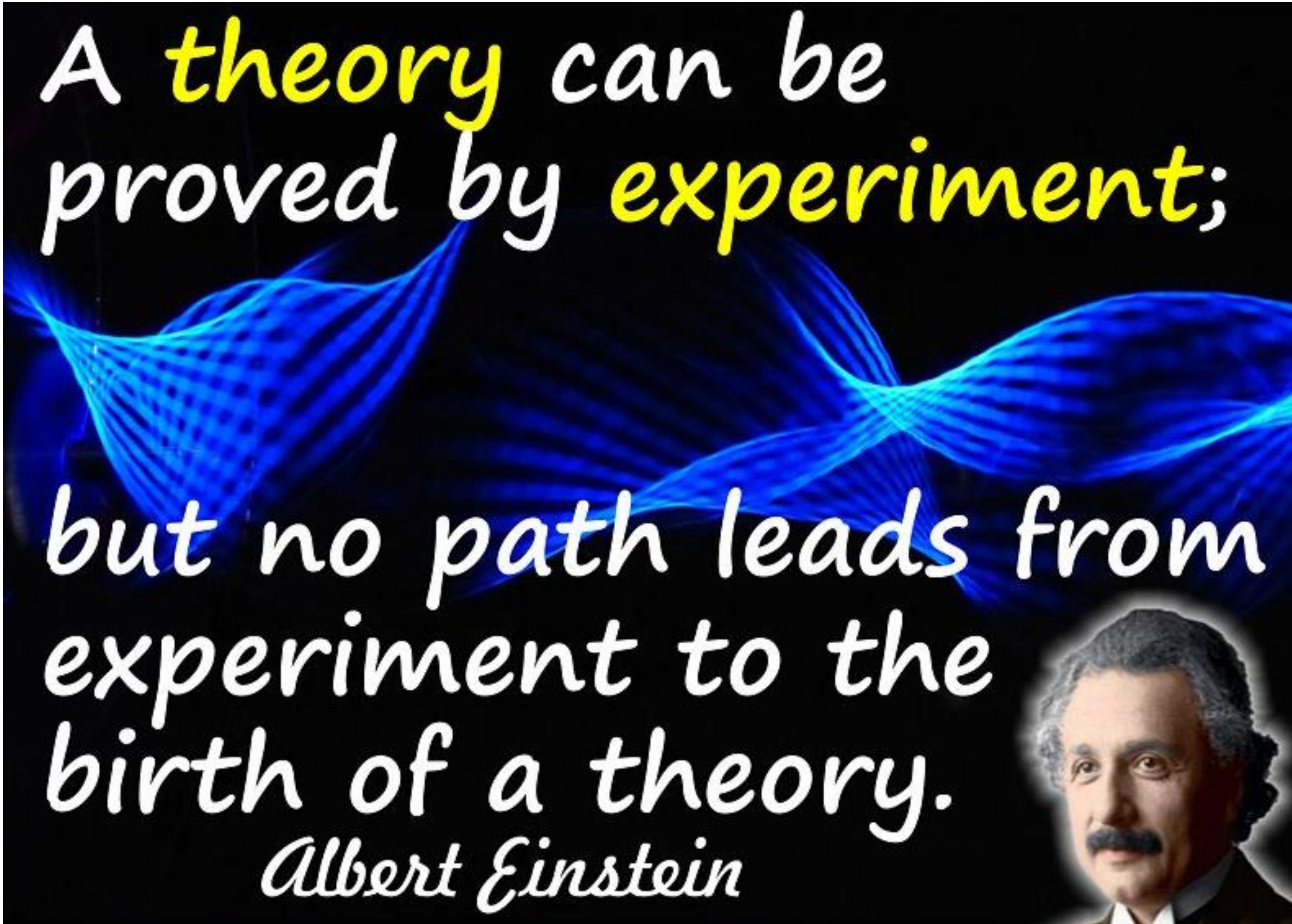
But, for massless particles like photon, neutrinos rest mass $m_o = 0$

$$\Rightarrow \boxed{E = pc}$$



Twin Paradox


When You Are Courting A Nice Girl
An Hour Seems Like A Second
When You Sit On A Red-Hot Cinder
A Second Seems Like An Hour
That's Relativity
- Albert Einstein
LinesCafe
LinesCafe


A theory can be proved by experiment;
but no path leads from experiment to the birth of a theory.
Albert Einstein