A Review on the Design and Optimization of Antennas Using Machine Learning Algorithms and Techniques

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REVIEW ARTICLE

A review on the design and optimization of antennas using machine learning algorithms and techniques

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Abstract

This paper presents a focused and comprehensive literature survey on the use of machine learning (ML) in antenna design and optimization. An overview of the conventional computational electromagnetics and numerical methods used to gain physical insight into the design of the antennas is first presented. The major aspects of ML are then presented, with a study of its different learning categories and frameworks. An overview and mathematical briefing of regression models built with ML algorithms is then illustrated, with a focus on those applied in antenna synthesis and analysis. An in-depth overview on the different research papers discussing the design and optimization of antennas using ML is then reported, covering the different techniques and algorithms applied to generate antenna parameters based on desired radiation characteristics and other antenna specifications. Various investigated antennas are sorted based on antenna type and configuration to assist the readers who wish to work with a specific type of antennas using ML.

KEYWORDS

antenna design, computational electromagnetics, machine learning, neural networks, regression models

1 | INTRODUCTION

Over the past few decades, the art of machine learning (ML) has taken the world by storm with its pervasive applications in automating mundane tasks and offering disruptive insights across all walks of science and engineering. Though arguably still in its infancy, ML has all but revolutionized the technology industry. ML practitioners have managed to alter the foundations of countless industries and fields of study, including lately the design and optimization of antennas. In the light of the Big Data era the world is experiencing, ML has garnered a lot of attention in this field. ML shows great promise in the field of antenna design and antenna behavior prediction, whereby the significant acceleration of this process can be achieved while maintaining high accuracy.

Known for their complex shapes, antennas typically do not have closed-form solutions. Computational Electromagnetics (CEM)[1-3] are applied to model the interaction of electromagnetic fields with antennas using Maxwell's equations. Approximate solutions are usually used to gain physical insight into the design of the antenna. With the advancements in numerical methods, integral equations were used to solve linear antennas. Later on, with the advancements in computers, it became possible to solve Maxwell's equations using integral and differential equation solvers. Method of moments (MoM)[4] was then introduced to also solve the integral equations. For a more complicated antenna structure, additional unknowns are added to the equations. Differential equation solvers were then developed with a simpler implementation even though they contain a larger number of unknowns. Memory and CPU usages are among the main drawbacks of the integral and differential equation solvers since they scale with the size of the antenna. Fast integral equation solvers were then developed, for which the integral equations are solved using iterative methods, with reduced memory requirements.

The most widely known CEM methods in antenna design can be classified into numerical methods and high frequency methods. Three numerical analysis methods that are commonly used in antenna simulations and testing are namely: finite difference time domain (FDTD),[5-7] finite element method (FEM),[8,9] and MoM.[10,11] Using physical optics approximation method, the radiation field of high frequency reflector antennas can be also obtained. Typically, most of the work involving antenna simulations require solving partial differential equations, with defined boundary conditions, using computers. High frequency methods include current based Physical optics (PO)[12] and field based Geometric optics (GO).[13] Other methods are also found, such as generalized multipole technique (GTM), multiple multipole program (MMP), conjugate gradient method (CGM), and transmission line matrix method (TLM).[14]

The most widely used commercial CEM software for antenna design and simulations are ADS, HFSS, CST, and IE3D. These software tools also lack several important features. For instance, 3D structures cannot be modeled using ADS, structures with finite details cannot be simulated using IE3D, and the execution time of HFSS and CST is high and increases as the size of the antenna structure is enlarged.

Due to their inherent nonlinearities, ML has been considered thoroughly as a complimentary method to CEM in designing and optimizing various types of antennas[15-18] for several advantages, as will be discussed further in this paper. ML is a large area within artificial intelligence (AI), as shown in Figure 1, that focuses on getting useful information out of data, thus explaining why ML has been frequently associated with statistics and data science. Indeed, the data-driven approach of ML has allowed us to design systems like never before, taking the world steps closer to building truly autonomous systems that can match, compete, and sometimes outperform human capabilities and intuition. However, the success of ML approaches relies heavily on the quality, quantity, and availability of data, which can be challenging to obtain in certain cases. From an antenna design perspective, this data need to be acquired, if not already available, since no standardized dataset for antennas, such as the ones available for computer vision, are yet available. This can be achieved by simulating the desired antenna on a wide range of values using CEM simulation software. Based on the obtained results, a dataset can be created and divided into a train, cross-validation, and test sets, for the purpose of training a ML model and validating whether this model succeeds in generalizing on new inputs. At this point, it is up to the designer's clairvoyance and expertise to know how to diagnose the model to improve performance. Some common steps to follow in this regard would be to plot the learning curves and to

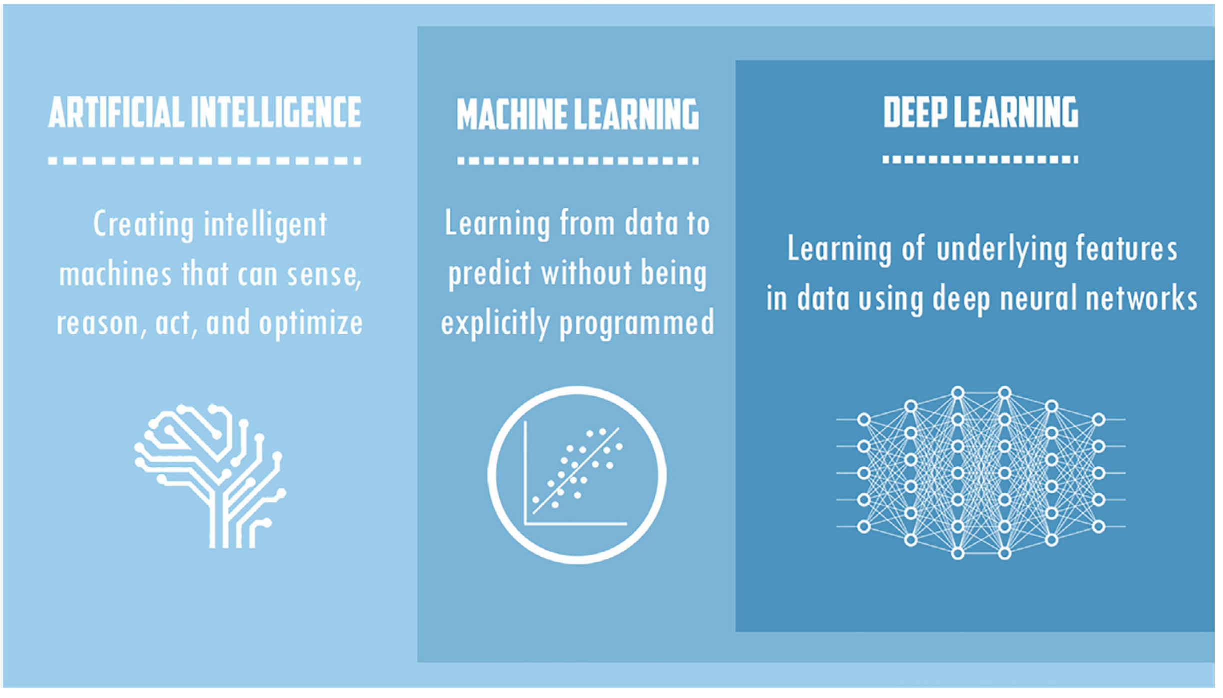

FIGURE 1 Relationship between artificial intelligence, machine learning, and deep learning

check the values for the bias and variance. Typically, a large part in optimizing a model's performance depends on the intuition of the designer, specifically when using neural networks, where the best possible architecture and hyper-parameters need to be found out for optimal performance.

This paper presents and investigates the use of ML in antenna design and optimization and provides a comprehensive survey of all the antennas designs found in the literature that have employed different ML techniques. It serves as a guide to researchers in the antenna community with minimal ML expertise seeking to employ this technology in their work. The different antenna design papers investigated are sorted according to the type and category of the antenna, which makes it simpler for readers interested in beginning research on antenna design and optimization using ML.

The rest of this paper is organized as illustrated in Figure 2 and as follows: a detailed overview of the CEM methods is presented in Section 2. Section 3 presents an overview on ML covering the different categories of learning, in addition to ML frameworks and applications. Section 4 investigates the regression models built with ML algorithm and used for antenna design. Section 5 presents the in-depth overview on the different works in the literature discussing the design and optimization of antenna parameters using ML. Section 6 presents another aspect of the literature, where ML was used to enhance different types of optimization algorithms in designing antennas. Concluding remarks, challenges, and future directions, follow in Section 7. A list of most of the acronyms used in paper is also presented in Table 1.

2 | CEM OVERVIEW

Using central-difference approximations, FDTD is based on discretizing the time-dependent Maxwell's equations to the space and time partial derivatives.[19,20] It basically contains a grid of points containing the computational domain with boundary conditions. Field equations are used to find physical quantities using post processing.[21] In FEM, linear equations are formed by meshing computational domain problems using weighted residual method.[22,23] As for MoM, the computational area is split into various segments. Each segment is then meshed and evaluated using basis functions.[24-26] The current of each segment and the strength of each moment are studied using Green's functions.

Nevertheless, all of these methods suffer from several drawbacks that affect their results. For FDTD, the accuracy of computation is affected by the reflection from the boundary. Truncation techniques can be used to reduce these reflections; however, the truncation also affects the

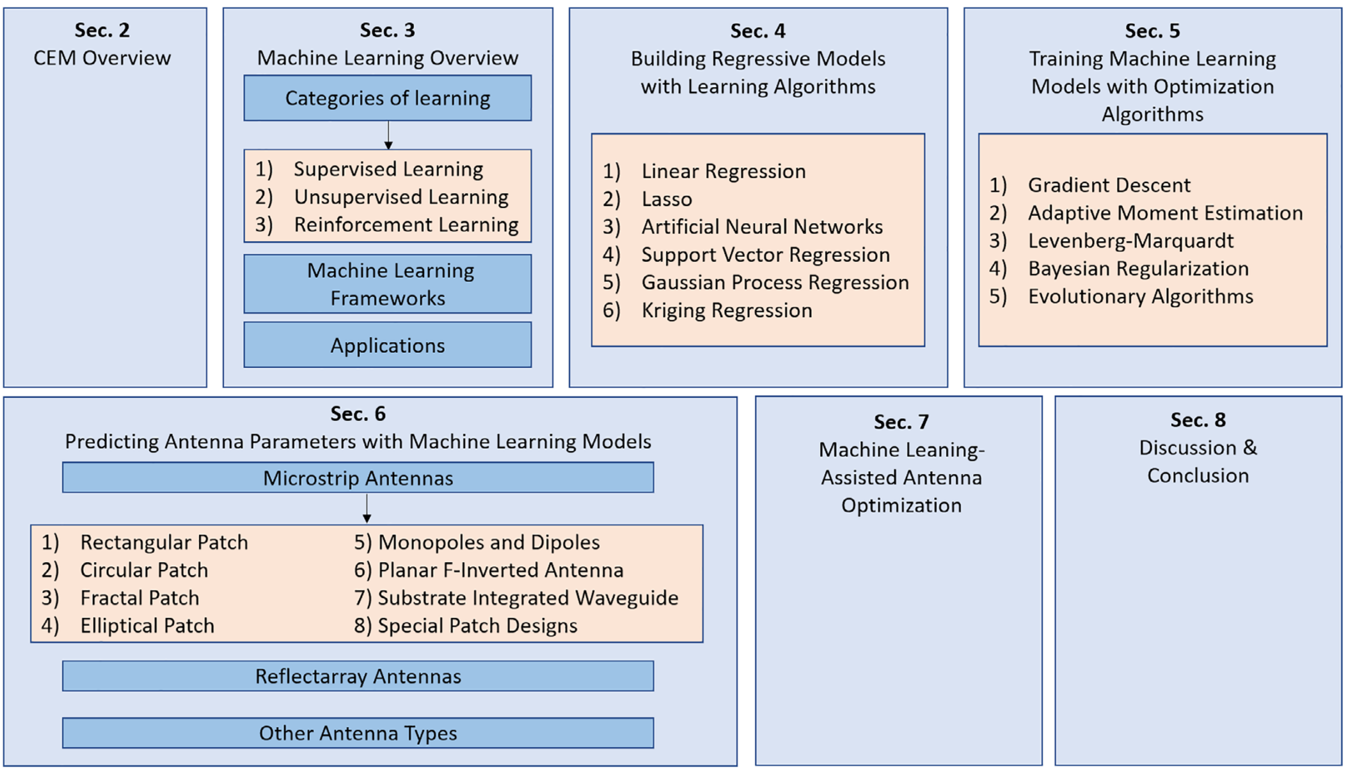

FIGURE 2 Diagrammatic view of the organization of this survey

FDTD

TABLE 1 List of acronyms

Acronym	Definition
ANN	Artificial neural network
BR	Bayesian regularization
BRANN	Bayesian regularized artificial neural network
CEM	Computational electromagnetics
DE	Differential evolution
FDTD	Finite difference time domain
FEM	Finite element method
FFBP	Feed forward backpropagation
GA	Genetic algorithm
GD	Gradient descent
GPR	Gaussian process regression
K-NN	K-Nearest neighbors
LASSO	Least absolute shrinkage and selection operator
LBE	Learning-by-example
LM	Levenberg–Marquardt
LR	Linear regression
ML	Machine learning
MLP	Multi-layer perceptron
MoM	Method of moments
MoM-LP	Method of moments based on local periodicity
MSE	Mean squared error
PIFA	Planar inverted-F antenna
PSO	Particle swarm optimization
RBF	Radial basis function
RPROP	Resilient backpropagation
SDG	Stochastic gradient descent
SIW	Substrate integrated waveguide
SOM	Self-organizing map
SVM	Support vector machines
SVR	Support vector regression
VSWR	Voltage standing wave ratio

