MA323(Lab-06)

Jatin Dhingra
Roll no. 180123060
Mathematics and Computing

Problem 01

First of all Box-Muller method is used to generate 1000 values of Z= (Z1, Z2) which corresponds to N(0, I). 0 and I(Identity matrix) are matrix of size (2x1) and (2x2) respectively.
Then for all given values of a, all the details are shown in below table:

Value of a	-0.5	0	0.5	1
Expectation	[5, 8]	[5, 8]	[5, 8]	[5, 8]
Variance Covariance	[[1, -1], [-1, 4]]	[[1, 0], [0, 4]]	[[1, 1], [1, 4]]	[[1, 2], [2, 4]]
Correlation Coefficient	-0.5	0	0.5	1

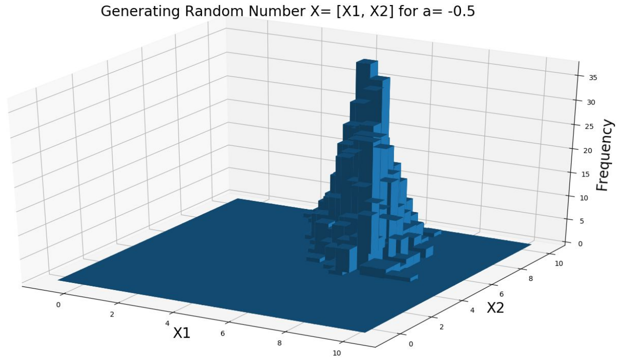
For all the above mentioned values of a(corresponding Expectation and Variance Covariance matrix), Random numbers are generated X= (X1, X2) using below mentioned formulas which corresponds to N(Expectation, Variance Covariance).

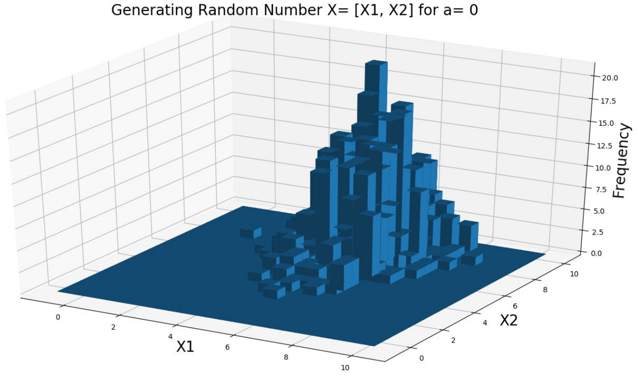
$$X1 = \mu 1 + (\sigma 1 * Z\ 1)$$

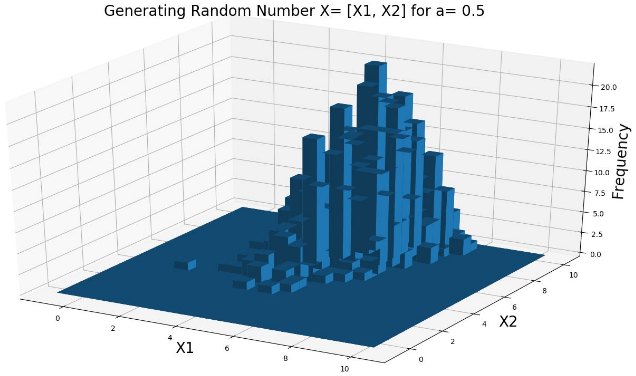
$$X2 = \mu 2 + (\rho * \sigma 2 * Z1) + (\mathrm{sqrt}(1-\rho * \rho) * \sigma 2 * Z2)$$

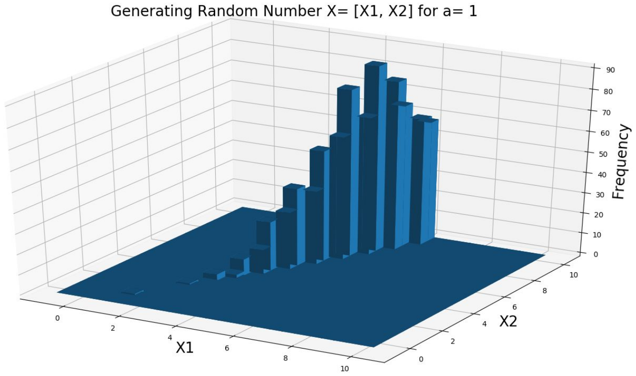
Problem 02

For each value of a, Frequency distribution histogram is plotted with X1 on x axis, X2 on y axis and frequency on z axis.
It is clear from below graphs that maximum frequency(for all values of a) is at x1= 5, x2= 8.


