How a Bayesian Hierarchical Model makes our roads safer

Project Report

A project on the paper "A novel Bayesian hierarchical model for road safety hotspot prediction" by Fawcett, Thorpe, Matthews & Kremer (2017)Link :-
[A novel Bayesian hierarchical model for road safety hotspot prediction - ScienceDirect]()

Aim of the paper is to take proactive road safety measures at specific sites by predicting the number of accidents and ranking them according to their safety hotspot potentials. In contrast to previous models, the confounding effects of regression-to-mean and trend are met.
Aim of this project was to analyze the model and to replicate it based on historical data of accident counts from the German city of Halle. As indicated throughout the report, we were able to successfully replicate the model

This model helps us to study the potentials and implementations of Bayesian models as well as their practical applications.

Introduction

Fawcett, Thorpe, Matthews & Kremer (2017) have developed a Bayesian Hierarchical Model that they proclaim to be well-suited to predict road safety hotspots and to be meeting the demands of both effectiveness and efficiency. Their model based on a past accident data is special as it accounts for regression-to-mean (RTM) and trend Also, the ability for more recent accident counts to inform model-based estimates of safety with greater precision by giving higher weights to recent years for prediction.

Challenges

Regression to mean:- A recent peak in accidents at a site, e.g. in the previous year can either be due to a severe safety lack or due to a random increase. A random increase would lead to a decrease in accidents (regression to the mean) in the following year anyway—no matter whether measures were implemented or not . So we don't want to allocate the budget to the locations where there was just a random increase but instead focus on those which have risk

Problem of trend:- If a measure has been implemented, accidents might decrease due to either the measure itself—or due to site-specific or global trends, i.e. that people drive slower in general. So there may be a short term trend which is developing at a particular region which may not be true for the long term

We will try to tackle these challenges using the approach given in the paper

Data

We use the same data set as used in the paper an open-source road network of the German city of Halle. The available data consists of accident counts at 734 sites in Halle for the years 2004–2012 inclusive, which are the so-called observations. We have historical points in time which are represented by {yj (t);t = −7, ..., −1, 0; j = 1, ..., 734} for the years 2004, ..., 2011 and a future time period {yj (t);t = +1; j = 1, ..., 734} for the year 2012. We keep the year 2012 for validating the model's predictions. For each site j = 1, ..., 734, we have covariates which include amongst others traffic volume, speed limit, or whether a respective site is located in an urban area.