accuracy of the computations.[27] For high order absorbing boundary conditions, the time and memory resources needed get higher as the computational domain is larger. Many methods were developed to remedy some of these drawbacks. For instance, perfectly matched layer (PML) can be used to decrease the reflections by absorption of EM. However, this comes at the cost of increasing the required CPU time and computational domain.[28] Staircase approximation was also proposed for discretization, but also increases the reflection and affects the accuracy of computation.[29] Another approach is to use 3D FDTD which employs two different time step increments; however, strong electric fields largely affects the stability of this method.[30] Other methods are also found, such as semi implicit schemes (SIS),[31] sub-cell algorithm,[32,33] FDTD-alternating direction implicit method,[34] one-dimensional-finite-difference time-domain method,[35] domain decomposition-Laguerre-FDTD Method,[36] and Runge-Kutta Higher Order FDTD.[37] Knowing that each one of these methods has its own advantages, such as the enhanced accuracy, and the reduced CPU time, most of these methods are still considered as time consuming. Also, they have difficulties in modeling thin wires, frequency dependent materials, and have dispersion errors.[38]

FEM

As for FEM, which is widely used in modeling waveguides, Yagi-Uda antennas, horn antennas, and vehicular antennas, it also suffers from certain drawbacks. For instance, as a result of its unstructured mesh, large radiation problems are difficult to be modeled using FEM, as they require excessive computation that could result in computational errors. Several methods have been proposed with FEM to remedy some of its drawbacks. For instance, Direct FE solver has been proposed for better accuracy with 3D structures, but also suffers from CPU time and memory storage requirements.[39] Dual prime,[40] which can be used with 3D structure problems, Vivaldi arrays, and other array problems, has a faster convergence time but also suffers from a trade-off between accuracy and computational cost.[41] Element Tearing and interconnecting full-dual-primal have been also proposed for the analysis of 3D large-scale problems, but also suffer from memory and CPU time requirements.[42] Despite its parallelization difficulty, finite element-boundary integral-multilevel fast multipole (FE-BI-MLFMA) algorithm method has been also proposed and used in biomedical and space applications, in addition to antenna arrays, as a result of its efficiency and accuracy.[43] Other methods are also found, such as non-conforming FETI, and domain decomposition based preconditioner (FE-BI-MLFMA) algorithm, but also suffer from memory requirements, and have difficulty working with lossless 3D objects with high permittivity and permeability.[44,45] Additionally, FEM has also difficulties modeling thin wires.

MoM

As for MoM, errors can occur as a result of the choice of the testing and basis functions.[46] Typically, many issues are associated with MoM, such as low-frequency breakdown and singularity.[47] In addition, MoM is not efficient to inhomogeneous and composite structures.[47] Although some solutions are found for several drawbacks of MoM, such as the use of pre-conditioners to solve the low-frequency breakdown,[47] recovery to solve the charge cancelation problem,[48] and the multi-resolution approach to improve the spectrum of MoM,[49] the computational cost, CPU memory and timing required can be further enhanced. MoM is also considered as computationally expensive since it requires dense systems of equations to solve the integral equations.

3 | MACHINE LEARNING OVERVIEW

In conjunction with the standard CEM methods, artificial neural networks (ANNs) can be used to minimize the energy function obtained by FEM.[50-52] Due to their stability, ANNs have also been used as a solution to MoM in Reference 53. Taking advantage of today's advances in distributed computing, ANNs can be used to efficiently solve large and complex EM problems, as well as integral equations, due to their parallel and distributed processing capabilities.[54] To speed up the solution of EM problems, ANNs were also used with FDTD and proved to increase the computational speed. For instance, they were used in Reference 55 to provide a global modeling approach for Microwave and Millimeter-Wave Circuits design, in a much faster approach than the traditional FDTD.

Generally speaking, ANN models possess advantageous characteristics that are beneficial in solving EM problems. They are characterized by their ability to approximate nonlinear input-output mappings which optimizes the relation between the input data and the required output, their adaptivity to changes in the environment, their uniformity of analysis and design, and neurobiological analogy.[56]

One of ML's major advantages in this field is the reduction of the large computational times found in the presented CEM techniques, especially when several parameters are to be optimized, or when a large structure is to be designed. The formulations of several antenna geometries, especially those with innovative structuring, complex geometries, or nonlinear loads, are still difficult to be treated analytically with known antenna theories, especially that some of them still suffer from low accuracy.[57] ML can be applied to model and predict scattering problems and analyze and optimize antennas in real-time.[58,59] ANNs can be easily realized using several available frameworks, implemented on high-performance computers, and can efficiently model electromagnetic structure in much less time with very low computational resources, and negligible degrees of errors.[60,61] In the antenna design sense, where closed-form solutions are hard to be found, ML can be the perfect solution to eliminate the time consumed in trial-and-error simulations when optimizing geometrical parameters to achieved some specific design requirements such as the desired radiation characteristics, especially if some of these characteristics are to be modified in real time.

Although the idea behind ML dates back to the 1950s,[62] recent times have witnessed an unanticipated surge of interest in ML algorithms. This interest has been stimulated by the large availability of data in the digital age the world has been witnessing, the access to high performance computing, and the better mathematical formulation and comprehension of learning techniques. Having revolutionized many aspects in research and industry, multiple breakthroughs in ML have occurred such as deep reinforcement learning[63]and generative adversarial networks (GANs).[64] Although some ML algorithms, specifically deep neural networks (DNNs), are perceived as "Black Box" tools, they work very well in practice and have outperformed some well-disciplined approaches.

3.1 | Categories of learning

ML can be generally divided to three key categories: supervised learning, unsupervised learning, and reinforcement learning, shown in Figure 3.

3.1.1 | Supervised learning

It is a learning task in which a model generalizes on a set of labeled input-output pairs to consequently make predictions on unseen input. There is a distinction between training and testing data in supervised learning, where training samples are associated with labels or targets which the test samples are missing. Supervised learning can be divided into parts:

- Regression: It is a supervised learning problem in which data are used to predict real-valued labels of unseen data. Regression algorithms include linear regression (LR),[65-67] kernel ridge regression,[68] support vector regression (SVR),[69-71] and least absolute shrinkage and selection operator (LASSO).[72]
- Classification: In classification, the goal is to label data from a finite set of classes. Binary classifications refer to classification based on a set of two classes, and multi-class classification refers to classification based on a set of three or more classes.

3.1.2 | Unsupervised learning

After receiving an unlabeled dataset, an unsupervised learning model then predicts certain labels for new data. Unlike the case in supervised learning, there is no

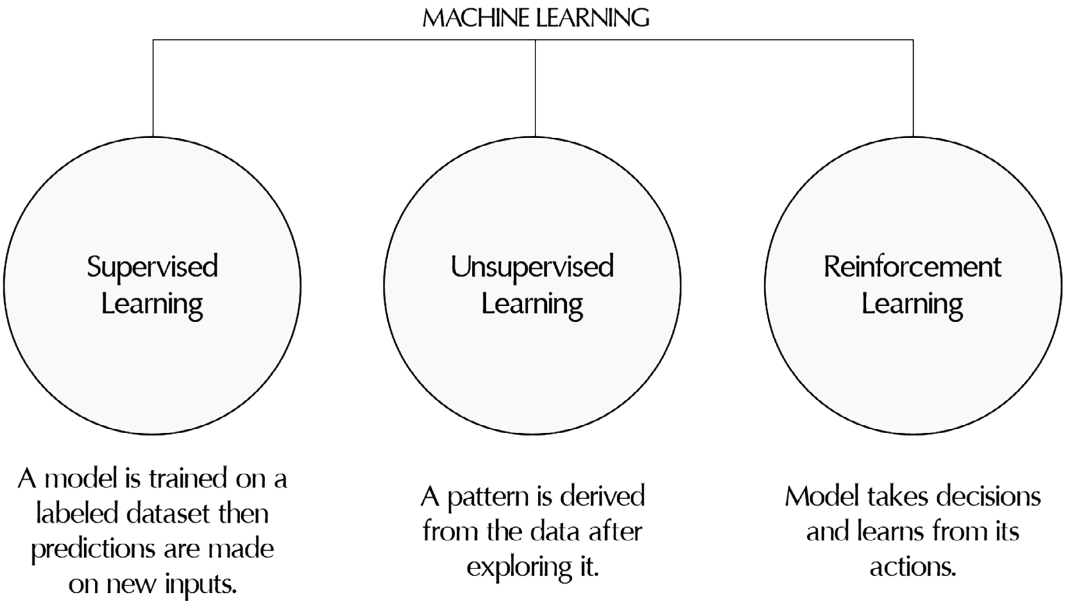

FIGURE 3 The three main categories of ML. ML, machine learning

distinction between train and test data in unsupervised learning.[73] Two learning problems are recognized in unsupervised learning:

- Clustering: Often used for large datasets, clustering is a learning problem that aims to identify regions or groups within these datasets.
- Dimensionality reduction: Also known as manifold learning, it is the process of reducing the dimensions in which data are represented while maintaining some principal features of the initial representation.

3.1.3 | Reinforcement learning

It is a learning paradigm in which the learner, also referred to as the agent, actively interacts with the learning environment to achieve a common goal. Used in control theory, optimization, and cognitive sciences, this paradigm depends on the notion of rewards given to the agent in amounts proportional to the achievements of the agent, which he aims to maximize. A model that is widely adopted in this field is Markov decision processes (MDPs) which represents the environment and the interactions with it. Since the transition and reward probabilities do not rely on the entire history of the model and only on its current state, the model is considered Markovian.[74]

3.2 | Machine learning frameworks

Numerous open-source frameworks are available to apply machine and deep learning concepts for solving real world problems. These platforms that are based on optimized codes written in Python, R, Java, or any other programming language, offer flexible and fast usage of several algorithms, thus making them essential and critical tools in research and development. These libraries include but are not limited to: Tensorflow,[75] Scikit-Learn,[76] ApacheSpark,[77] CAFFE,[78] Microsoft CNTK,[79] LIBSVM,[80] and many others. In addition to these, off-the-shelf tools such as the WEKA software[81] are available for people with domain-expertise but minimal ML experience where they would only have the task of acquiring data and tuning the hyperparameters.

3.3 | Applications

There is an abundance of areas where ML can have an impact ranging from molecular dynamics for predicting atomic behavior,[82] to serving as an analysis tool in bioinformatics,[83] or building reliable financial predictors.[84] In the realm of electrical and computer engineering, a plethora of works presented in the literature can be found where ML has contributed to the enhancement of previous systems, or in finding new approximate solutions to recurring problems. These techniques have also been widely employed in communication technology, whether it be at the physical layer or the upper layers, among which we mention: deep learning based detection and decoding,[85] antenna selection in MIMO,[86] wireless and cellular networks,[87] cognitive radios,[88] wireless sensor networks,[89] cybersecurity,[90] and others.[91]

4 | BUILDING REGRESSION MODELS WITH LEARNING ALGORITHMS

Regression algorithms are the essential tools needed when applying ML in the design of antennas. By using these algorithms and dataset of a considerable size, a model representing the mapping function of the non-linear relationship between the antenna's geometrical parameters and characteristics can be derived. The most widely used ML algorithms for antenna design are ANNs[92-95] and SVR.[96,97] Other regression methods that are less widely used are LR, LASSO, Gaussian process regression (GPR),[98,99] and Kriging Regression.[100,101] This section provides a mathematical briefing of these ML algorithms that are applied in antenna design.