Generating Random Number X= [X1, X2] for a= -0.5
Frequency
X1
X2
0 5 10 15 20 25 30 35
0 2 4 6 8 10
0 2 4 6 8 10

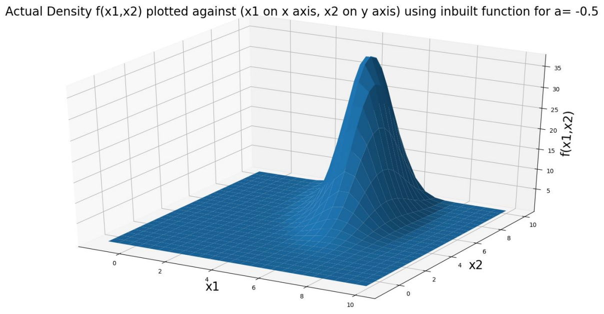

Generating Random Number X= [X1, X2] for a= 0
Frequency
X1
X2
0.0 2.5 5.0 7.5 10.0 12.5 15.0 17.5 20.0
0 2 4 6 8 10
0 2 4 6 8 10

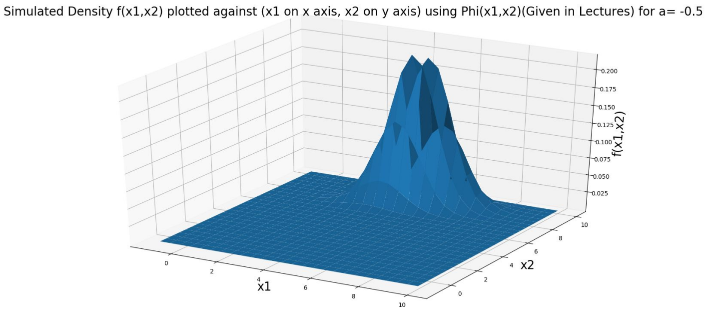

Generating Random Number X= [X1, X2] for a= 0.5
Frequency
X1
X2
0.0 2.5 5.0 7.5 10.0 12.5 15.0 17.5 20.0
0 2 4 6 8 10
0 2 4 6 8 10

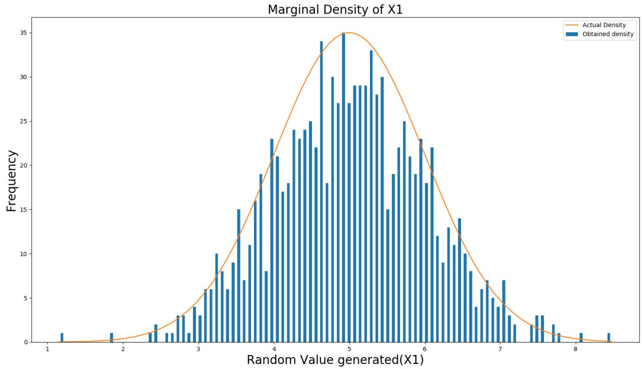

Generating Random Number X= [X1, X2] for a= 1
Frequency
X1
X2
0 10 20 30 40 50 60 70 80 90
0 2 4 6 8 10
0 2 4 6 8 10

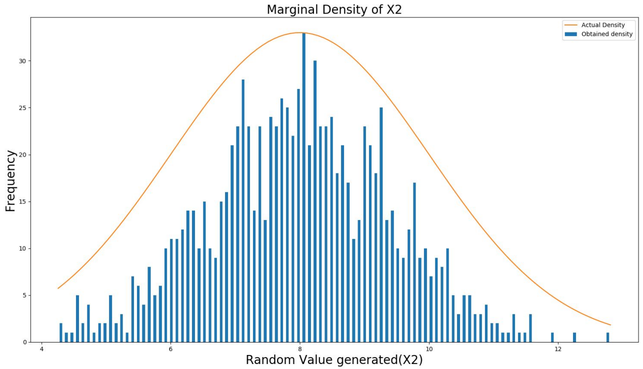
Problem 03: For each value of a(except 1), Actual and Simulated density are plotted(after scaling).
For plotting Actual density curves, inbuilt function is used and for plotting Simulated density curves, Given function Phi(x1, x2) (given in lecture 6) is used.
And then Marginal densities are plotted. It is clear from figure that Obtained densities converges to actual densities.

