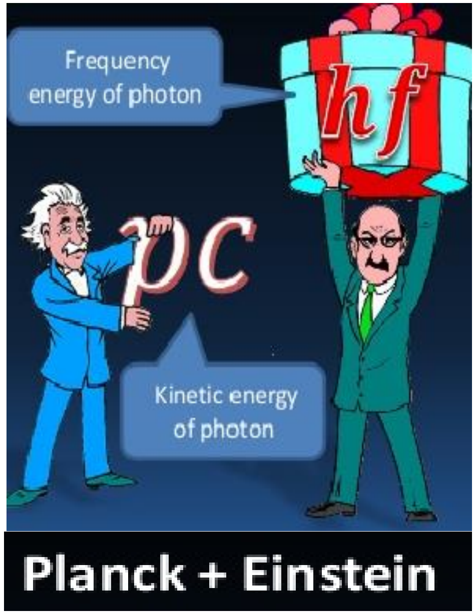
SECTION-B

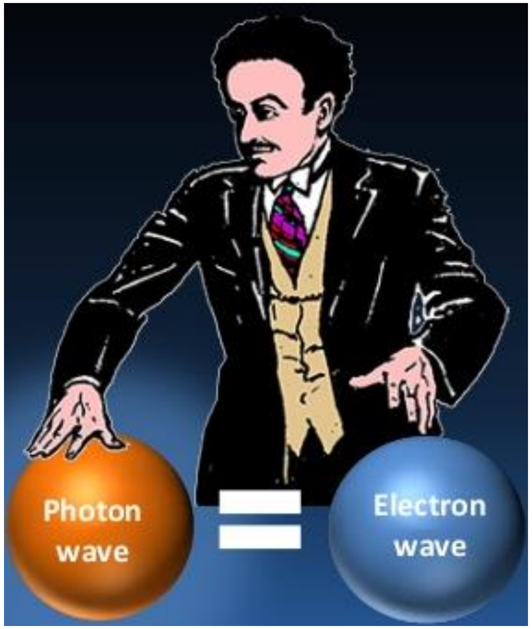
Elements of Quantum Mechanics

Lecture Notes:- Dr. Shyam

Wave Particle Duality
(Concept of de Broglie waves)

Born	15 Aug.1892 Dieppe, France
Died	19 March 1987 (aged 94)
Nationality	French
Alma mater	University of Paris (BA in History, 1910; BA in Sciences, 1913; PhD in physics, 1924)





Like Radiations, particles also may exhibit Dual Nature- wave as well as particle. The Waves associated with particles are called as de Broglie waves or Matter waves

Relativistic relation for Energy

$$E = \sqrt{p^2c^2 + m_o{}^2c^4}$$

For photons, $m_o = 0$

$$\boxed{E = pc}$$

Further,

$$E = hv = \frac{hc}{\lambda}$$

$$\boxed{E = \frac{hc}{\lambda}}$$

$$pc = \frac{hc}{\lambda}$$

$$\boxed{\lambda = \frac{h}{p}}$$

For a particle of mass *m* moving with speed *v*

$$p = mv$$

Hence de-Broglie wave associated should be

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

The de-Broglie wave length ⇨

$$\boxed{\lambda = \frac{h}{mv}}$$

Speed of de Broglie waves

$$\boxed{\lambda = \frac{h}{mv}}$$

$$E = h\nu$$

$$mc^2 = h\nu$$

$$\boxed{\nu = \frac{mc^2}{h}}$$

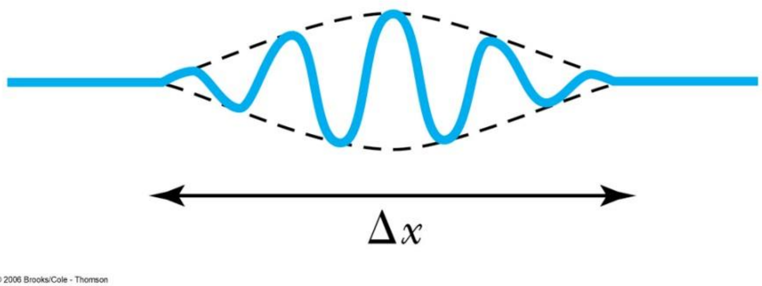
$$v_w = \lambda\nu$$

$$v_w = \frac{h}{mv}.\frac{mc^2}{h}$$

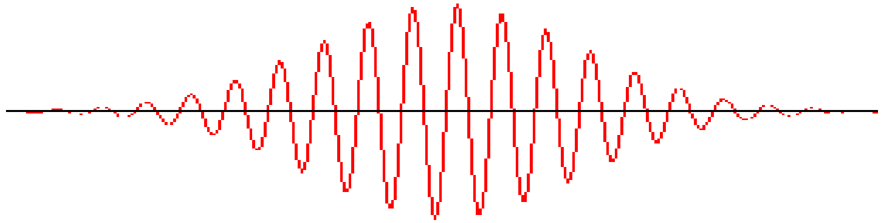
$$\boxed{v_w = \frac{c^2}{v} > c}$$

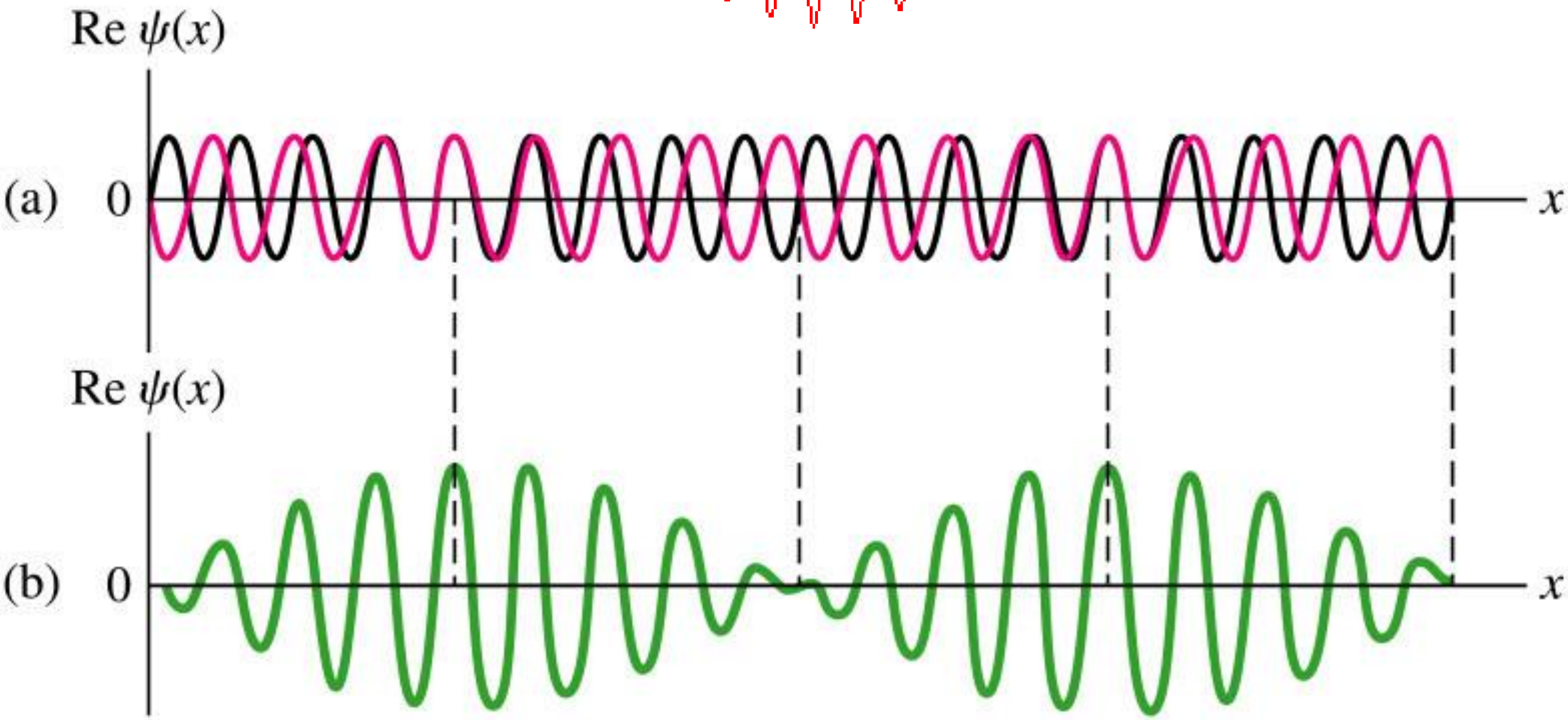
How?

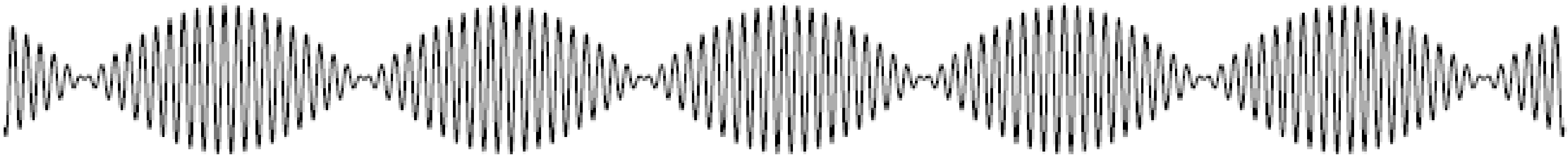
Since $v_w > c$ hence it was proposed that, de Broglie wave is not a single wave but consists of group of waves of slightly different wavelengths, which superimpose constructively only over a small region and destructively beyond that, so as to form a wave group called Wave-packet



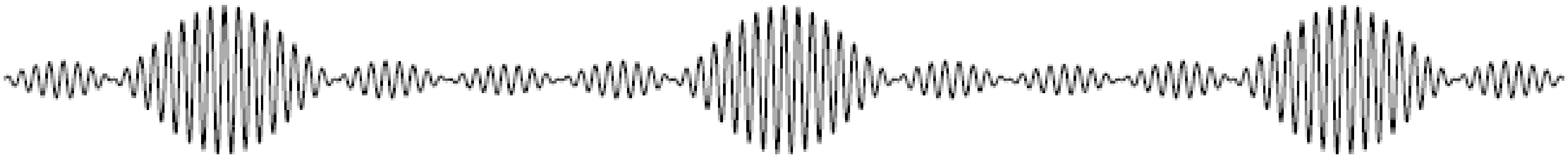
Formation of Wave-packet



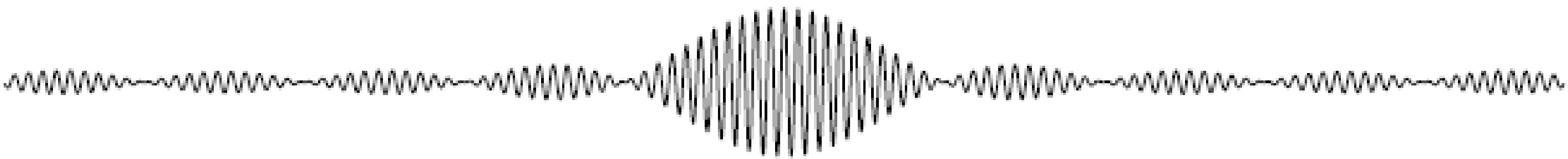




(a) $\cos(5x) + \cos(5.25x)$.