4.1 | Linear regression

Considered to be one of the simplest regression algorithms, LR is a statistical tool used to trace a linear relationship between some variables and their respective numeric target values. For an unknown stochastic environment, we consider a set of labeled examples $\{(x_i, y_i)\}_{i=1}^{N}$ for the goal of building a model[102]:

$$f_{\{w,b\}}(x) = wx + b \quad (1)$$

where N is the size of the set, $x_i$is a D-dimensional vector of example $i = 1, ..., N$, $y_i \in R$ is the numeric target value, w_i is a D-dimensional vector of unknown, but fixed parameters, and b and D are real numbers.[102] For the model to produce the most accurate prediction of y, the optimal values of w and b need to be reached. To that end, we consider the following cost function to be minimized:

$$l(w,b) = \frac{1}{N}\sum_{i=1}^{N}\left(f_{w,b}(x_i) - y_i\right)^2 \quad (2)$$

This squared error loss function represents the average loss, or empirical risk, obtained after applying the model to the training data. It accounts for the average penalties for misclassification of examples $i = 1, ..., N$. Gradient descent optimization algorithm (GD),[103] is used to minimize the cost function. GD is used in LR to iteratively find the minimum of the function by gradually taking steps toward the negative of the gradient. The first step of GD is calculating the partial derivative of every parameter in the cost function as follows:

$$\frac{\partial l}{\partial w} = \frac{1}{N}\sum_{i=1}^{N} -2x_i(y_i - (wx_i + b)) \quad (3)$$

$$\frac{\partial l}{\partial b} = \frac{1}{N}\sum_{i=1}^{N} -2(y_i - (wx_i + b)) \quad (4)$$

where the partial derivatives were calculated using the chain rule. The parameters w_0 and b_0 are initialized by zero. It is worth noting that the correct initialization of parameters is integral in the success of the optimization algorithm. After initializing the parameters, training data (x_i, y_i) are iterated through, where in each iteration the parameters are updated as follows:

$$w_i = \alpha\frac{-2x_i(y_i - (w_{i-1}x_i + b_{i-1}))}{N} \quad (5)$$

$$b_i = \alpha\frac{-2(y_i - (w_{i-1}x_i + b_{i-1}))}{N} \quad (6)$$

where α denotes the learning rate, and w_i and b_i denote the respective values of w and b after using the training example (x_i, y_i). The algorithm stops iterating when the values of the parameters remain relatively constant upon the end of an epoch, where an epoch is a pass over all training examples.

4.2 | Least absolute shrinkage and selection operator

LASSO algorithm, also known as Sparse Linear Regression, integrates L1 regularization and mean-squared error with a linear model.[104] L1 regularization is known to result in a sparse solution, where sparsity refers to having parameters with an optimal value of zero. Thus, this algorithm can be used for feature selection. The LASSO estimate is defined by[105]:

$$\sum_{i=1}^{N}(wx_i + b - y_i)^2 + \lambda\|w\|_1 \quad (7)$$

where λ is the regularization parameter, and $\|w\|_1$ is the L1 norm obtained by $\sum_{i=1}^{d}|w_i|$.

4.3 | Artificial neural networks

A neurobiological analogy of the brain, an ANN is a ML technique that derives its computing power from the massive interconnections between its "neurons," which are the computing cells, and from its ability to generalize based on experiential knowledge. ANNs are known to be great function approximators[106] and are widely used for regression problems. In general, an ANN consists of an input layer of nodes that is not counted since no computations occur at this layer, an output layer of computation nodes, and zero or more hidden layers whose computation nodes are referred to as hidden nodes. An example is shown in Figure 4 where a deep neural network with two hidden layers is sketched. The architecture of the network when it comes to the number of layers and the number of nodes at each layer depends on the algorithm used in the learning process and the desired output of the network.[107]

Consider the following nested function that represents an ANN:

$$y = f_{\mathrm{NN}}(x) \tag{8}$$

The internal functions of layer indices l of the nested function have the following form:

$$f_l(z) = g_l(W_l z + b_l) \tag{9}$$

where g_l represents an activation function. Activation functions are fixed non-linear functions used as tools to compute the output of a computation node, which is then fed as input to the subsequent nodes.[108] We present three types of commonly used activation functions:

Logistic function: Also known as the sigmoid function, the logistic function is defined as follows[108]:

$$g(x) = \frac{1}{1+e^{-x}} \tag{10}$$

As shown in Figure 5 the logistic function saturates and becomes less sensitive to input at high or low values of x, while they exhibit sensitivity for values of x near zero.

Hyperbolic tangent function: Also known as tanh function, shown in Figure 6. It is characterized as[108]:

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} \tag{11}$$

ReLU Function: Typically used in all hidden layers, the ReLU function is a rectified linear unit function shown in Figure 7 and defined as follows[108]:

$$\mathrm{relu}(x) = \begin{cases} 0 \text{ if } x < 0 \\ x \text{ otherwise} \end{cases} \tag{12}$$

To non-linearly estimate the gradient of the cost function of an ANN, which is the cross-entropy loss, we consider a popular, widely used training algorithm called backpropagation.[109]

Based on GD, Backpropagation is a computational iterative procedure that aims to find a local minimum of the cost function. It consists of forward and backward passes. During the forward passes, the outputs of the activation functions are computed and stored to be used in the following pass.

During backward passes, partial derivatives of the cost function are calculated using the chain rule starting from the final layer to eventually update the parameters. The error is said to be "back propagated" from layer to layer. It is worth noting that the non-convex nature of the cost function in this case implies that a local rather than a global optimum is reached.[110] In Backpropagation, the change $\Delta w_{ji}(k)$ in the weight of a connection between two neurons i and j is given by the following:

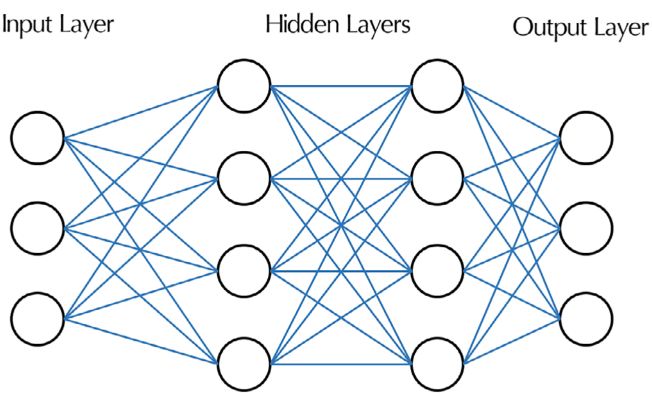

FIGURE 4 Schematic of a deep neural network with two hidden layers

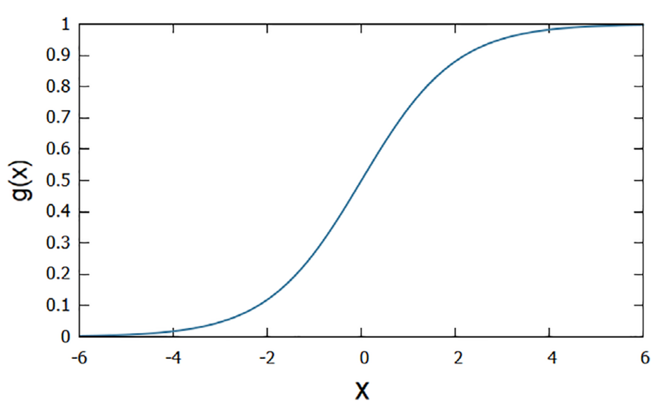

FIGURE 5 Sigmoid function

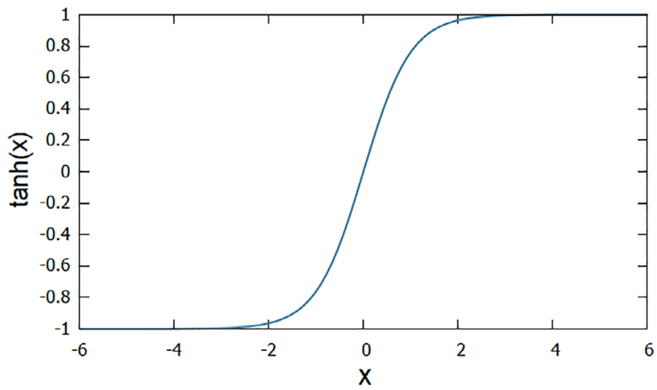

FIGURE 6 Tanh function

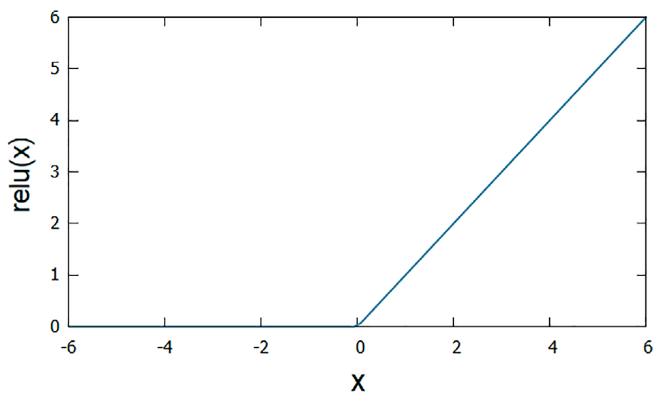

FIGURE 7 ReLU function

$$\Delta w_{ji}(k) = \alpha \delta_j x_i + \mu \Delta w_{ji}(k-1) \tag{13}$$

where the input is x_i, α is the learning rate, δ_j determines whether the neuron j is a hidden neuron or an output neuron, and μ is the momentum coefficient.

4.4 | Support vector regression

Support vector machines (SVM), a widely popular modern ML algorithm used for classification has inspired another algorithm used for regression: SVR.[111] Similar to its classification counterpart, the idea behind SVR is to separate the data points into two sets: points which fit within a predefined tube of width $\epsilon > 0$ and which are not penalized, and points which fall outside this boundary and are thus penalized as shown in Figure 8.

For a set of linear hypothesis functions[111]:

$$H = \{x \mapsto w.\Phi(x_i) + b : w \in R^N, b \in R\} \tag{14}$$

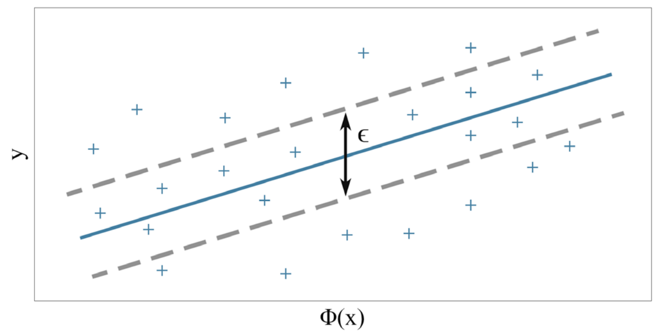

FIGURE 8 SVR epsilon-bounded data. SVR, support vector regression

where Φ is the feature mapping corresponding to a positive definite symmetric kernel function K, and $w.\ \Phi(x_i)$ is the dot product of the feature mapping $\Phi(x_i)$ and w. Training this model involves reaching optimal values of w and b by minimizing the corresponding cost function. The cost function to be minimized is as follows[111]:

$$\frac{1}{2}|w|^2 + C\sum_{i=1}^{m} |y_i - (w.\Phi(x_i) + b)|_\varepsilon \tag{15}$$

where $|.|_\epsilon$ denotes the ϵ-insensitive loss as shown in Figure 8.

It is worth noting that the choice of the parameter ε plays a role in determining the sparsity and accuracy of the model, where assigning large values to ε results in sparser solutions.[111]

Gaussian kernels, otherwise known as radial basis function (RBF), is a kernel K defined over R^N as:

$$\forall x, x' \in R^N, K(x, x') = \exp\left(-\frac{|x - x'|^2}{2\sigma^2}\right) \tag{16}$$

for any constant $\sigma > 0$. These kernels are the most commonly used kernels in this and other applications.[111]

4.5 | Gaussian process regression

In GPR, the objective function is considered as a sample of a Gaussian stochastic process. By using the available data samples, the distribution of the function value for new samples can be predicted. Considering a set of labeled examples $\{(x_i, y_i)\}$, new predictions at a certain input x^* can be obtained by the following[112]:

$$\hat{y}(x^*) = \mu + r^T R^{-1}(y - I\mu) \tag{17}$$

where I is a $n \times 1$ vector of ones, μ is the mean of the predictive distribution, $R_{i,\ j} = \mathrm{Corr}(x_i, x_j)$ is a correlation function with $i, j = 1, 2, ..., n$, and $r = [\mathrm{Corr}(x^*, x_1), \mathrm{Corr}(x^*, x_2), ..., \mathrm{Corr}(x^*, x_n)]$.