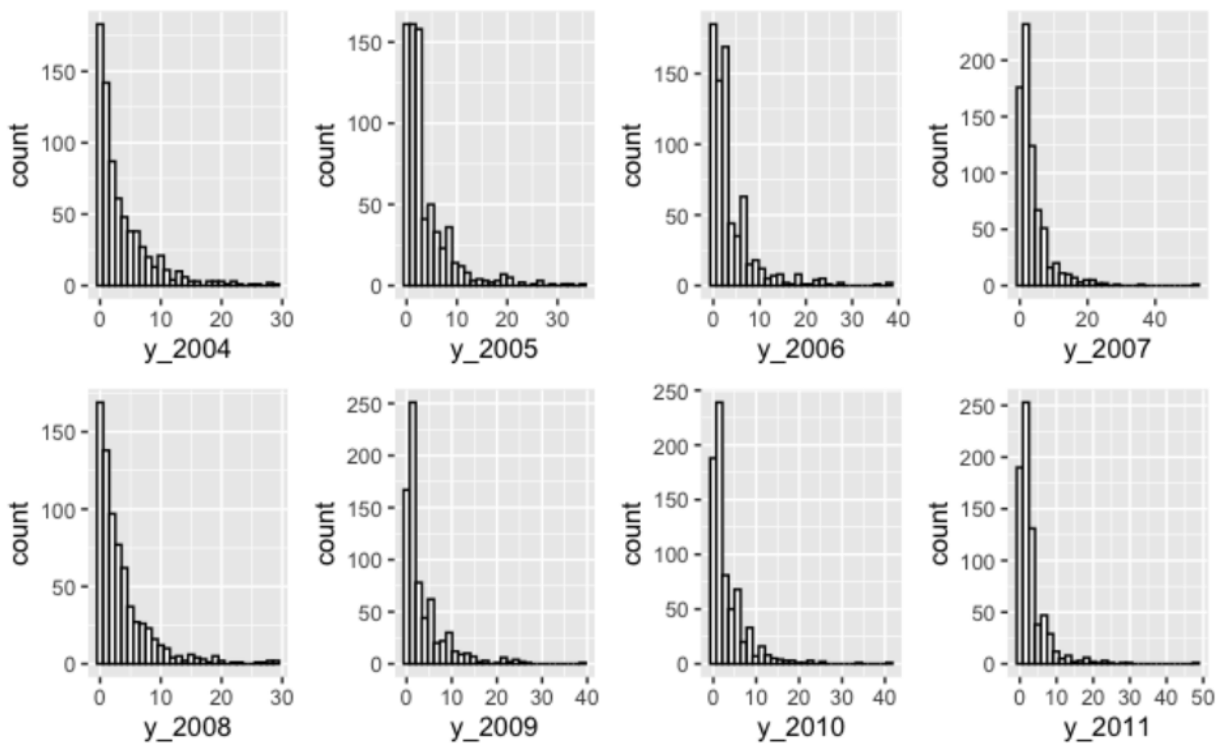
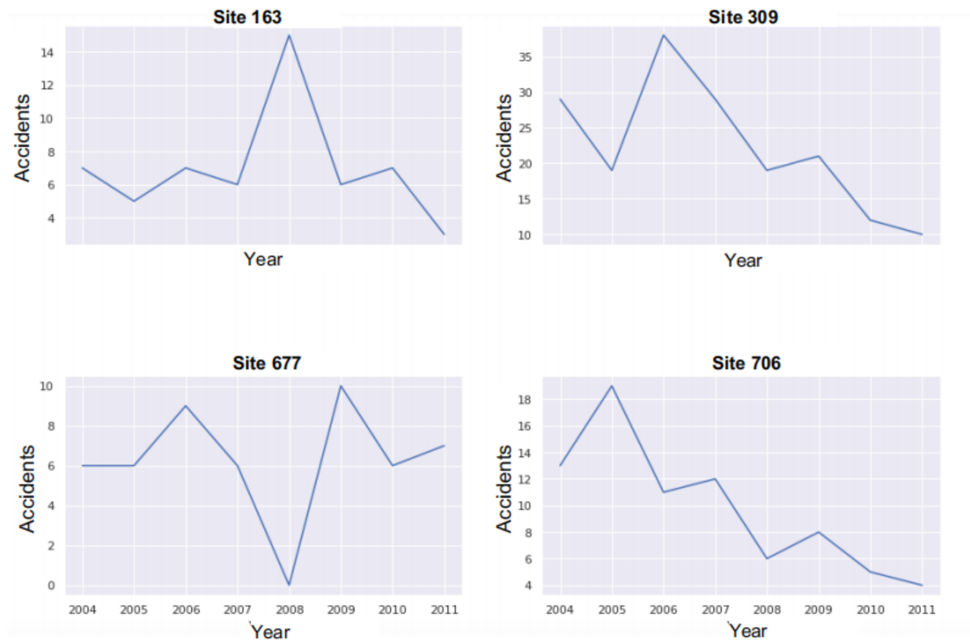


Figure 1: Distributions of yearly accident totals $y(t)$

Understanding the Challenges



RTM:-While the yearly accident count was between zero and eight for the site 163 over the eight-year recording period (2004–2011 inclusive), more than fifteen accidents were observed for the same site in 2008. Similarly, yearly accident counts in site 677 ranged from six to eleven but fell to zero in 2008. Both sites 163 and 677 of Figure 3 show that after a random and temporary change in the year 2008 accident counts reverted back to an average level. This indicates that an RTM effect may have occurred after 2008: the number of accidents naturally may have reverted back to an average level in 2009.

Trend:-Sites 309 and 706 suggest evidence of a temporal trend. In both plots of site 309 and 706 in Figure 3, there is an indication of a trend that the yearly accident counts decrease. Figure 3 proves that trend can vary at a local level since downward-sloping is observed only for site 309 and 706 (and not the other two sites displayed). Therefore, it is crucial that our model should include both the effects of the local and global trend for road safety hotspot prediction

Model Used by Fawcett et al

The main idea is to include data from multiple time periods for training and analysis. For the model, t = 0 represents current time, t < 0 represents the before period and t > 0 is reserved for the after period. The assumption is that current and future accidents counts are Poisson distributed with variance λj . The historical counts with variance λj (t)*c(t) We include c(t) is to give more weight to recent accidents.