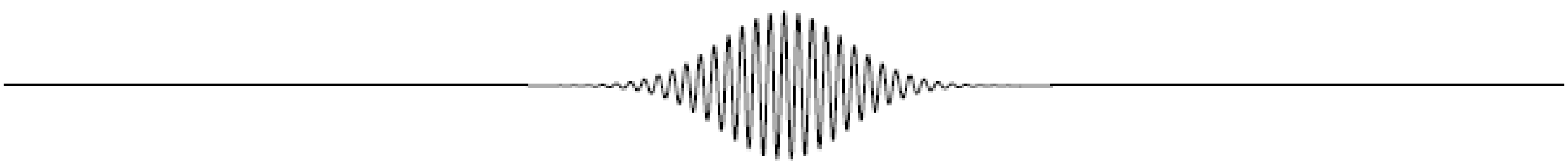


$\cos(4.75x) + \cos(4.875x) + \cos(5x) + \cos(5.125x) + \cos(5.25x)$.



(c)

$\cos(4.8125x) + \cos(4.875x) + \cos(4.9375x) + \cos(5x) + \cos(5.0625x) + \cos(5.125x) + \cos(5.1875x)$



(d)

An integral over a continuous range of wave numbers produces a single wave packet.

Mathematical Treatment

$$y_1 = A\cos(\omega t - kx)$$

$$y_2 = A\cos[(\omega + d\omega)t - (k + dk)x]$$

$$y = y_1 + y_2$$

$$y = 2A\cos\left[\frac{(2\omega + d\omega)t}{2} + \frac{(2k + dk)x}{2}\right]\cos\left[\frac{(d\omega)t}{2} - \frac{(dk)x}{2}\right]$$

With $d\omega << \omega, dk << k$

$$y \cong 2A\cos\left[\frac{d\omega}{2}t - \frac{dk}{2}x\right]\cos[\omega t - kx] ------(1)$$

This equation represents a wave of frequency *ω* and wave number *k* modulated by a wave of frequency *$d\,\omega$* and number *dk*. We can see two types of velocities here

$$v_p = \frac{\omega}{k}$$

$$v_g = \frac{d\omega}{dk}$$

Phase velocity and Group velocity

- The velocities of the individual waves which superpose to produce the wave packet representing the particle are different - the wave packet as a whole has a different velocity from the waves that comprise it
- **Phase velocity:** The rate at which the phase of the wave propagates in space or the velocity of individual de-Broglie waves

$$\boxed{v_p = \frac{\omega}{k} = \frac{c^2}{v}}$$

- Group velocity: The rate at which the envelope of the wave packet propagates. Its value is equal to velocity of particle

$$\boxed{v_g = \frac{d\omega}{dk} = v}$$

Proof:

$$\omega = 2\pi\nu \quad = 2\pi\frac{E}{h}$$

$$\omega = 2\pi\frac{mc^2}{h}$$

$$\boxed{\omega = 2\pi\frac{m_0c^2}{h\sqrt{1-\frac{v^2}{c^2}}}}$$

$$k = \frac{2\pi}{\lambda} \quad = \frac{2\pi}{h}mv$$

$$\boxed{k = \frac{2\pi}{h}\frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}}$$

$$v_g = \frac{d\omega}{dk} = \frac{d\omega/dv}{dk/dv}$$

$$d\omega/dv = \frac{2\pi m_0 c^2}{h}(-\frac{1}{2})\left(1-\frac{v^2}{c^2}\right)^{-3/2}\left[-2v/c^2\right]$$

$$\boxed{d\omega/dv = \frac{2\pi}{h}\frac{m_0 v}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}}$$

$$dk/dv = \frac{2\pi m_0}{h}\left[\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} + (-\frac{1}{2})\left(1-(v/c)^2\right)^{-3/2}\left[-2v/c^2\right]\right]$$

$$dk/dv = \frac{2\pi m_0}{h}\left(1-(v/c)^2\right)^{-3/2}\left[1-\frac{v^2}{c^2}+\frac{v^2}{c^2}\right]$$

$$\boxed{dk/dv = \frac{2\pi m_0}{h}\frac{1}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}}$$

Clearly

$$\boxed{v_p = \frac{\omega}{k} = \frac{c^2}{v}} \qquad \boxed{v_g = \frac{d\omega}{dk} = v}$$

More relations for de-Broglie wave length

$$\lambda = \frac{h}{mv} = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqV}}$$

For an electron accelerated through V potential

$$\lambda = \frac{h}{\sqrt{2m_e eV}} = \frac{12.27}{\sqrt{V}} A^o$$

Quantitative Confirmation of de Broglie's Relation:

Crystal Diffraction of Electron Beam

<u>Davisson-Germer Experiment</u>

Second Series *December, 1927* *Vol. 30, No. 6*

THE

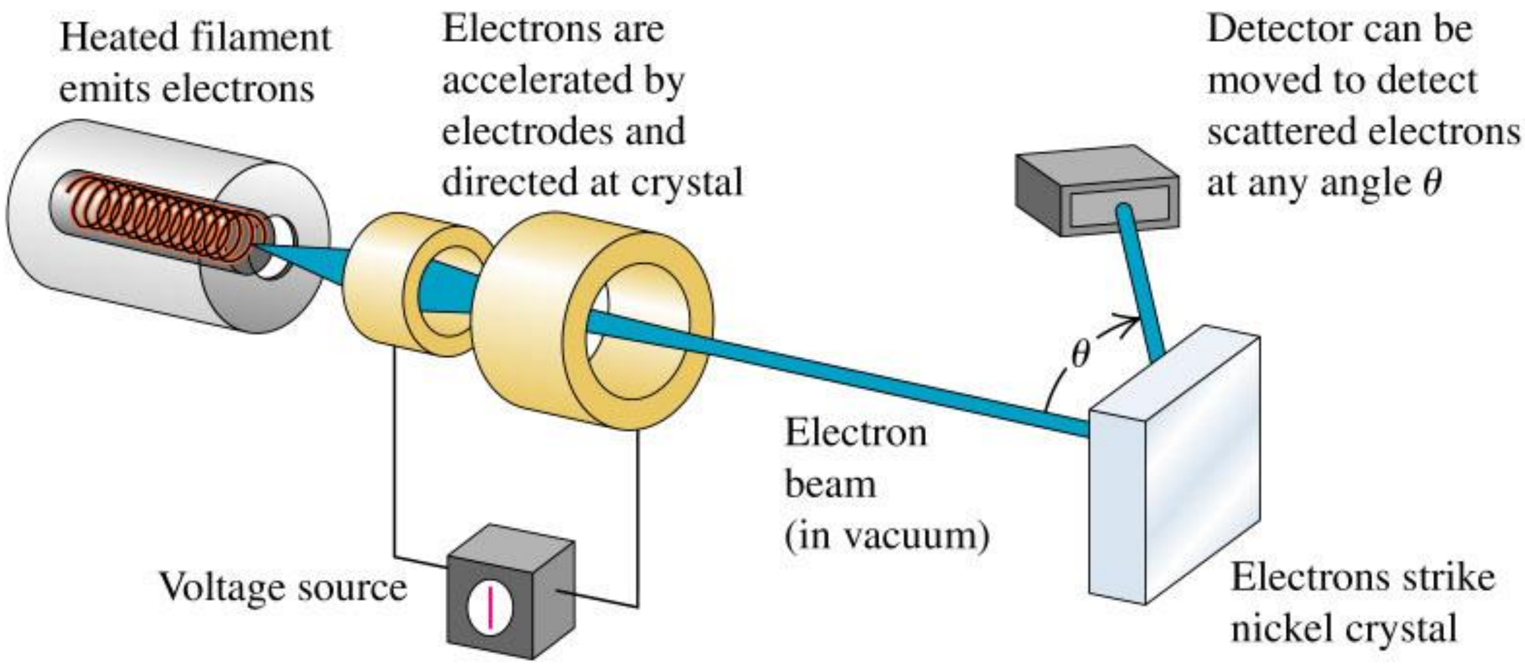
PHYSICAL REVIEW

DIFFRACTION OF ELECTRONS BY A CRYSTAL OF NICKEL

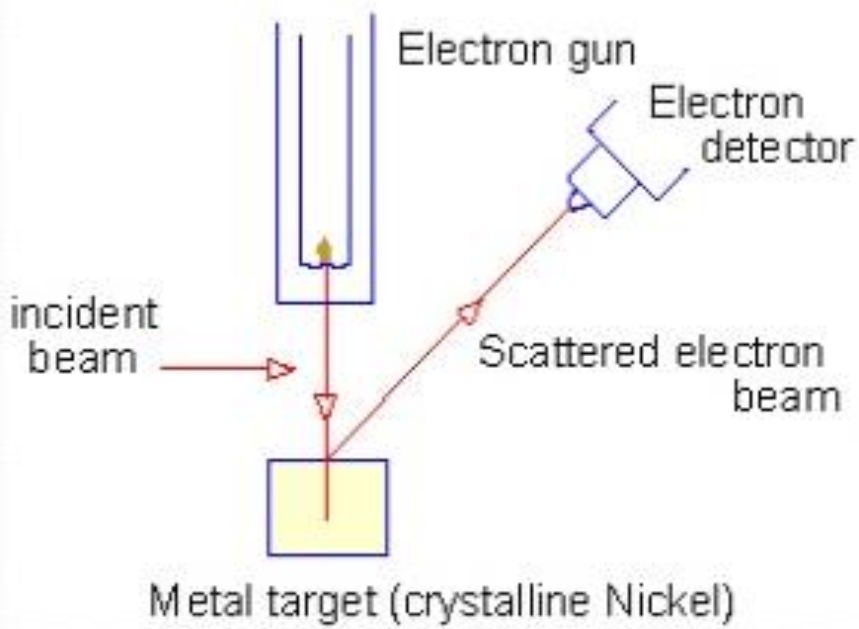
BY C. DAVISSON AND L. H. GERMER

ABSTRACT

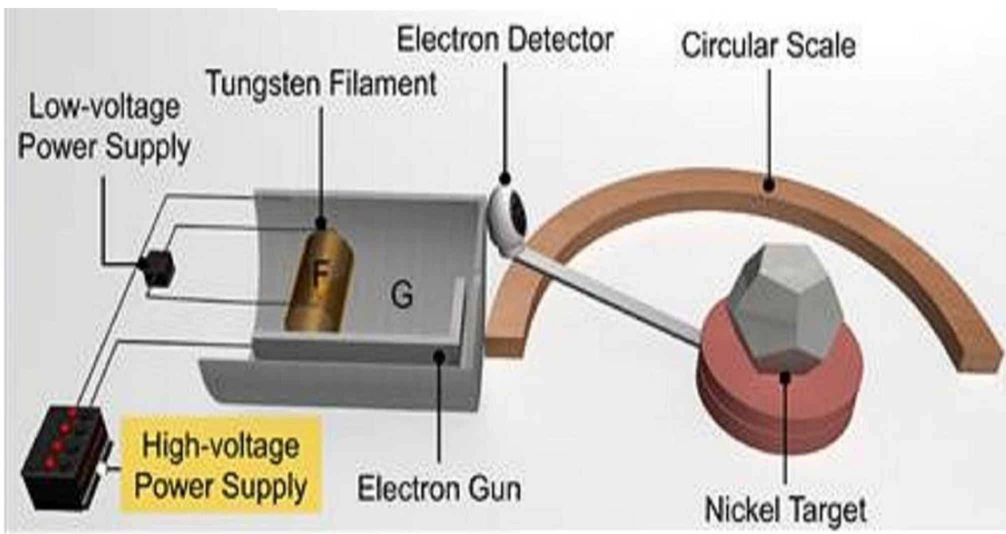
The intensity of scattering of a homogeneous beam of electrons of adjustable speed incident upon a single crystal of nickel has been measured as a function of direction. The crystal is cut parallel to a set of its {111}-planes and bombardment is at normal incidence. The distribution in latitude and azimuth has been determined for such scattered electrons as have lost little or none of their incident energy.



Heated filament emits electrons
Electrons are accelerated by electrodes and directed at crystal
Detector can be moved to detect scattered electrons at any angle θ
θ
Electron beam (in vacuum)
Voltage source
Electrons strike nickel crystal

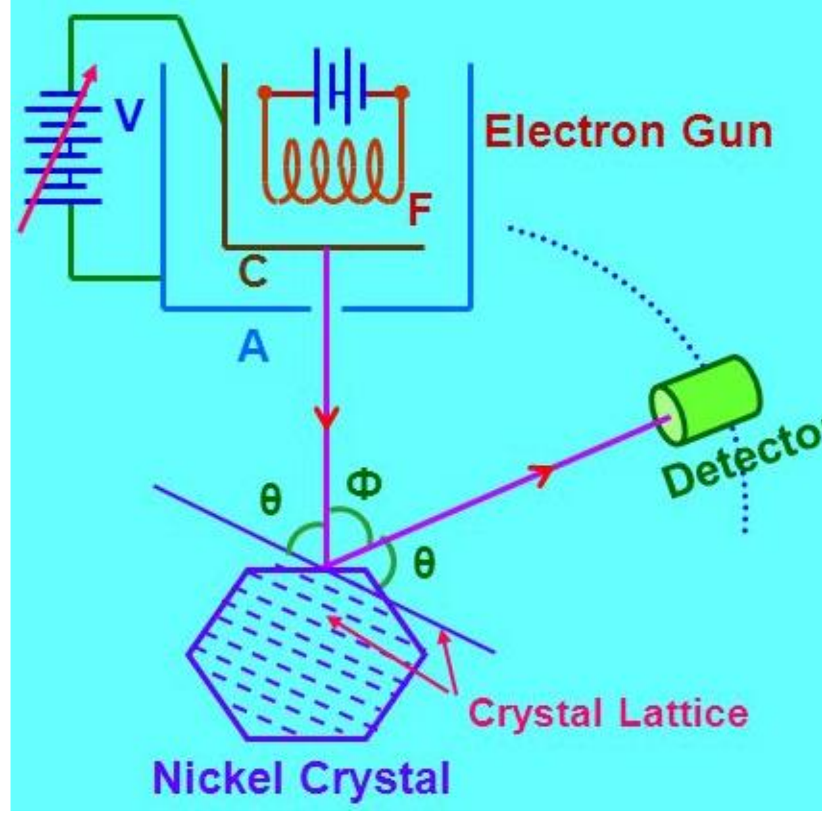


Electron gun
Electron detector
incident beam
Scattered electron beam
Metal target (crystalline Nickel)



Electron Detector
Circular Scale
Tungsten Filament
Low-voltage Power Supply
F
G
High-voltage Power Supply
Electron Gun
Nickel Target

A beam of electrons emitted by the electron gun is made to fall on Nickel crystal cut along cubical axis at a particular angle.



The scattered beam of electrons is received by the detector which can be rotated at any angle.

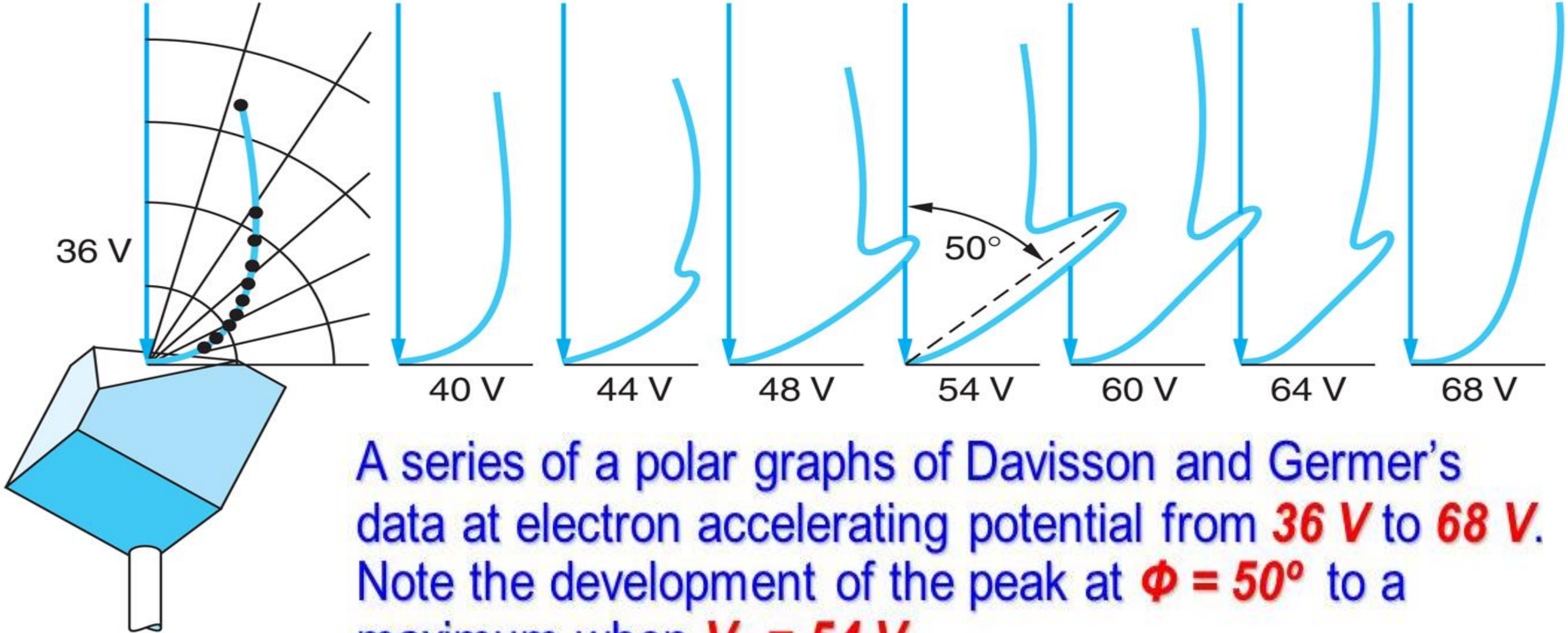
The energy of the incident beam of electrons can be varied by changing the applied voltage to the electron gun.

Intensity of scattered beam of electrons is found to be maximum when angle of scattering is 50° and the accelerating potential is 54 V.

$\theta + 50° + \theta = 180°$ i.e. $\theta = 65°$

For Ni crystal, lattice spacing $d = 0.91$ Å

For first principal maximum, $n = 1$



A series of a polar graphs of Davisson and Germer's data at electron accelerating potential from **36 V** to **68 V**. Note the development of the peak at $\Phi = 50^o$ to a maximum when $V_0 = 54$ V.

Electron diffraction is similar to X-ray diffraction.