4.6 | Kriging regression

A less widely used regression method in the design of antennas is the Kriging Regression algorithm. In this method, the relationship between auxiliary variables and a target is modeled using known values of auxiliary variables. This algorithm can be defined as follows[113]:

$$\hat{z}(s_0) = \sum_{k=0}^{p} \hat{\beta_k} q_k(s_0) \tag{18}$$

where $\hat{\beta_k}$ are the regression coefficients, p is the number of auxiliary variables q, and $\hat{z}(s_0)$ is the predicted value of a target variable given an input s_0.

5 | TRAINING MACHINE LEARNING MODELS WITH OPTIMIZATION ALGORITHMS

Optimization algorithms are an important aspect in ML, since they allow to find the optimal weight and bias parameters of the ML model. Specifically, these algorithms are not ML algorithms but are used in the training process of a ML model to minimize the cost function and find the optimal values for the parameters. The optimization algorithm used has a direct impact on the performance of the ML model that results after training, and the choice of this optimizer is based purely on the type and amount of data available and on the designer's intuition. The most commonly used optimizers in antenna design can be listed as follows:

5.1 | Gradient descent

GD algorithm, also known as batch GD, is known to be slow since it updates the parameters once after calculating the gradient of the whole dataset. Another drawback of GD is its vulnerability to being stuck in local minima before converging to the global minimum in a non-convex surface. In the era of Deep Learning, where we may have millions of data samples, vanilla GD would not do. Hence, several optimization algorithms are available to use inside the architecture of a ML algorithm. Alternatives include Stochastic Gradient Descent (SGD),[114] where the parameters would be updated for each training example, and mini-batch GD,[115] where the gradient of a small amount of data samples are computed before performing updates.

5.2 | Adaptive moment estimation

A more recent, computationally efficient, and faster algorithm is the adaptive moment estimation (ADAM) algorithm, where the learning rates are computed for each parameter.[116] This algorithm is especially useful in the case of optimization problems with relatively huge amounts of data or with big numbers of parameters.

5.3 | Levenberg-Marquardt algorithm

Used for nonlinear least-squares estimation problems, the Levenberg-Marquardt (LM) algorithm is a batch-form trust region optimization algorithm that is widely used in a variety of disciplines to find the local minimum of a function.[117] The LM algorithm is a mix between Gauss-Newton iterations and GD, making it faster in convergence than vanilla GD. It is most efficient for usage in cases of small or medium sized patterns and offers a solution for nonlinear least squares minimization.[118]

5.4 | Bayesian regularization

Bayesian regularization (BR) is mostly used to train ANNs instead of error backpropagation, with the main advantage of bypassing the need for lengthy cross-validation.[119] Bayesian regularized artificial neural networks (BRANNs) are known to be difficult to over-train and over-fit making them an attractive choice for usage.

5.5 | Evolutionary algorithms

Evolutionary algorithms are a category of algorithms that are inspired by the biological behavior and evolutionary process of living creatures.[120] This class of algorithms, that contains genetic algorithms (GA), differential evolution (DE), particle swarm optimization (PSO), and others, is usually used in global optimization, and has been extensively used in electromagnetic optimization[121-123] and can also be used to train ML models in the case of antenna design.

6 | PREDICTING ANTENNA PARAMETERS WITH MACHINE LEARNING MODELS

A large body of literature exists where ML has been used to design and optimize antennas. Most of these works have employed the usage of ANNs to find direct relationships between different antenna parameters, such as between the geometrical properties of the antenna and the antenna characteristics. As the complexity of an antenna's structure increases, the number of geometrical parameters increase, and it becomes hard to derive relationships between these parameters and values for the resonant frequency and other radiation characteristics. The usual approach for optimizing a design is simulating the antenna to finally reach the desired values, a process described as computationally heavy and time demanding. Instead, ML can accelerate the design process by providing a mapping between whatever the desired inputs and outputs may be. In general, the following procedure can be adopted:

1. Numeric values corresponding to the desired inputs with their respective outputs are obtained by simulations and are stored in a database
2. Once this dataset is created, it is split into training, cross-validation, and test-sets, where the percentage of each depends on the amount of data samples
3. A ML algorithm is chosen to learn from this data. The choice of the algorithm relies on the complexity of the problem, the amount of data at hand, and the mathematical formulation of the algorithm
4. After training and testing the model, it can be used to predict output values for the desired inputs

Although this process demands going into simulations to create a dataset for training, once a model is obtained, predictions can be made for any desired inputs at very high speeds, and within very low error margins compared to simulated results. Several metrics have been in the literature to quantify this error, among which are:

The Output Error: obtained by calculating the difference between the output obtained by simulations and the output predicted by the ML model. The unit of this error depends the parameter being predicted and could be in dB, Hz, mm, or any other unit. It is expressed by:

$$e_i = y_\mathrm{d} - y_i \tag{19}$$

where e_i is the output error, y_d is the desired output, and y_i is the output predicted by the ML model.

The mean squared error (MSE) expressed by:

$$MSE = \frac{1}{N}\sum_{i=1}^{N}(e_i)^2 \tag{20}$$

where N is the size of the training samples.

The error percentage is obtained by the following:

$$\mathrm{Error}\% = \left|\frac{y_\mathrm{d} - y_i}{y_\mathrm{d}}\right| \times 100 \tag{21}$$

In this section, we investigate the different papers found in the literature on the design and optimization of antennas using ML procedures. These papers are sorted according to the type and configuration of antennas, starting with the typical rectangular and circular patch antennas, fractal shape antennas, elliptical shape antennas, monopole and dipole antennas, planar inverted-F antenna (PIFA), substrate integrated waveguide (SIW), special patch design, reflectarray antennas, in addition to some other types of antennas.

6.1 | Microstrip antennas

6.1.1 | Rectangular patch

The simplest form of antenna design using ML is the design of rectangular patch antennas. In References 124 and 125, multi-layer perceptron (MLP) neural networks have been used for the synthesis and analysis of rectangular microstrip antennas. During the synthesis phase, the height and permittivity of the substrate, denoted by H and ϵ_r in Figure 8, in addition to the resonance frequency of the antenna, are used to generate the length and width of the rectangular patch, denoted by L and W in Figure 9. During the analysis phase, the width and length, in addition to the height and effective permittivity of the substrate are used to generate the resonance frequency. The obtained neural network results were compared to those available in the literature where an MSE

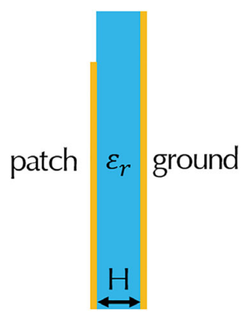

FIGURE 9 Substrate configuration

of 10^{-5} was obtained, showing good agreement. In Reference 125, RBF networks were used in the proposed approach, where results showed that the RBF network gave the best results with an error percentage of 0.91% compared with the MLP approach that reached 3.47%.

Other works have employed SVRs in the design of rectangular microstrip antennas. The optimization of the resonant frequency f_r, operation bandwidth (BW), and input impedance R_{in} of a rectangular microstrip patch antenna using SVR was presented in References 126 and 97. The results obtained from the proposed approach were compared to those obtained from an ANN-based approach. It was determined that SVR computed the above design parameters with higher accuracy than the ANN approach. SVR error percentages reached 1.21% for f_r, 2.15% for BW, and 0.2% for R_{in} while the ANN approach achieved accuracy percentages of 1.67% for f_r,1.19% for BW, and 1.13% for R_{in}.

Similarly, a rectangular patch antenna was designed using SVR with a Gaussian Kernel in Reference 127. The training and test sets were obtained by FDTD simulations where accurate values of the antenna's performance parameters such as the resonant frequency, gain, and voltage standing wave ratio (VSWR) were obtained with the corresponding values for the width and length of the rectangular patch. This data was then used to train the SVM, where the geometrical properties of the antenna are predicted based on desired values for the performance parameters that are given as the input.

In Reference 128, the resonance magnitude of a rectangular patch antenna with a two-section feed was predicted using SVR. The patch antenna, which has dimensions of 50.7×39.4 mm^2 and an operating frequency of 1.8 GHz, has two feeds of about 20 mm in length. A total number of 23 samples has been obtained through varying the widths of the feeds, out of which 21 were used for training and 2 for testing. During training, the two width values were taken as input parameters, and the resonance frequency as the output. Different kernel configurations including linear, polynomial of order 3, sigmoid and radial kernels were tested, and the radial kernel was used. It was shown that the average predicted error between the simulations results and the predicted value was around 3 dB on average.

In Reference 129, the slot-position and slot-size of rectangular microstrip antenna were predicted using a SVR model and ANN model. Two asymmetrical and two symmetrical slots were inserted on the radiating and grounding surface respectively after which the models were used to predict the slot-size and slot-position. Analytical results showed that SVR was more accurate and time efficient than the ANN, where the SVR was ~10% more accurate and had a speedup rate of 416 times.

Using ANN, the radiation characteristics of a slotted rectangular patch antenna, including its resonance frequency, gain, and directivity, have been used to generate the required slot-size and substrate air-gap dimensions in Reference 130. Multiple optimization algorithms have been tested to train the ANN with the LM algorithm proving to be the most efficient by providing the most accurate results in the shortest training time and least number of iterations. A prototype antenna was also fabricated to validate the accuracy of the obtained model, where the measured results showed great agreement with the simulated and predicted ones where a low percentage error of 0.208% was achieved.

More recently in Reference 131, rectangular patch antenna was designed using a new ANN architecture. ANNs, based on feed forward backpropagation (FFBP) algorithm, resilient backpropagation (RPROP) algorithm, LM algorithm, and RBF, were trained and tested using MATLAB. The input parameters of the models were the dielectric constant, width and length of the patch, and the substrate thickness, with the output being the resonance frequency of the antenna. After comparing the performance error of the four algorithms used, it was concluded that the RBF-based network produced the most accurate results with a value of 3.49886×10^{-14} for the error.

In Reference 132, PSO was used to train an ANN in the design of rectangular patch antennas. The ANN used the sigmoid function as an activation function, with input units representing the resonance frequency, the height, and permittivity of the substrate material, and output units representing the dimensions of the patch. It was shown that training required less than 5 minutes of computer work. In addition, an RBF ANN was also used to produce the value of inset feed distance d, shown in Figure 10, corresponding to the suitable normalized input resistance. The results produced by the proposed approach and those obtained from conventional simulations were determined to be in good agreement with an MSE value of 0.104.

In Reference 133, a GA was used to train an ANN instead of backpropagation for the purpose of optimizing a rectangular microstrip antenna. The ANN was able to predict the resonant frequency of the antenna, having the substrate dielectric constant ε_r, the width W and length L of the patch, and the shorting post position as inputs. Although the results obtained showed good.

agreement with the experimental ones with an average error of 0.013545 GHz for the resonant frequency, it was concluded that despite optimizing the parameters accurately, using GA to train the ANN was not very time efficient and could have been achieved in less time by employing backpropagation instead.

