Inclusion-Exclusion and its various Applications

Difficulty Level : Easy • Last Updated : 17 Aug, 2021

In the field of Combinatorics, it is a counting method used to compute the cardinality of the union set. According to basic **Inclusion-Exclusion principle**:

- For 2 finite sets $A_1$and A_2, which are subsets of Universal set, then $(A_1 - A_2), (A_2 - A_1)$and $(A_1 \bigcap A_2)$are disjoint sets.

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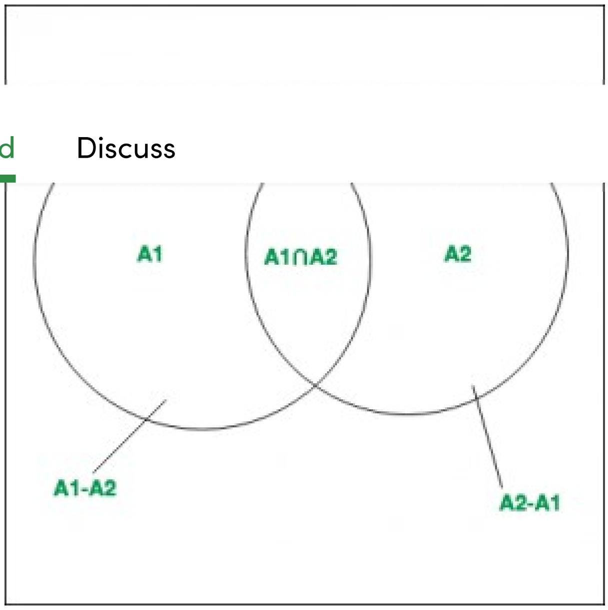

- Hence it can be said that,
  $|(A_1 - A_2) \bigcup (A_2 - A_1) \bigcup (A_1 \bigcap A_2)| = |A_1| - |A_1 \bigcap A_2| + |A_2| - |A_1 \bigcap A_2| + |A_1 \bigcap A_2|$

  $|A_1 \bigcup A_2| = |A_1| + |A_2| - |A_1 \bigcap A_2|$.
- Similarly for 3 finite sets A_1, $A_2$and A_3,
  $|A_1 \bigcup A_2 \bigcup A_3| = |A_1| + |A_2| + |A_3| - |A_1 \bigcap A_2| - |A_2 \bigcap A_3| - |A_1 \bigcap A_3| + |A_1 \bigcap A_2 \bigcap A_3|$

sets – Sum of all 2-set intersections + Sum of all the 3-set

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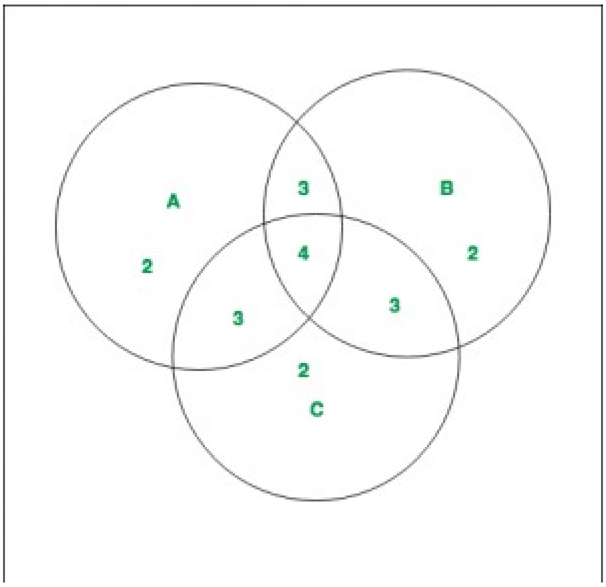
In general it can be said that,

$$|A_1 \bigcup A_2 \bigcup A_3 .. \bigcup A_i| = \sum_{1 \le k \le i} |A_k| + (-1) \sum_{1 \le k_1 < k_2 \le i} |A_{k_1} \bigcap A_{k_2}| + (-1)^2 \sum_{1 \le k_1 < k_2 < k_3 \le i} |A_{k_1} \bigcap A_{k_2} \bigcap A_{k_3}| .. + (-1)^{i+1} \sum_{1 \le k_1 < k_2 < k_3 < .. k_i \le i} |A_{k_1} \bigcap A_{k_2} \bigcap A_{k_3} .. \bigcap A_{k_i}|$$

Properties :

1. Computes the total number of elements that satisfy at least one of several properties.
2. It prevents the problem of double counting.

Example 1:
As shown in the diagram, 3 finite sets A, B and C with their corresponding values are given. Compute $|A \bigcup B \bigcup C|$.



The values of the corresponding regions, as can be noted from the

$|A \bigcap C| = 3, |A \bigcap B \bigcap C| = 4$

By applying Inclusion-Exclusion principle,

$|A_1 \bigcup A_2 \bigcup A_3| = |A_1| + |A_2| + |A_3| - |A_1 \bigcap A_2| - |A_2 \bigcap A_3| - |A_1 \bigcap A_3| + |A_1 \bigcap A_2 \bigcap A_3|$

$|A_1 \bigcup A_2 \bigcup A_3| = 2 + 2 + 2 - 3 - 3 - 3 + 4 = 1$

Applications :

- **Derangements**
  To determine the number of derangements(or permutations) of n objects such that no object is in its original position (like Hat-check problem).
  As an example we can consider the derangements of the number in the following cases:
  For i = 1, the total number of derangements is 0.
  For i = 2, the total number of derangements is 1. This is 21.
  For i = 3, the total number of derangements is 2. These are $231$and 3 1 2.

Theory | Set 1 (Introduction)

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