∴ Bragg's equation $2d\sin\theta = n\lambda$ gives

$$\boxed{\lambda = 1.65\ \text{Å}}$$

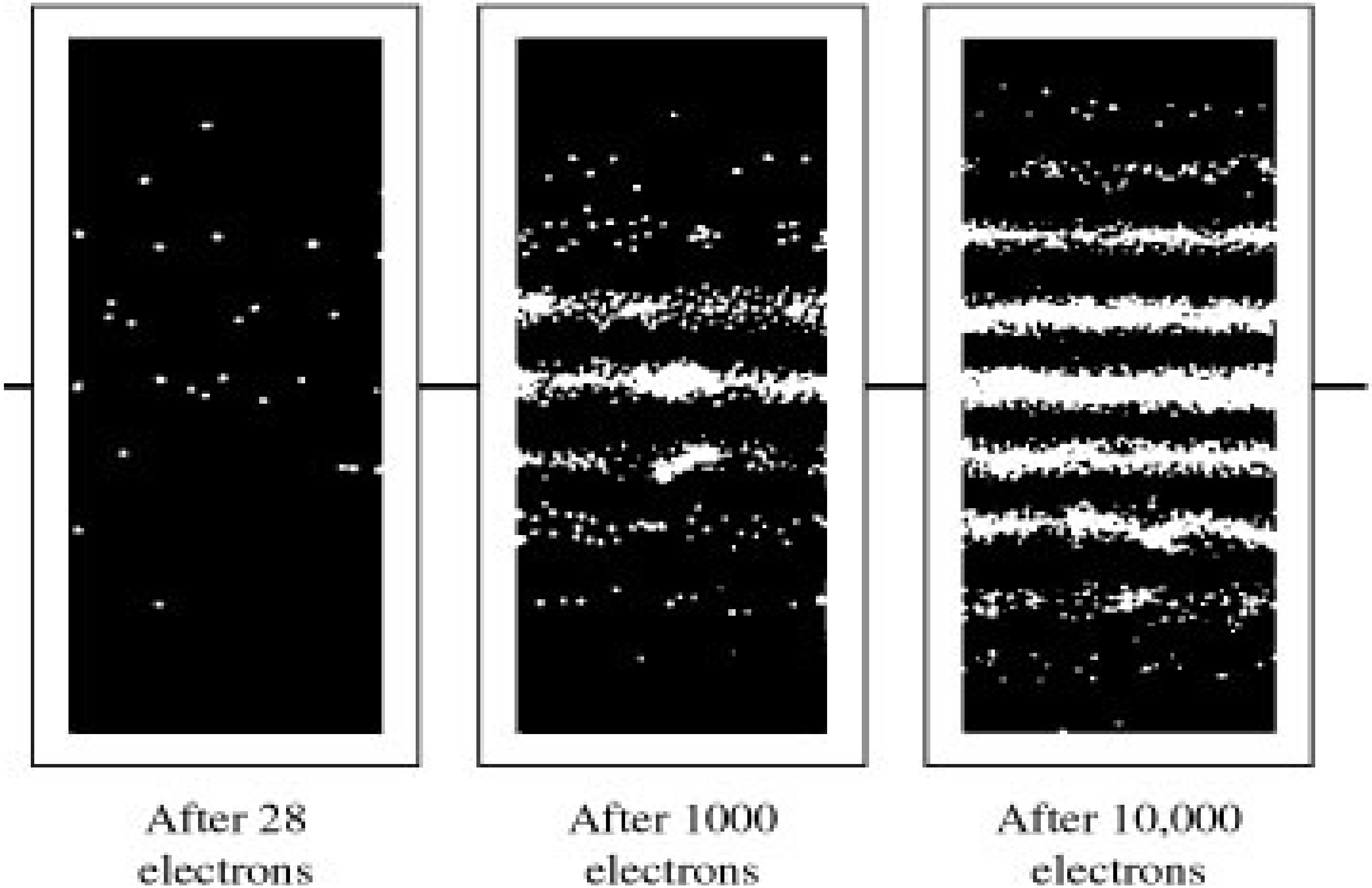
Further for an e- accelerated through V potential, de- Broglie λ is

$$\lambda = \frac{12.27}{\sqrt{V}} A^o = \frac{12.27}{\sqrt{54}} A^o$$

$$\boxed{\lambda = 1.669\ A^o}$$

The above two results are in good agreement with each other. Hence the experiment was able to demonstrate the wave nature of electrons and hence the existence of de-Broglie waves

Electrons diffracting through 2 slits



(b)

The Heisenberg's Uncertainty Principle

" It is impossible to know simultaneously and with desired exactness both the position and the momentum of the fundamental particles"

If the uncertainties in position and momentum of any microscopic particle are Δx and Δp respectively, then :

$$\Delta x \cdot \Delta p \geq \hbar \qquad \hbar = \frac{h}{2\pi} \qquad (h = 1.054\times 10^{-34}\, J.s)$$

$$\hbar = 1.054\times 10^{-34}\, J.s$$

Other forms of Uncertainty Relations :

$$\Delta E \cdot \Delta t \geq \hbar$$

$$\Delta\theta \,.\, \Delta J \geq \hbar$$

Problem: An electron is moving with the speed of 2×10^{6} m/s (known with a precision of 0.50%). What is the minimum uncertainty with which we can simultaneously measure its position?

Problem: In an experiment, an electron is determined to be within 0.1*mm* of a particular point. If we try to measure the electron's velocity, what will be the minimum uncertainty?

Problem: A sodium atom is in one of the states labeled ''Lowest excited levels''. It remains in that state for an average time of 1.6×10^{-8} s before it makes a transition back to a ground state, emitting a photon with wavelength 589.0 *nm* and energy 2.105 *eV*. What is the uncertainty in energy of that excited state? What is the wavelength spread of the corresponding spectrum line?

Problem: Explain how does uncertainty principle prove the non-existence of electrons in a nucleus?

Hint: Radius of nucleus is 3*fm* (approx) and maximum energy of nuclear radiation emitted in nuclear decay is below 2 *Mev*.

The de-Broglie waves and concept of wave-function (ψ)

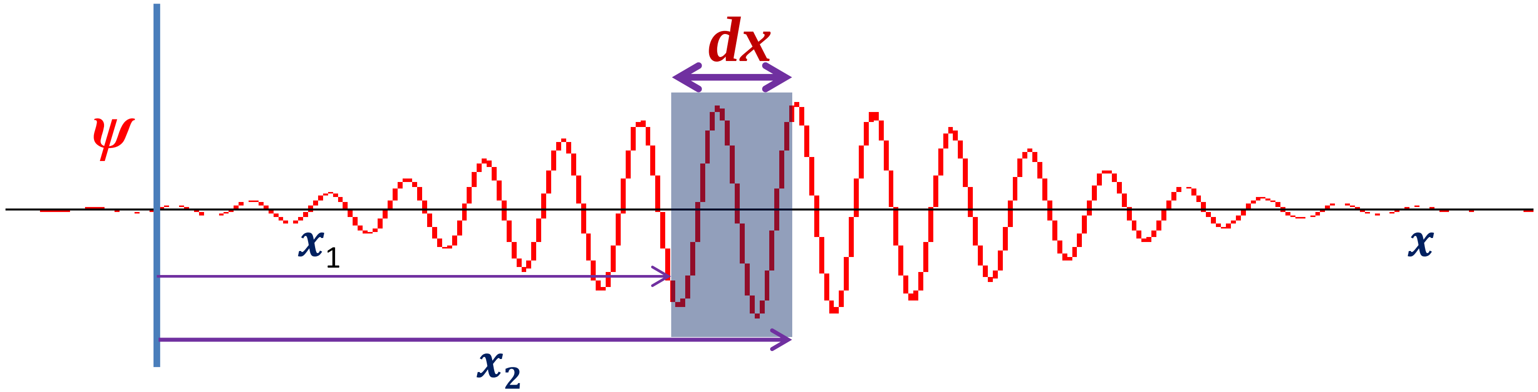
The waves associated with material particles, consist of oscillations of a purely mathematical quantity called wave-function (ψ)

<u>Born interpretation of wave-function or physical significance of (ψ)</u>

The wave-function (ψ) is a purely mathematical quantity and a complex function, which it self has no meaning and is not a physically observable quantity. But, its square ($\psi^* \psi = |\psi^2|$) gives the probability density of finding the particle within a given space. Though ψ itself is meaning less yet it contains entire description of the system under consideration

Note: ψ *is a* complex quantity. It can be written in the form $\psi = A + i\,B$ where A and B are real functions. Complex conjugate of ψ is defined as $\psi^* = A - iB$ and $\psi^*\,\psi = A^2 + B^2$.

Therefore. $\psi^*\,\psi = |\psi^2|$ is always positive and real.



$$\frac{dP}{dx} = \psi^* \psi.$$

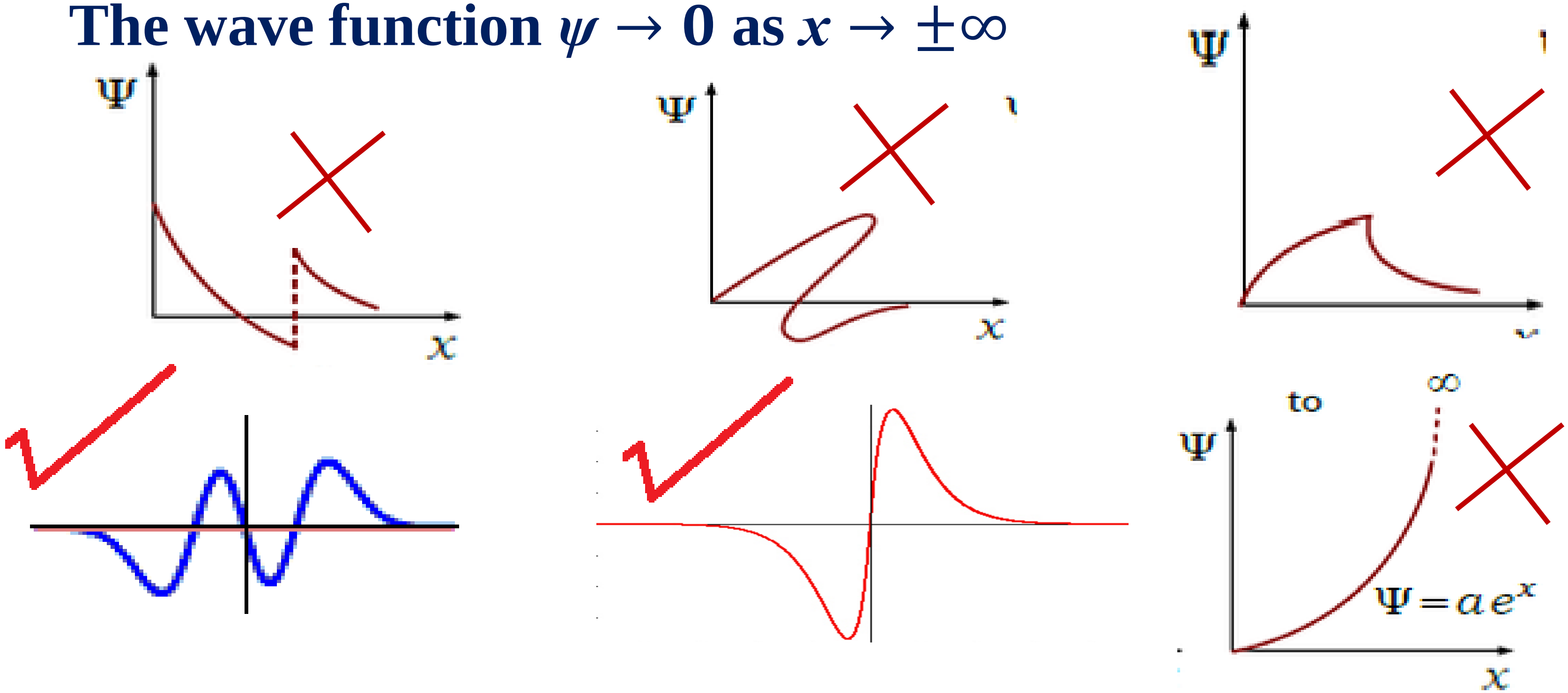
Total Probability of finding the particle between x_1 and x_2

$$P_{1,2} = \int_{x_1}^{x_2} \psi^* \psi . dx$$

Properties of wave-function (ψ):