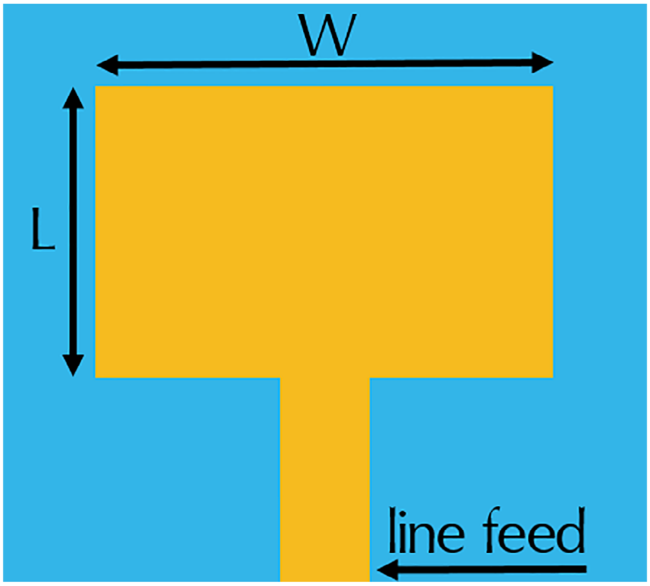

FIGURE 10 Rectangular patch antenna with line feed

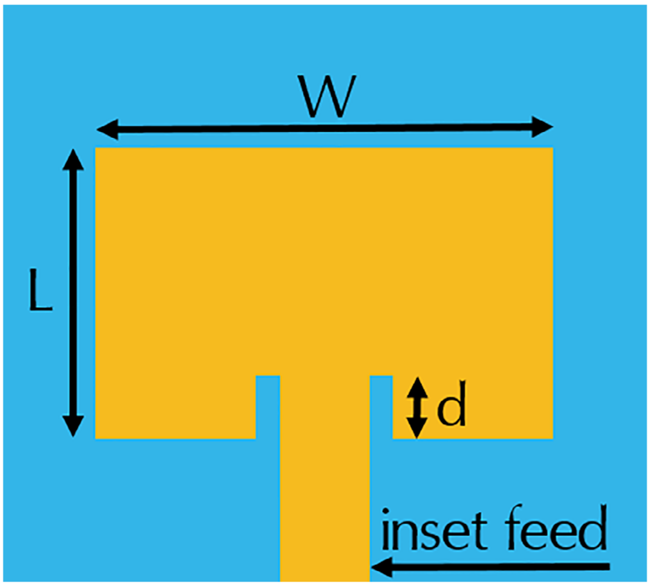

FIGURE 11 Rectangular patch antenna with insert feed

6.1.2 | Circular patch

Another well-known and simple type of microstrip antennas is the circular patch antenna, shown in Figure 11. The design of a circular patch antenna with thin and thick substrates, using ANN, was presented in Reference 134. The ANN took as input the radius of the patch, the height and permittivity of the substrate, to generate the resonance frequency using MLP and RBF networks. The effectiveness of five learning algorithms in the training of MLPs was investigated, the delta-bar-delta (DBD), the extended delta-bar-delta (EDBD), the quick-propagation (QP), the directed random search (DRS), and the GA. After comparing train, test, and total errors of the mentioned algorithms, it was deduced that EBDB attained the best results with a test error of 2 MHz compared with 13, 142 and 271 MHz in error for the DBD, DRS, and GA approaches respectively. As for the RBF-based network, its learning strategy was used for training. Additionally, a neural network trained by EDBD and backpropagation was used to compute the characteristic impedance and the effective permittivity of asymmetric coplanar waveguide (ACPW) backed with a conductor.

In Reference 135, ANNs were used in the design and determination of feed position for a circular microstrip antenna. The first network, an MLP neural model with two hidden layers was used to predict the radius a, effective radius, and directivity of the patch. The inputs were thickness of the substrate h, relative dielectric constant of the substrate, and resonant frequency, and the optimization algorithm used was LM algorithm. The trained network was tested on 45 various samples, and the reported MSE was 9.70×10^{-4}, 9.80×10^{-4}, and 7.76×10^{-4} for the respective inputs. The second network, an RBF neural model with one hidden layer, was used to predict the input impedance. The input was a representation of the various radial distances from the center of the patch. The network, which was trained with 200 input-output pairs, had an MSE of 2.69×10^{-4} upon testing.

An MLP ANN was used in Reference 136, to model, simulate, and optimize multilayer circular microstrip antennas. Chosen from 11 tested learning algorithms, LM algorithm was used to train the network. The resonance frequency was calculated for any arbitrary values of the patch radius, dielectric constant of different layers and their thickness. The results showed good agreement with reference results, where the average error percentage of the resonant frequency was 0.35%, 0.065%, 0.43%, and 0.066% for circular microstrip antenna with and without cover, spaced dielectric antenna, and microstrip antenna with two superstrates respectively.

In Reference 137, the resonance frequency of a circular patch antenna was also modeled using a conjugate gradient model of an ANN. A closed form expression of the resonance frequency of the antenna, based on the circular patch radius, height, and permittivity of the substrate, was used to generate data for ANN modeling and testing. Forward modeling and reverse modeling were used to either predict the resonance frequency of the antenna or the circular patch radius. A comparison between the simulated results and the results predicted by the ANN showed 0.10721% error for the resonant frequency and 0.1956% error for the patch radius.

The optimization of various design parameters of a circular microstrip patch antenna using ANN trained with LM algorithm was presented in Reference 138. A FFBP neural network was used to estimate the following seven parameters: return loss (RL), VSWR, resonance frequency, BW, gain, directivity, and antenna efficiency. The input parameters were the patch radius, in addition to the height and permittivity of the substrate. Results from testing the model were in good agreement with simulated results and achieved an MSE of 9.96×10^{-7}.

6.1.3 | Fractal patch

Fractal patch antennas are another type of microstrip antennas that have their design procedure dominated by ANNs. In Reference 139, the resonance frequency, RL, and the gain of a coaxial fed elliptical fractal patch antenna were calculated using an ANN with backpropagation algorithm. IE3D software was used to generate the dataset for different values of feed positions and for different iterations of the antenna fractal shape. These values were used for training a model to find the position of the feed point of the coaxial feed for optimized impedance matching of the antenna.

In another work, the optimization of a square fractal antenna using ANN was presented in Reference 140. With the aim to reduce the size of the broadband resonance antenna, selected iterated structures resulting from the ANN were simulated in HFSS to obtain optimal resonance characteristics. Also, the design of quasi-fractal patch antennas using ANNs has been presented in Reference 141. Several values of the antenna parameters that allow operation at a specific resonance frequency have been obtained by simulations. The dataset was then used to train the network, which resulted in a prediction model that provides a mapping between parameters and frequency of operation.

6.1.4 | Elliptical patch

ANNs were used in two works for the design of elliptical patch antennas. In Reference 142, ANNs using RBF were utilized in the design of elliptical microstrip patch antenna. The resonance frequency for even mode, substrate height and permittivity, and the eccentricity of elliptical patch were used as input parameters to compute the resonance frequency for odd mode and the semi-major axis. When comparing the obtained results with the results of conventional simulations, the error percentage reached as low as 0.006% and 0.043%.

In a different approach, the design and modeling of an elliptical microstrip patch antenna using ANN was presented in.[143] For the purpose of computing the RL and the gain of the antenna, a FFBP neural network was trained in MATLAB using the three major axes of the connected ellipses as input parameters. A dataset was obtained from CST simulations. The obtained results were compared to those of the simulated and measured results of a fabricated antenna, and a good agreement was revealed with error values as low as 0.0202 dB for the Gain and 0.2014 dB for the RL.

6.1.5 | Monopole and dipole antennas

The design of a circular monopole antenna was facilitated using an ANN in Reference 144. The feed-gap, a design parameter required for the antenna to operate within a specific frequency band, was calculated using an ANN trained with dataset obtained from IE3D simulations. The model was later tested with five input-output pairs, and the resultant error percentage was determined to be within 0.4 and 4.6%.

The LASSO technique, a sparse LR method, was used in Reference 145, to design a reference dual band double T-shaped monopole antenna. Five design parameters that cooperatively represent the shape and structural geometry of the antenna were repeatedly obtained from HFSS simulations. After the model was fitted using the LASSO method, which is characterized by variable selection and regularization, optimum predicted design parameters were reached. The resulting model was able to predict values 495 616 design points after being trained on a dataset that consists of only 450 training samples. Even given the relatively small size of the dataset, the model was able to analyze an exponentially higher number of data points in a very brief amount of time without the need to perform further electromagnetic simulations. This work was later developed in Reference 146, where two more ML techniques, namely ANNs and the k-nearest neighbor (k-NN) algorithm, were used to optimize the same antenna. It was shown that by using ANNs or LASSO, better predictions can be achieved than the k-NN approach that had an error percentage of 2.90%.

The first took the microstrip line impedance Z as input, and the line width as output. The second took Z and the substrate dielectric constant as input, and the width and height of the substrate as output. The third took Z as input, the substrate dielectric constant, width, and height, as output. The synthesis ANN was further tested on a printed dipole antenna with integrated balun, where the results of the proposed neuro-computational model were compared to those of a developed FDTD analysis tool. The input parameters to the model were three voltage standing-wave ratio numbers and two frequencies, and the output was two geometric parameters.

6.1.6 | Planar inverted-F antenna

The optimization of design parameters of a PIFA with magneto dielectric nano-composite substrate using ANNs trained with the BR algorithm was presented in Reference 147. The model was trained on two databases obtained from CST simulations. Taking as inputs the particle radius and volume fraction of the nano-magnetic material, different antennas parameters, such as gain, BW, radiation efficiency, and resonance frequency can be generated using neural networks with error percentages close to zero.

Working with the same antenna, the algorithm used in Reference 147 has been further optimized in[148] for the same input and output parameters. In addition, a reverse technique has been also addressed using ML, for which the corresponding design space of possible material parameters can be generated based on given antenna parameters.

6.1.7 | Substrate integrated waveguide

ANNs were used to predict the geometrical parameters of a SIW patch antenna in Reference 149, taking as inputs the desired resonance frequency and the RL. Feed-forward MLP and backpropagation were used for training the ANN in MATLAB, using dataset obtained from HFSS simulations.

In Reference 150, the design of a broadband millimeter-wave SIW cavity-backed slot (CBS) antenna using ML was presented. A ML assisted optimization method (MLOM) was used as the reference method compared to a proposed ML assisted method with additional feature (MLOMAF), which utilizes the population-based metaheuristic optimization method. HFSS was used in the design and analysis process of the antenna structures. Following database initialization, the initial training set was sampled using the Latin hypercube sampling (LHS) and the resultant data was exploited in building a Gaussian process (GP) surrogate model. It was shown that this algorithm was able to reach the stopping criterion 12 iterations before the MLOM did, and that the proposed antenna exhibited notable features related to BW and ease of fabrication.

6.1.8 | Special patch designs

Other types of special patch designs have been also designed in the literature using different ML techniques. The analysis and design of a frequency re-configurable planar antenna using a MLP ANN and a self-organizing map (SOM) neural network respectively was presented in Reference 151. In the analysis phase, the operational frequency bands of the antenna at different reconfigured conditions were located by the trained MLP network. In the design phase, switches to be turned on for a specific desired frequency response were identified using a SOM neural network, trained using Kohonen learning algorithm. Frequency responses from the output of the MLP network were used to feed the input layer, whereas the output of the networks was four clusters of frequency responses, used to approximate the position of the switches to be turned on.

In Reference 152, the design of a three-layer annular microstrip ring antenna with pre-specified operational features was facilitated using ANNs, where structural design parameters were computed. A dataset of reflection coefficient vs frequency was used to train an MLP ANN to generate the geometrical properties of the patch as well as the physical properties of the substrate. Upon testing, the root MSE of the model was determined to be 1%.

In Reference 153, the design of a two-slot rectangular microstrip patch antenna was facilitated using MLP and RBF, based on an ANN trained with different learning algorithms. The MLP-based networks were trained using five learning algorithms: LM algorithm, scale conjugate gradient backpropagation, Fletcher Powell CG backpropagation, gradient decent with momentum, and adaptive gradient decent. It was determined that the LM algorithm resulted in the least MSE compared to the other MLP-based algorithms, but the RBF-based network resulted in a lower error percentage of 0.09%.

A spiral microstrip antenna has been designed in Reference 154 using ANN. Antenna parameters were mapped to characteristics such as the resonance frequency, RL, and VSWR. After training the network on data samples obtained by the simulator, it was shown that accurate prediction results may be obtained which allows bypassing the computational burdens of conventional simulation methods.

In Reference 155, the design of an aperture-coupled microstrip antenna using an ANN with a hybrid network architecture was presented. The RBF and the backpropagation algorithm were combined to develop the hybrid network. Using the hybrid ANN, the antenna parameters including the dimensions of the ground plane, the aperture, and the radiating element, in addition to the dimensions of the feed and its position, were determined based on different resonance frequencies. The obtained ANN results were compared with those using backpropagation and RBF models, which showed performance superiority after showing an error percentage of 0.27%.

The design of single-feed circularly polarized square microstrip antenna (CPSMA) with truncated corners, has been facilitated in Reference 156 using ANN synthesis model. A total of 5000 data samples were generated by calculating the resonance frequency, in addition to the Q-factor of the antenna by analytical formulations, out of which 3500 were used for training the model. The LM algorithm was used for training, which resulted in a faster and simpler structure. During the synthesis process, for a desired operating frequency and a given substrate, the size of the truncated corners can be obtained for CP operation. An ANN with three hidden layers was found to give the highest accuracy. The average relative error and the maximal relative error have been calculated to test the accuracy of the model. The proposed model was compared with simulation results. It was shown that the antenna with the calculated parameters achieved a circular polarization with less than 2 dB of axial ratio, with some discrepancy in the frequency of operation of less than 5%. Eight different CPMAs were also fabricated and tested, for which the measured results showed an axial ratio of less than 1 dB, with a discrepancy between the measured and synthesis values of 3.6% for the physical dimensions, and 2.3% for the frequency of operation, with an average relative error of less than 1%.

In Reference 157, the optimization of design parameters of a tulip-shaped microstrip patch antenna using ANN was presented. Taking as input the resonance frequency band and the RL of the lower and higher resonance frequencies, the ANN was used to generate the patch dimensions. Backpropagation was used to train the ANN in MATLAB using a dataset obtained from HFSS simulations.