For the before time period (t < 0), we are assuming a Negative Binomial distribution (NegBin). For the after time period, accident counts (t ≥ 0) are assumed to be Poisson distributed. The Negative Binomial distribution allows for different mean and variance, whereas for the Poisson distribution mean and variance are assumed to be the same. We use these particular distributions as we have discrete data and the events occurring are independent of the time since the last event this property is obeyed by both the distributions. The c(t) has weight which is decaying exponentially with time thereby giving higher weight to more recent data.

$$y_j(t)|\lambda_j(t)) = \begin{cases} Poisson(\lambda_j(t)), & t \geq 0 \\ NegBin\left(r = \frac{\lambda_j(t)}{c(t)-1} \ , \ p = \frac{1}{c(t)} \right), & t < 0 \end{cases}$$

$$c(t) = exp\{-t\tau\}, t < 0, \tau > 0$$

The mean accidents are built as a log-linear model with coefficients and site details as covariates

$$\mu_j = exp\left\{ \beta_0 + \beta_t t + \sum_{p=1}^{n_p} \beta_p x_{p,j} \right\}$$

the mean accident rate (λj(t)) is dependent on mean accidents represented by μj (t),aj which accounts for random effects (RTM) and bj which tackles the effects of local trend.Afterwards, when the model is trained, each site will have coefficients for random effects and local trend.

$$\lambda_j(t) = a_j \mu_j(t) exp(b_j t), a_j > 0; -\infty < b_j < \infty; t < 0$$

Priors

Following priors were used in the paper

$$\tau \sim \text{Gamma}(2, 20)$$

$$a_j \sim \text{Gamma}(\gamma, \gamma) \quad b_N \sim N(0, 0.1) \quad b_Z \sim \text{Bernoulli}(0.5) \; b_j = b_N \text{b}_Z$$

τ is used for higher weightage given to recent accidents using Gamma(2,20)

aj also helps us to capture causes we have not captured in our observations and Regression to Mean

bj is the local trend bz tries to penalise those trends which have already been accounted by the global trend.

MCMC

Computing the conditional distributions of the respective parameters we would run MCMC for 10000 times

$$\theta_j^{(i)} = \{a_j, b_j, \tau_j, \lambda_j(t)\}^{(i)}, j = 1, ..., 734; t = -7, ..., 0$$

Detailed Approach:-

Global Accident Prediction Model (APM)-global APM to get the µj values for all sites and every single year, we first had to rearrange the data towards a horizontal representation. The original data has site-wise accident counts for each year in different columns, which made it impossible to get one time coefficient to estimate the global APM without rearrangement. In the horizontal way, the respective years t1, t0, ..., t-7

Now we can fit the Negative Binomial Model, we get coefficients for all covariates as well as time and intercept.When these parameters are obtained for all the variables, we can calculate the respective mean accidents µj for each year and site.

the mean accident rate yt(t) is learned by taking the mean accidents µj as a prior ,now we include the priors that account for RTM and global as well as local trend.All other priors/hyperpriors are initialized as has been specified by Fawcett et al.

We could evaluate the final prediction using the following formula

$$\lambda_j(t) = a_j \mu_j(t) exp(b_j t), a_j > 0; -\infty < b_j < \infty; t < 0$$

