Portfolio Optimization

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Video Link: https://youtu.be/i1DIpljeJnQ

The expected return is calculated as follows [1]:

$$E_r = w_1 \times R_1 + w_2 \times R_2 + \ldots + w_n \times R_n$$

The covariance matrix is calculated as follows [2]:

$$\text{Covariance matrix} = \underbrace{\begin{bmatrix} \sigma_1 & 0 & \cdots & 0 \\ 0 & \sigma_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & \sigma_n \end{bmatrix}}_{\substack{\text{Diagonal matrix with} \\ \text{standard deviations in} \\ \text{the diagonal (and} \\ \text{zeros in the other} \\ \text{cells)}}} \text{x} \underbrace{\begin{bmatrix} 1 & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & 1 & \cdots & \rho_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{n1} & \cdots & \cdots & 1 \end{bmatrix}}_{\text{Correlation matrix}} \text{x} \underbrace{\begin{bmatrix} \sigma_1 & 0 & \cdots & 0 \\ 0 & \sigma_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & \sigma_n \end{bmatrix}}_{\substack{\text{Diagonal matrix with} \\ \text{standard deviations in} \\ \text{the diagonal (and zeros} \\ \text{in the other cells) –} \\ \text{same as the one before}}}$$

The risk is calculated as follows [2]:

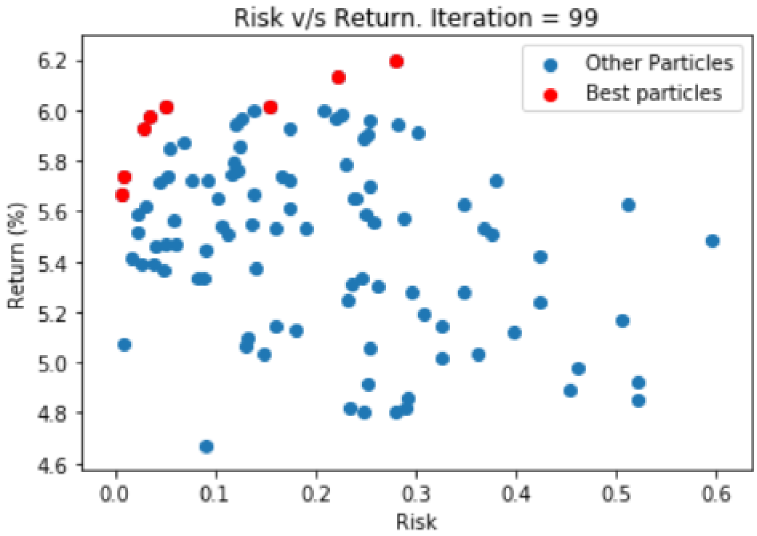
$$\text{Portfolio variance} = \begin{bmatrix} w_1 & \cdots & w_n \end{bmatrix} \text{x} \begin{bmatrix} \sigma_1^2 & \cdots & \sigma_{1n} \\ \vdots & \ddots & \vdots \\ \sigma_{n1} & \cdots & \sigma_n^2 \end{bmatrix} \text{x} \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix}$$

Algorithm:

- A particle is defined to have a position, velocity, cost, best position and best cost.
- Initially all particles are initialized with random positions that satisfy the constraints mentioned.
- A function is used to check if particle A dominates particle B. It returns true if Return of particle A is more than particle B and risk is less.
- A new velocity is determined for each particle as per PSO algorithm. If the new position satisfies constraints, the new velocity and new position are assigned to the particle.
- The new cost is computed and if it dominates the particle’s previous best, the best is updated.
- The process is repeated in each iteration and one of the best particles is also maintained.
- Non-domination sorting is used to find the pareto front of the best particles after the iterations.

Result:

The pareto front of the best particles:



One of the optimal solutions:

Portfolio Weights in % = [12.54281999 1.70996579 1.85738441 4.72019995 0.46593829 7.95363571 5.33384295 13.69431704 1.03606766 2.66582961 3.46111848 4.37506893 5.43596955 0.34328826 7.39701638 4.85949 0.4812174 13.81138902 1.00461385 6.85082672]

Expected Return (%) = 6.011 %

Risk = 0.0512

These weights satisfy the constraints.

References:

1. https://xplaind.com/730404/risk-and-return#toc-portfolio-risk-and-return
2. https://www.riskprep.com/all-tutorials/36-exam-22/58-modeling-portfolio-variance
3. Assignment 1 – PSO algorithm
4. Assignment 2 – Non-domination sorting function