Physically acceptable wave function is the one which obeys following conditions:

- **The wave function and its first derivative must be finite and single valued**
- **The wave function and its first derivative must be continuous**
- **The wave function $\psi \rightarrow 0$ as $x \rightarrow \pm\infty$**



Normalization of wave-function (ψ)

$$\boxed{\int_{-\infty}^{+\infty} \psi * \psi . dx = 1}$$

By normalizing the wave-function, we are able to get its amplitude

Expression for wave-function (ψ) in 1-D

$$\psi = Ae^{-i\,(\omega t - kx)}$$

$$\psi = Ae^{-i\,(2\pi \upsilon t - \frac{2\pi}{\lambda}x)}$$

$$\psi = Ae^{-i\left[2\pi\frac{E}{h}t - \frac{2\pi}{h}px\right]}$$

$$\psi = Ae^{-\frac{i}{\hbar}[Et - px]}$$

$$\psi(x,t) = Ae^{-\frac{i}{\hbar}[Et - px]}$$

For 1-D

$$\psi(r,t) = Ae^{-\frac{i}{\hbar}[Et - \vec{p}.\vec{r}]}$$

For 3-D

Operators in Quantum Mechanics

$$\psi = Ae^{-\frac{i}{\hbar}[Et-px]}$$

$$\frac{\partial\psi}{\partial x} = Ae^{-\frac{i}{\hbar}[Et-px]}\left(-\frac{i}{\hbar}[-p]\right)$$

$$-i\hbar\frac{\partial\psi}{\partial x} = (p)\;Ae^{-\frac{i}{\hbar}[Et-px]}$$

$$-i\hbar\frac{\partial\psi}{\partial x} = p\;\psi$$

$$\hat{p}\;\psi = p\;\psi$$

Momentum Operator → $\boxed{\hat{p} = -\,i\hbar\frac{\partial}{\partial x}}$

$$\psi = Ae^{-\frac{i}{\hbar}[Et-px]}$$

$$\frac{\partial\psi}{\partial t} = Ae^{-\frac{i}{\hbar}[Et-px]}\left(-\frac{i}{\hbar}[E]\right)$$

$$i\hbar\frac{\partial\psi}{\partial t} = (E)\;Ae^{-\frac{i}{\hbar}[Et-px]}$$

$$i\hbar\frac{\partial\psi}{\partial t} = E\;\psi$$

$$\hat{E}\;\psi = E\;\psi$$

Energy Operator → $\boxed{\hat{E} = i\hbar\frac{\partial}{\partial t}}$

Velocity Operator

$$\hat{v} = \frac{\hat{p}}{m} = -\frac{i\hbar}{m}\frac{\partial}{\partial x}$$

Kinetic energy Operator

$$\widehat{E_k} = \frac{\hat{p}^2}{2m} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$$

Hamiltonian Operator

$$\widehat{E_k} + V = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \mathrm{V}$$

Expectation value :

Def: Average value of any physical observable

For any physical observable A having $\hat{A}$ as corresponding Operator, the expectation value is denoted as $\langle A \rangle$ and is calculated as:

$$\langle A \rangle = \int_{x_1}^{x_2} \psi * \hat{A}\, \psi \,.\, dx$$

Eigen Value and Eigen Function

For any physical observable A having $\hat{A}$ as corresponding Operator, if the condition $\hat{A}\,\psi = A\,\psi$ is satisfied, then the Wave function ψ is called Eigen function and A is called as Eigen value of given physical observable.

Postulates of Quantum Mechanics

1. Every quantum mechanical system is associated with a wave function ψ which contains all the information of system
2. Every physical observable of system has corresponding operator
3. Only those values of observable A are possible which satisfy the condition $\hat{A}\,\psi = A\,\psi$
4. Probability of finding the particle in a given space is given by

$$P_{1,2} = \int_{x_1}^{x_2} \psi * \psi . dx$$

5. Expectation value of any physical observable is calculated as

$$\langle A \rangle = \int_{x_1}^{x_2} \psi * \hat{A}\, \psi \, . \, dx$$

Schrodinger Wave Equations (SWE)

1. Time dependent SWE:

Let a quantum mechanical system is associated with a wave function ψ given as:

$$\psi = Ae^{-\frac{i}{\hbar}[Et-px]}$$

$$\frac{\partial^2\psi}{\partial x^2} = Ae^{-\frac{i}{\hbar}[Et-px]}\left(-\frac{i}{\hbar}[-p]\right)^2$$

$$\boxed{-\hbar^2\frac{\partial^2\psi}{\partial x^2} = p^2\,\psi}$$

$$\frac{\partial\psi}{\partial t} = Ae^{-\frac{i}{\hbar}[Et-px]}\left(-\frac{i}{\hbar}[E]\right)$$

$$\boxed{i\hbar\frac{\partial\psi}{\partial t} = E\,\psi}$$

But $E = \frac{p^2}{2m} + V(x)$

$$E\psi = \frac{p^2\psi}{2m} + V(x)\psi$$

$$\boxed{i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x)\psi}$$

Time dependent SWE

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + V(x)\psi(x,t)$$

In a force free space $V(x) = 0$

$$\boxed{i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}}$$

2. Time Independent SWE:

Let a quantum mechanical system is associated with a wave function ∅ given as:

$$\emptyset(x,t) = Ae^{-\frac{i}{\hbar}[Et-px]} = Ae^{-\frac{i}{\hbar}[Et]}\, e^{\frac{i}{\hbar}[px]}$$

$$\boxed{\emptyset(x,t) = Ae^{-\frac{i}{\hbar}[Et]}\,\psi(x)}$$

$$i\hbar\frac{\partial \emptyset(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \emptyset(x,t)}{\partial x^2} + V(x)\emptyset(x,t)$$

$$i\hbar\frac{\partial}{\partial t}\, Ae^{-\frac{i}{\hbar}[Et]}\,\psi(x) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\, Ae^{-\frac{i}{\hbar}[Et]}\,\psi(x) + V(x)\, Ae^{-\frac{i}{\hbar}[Et]}\,\psi(x)$$

$$i\hbar\left(-\frac{i}{\hbar}E\right) \cancel{Ae^{-\frac{i}{\hbar}[Et]}}\,\psi(x) = -\frac{\hbar^2}{2m}\, \cancel{Ae^{-\frac{i}{\hbar}[Et]}}\,\frac{\partial^2}{\partial x^2}\psi(x) + V(x)\, \cancel{Ae^{-\frac{i}{\hbar}[Et]}}\,\psi(x)$$

$$E\psi(x) = -\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + V(x)\psi(x)$$

$$\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + [E-V(x)]\psi(x) = 0$$

$$\boxed{\frac{\partial^2\psi(x)}{\partial x^2} + \frac{2m}{\hbar^2}[E-V(x)]\psi(x) = 0}$$

Time Independent SWE

$$\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}[E-V]\psi = 0$$

Physical Significance of SWE

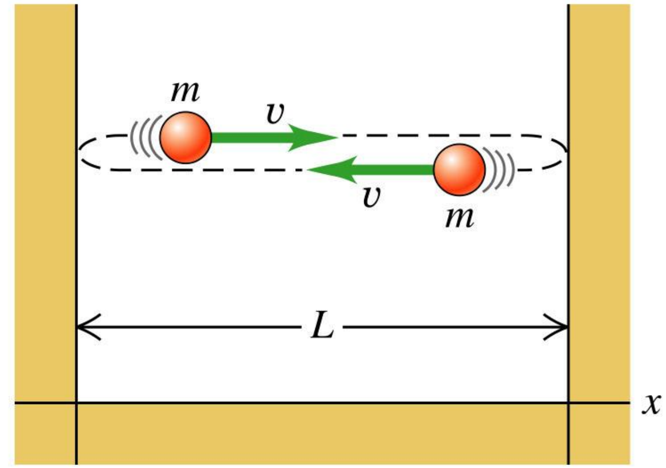
- SWE is a fundamental equation in Quantum Mechanics just like <u>Newton's laws in Classical Mechanics</u> and <u>Maxwell's Equations in Electricity and Magnetism</u>

Erwin Schrödinger (Austrian)
1887 –1961
Won Nobel Prize for Physics in 1933

- It is a fundamental in itself and hence can not be derived from any other equations.
- Its significance lies in the fact that it can be solved for wave function of a given system, which contains all the information about system.
- Some times solution consists of <u>discrete set of multiple wave functions</u> and as a result the discreteness in physical observables appears naturally

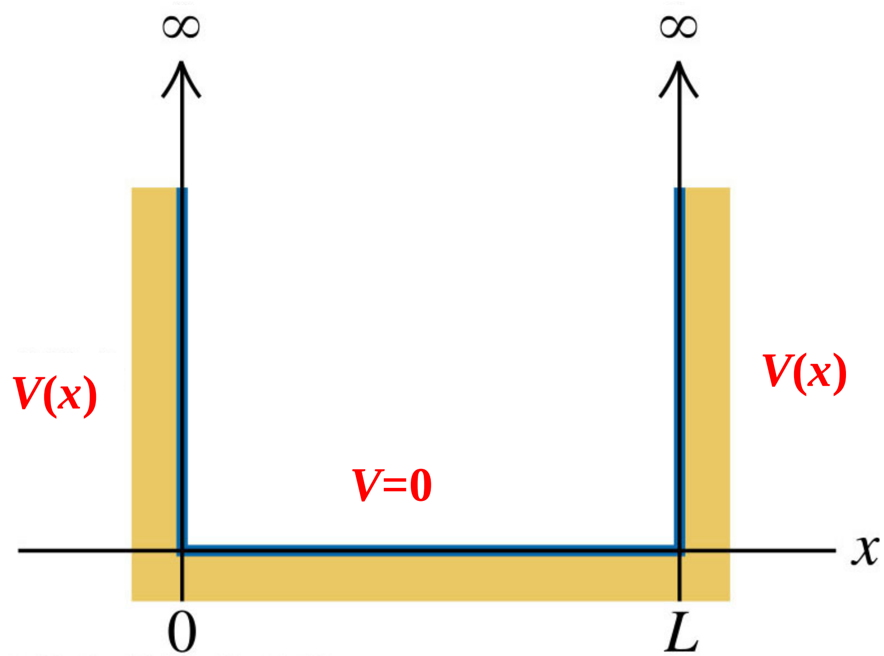
The Particle in A Box

Consider a particle moving along the x-axis between the points $x = 0$ and $x = L$, where L is the length of the "box." Inside the box the particle is free; at the endpoints, however, it experiences strong forces that serve to confine it within the boundary of box