The design of a pentagonal-shaped flexible antenna, shown in Figure 12, for ultra-wide band (UWB) wireless applications such as WLAN, 5G, and WiMAX applications using ANN was presented in Reference 158. The aim was to determine the two frequencies representing the structure BW from the radius.

of the pentagonal shape. This approach was used due to the complexity behind finding the non-linear relationship between these parameters and representing it in an equation, and due to some time and cost concerns. For this, an ANN based on LM algorithm was trained and tested using a dataset obtained from Ansoft simulator. The error resulting from learning, validation, and testing was 6%.

In Reference 159, the resonance frequency of rectangular patch antennas printed on isotropic or uniaxially anisotropic substrate, with or without air gap, was modeled using ANN. Spectral dyadic Green's function was used in conjunction with a developed single neural network. To reduce the computational complexities, required time, and amount of data needed to maintain the accuracy of the ANN model, a single matrix was used to present the effective parameters. Two types of antennas were tested with two ANN models: a circular patch antenna where the radius of the patch has been determined based on the resonance frequency, substrate thickness, and permittivity, in addition to a modified PIFA antenna with a chip resistance where the feed position was determined based on the input impedance.

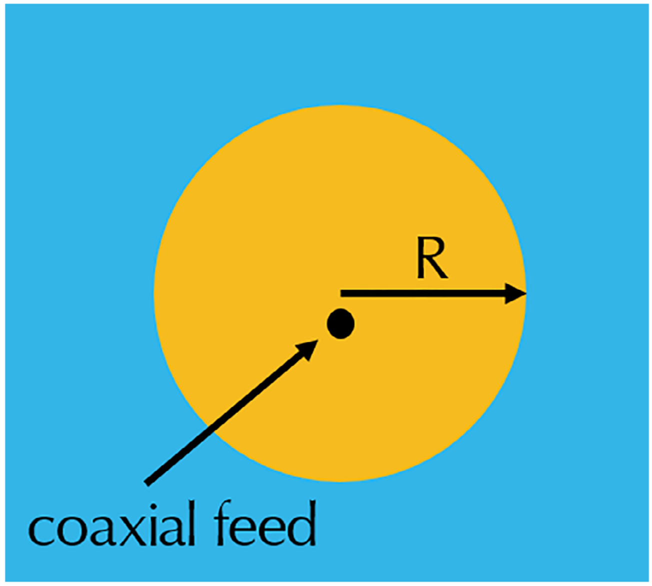

FIGURE 12 Circular patch antenna

In Reference 160, an ANN has been used in the analysis and synthesis of Short-Circuited Ring-Patch Antennas (SCRP). The importance of the ANN in this work was to solve the drawbacks of the analytical calculations that do not accurately model the effect of the thickness and permittivity of the substrate on the resonance frequency of the antenna in TM_{10} mode. For the training process, the internal and external radius, in addition to the substrate permittivity and thickness, were varied to obtain 275 training samples from simulations. During the training phase that required less than 1-minute, least squares cost function with a backpropagation method have been used. In the analysis case, the resonance frequency was obtained from the ANN given the other patch dimensions for 30 test cases. The comparison between the frequency obtained by the trained ANN and that obtained by simulations showed a percentage error of less than 0.3%. In the synthesis case, the importance of using ANN for this type of antennas was seen in estimating the value needed for the external radius from other parameters and for a desired resonance frequency. Usually, the external radius is estimated via analytical formulations and adjusted by trial and error using the simulation software. In this case, the comparison between the external radius obtained by the trained ANN and the simulations achieved an error of less than 3.2%. A SCRP has been also fabricated based on the trained ANN, for which the measured results showed closer results to those estimated by the ANN than the simulated ones.

Some works have explored the design of multiple patch types at the same time. In Reference 161, a combination of ANN and adaptive-network-based fuzzy inference system (ANFIS) was used to calculate the resonance frequencies of rectangular, circular, and triangular microstrip antennas. The MLP ANN, trained with the BR algorithm, was utilized to compute the resonance frequencies of the antennas. As for the ANFIS, it was trained using the hybrid learning algorithm, which is a combination of least square method and backpropagation. The inputs to this hybrid model were the geometrical parameters of the patch and the dielectric constant of the substrate, whereas the calculated output was the resonant frequencies. The MLP ANN was used in computing the resonant frequencies, and the ANFIS was used in compensating for the inaccuracies in the ANN results. Finally, the results were compared to those of the single neural models, conventional methods, and approaches based on GA and Tabu search algorithm (TSA). It was determined that the proposed hybrid method provided results of higher accuracy.

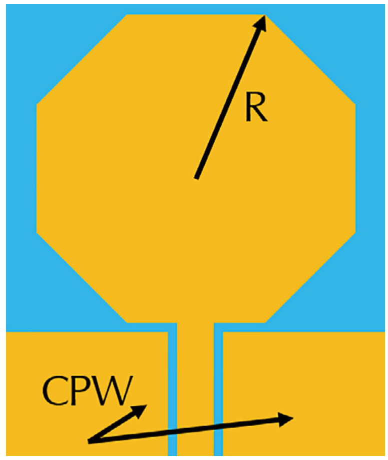

FIGURE 13 Pentagonal-shaped CPW antenna. CPW, coplanar waveguide

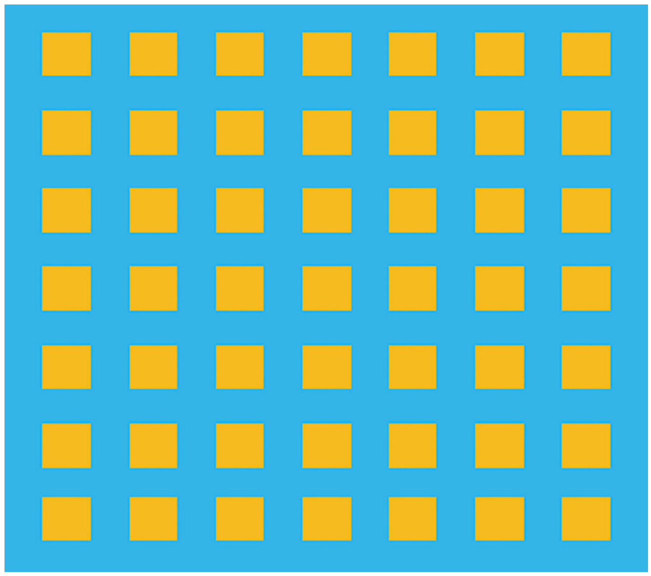

FIGURE 14 Reflectarray with unit cells

6.2 | Reflectarray antennas

Several works focusing on the accelerated design and analysis of very large reflectarrays, as the one shown in Figure 13, using ML, have been presented in the literature. Among these, many have employed ANNs as the main design and analysis tool. In Reference 162, an ANN was utilized in the optimization of microstrip patches unit cell parameters of broadband reflectarray antennas with Malta cross unit cell configuration, as shown in Figure 14. To this end, the ANN was used to accurately characterize the non-linear relationship between the phase behavior of patch radiator and its geometric parameters. The proposed network used MLP. The hyperbolic tangent was chosen as the activation function for the two-layered network. The model was trained using the error backpropagation algorithm. It was shown that, when compared to direct evaluation, the ANN approach had results of similar accuracy that were attained with an enhanced speed.

This work was later expanded in Reference 163 where the results of a modified ANN were compared with those of a full-wave method of moments based on local periodicity (MoM-LP) approach. The sigmoid function was used here as the activation function. The reflection coefficient corresponding to any re-radiating element of a reflectarray approximated by the two approaches were in agreement, further showing that the ANN method can maintain the desired level of accuracy while significantly reducing the computational cost.

In Reference 164, an ANN was used to characterize the elements of a rectangular planar surface reflectarray composed of 70×74 elements, for satellite applications covering the Eutelsat footprint. Each ANN took as input the angle of incidence and patch dimensions, in addition to the resonance frequency as a constant parameter. Feed-forward MLP topology was used, along with backpropagation training algorithm, to optimize the reflectarray patch dimensions by training the model on a dataset obtained from MoM-LP-based computations. The obtained ANN results of the gain pattern and phase distribution of the reflectarray were compared with those obtained from MoM-LP computations and showed good agreement while having a speed up factor of 2×10^2.

An MLP-ANN has been used in Reference 165 to optimize and speed up the design and analysis of reflectarrays. The reflection phase characteristics of the unit cell element was first trained and tested using CST. A dataset of 990 samples was used for training, while 660 data samples were used for testing. In the analysis model, the edge length of the patch, the ratio of the cavity to the edge length, the substrate thickness and resonance frequency, were taken as input in the analysis model. The LM algorithm was used for training. The MSE was calculated to be 3.5992×10^{-4} for training and 4.0192×10^{-4} for testing. The reconstructed phase variations and the target ones were of high similarity, validating the efficiency of the proposed model. A reflectarray with the optimized Minkowski elements was then tested to validate the overall optimized performance of the antenna (Figure 15).

The design of reflectarrays composed of second-order phoenix cells was presented in Reference 166. Fast characterization of these cells was made possible by using ANNs, allowing to obtain a spherical mapping that complies with the results obtained by full-wave simulations with the local periodicity assumption.

A reflectarray antenna using modified Malta-Cross cells was designed using an ANN in Reference 167. Starting by a dataset obtained through full-wave simulations, the ANN was trained using error backpropagation, resulting in a model that allows the computation of reflection coefficients from any input value for the geometrical and re-radiating field parameters of the reflectarray in both cases of horizontal and vertical polarizations. The obtained ANN model allows high accuracy predictions for lower memory usage with less computation time and load.

A contour-shaped reflectarray antenna was analyzed in Reference 168. A trained ANN is used to predict the complex reflection coefficient's amplitude and phase by taking six geometrical parameters, the incident angle in terms of azimuth and elevation, and the frequency as inputs. The results were compared to full-wave electromagnetic computations and showed great agreement while having a speed up factor of 700.

Other techniques have been also employed in the design of reflectarray antennas. Using an advanced learning-by-example (LBE) method, namely the Kriging method, the design of high-performance reflectarrays

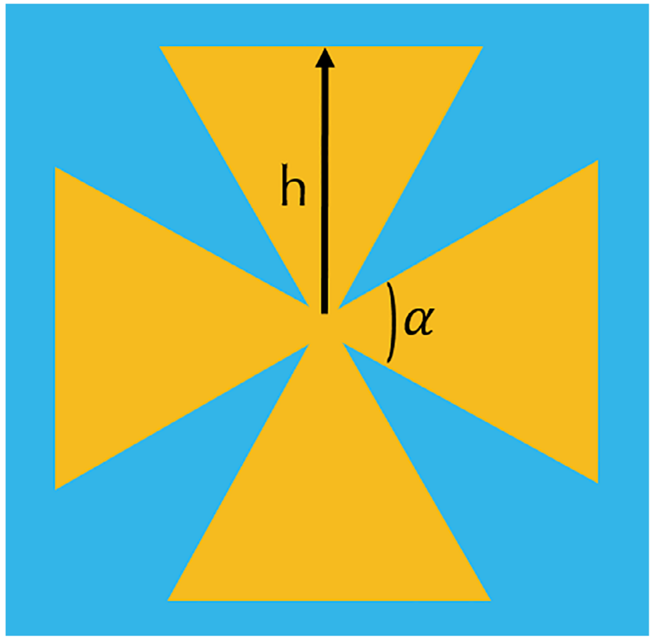

FIGURE 15 Malta cross patch

was presented in Reference 169. The problem of predicting the scattering matrix of complex reflectarray elements was addressed using this LBE algorithm that, if trained on a set of known input-output relationships, can accurately predict the output of new input-output pairs. The Kriging method is not only proficient in dealing with deterministic noiseless processes but can also facilitate vectorized outputs. A set of preliminary numerical results were used to validate the accuracy and time-efficiency of the proposed model. It was confirmed that this method, while maintaining a prediction error below 5%, allowed for a 99.9% time saving percentage when compared to standard full-wave approaches.

The prediction of the electromagnetic response of complex-shaped reflectarray elements was presented in Reference 170, where the authors presented an innovative LBE method based on Ordinary Kriging to obtain reliable predictions. Full-Wave simulations were used in order to generate a training set composed of the elevation angle, azimuth, operating frequency, and the degrees-of-freedom (DoF) for each array element, along with the corresponding field distribution. The relationship between such parameters is highly non-linear which encourages the usage of ML techniques to find an accurate input/output mapping of these parameters without having to go through simulations. The authors compared the performance of their approach to the performance of SVR and Augmented RBF neural networks used for the same problem. In addition, several unit-cell shapes have been considered such as the several cross-slot, ring-slot, and square/rectangular Phoenix shapes. Results showed that the customized LBE approach achieved lower error rates for the same number of training examples compared with SVR and Augmented RBF neural networks.