All the computation was done using pymc3 Library

Comparison with the original Paper

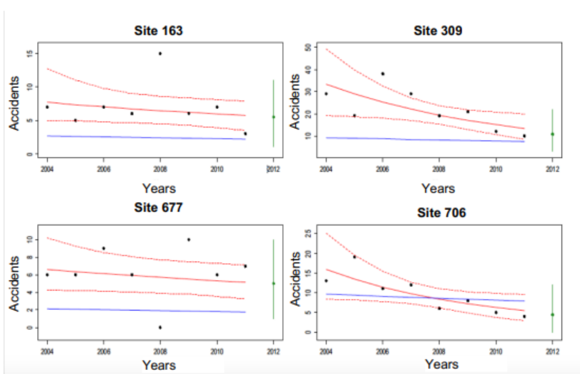
Comparison of posterior results

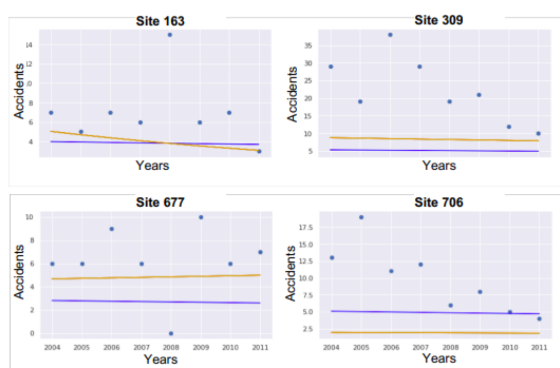
site	a_j	b_j	2008 Observed	2008 $\mu_j(t=-4)$ (APM)	2008 $\lambda_j(t=-4)$	2011 Observed	2011 $\mu_j(t=0)$ (APM)	2011 $\lambda_j(t=0)$	2012 Prediction $(t=1)$
163	2.62 (1.43, 3.64)	−0.01 (−0.13, 0.02)	15	2.37	6.44 (4.56, 8.57)	3	2.17	5.72 (3.46, 7.87)	5.50 (1, 11)
309	1.61 (0.94, 2.61)	−0.10 (−0.20, 0.00)	19	8.23	19.40 (15.49, 23.60)	10	7.54	13.30 (8.55, 19.69)	10.93 (3, 22)
677	2.96 (1.80, 4.16)	−0.01 (−0.10, 0.05)	0	1.90	5.71 (3.95, 7.73)	7	1.75	5.18 (3.32, 7.17)	5.07 (1, 10)
706	0.63 (0.29, 1.20)	−0.12 (−0.25, 0.00)	6	8.61	8.39 (6.05, 11.03)	4	7.90	5.50 (2.89, 9.44)	4.42 (0, 11)

site_id	aj	bj	observed_2008	mj_2008	lamda_2008	mj_2011	observed_2011	lamda_2011	prediction_2012
162	0.832686	-0.059461	15.0	3.571223	3.795883	3.690532	3.0	3.073054	2.864107
308	1.601160	-0.004202	19.0	4.760884	8.244046	4.919937	10.0	7.877606	7.759115
676	1.922416	0.020133	0.0	2.504303	4.840023	2.587968	7.0	4.975151	5.021027
705	0.393558	0.002947	6.0	4.544583	1.893249	4.696410	4.0	1.848311	1.833571

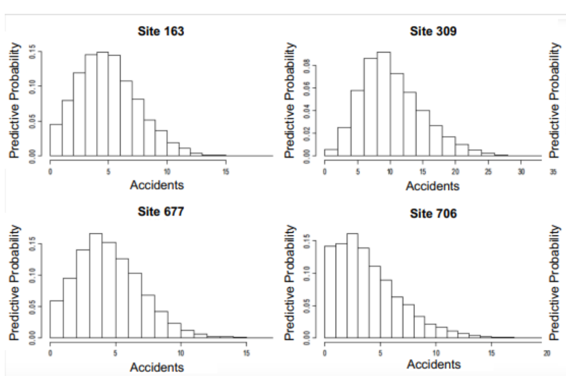
The table on the top is from the paper and the table below is from our computation

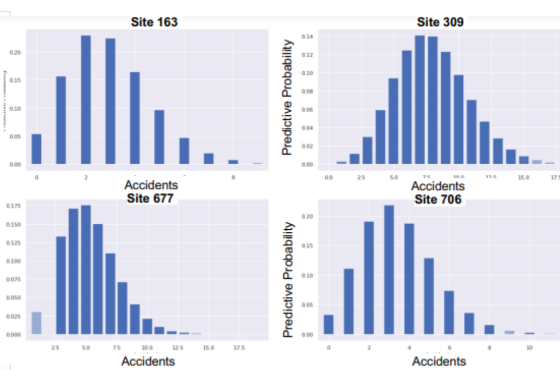
Comparison of (predicted) accidents at sites over years





Comparison of posterior samples





The charts on the left are from the paper and the charts on the right are from our model