Type: MCQ

Q1. A and B are equally likely, independent events in a sample space. If P(A ∪ B) = 8/9, then P(A) = ______. (0.5)

1. 1/3
2. **2/3
3. 4/3
4. 1/9

Q2. If $\mu = 4$ and $\sigma = \frac{2}{\sqrt{3}}$, then $P(|X-4| \geq 2) \leq$ ____. (0.5)

1. **1/3
2. $\frac{1}{\sqrt{3}}$
3. $\frac{1}{2\sqrt{3}}$
4. $\frac{2}{\sqrt{3}}$

Q3. A box contains **2 red**, **2 green**, and **3 blue** marbles. If **3** marbles are drawn simultaneously at random from the box, the probability that **not all of three are of the same colour** is ______ (0.5)

1. **20/21
2. 12/21
3. 9/21
4. 4/21

Q4. If $f(x) = \begin{cases} \frac{3x}{a}; & 0 \leq x \leq 1 \\ 0; & otherwise \end{cases}$, then $a =$______. (0.5)

1. **3/2
2. 2/3
3. 3
4. 2

Q5. X is uniformly distributed in [-1, 1]. Then $P[X^2 > 1/9]$ is ______. (0.5)

1. 1/3
2. **2/3
3. 1/9

4. 2/9

Q6. A drawer contains 8 different pairs of socks. If 6 socks are selected successively at random and without replacement. Compute the probability that there is at least one matching pair among the 6 socks. (0.5)

1. $\frac{32}{143}$
2. **$\frac{111}{143}$
3. $\frac{1}{143}$
4. $\frac{1}{15}$

Q7. A random variable X has the pdf $p(x) = \frac{x}{5050}, x = 1,2, \dots, 100$. Find $E(X)$. (0.5)

1. 1
2. **67
3. 50
4. 5050

Q8. A random variable X is uniformly distributed in the interval $(-1,3)$. Find the probability $P(|X| > 2)$. (0.5)

1. $\frac{3}{4}$
2. **$\frac{1}{4}$
3. 0
4. $\frac{1}{2}$

Q9. Which of the following is a valid value of correlation coefficient? (0.5)

1. $\frac{\pi}{e}$
2. **$\frac{e}{\pi}$
3. π^e
4. e^π

Q10. A random variable X has probability mass function $f(x) = \frac{c}{2^k}, k = 0,1,2, \dots$. Then the value of c is (0.5)

1. 1
2. **$\frac{1}{2}$
3. 2

4. $\frac{3}{2}$

Type: DES

Q11. A box contains 2 fair coins and 1 two-headed coin. One coin is taken from the box at random and tossed once. Then it is **put back into the box** and a coin is again drawn from the box and tossed. If the result is heads on both tosses, what is the probability that the two-headed coin was taken both times?. (2)

Q12. Suppose that a 2 dimensional random variable (X, Y) is uniformly distributed over the triangular region $R = \{(x, y) \mid 0 < x < y < 1\}$. Find the covariance. (2)

Q13. Given $f(x,y) = \begin{cases} e^{-(x+y)}; & x \geq 0, y \geq 0 \\ 0; & otherwise \end{cases}$. Then evaluate $P(X > Y)$, $P(X + Y \leq 1)$ and marginal pdf of X. (2)

Q14. Two dice are rolled. Let X be the random variable which takes the greatest common divisor of the two numbers appearing on the dice. Find E(X) and V(X). (2)

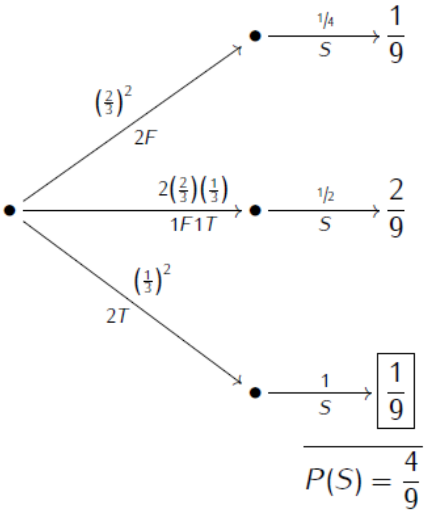
Q15. Suppose that the 2D random variable (X,Y) has joint pdf

$$f(x,y) = kx(x-y), 0 < x < 2, |y| < x$$

Then find (a) k (b) marginal pdf of Y. (2)

11)

Ans. Let $F \equiv$ Fair coin is taken in one draw, $T \equiv$ Two-headed coin is taken in one draw, and $S \equiv$ Both tosses result in heads.



Thus, by Bayes' theorem, $P(T \mid S) = \dfrac{1}{4}$.

(0.5 marks for partitioning+1 Marks for P(S)+0.5= 2Marks)

Alternative method.

The total probability of getting a head in a single toss is $\frac{2}{3} \times \frac{1}{2} + \frac{1}{3} \times 1 = \frac{2}{3}$.

Therefore, the total probability of getting heads in both tosses is $P(S) = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$.

Then $P(T \mid S) = \frac{P(T \cap S)}{P(S)} = \left(\frac{1}{3}\right)^2 \times 1 \Big/ \left(\frac{4}{9}\right) = \frac{1}{4}$.

(0.5 marks for partitioning+1 Marks for P(S)+0.5= 2Marks)

....

12)

The joint pdf is $f(x,y) = \begin{cases} 2 & if\ (x,y) \in R \\ 0 & else \end{cases}$ ....(0.5 Marks)

Marginal pdf of $x, g(x) = \int_x^1 2dy = 2(1-x), 0 < x < 1$

Marginal pdf of $y, h(y) = \int_0^y 2dx = 2y, 0 < y < 1$......................(0.5 Marks)

$E(x) = \int_0^1 2x(1-x)dx = \frac{1}{3}$ and

$E(y) = \int_0^1 2y^2dy = \frac{2}{3}, E(xy) = \int_0^1 \int_0^x 2\,xydy\,dx = \frac{1}{4}$

Cov(x,y) = (1/4)-(2/9)=(1/36)=0.0278 ....(1 Marks)

(consider alternative methods with appropriate weightage)

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13) $P(X > Y) = \int_0^\infty \int_0^x e^{-(x+y)}\,dydx = 0.5.$ ....(0.5 Marks)

$P(X + Y \leq 1) = \int_0^1 \int_0^{1-x} e^{-(x+y)}\,dydx = 0.2642$..................(1 Marks)

Marginal pdf of X is $g(x) = \int_0^\infty e^{-(x+y)}\,dy = e^{-x}; 0 < x < \infty.$

....(0.5 Marks)

....

14) The pdf is given by

$X = x$	1	2	3	4	5	6
$p(x)$	$\frac{23}{36}$	$\frac{7}{36}$	$\frac{3}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$

(1 Mark)

$\boldsymbol{E(X)} = \frac{61}{36}\, E(X^2) = \frac{155}{36}$ and $\boldsymbol{V(X)} = \mathbf{1.4345}$ **.................(1 Mark)**

....

15) k = (1/8) ....(1 Mark)

g(y) = (16+y^3-12y)/48, 0<y<2
g(y)= (16+5y^3-12y)/48 -2<y<0 ....(1 Mark)

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