<u>a= -0.5(shown on this page)</u>

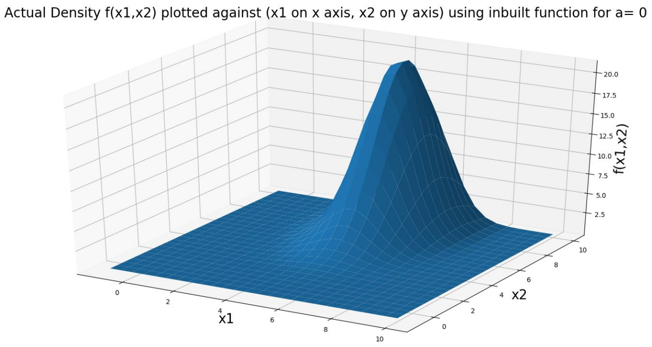


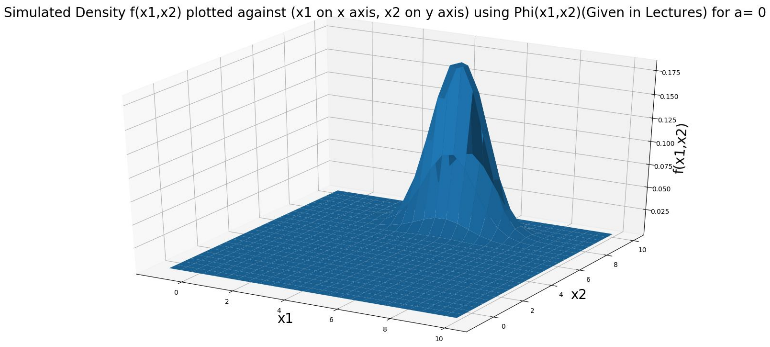


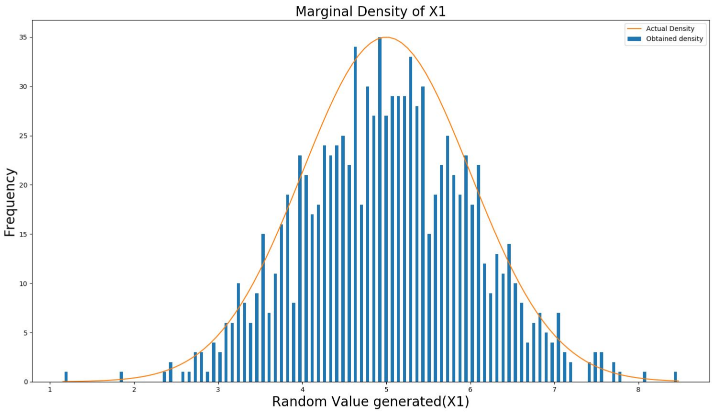


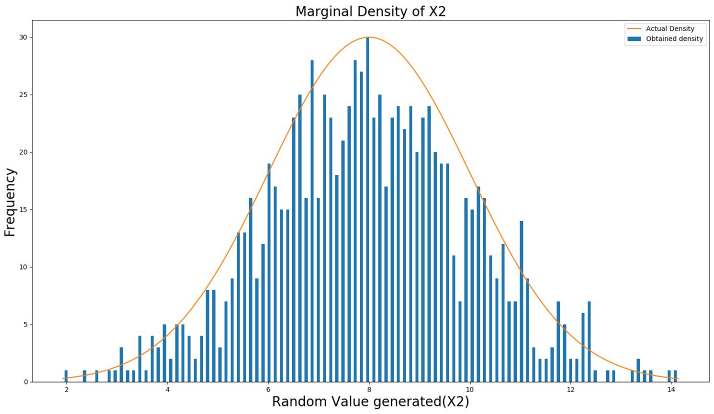


a= 0(shown on this page)

