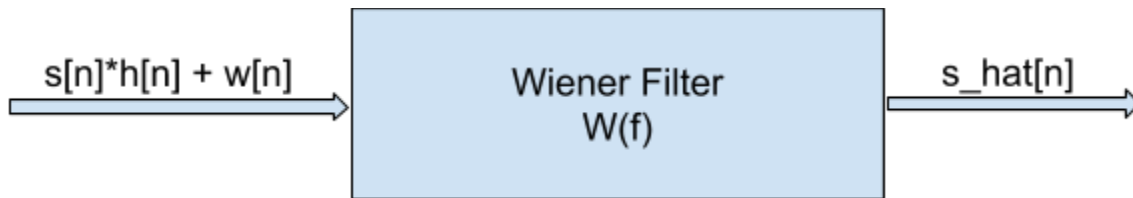


Image Restoration using Wiener Filter

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Wiener filtering is used to restore a signal which has additive noise, by minimising the Mean Squared Error, between the output and the original signal.



Here, $s[n]$ - Original Image

$h[n]$ - Kernel used for defocus blur

$w[n]$ - Additive Gaussian Noise

$s_hat[n]$ - Output

Let, $X[n] = s[n]*h[n] + w[n] = \text{Input to Wiener Filter}$

By minimising the Mean Square error between $s[n]$ and $s_hat[n]$, the following expression is obtained for the Wiener filter:

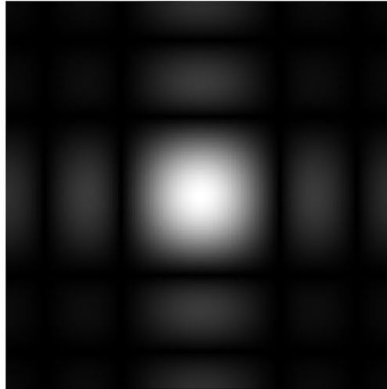
$$W(f) = \frac{H^*(f)}{|H(f)|^2 + K}$$

Here, $W(f)$ - Frequency Response of Wiener Filter

$H(f)$ - Fourier Transform of the kernel used for defocus blur

$K = S_w(f) / S_s(f)$, Ratio of power spectral densities of additive noise and original image.

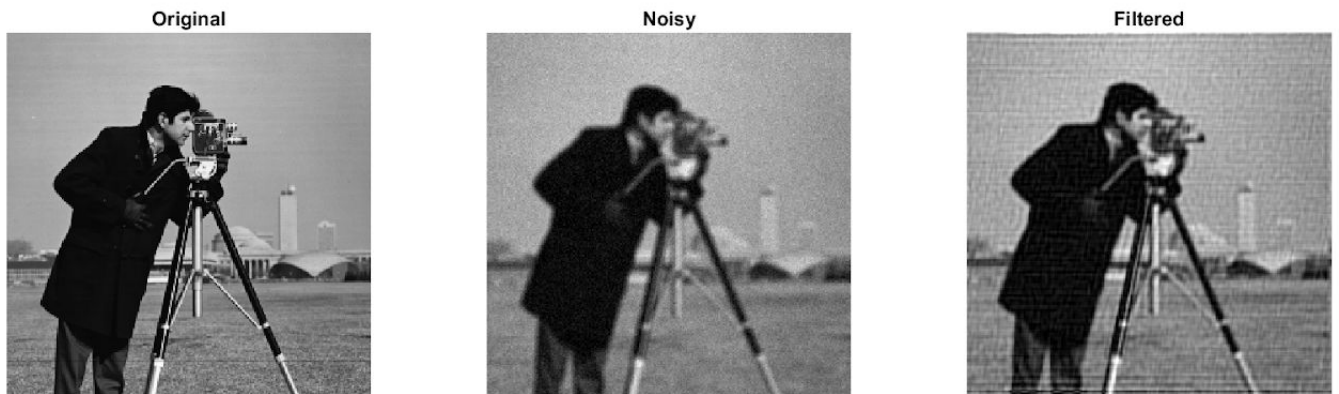
- A set of 10 training images are chosen, from the MATLAB demo images, which are used to determine $W(f)$ (Frequency Response of Wiener Filter).
- They are converted to grayscale and resized to (256,256) pixels, as their filters should have same size.
- A Gaussian kernel is used which is a rotationally symmetric low-pass filter to blur the image. The kernel is chosen of size (5,5) and standard deviation = 5. It is convolved with the original images for blurring. The fourier transform of the kernel is shown below:



- Additive Gaussian noise is added to the resultant images with 0 mean and 0.01 variance.
- The fourier transform of the kernel used for blurring is computed and the fourier transform of the original images is computed, which are used to make the frequency response of the filter. The power spectral density of the original images is computed from their fourier transform and the power spectral density of additive noise is equal to σ^2 .
- Finally, the Wiener filter is determined by taking the average of the filters obtained in the 10 training images.
- A set of 5 testing images is considered, which are blurred with the same kernel and gaussian noise is added.
- Their fourier transform is calculated and multiplied with the frequency response of the filter obtained above. The resultant is operated by inverse fourier transform, to obtain the restored image.

Results:

An example of the original image, corrupted image and image filtered by Wiener filter is shown below:



The PSNR values of the test images are shown below:(w.r.t their original images)

Image Name	PSNR (noisy image)	PSNR (filtered image)
Coins	19.2744	21.5782
Rice	19.1184	20.2302
Moon	21.2761	22.7752
Football	19.6754	25.0954
Canoe	16.5519	17.0072
Average	19.172	21.3372

Hence, it is observed that the PSNR of the filtered images is higher than the input images and quality has improved.

References

- http://shodhganga.inflibnet.ac.in/bitstream/10603/188041/16/15_chapter_5.pdf
- <http://www.owl.net.rice.edu/~elec539/Projects99/BACH/proj2/wiener.html>
- <https://www.math.uni-bielefeld.de/~rehmann/ECM/cdrom/3ecm/pdfs/pant3/strela.pdf>
- <https://www.ijcsmc.com/docs/papers/June2014/V3I6201467.pdf>
- <https://www.sciencedirect.com/science/article/pii/S0263224110002174>