The potential energy $V(x)$ for this situation is:

$$V(x) = \begin{cases} 0 & ;\ 0 < x < L \\ \infty & ;\ \textit{outside} \end{cases}$$



Classical view:

The particle can exist anywhere in the box and follows a path in accordance to Newton's Laws. All parameters of particle will have continuous spectrum

Quantum Mechanical view:

The particle and its properties are contained in a wave function which satisfies Schrodinger wave equation:

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m(E-V)}{\hbar^2}\psi(x) = 0$$

$$V(x) = 0 \qquad 0 < x < L$$

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0 \qquad k^2 = \frac{2m}{\hbar^2}E$$

Solution is

$$\psi = A\sin kx + B\cos kx$$

The boundary conditions applicable are:

(a) At $x = 0$, $\psi = 0$ $\Rightarrow$ $B = 0$

Solution reduces to

$$\boxed{\psi = A\sin kx}$$

(b) At $x = L$ $\psi = 0$

$$0 = A\sin kL$$

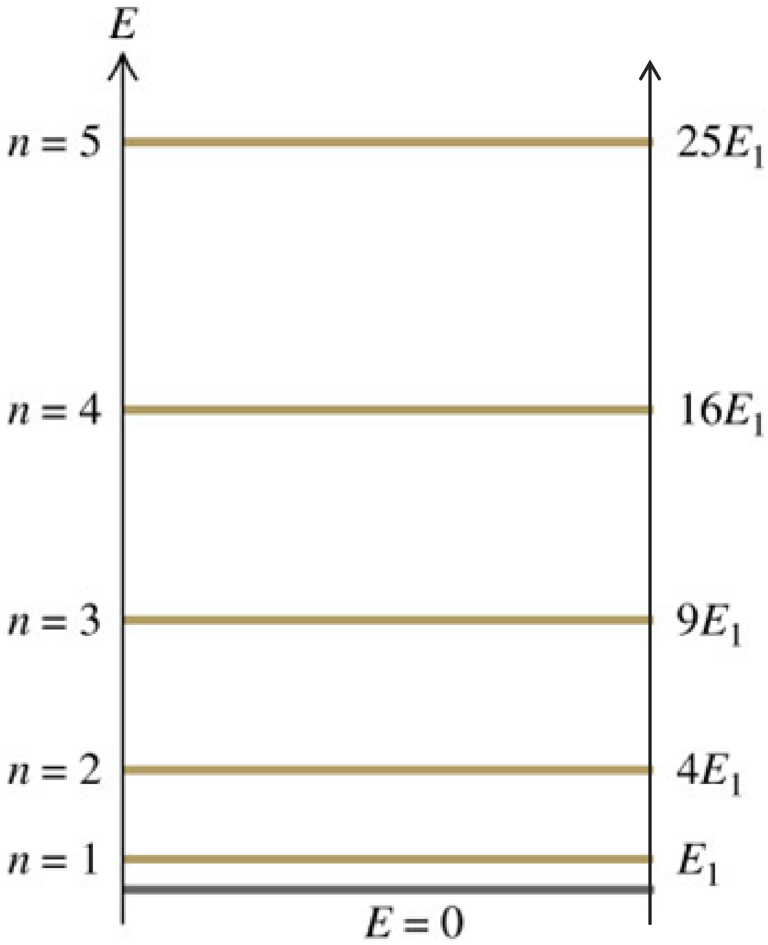
$$kL = n\pi$$

where n=1,2,3…..

$$k^2 L^2 = n^2 \pi^2$$

$$\frac{2m}{\hbar^2} E L^2 = n^2 \pi^2$$

$$\boxed{E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}}$$



Energy is quantized. It depends on variable *n* called principle quantum number having discrete values only

Normalization of wave-function (ψ)

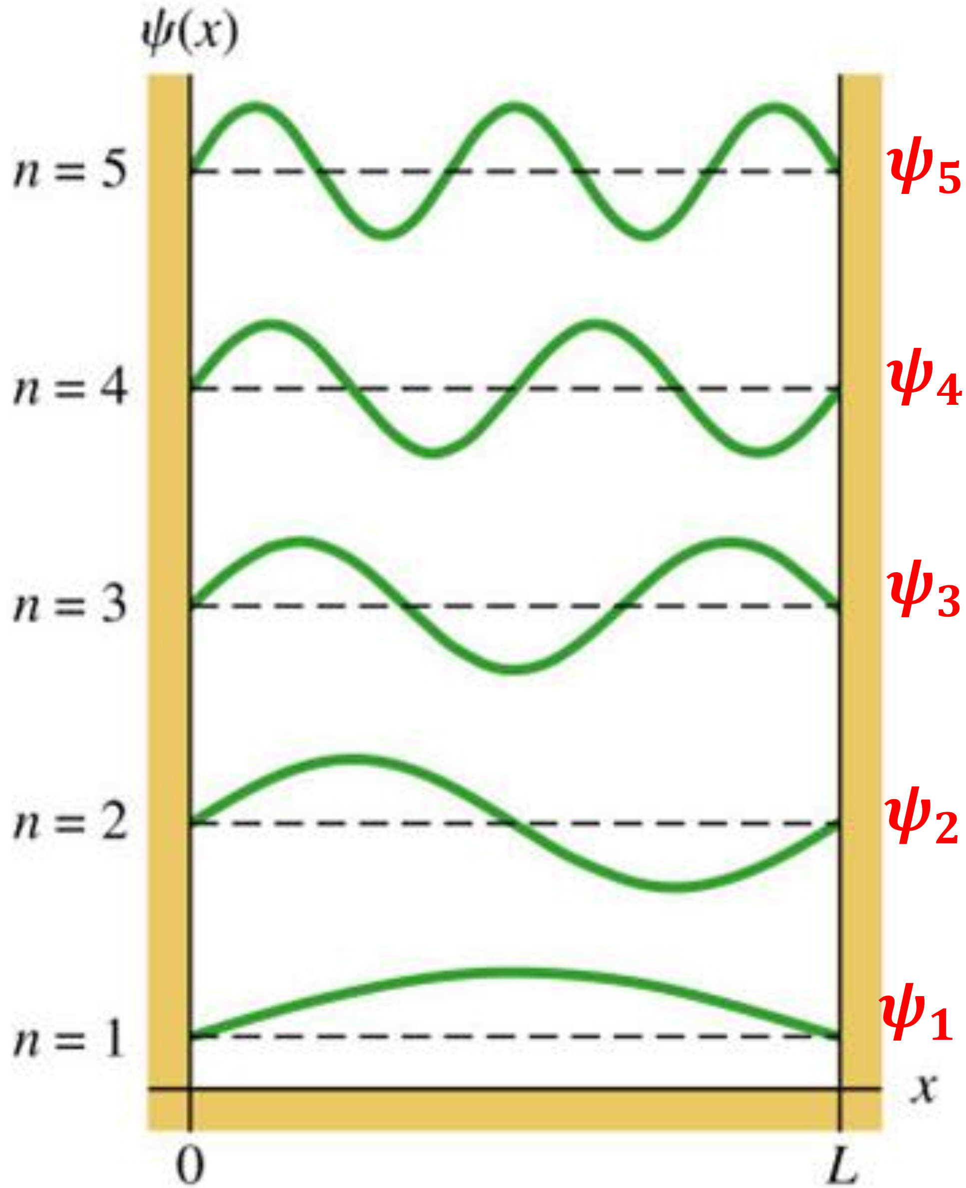
$$\int_0^L \psi^* \psi . dx = 1$$

$$\int_0^L A^2 \sin^2\left(\frac{n\pi}{L}x\right) = 1$$

$$\int_0^L A^2[1-\cos 2\left(\frac{n\pi}{L}x\right)]/2 = 1$$

$$|A|^2\left[\frac{L}{2} - \frac{\sin 2\frac{n\pi}{L}L}{4\frac{n\pi}{L}}\right] = 1$$

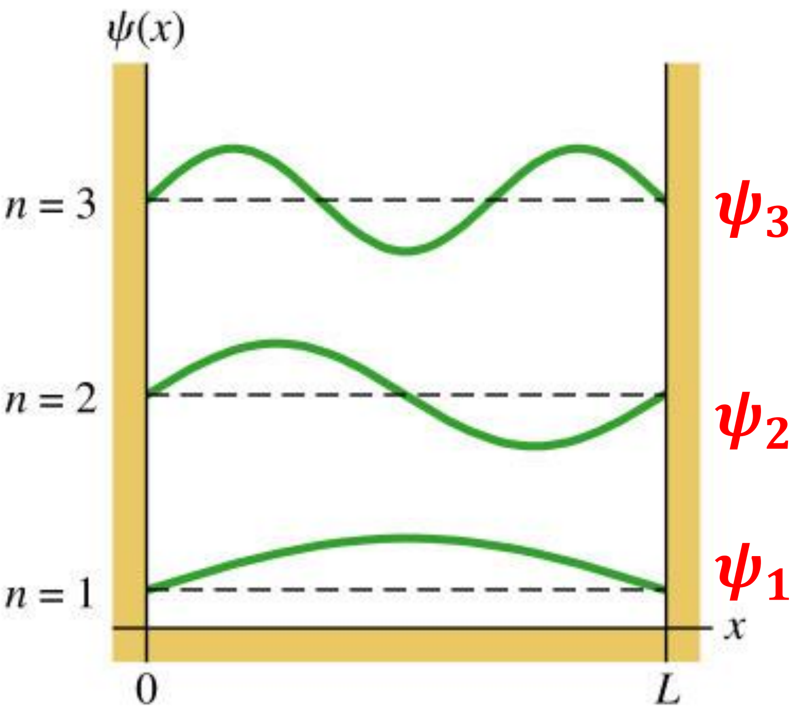
$$|A|^2\left(\frac{L}{2}\right) = 1 \qquad |A| = \sqrt{\frac{2}{L}}$$