A framework that employs a ML technique in its architecture was proposed in Reference 171, where the algorithm focuses mainly on improving the antenna performance. A surrogate model was obtained using the SVMs. SVM was used in References 172-174 to design shaped-beam reflectarrays and for modeling dual-polarized reflectarray unit cells in Reference 175. Using SVM, the computational burden resulting from the use of Full-Wave Local-Periodicity for the design and analysis was reduced. This has been tested and proven where an acceleration factor of 880 was achieved compared to simulations based on the MoM-LP, with error percentages as low as 0.43%.

6.3 | Other antenna types

The indirect use of an ANN for predicting the input impedance of broadband antennas through a parametric frequency model was proposed in Reference 176. While the antenna geometry parameters and frequency are routinely used as inputs of the ANN, the resistance was initially parametrized by a Gaussian model and the ANN was later used to reach an approximate non-linear relationship between the antenna geometry and the model parameters. This novel method was used to obtain a smaller network size with a smaller number of hidden units for an ultimately faster training time. For testing, a loop-based broadband antenna with three tuning arms was used, where the results were compared to those of a direct approach. It was found that the proposed model was considerably more time efficient as it required 10 times less the amount of electromagnetic computations when training the ANN.

The design of a loop antenna was facilitated using competitive learning ANN in Reference 177. The aim was to determine the physical dimensions for frequencies in the range of 200 to 300 MHz by calculating the best combination of conductor thickness and loop radius using a SOM. 11 sets of efficiency values corresponding to frequencies related to frequencies for 11 pairs of loop radius a and wire radius b were used to train the SOM. The SOM was later used to produce the desired set of (a, b) that has the required radiation efficiency, which was verified by comparison to theoretical results. The design was shown to respond well to input parameter changes of 50%.

An MLP-ANN was used to model and predict the radar cross section of a non-linearly loaded antenna in Reference 57. After training the MLP-ANN with backpropagation, the slope information of the resulting model was used to optimize the antenna. Theoretical formulations have been proposed for this aim, verified by numerical simulation results. A nonlinear loaded dipole antenna was used for simplicity as an example. The harmonic balance technique[178] was used to calculate 101 data samples for training, and 100 data samples for testing. Comparing the predicted values by extension of MLP-ANN with those calculated from the harmonic balance technique, it was concluded that the proposed method is accurate and can obtain the required results in less time.

A multi-grade ANN model was proposed in Reference 179 for the design of finite periodic arrays. To take into consideration the mutual coupling and the array environment, this approach introduced an innovative approach where two sub ANNs were used. The first-grade ANN is called the element-ANN that can provide the non-linear relationship between the geometrical parameters and electromagnetic behavior, represented by a certain transfer function (TF) coefficients, of the array element without considering mutual coupling. The output of this element-ANN is then fed as the input to the second-grade ANN called the array-ANN. The array-ANN is then capable of producing outputs of the electromagnetic behavior of the whole array, with mutual coupling considered. This approach allows to obtain the mapping between the geometrical parameters of the element and the electromagnetic response of the whole array, while separating array and element information. Several arrays types were used to verify the effectiveness of the proposed approach including a linear phased array, a six-element printed dipole array, and a U-slot microstrip array. Results showed training and testing errors smaller than previous approaches that do not use the multi-grade ANN approach.

In Reference 180, ANNs were used to optimize the parameters of a pyramidal horn antenna. The ANN used RBF as the activation function in its layers and was trained on data obtained by a full wave simulator. Taking as inputs the desired frequency of operation and gain, the ANN generated the required antenna dimensions such as the height and width of the flared end, the height and width of the waveguide, and the length of the horn antenna. Results showed that the trained model can give very accurate results compared to those obtained by a simulator with an error percentage as low as 1.3%.

In Reference 181, an ANN was used for analysis and synthesis simulations of profiled corrugated circular horns, and then compared with the conventional mode matching- combined field integral equation (CFIE) technique. During analysis, the ANN takes as inputs the aperture radius, the horn length, the corrugation height, the metal-void ratio, in addition to the number of corrugations per wavelength. The output of the ANN was the RL, in addition to the co- and cross-polar patterns limited to 0° to 40°range, with 2° step. To accelerate the analysis, several ANNs are used. The input space is formed of 10 hypercubes, with each single hypercube mapped to a subspace of the output space. This approach has been also presented in Reference 182. The ANN was then trained in the synthesis procedure to approximate the function that can relate the main beam width and the maximum level of the cross-polar level, to the corrugated horn geometrical parameters. The RL was not taken into consideration as an input to the ANN during analysis, since with this type of horn, low levels of RL can be easily obtained. In addition, some of the geometrical parameters were assumed to be constant, and not varied during the synthesis process. An example of an optimized profiled corrugated horn using the proposed ANN has been also fabricated and measured. The results have been compared with the traditional electromagnetic analysis. The results showed less accuracy when compared with those obtained from a careful optimization process. Nevertheless, the cost and the time needed to design the antenna as per the required parameters were highly reduced.

ANNs were also used in the design of a W-Band slotted waveguide array antenna in Reference 183. The model was trained, cross-validated, and tested on dataset obtained by HFSS simulations. The seven design parameters that were used as input were the lengths and orientation angles of the coupling slots, in addition to the length of the radiating slots. The antenna was later fabricated using Stereolithography 3D printing techniques, and the measured and simulated results were compared, which were in good agreement with slight errors.

In Reference 184 a novel multibranch ANN modeling technique was proposed as a solution to the non-uniqueness problem in the design of antenna arrays. The nonuniqueness issue can be defined as the case where the desired output can be mapped to several inputs, resulting in conflicting output values for similar inputs in a dataset, and leading to a large training error and poor ML model accuracy. This work presented a novel technique based on calculus to provide a solution for the non-uniqueness problem, where the training data is separated into several groups after obtaining the data boundary locations and monotonicity of this data. These groups can be used separately to train different ANN branches, forming one multibranch ANN model that can predict the antenna's geometrical or physical parameters based on the desired input EM characteristics.

This technique was tested on Short Dipole Planar Array where the obtained model had a train and test set errors of 0.25% and 0.28% respectively, considered to be a significant improvement on the non-multibranch conventional approach that has a 20.38% training set error and a 20.40% test set error. Further testing was made on sparse linear dipole array resulting in 0.37% and 0.68% training and test set errors respectively.

As a summary to the in-depth investigation of the different antenna design papers using ML presented in the literature and studied in this work, Table 2 lists the different papers investigated sorted by ML and as per the antenna type and configuration.

7 | MACHINE LEARNING-ASSISTED ANTENNA OPTIMIZATION

Another line of researcher focuses on embedding ML models inside an optimization algorithm that is used to reach optimal parameters and performance of an antenna. By integrating a ML model within the optimizer, the design and optimization process would speed up since less simulations would be required. This section presents the work that has been done in this regard, along with the various results obtained. A Summary of these antennas along with the used algorithms can be found in Table 3.

Interpolation combined with GA used for the design of an UWB ring monopole antenna was presented in References 186 and 187 where fitness function behaviors such as the BW, the RL, and the central frequency division (CFD) were estimated. After optimizing those parameters, comparison was held between a simulated antenna and a real prototype manufactured from the obtained values.

Different numbers of datasets were used in training the model and it was determined that the perception on the behavior of the objectives (BW, RL, and CFD) increases as the size of the dataset increases.

The design of stacked patch antennas using ANNs was presented in References 188-191, where a trained ANN embedded in PSO was used to obtain multi-band characteristics. After having decided upon the geometrical parameters of the antenna by the PSO, a function mapping "black-box" was built by the ANN, and the frequencies and associated bandwidths were related to the dimensional antenna parameters. The obtained ANN results were then compared with measured results of a fabricated antennas, where good agreement has been revealed with an error of order 10^{-5}.

In Reference 192, DE and the Kriging algorithm were used in the design optimization of an E-shaped antenna. Six antenna variables were optimized, which were feed position, the slot position, the length and width of the

TABLE 2 Investigated antennas designed using ML

ML algorithm	Antenna type	References
ANN	Rectangular patch	[124,125,130-133]
	Circular patch	[134-138]
	Fractal patch	[139-141]
	Elliptical patch	[142,143]
	Monopole antenna	[144]
	Dipole antenna	[95,185]
	PIFA	[147,148]
	SIW	[149]
	Special patch structures	[151-161]
	Reflectarrays	[162-168]
	Broadband antenna	[176]
	Loop antenna	[177]
	Non-linearly loaded dipole antenna	[57]
	Antenna arrays	[179]
	Corrugated circular horn antenna	[181]
	Pyramidal horn antenna	[180]
	Slotted waveguide antenna array	[183]
SVR/SVM	Rectangular patch	[97,126-129]
	Reflectarrays	[171-175]
GPR	SIW	[150]
Kriging regression	Reflectarrays	[169,170]
LASSO	Monopole antenna	[145,146]

Abbreviations: ANN, artificial neural networks; GPR, Gaussian process regression; LASSO, least absolute shrinkage and selection operator; ML, machine learning; PIFA, planar inverted-F antenna; SIW, substrate integrated waveguide; SVR/SVM, support vector regression/support vector machines.

patch, and the slot width and length. Good prediction accuracy was exhibited by the model after reaching optimal solutions by the model. It was concluded that the proposed approach reduced the number of necessary simulations significantly.

In Reference 193, a new algorithm named (SADEA) based on surrogate model assisted (SMA-DE) and GPR was proposed. This method was found efficient in the design of antennas, where it has been tried on three types of antennas that are namely: an inter-chip antenna, a four-element linear array, and a 2-D array. It was shown that SADEA can speed up the design and optimization procedure by more than four times compared with DE.

Slots antennas were optimized in References 194 and 195 by using Space Mapping as an optimization engine. Computational costs were reduced by implementing Bayesian SVR (BSVR)[196] as the coarse response surface model instead of relying on electromagnetic simulations. The parameters of a CPW-fed Slot Dipole Antenna and a CPW-fed T-shaped Slot Antenna were optimized using this procedure which resulted in satisfactory designs.

8 | DISCUSSION AND CONCLUSION

The aforementioned discussion has highlighted the importance and usefulness of using ML techniques in the design and analysis of many antennas. However, many challenges arise when adopting this approach instead of relying on computational electromagnetics. The first challenge relates to the lack of standardized datasets for antenna structures that can be used directly to train a certain model and obtain results. Instead, data need to be generated by simulations beforehand to create a database of selected input and output variables. This can be a tedious and time-consuming task since the initial goal of using ML in the context of antennas is obtaining an

TABLE 3 Investigated antennas designed using ML assisted optimization

ML algorithm	Optimization algorithm	Antenna type	References
Interpolation	GA	Ring monopole antenna	[186,187]
ANN	PSO	Stacked patch antenna	[188-191]
Kriging	DE	E-shaped antenna	[192]
GPR	SMA-DE	Inter-chip antenna	[193]
		Four-element array	
		2-D array	
BSVR	Space mapping	Slots antenna	[194,195]

Abbreviations: ANN, artificial neural network; BSVR, Bayesian SVR; DE, differential evolution; GA, genetic algorithm; GPR, Gaussian process regression; ML, machine learning; PSO, particle swarm optimization; SMA-DE, surrogate model assisted differential evolution.

accelerated design and characterization process while maintaining high accuracy. Having to go through simulations to obtain a dataset also translates into a heavier computational load.

Another aspect to be considered is selecting the best model hyperparameters that can lead to the optimal results. It can be clearly deduced that ANNs have dominated this research area by being the most popular choice of ML technique with many frameworks and software packages available for their quick and efficient employment, and by showing resilience in providing highly accurate results compared to conventional CEM approaches. The importance of ANNs in antenna design becomes more recognizable as the complexity of the antenna structure increases. Therefore, it is necessary to investigate what type of training and optimization method, network architecture, regularization techniques, choice of activation functions, and similar factors that affect the model's performance, would be most suitable for each antenna type.

While ML stands out as an attractive antenna design and analysis tool that can perform predictions with high accuracy in a shorter period of time compared to simulation approaches, having to generate the training data would seem unattractive and demanding. For this reason, a good approach to address this issue is the development of an antenna design software based purely on ML models to replace simulators. Such a tool would of course be limited in terms of designer flexibility and would have to be targeted on specific antenna types and structures but may be extended to cover a large number of antennas. Having a fast, accurate, and optimized design tool would allow quick characterization of the selected antenna type, where the user would only need to input the design requirement to obtain the geometrical predictions. However, this software falls short in cases where a special structure is desired, which forces the designer of going through simulations.