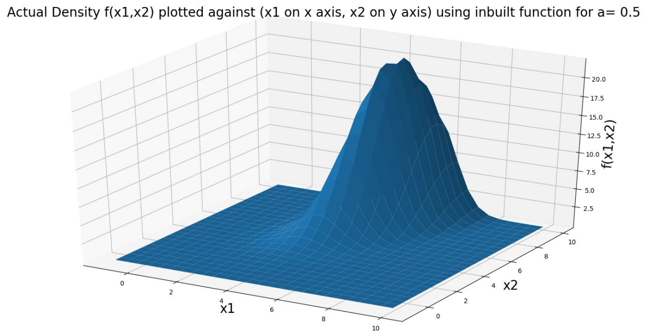


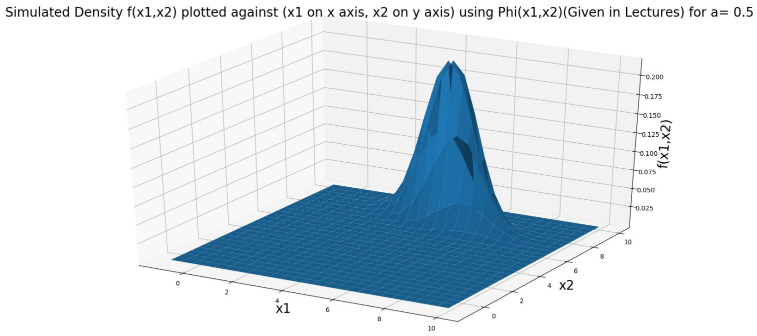


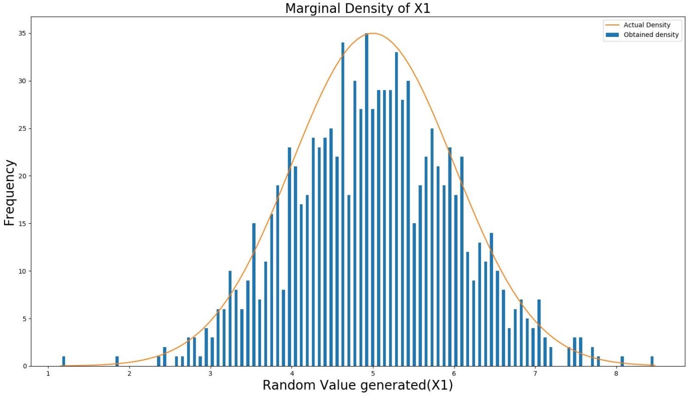


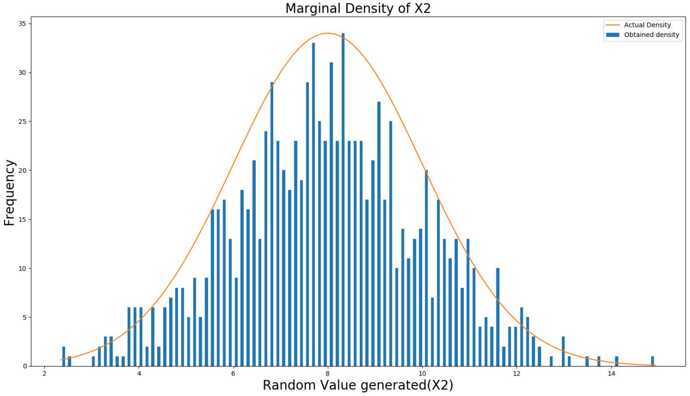


a= 0.5(shown on this page)









a= 1(shown on this page)

For a= 1, Variance Covariance matrix is non-Singular. Therefore, it is non-invertible and determinant is 0. So, actual and simulated densities can't be generated and plotted.

Variance Covariance= [[1, 2], [2, 4]]

Value of Correlation Coefficient is 1 which means X1 and X2 are completely correlated.

X1 = μ1 + (σ1*Z 1) X1= 5 + Z1

X2 = μ2 + (ρ*σ2*Z1) + (sqrt(1-ρ*ρ)*σ2*Z2) X2= 8 + 2*Z1

Solving above 2 equations, X2= 8 + 2*(X1-5)= 2*X1 - 2 (which means X1, X2 are linearlly dependent)

Below shown figures are Marginal densities of X1, X2.