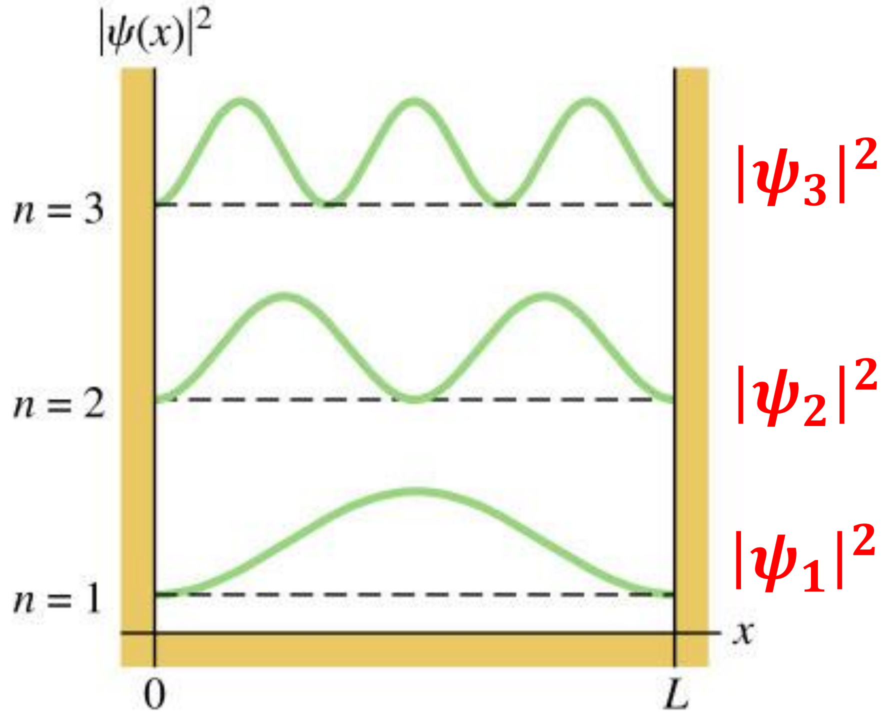
Thus normalized wave function is:

$$\psi_n = \sqrt{\frac{2}{L}} \sin \frac{n\pi}{L} x$$

Wave function **Probability distribution function**



(a)

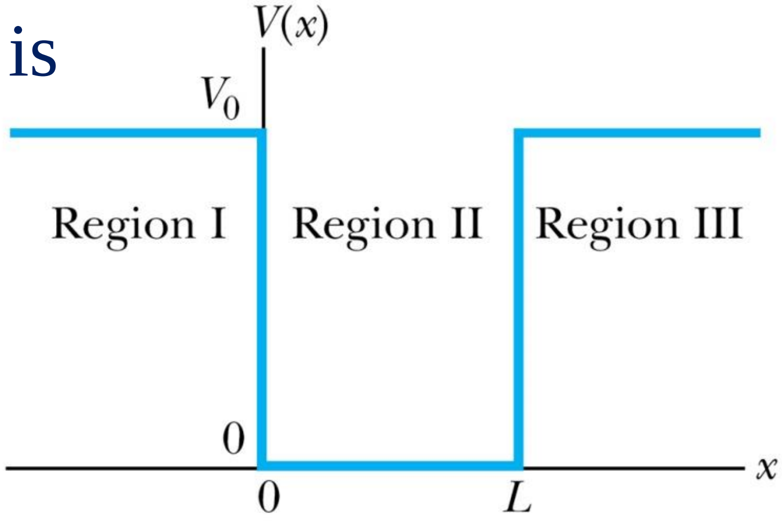


(b)

Finite Square-Well Potential

The finite square-well potential is

$$V(x) = \begin{cases} V_0 & x \le 0 & \text{region I} \\ 0 & 0 < x < L & \text{region II} \\ V_0 & x \ge L & \text{region III} \end{cases}$$



$$\frac{d^2\psi(x)}{dx^2} + \frac{2m(E-V)}{\hbar^2}\psi(x) = 0$$

$$V(x) = 0 \qquad 0 < x < L$$

In Region-II

$$\frac{d^2\psi_{II}}{dx^2} + k^2\psi_{II} = 0 \qquad k^2 = \frac{2m}{\hbar^2}E$$

Solution is $$\psi_{II} = A\sin kx + B\cos kx$$

In Region-I and Region-III

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m(E - V_o)}{\hbar^2}\psi(x) = 0$$

$$\frac{d^2\psi(x)}{dx^2} - \frac{2m(V_0 - E)}{\hbar^2}\psi(x) = 0$$

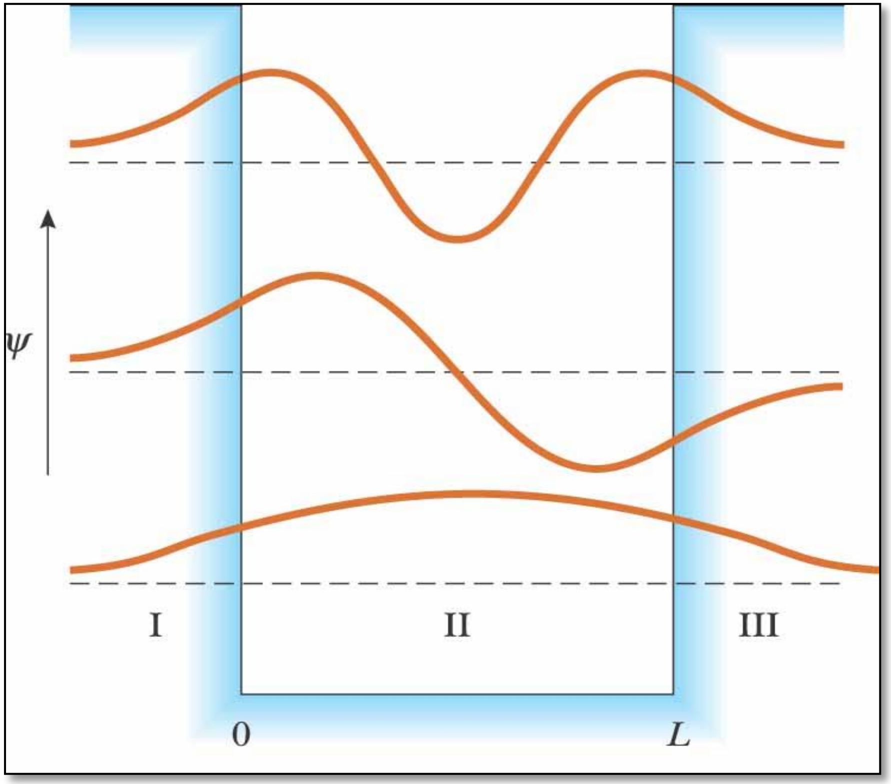
$$\frac{d^2\psi_I}{dx^2} - \alpha^2\,\psi_I = 0$$

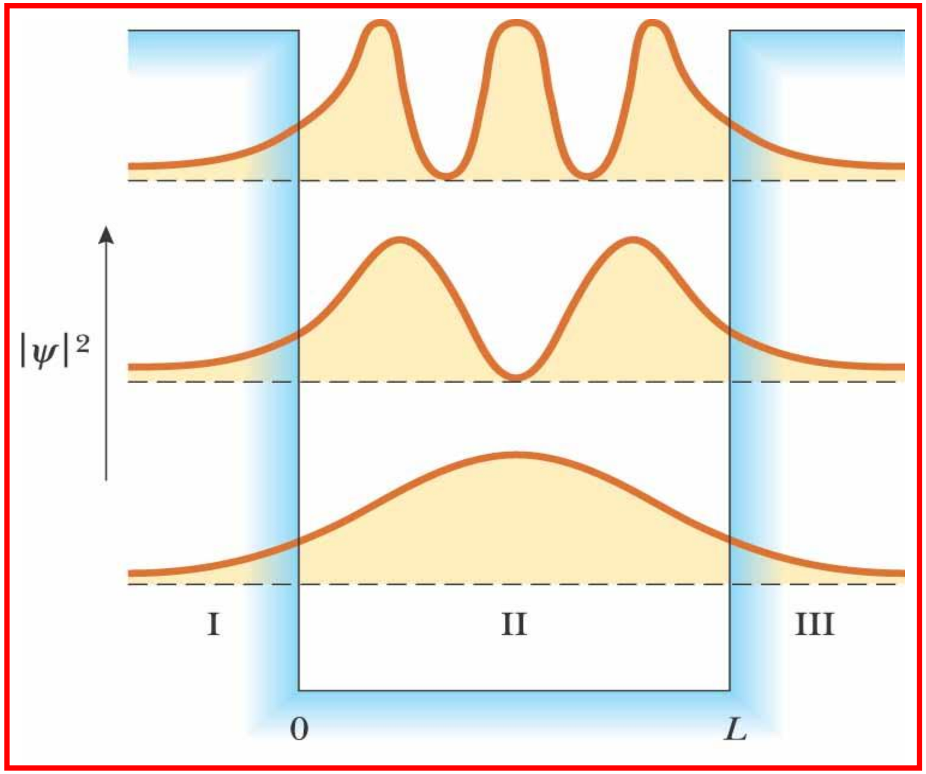
$$\alpha^2 = \frac{2m}{\hbar^2}(Vo - E)$$

$$\psi_{\mathrm{I}}(x) = Ae^{\alpha x} \qquad \text{region I, } x < 0$$

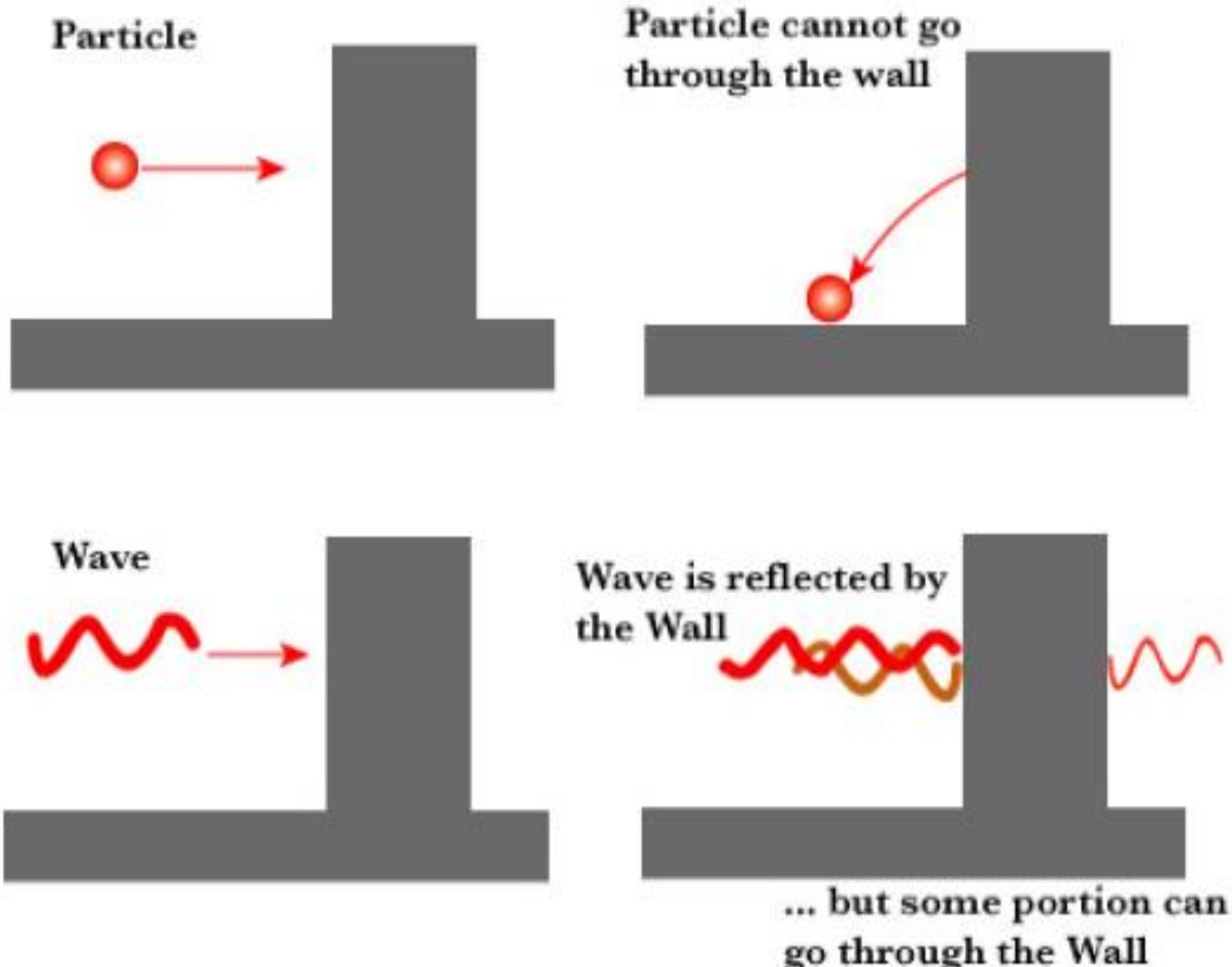
$$\psi_{\mathrm{III}}(x) = Be^{-\alpha x} \qquad \text{region III, } x > L$$

$$\psi_{II}(x) = A\sin kx + B\cos kx$$

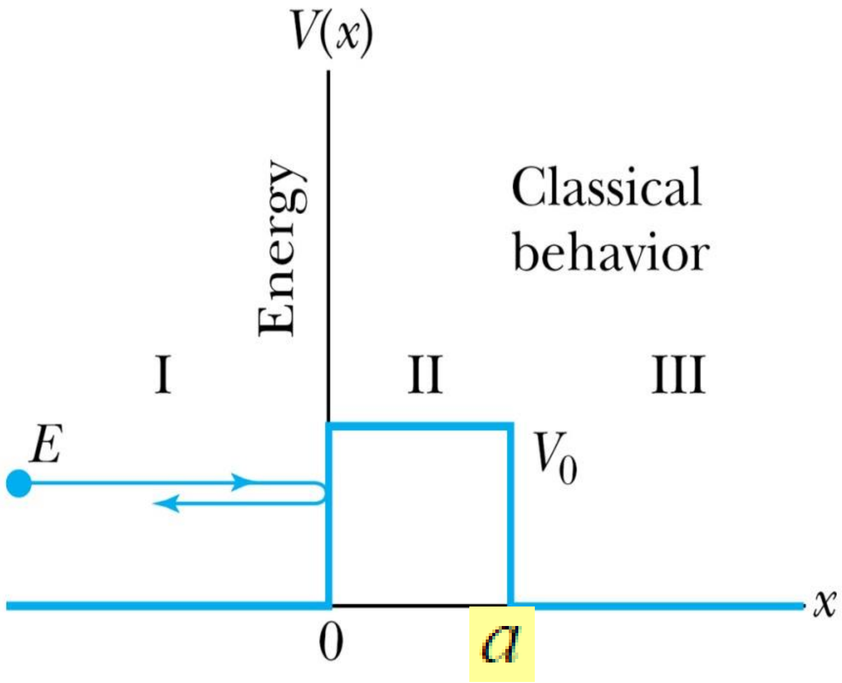


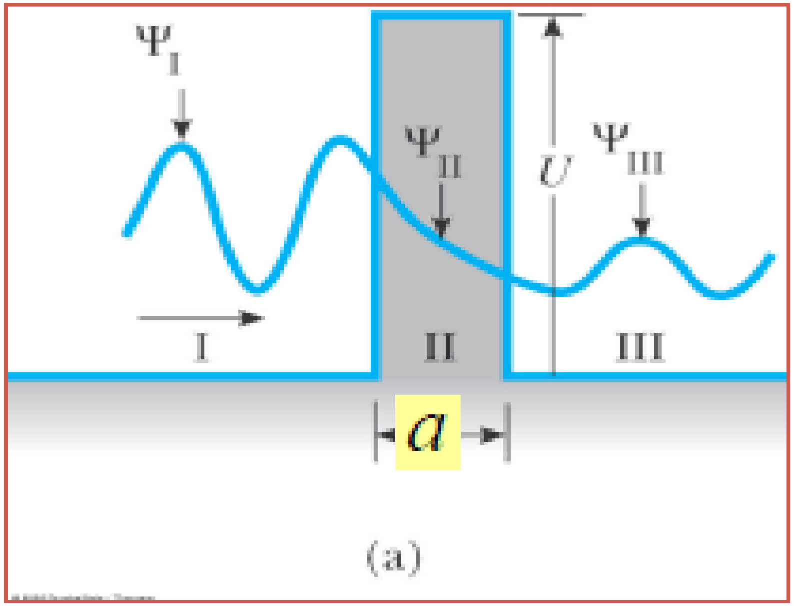


Tunnel Effect



Quantum Mechanical Tunneling





$$V(x) = 0, \quad \text{for } x < 0$$

$$V(x) = V_0, \quad \text{for } 0 \le x \le a$$

$$V(x) = 0, \quad \text{for } x > a$$

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m(E-V)}{\hbar^2}\psi(x) = 0$$

$$\Psi_1(x) = Ae^{ik_1x} + Be^{-ik_1x}$$

$$\Psi_2(x) = Ce^{ik_2x} + De^{-ik_2x}$$

$$\Psi_3(x) = Fe^{ik_3x}$$

$$k_1 = \frac{\sqrt{2mE}}{\hbar}$$

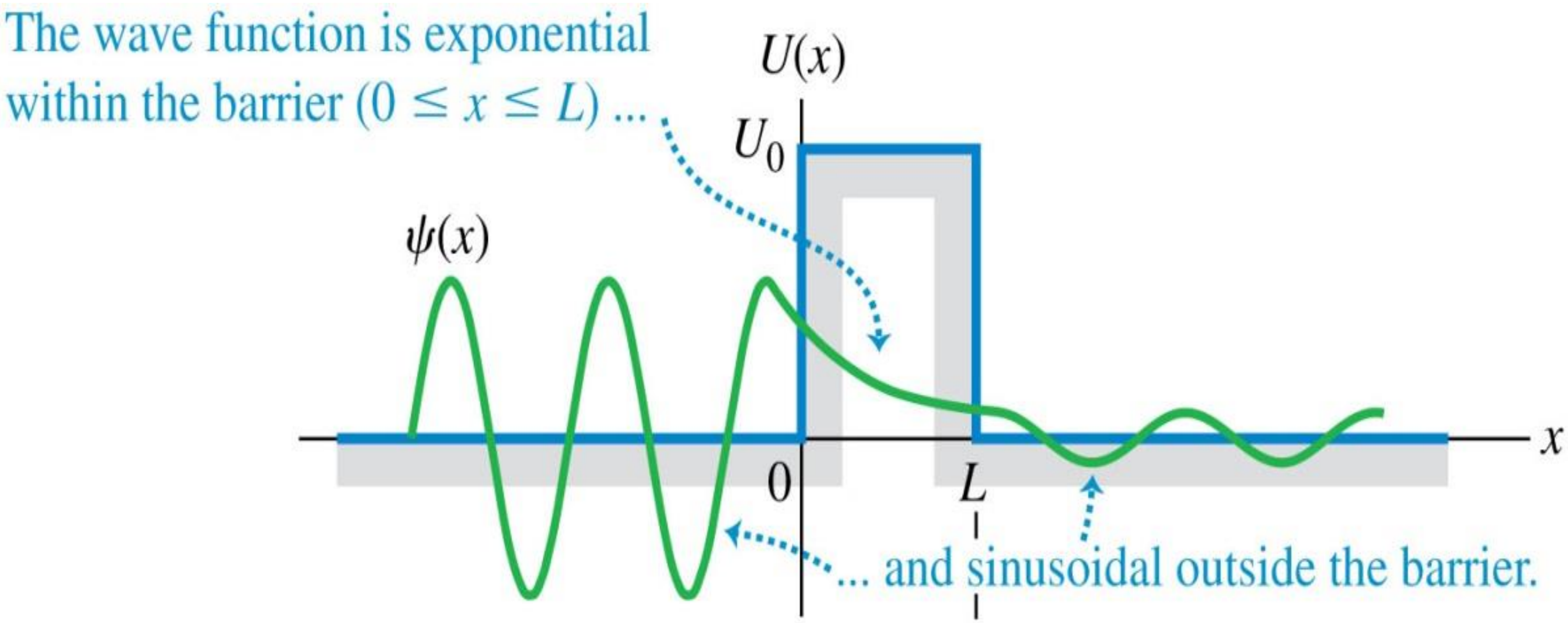
$$k_2 = \frac{\sqrt{2m(E - V_0)}}{\hbar}$$

When boundary conditions are applied then we can get the values of A,B,C,D and F.

$$\frac{F}{A} = \frac{4k_1k_2e^{i(k_2-k_1)a}}{(k_1 + k_2)^2 - (k_1 - k_2)^2 e^{i2ak_2}}$$

$$T = \frac{|\psi_{\text{III}}(\text{transmitted})|^2}{|\psi_{\text{I}}(\text{incident})|^2} = \frac{F^* F}{A^* A}$$

$$\boxed{T = \left|\frac{F}{A}\right|^2 = \frac{4E(E - V_0)}{V_0^2 \sin^2(k_2a) + 4E(E - V_0)}}$$



A particle incident on a barrier from the left has an oscillating wave function ψ.

But inside the barrier there are no oscillations ($E < V$).

If the barrier is not too thick, ψ is Non-zero at its opposite face, and oscillations begin again there.

- The tunnel probability increases with increase in the particle energy.
- A particle with energy E can tunnel through a barrier with height V_0.
- The tunneling probability T is greater for lower and thinner potential barriers.