This paper provided a comprehensive survey on the usage of ML in antenna design and analysis. ML is expected to reduce the computational burdens imposed by simulators and accelerate the design process. The different research papers presented in the literature that have employed ML algorithms in their design have been investigated. An overview on a variety of ML concepts has also been presented, thus enabling readers that are interested in antenna research but have minimal ML expertise with the basic and fundamental understandings needed to use these effective tools in their projects.

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REFERENCES

1. Volakis JL, Johnson RC, Jasik H. *Antenna Engineering Handbook*. New York: McGraw-Hill; 2007.
2. Sumithra P, Thiripurasundari D. Review on computational electromagnetics. *Adv Electromagn*. 2017;6:42-55.
3. Tayli D. *Computational Tools for Antenna Analysis and Design*. Electromagnetic Theory Department of Electrical and Information Technology, Lund University; 2018.
4. Gibson WC. *The Method of Moments in Electromagnetics*. CRC Press; 2014.
5. Reineix A, Jecko B. Analysis of microstrip patch antennas using finite difference time domain method. *IEEE Trans Antenna Propag*. 1989;37:1361-1369.
6. Tirkas PA, Balanis CA. Finite-difference time-domain method for antenna radiation. *IEEE Trans Antenna Propag*. 1992;40:334-340.
7. Maloney JG, Smith GS, Scott WR. Accurate computation of the radiation from simple antennas using the finite-difference time-domain method. *IEEE Trans Antenna Propag*. 1990;38:1059-1068.
8. Volakis JL, Chatterjee A, Kempel LC. *Finite Element Method Electromagnetics: Antennas, Microwave Circuits, and Scattering Applications*. Vol 6. John Wiley & Sons; 1998.
9. Lou Z, Jin JM. Modeling and simulation of broad-band antennas using the time-domain finite element method. *IEEE Trans Antenna Propag*. 2005;53:4099-4110.
10. Sarkar TK, Djordjevic AR, Kolundzija BM. Method of moments applied to antennas. *Handbook of Antennas in Wireless Communications*; 2000:239-279.
11. Rawle W, Smiths A. The method of moments: a numerical technique for wire antenna design. *High Freq Electron*. 2006;5:42-47.
12. Wu YM. The contour deformation method for calculating the high frequency scattered fields by the Fock current on the surface of the 3-D convex cylinder. *IEEE Trans Antenna Propag*. 2014;63:2180-2190.
13. Xu Q, Huang Y, Zhu X, Xing L, Duxbury P, Noonan J. Building a better anechoic chamber: a geometric optics-based systematic solution, simulated and verified [measurements corner]. *IEEE Antennas Propag Mag*. 2016;58(2):94-119.
14. Weston D. *Electromagnetic Compatibility: Principles and Applications*. 2nd ed. (Revised and Expanded) CRC Press; 2017.
15. Testolina P, Lecci M, Rebato M, et al. Enabling simulation-based optimization through machine learning: a case study on antenna design. *arXiv Preprint*. 2019;1908:11225.
16. Ledesma S, Ruiz-Pinales J, Garcia-Hernandez M, et al. A hybrid method to design wire antennas: design and optimization of antennas using artificial intelligence. *IEEE Antennas Propag Mag*. 2015;57:23-31.
17. Misilmani HME, Naous T. Machine learning in antenna design: an overview on machine learning concept and algorithms. Paper presented at: International Conference on High Performance Computing & Simulation; Dublin, Ireland; 2019.
18. Zhang Q-J, Gupta KC, Devabhaktuni VK. Artificial neural networks for RF and microwave design-from theory to practice. *IEEE Trans Microw Theory Tech*. 2003;51:1339-1350.

19. Yee K. Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media. *IEEE Trans Antenna Propag*. 1966;14:302-307.
20. Umashankar K, Taflove A. A novel method to analyze electromagnetic scattering of complex objects. *IEEE Trans Electromagn Compat*. 1982;EMC-24:397-405.
21. Warnick KF. *Numerical Methods for Engineering: An Introduction Using MATLAB and Computational Electromagnetics Examples*. SciTech Pub; 2011.
22. Turner MJ, Clough RW, Martin HC, Topp L. Stiffness and deflection analysis of complex structures. *J Aeronaut Sci*. 1956;23:805-823.
23. Taylor OZR. *The Finite Element Method*. McGraw-Hill Book Company, Inc; 1991.
24. Harrington RF. *Field Computation by Moment Methods*. Wiley-IEEE Press; 1993.
25. Burke GJ, Poggio A, Logan J, Rockway J. Numerical electromagnetic code (NEC). Paper presented at: IEEE International Symposium on Electromagnetic Compatibility; San Diego, CA; 1979.
26. Rockway JW, Logan JC, Tam DW, Li ST. *The MININEC System*. Boston, MA: Artech House; 1988.
27. Sankaran K. *Accurate Domain Truncation Techniques for Time-Domain Conformal Methods*. Switzerland: ETH Zurich; 2007.
28. Gedney SD. An anisotropic perfectly matched layer-absorbing medium for the truncation of FDTD lattices. *IEEE Trans Antenna Propag*. 1996;44:1630-1639.
29. Kaneda N, Houshmand B, Itoh T. FDTD analysis of dielectric resonators with curved surfaces. *IEEE Trans Microw Theory Tech*. 1997;45:1645-1649.
30. Samimi A, Simpson JJ. An efficient 3-D FDTD model of electromagnetic wave propagation in magnetized plasma. *IEEE Trans Antenna Propag*. 2014;63:269-279.
31. Giannakis I, Giannopoulos A. Time-synchronized convolutional perfectly matched layer for improved absorbing performance in FDTD. *IEEE Antennas Wirel Propag Lett*. 2014;14:690-693.
32. Xiong R, Chen B, Fang D. An algorithm for the FDTD modeling of flat electrodes in grounding systems. *IEEE Trans Antenna Propag*. 2013;62:345-353.
33. Xiong R, Gao C, Chen B, Duan YT, Yin Q. Uniform two-step method for the FDTD analysis of aperture coupling. *IEEE Antennas Propag Mag*. 2014;56:181-192.
34. Wang YG, Chen B, Chen HL, Yi Y, Kong XL. One-step leapfrog ADI-FDTD method in 3-D cylindrical grids with a CPML implementation. *IEEE Antennas Wirel Propag Lett*. 2014;13:714-717.
35. Hemmi T, Costen F, Garcia SG, Himeno R, Yokota H, Mustafa M. Efficient parallel LOD-FDTD method for Debye-dispersive media. *IEEE Trans Antenna Propag*. 2013;62:1330-1338.
36. Lai ZH, Kiang JF, Mittra R. A domain decomposition finite difference time domain (FDTD) method for scattering problem from very large rough surfaces. *IEEE Trans Antenna Propag*. 2015;63:4468-4476.
37. Zhu M, Cao Q, Zhao L. Study and analysis of a novel Runge-Kutta high-order finite-difference time-domain method. *IET Microw Antennas Propag*. 2014;8:951-958.
38. Niziolek M. Review of methods used for computational electromagnetics. Paper presented at: 2nd International Students Conference on Electrodynamic and Mechatronics; Silesia, Poland; 2009.
39. Zhou B, Jiao D. Direct finite element solver of linear complexity for analyzing electrically large problems. Paper presented at: 31st International Review of Progress in Applied Computational Electromagnetics (ACES); Williamsburg, VA; 2015.
40. Wan T, Zhang Q, Hong T, Fan Z, Ding D, Chen RS. Fast analysis of three-dimensional electromagnetic problems using dual-primal finite-element tearing and interconnecting method combined with H-matrix technique. *IET Microw Antennas Propag*. 2015;9:640-647.
41. Moon H, Teixeira FL, Kim J, Omelchenko YA. Trade-offs for unconditional stability in the finite-element time-domain method. *IEEE Microw Wirel Compon Lett*. 2014;24:361-363.
42. Voznyuk I, Tortel H, Litman A. 3-D electromagnetic scattering computation in free-space with the FETI-FDP2 method. *IEEE Trans Antenna Propag*. 2015;63:2604-2613.
43. Guan J, Yan S, Jin JM. An accurate and efficient finite element-boundary integral method with GPU acceleration for 3-D electromagnetic analysis. *IEEE Trans Antenna Propag*. 2014;62:6325-6336.
44. Lu ZQ, An X. Non-conforming finite element tearing and interconnecting method with one Lagrange multiplier for solving large-scale electromagnetic problems. *IET Microw Antennas Propag*. 2014;8:730-735.
45. Yang ML, Gao HW, Song W, Sheng XQ. An effective domain-decomposition-based preconditioner for the FE-BI-MLFMA method for 3D scattering problems. *IEEE Trans Antenna Propag*. 2014;62:2263-2268.
46. Hu FG, Song J, Yang M. Errors in projection of plane waves using various basis functions. *IEEE Antennas Propag Mag*. 2009;51:86-98.
47. Bautista ME, Francavilla MA, Vipiana F, Vecchi G. A hierarchical fast solver for EFIE-MoM analysis of multiscale structures at very low frequencies. *IEEE Trans Antenna Propag*. 2014;62:1523-1528.
48. Strydom WJ, Botha MM. Charge recovery for the RWG-based method of moments. *IEEE Antennas Wirel Propag Lett*. 2014;14:305-308.
49. Vipiana F, Francavilla MA, Andriulli FP, Pirinoli P, Vecchi G. Multi-resolution approach to three-dimensional method-of-moments problems. Paper presented at: Computational Electromagnetics International Workshop; Izmir, Turkey; 2009.
50. Hoole SRH. Artificial neural networks in the solution of inverse electromagnetic field problems. *IEEE Trans Magn*. 1993;29:1931-1934.
51. Yamashita H, Kowata N, Cingoski V, Kaneda K. Direct solution method for finite element analysis using Hopfield neural network. *IEEE Trans Magn*. 1995;31:1964-1967.
52. Kant JD, Le Drezen J, Bigeon J. Electromagnetic field parallel computation with a Hopfield neural network. *IEEE Trans Magn*. 1995;31:1968-1971.
53. Vegni L, Toscano A. Analysis of microstrip antennas using neural networks. *IEEE Trans Magn*. 1997;33:1414-1419.
54. Christodoulou C, Georgiopoulos M. *Applications of Neural Networks in Electromagnetics*. Artech House, Inc.; 2000.
55. Goasguen S, Hammadi SM, El-Ghazaly SM. A global modeling approach using artificial neural network. Paper presented

at: IEEE MTT-S International Microwave Symposium Digest; Anaheim, CA; 1999.

56. Haykin S. *Neural Networks: A Comprehensive Foundation*. Prentice Hall PTR; 1994.
57. Lee KC. Application of neural network and its extension of derivative to scattering from a nonlinearly loaded antenna. *IEEE Trans Antennas Propag*. 2007;55:990-993.
58. Lee KC, Lin TN. Application of neural networks to analyses of nonlinearly loaded antenna arrays including mutual coupling effects. *IEEE Trans Antenna Propag*. 2005;53:1126-1132.
59. Qj Z, Gupta KC. *Neural Networks for RF and Microwave Design*. Artech House, Inc.; 2000.
60. Creech GL, Paul BJ, Lesniak CD, Jenkins TJ, Calcatera MC. Artificial neural networks for fast and accurate EM-CAD of microwave circuits. *IEEE Trans Microw Theory Tech*. 1997;45:794-802.
61. Wang F, Zhang QJ. Knowledge-based neural models for microwave design. *IEEE Trans Microw Theory Tech*. 1997;45:2333-2343.
62. Friedberg RM. A learning machine: part I. *IBM J Res Dev*. 1958;2(1):2-13.
63. Mnih V, Kavukcuoglu K, Silver D, et al. Playing atari with deep reinforcement learning. *arXiv Preprint*. 2013;1312.5602.
64. Goodfellow I, Pouget-Abadie J, Mirza M, et al. Generative adversarial nets. *Advances in Neural Information Processing Systems*. 2014;2672-2680.
65. Weisberg S. *Applied Linear Regression*. Vol 528. John Wiley & Sons; 2005.
66. Montgomery DC, Peck EA, Vining GG. *Introduction to Linear Regression Analysis*. Vol 821. John Wiley & Sons; 2012.
67. Seber GA, Lee AJ. *Linear Regression Analysis*. Vol 329. John Wiley & Sons; 2012.
68. Vovk V. Kernel ridge regression. *Empirical Inference*. Springer; 2013.
69. Drucker H, Burges CJ, Kaufman L, Smola AJ, Vapnik V. Support vector regression machines. *Adv Neural Inf Process Syst*. 1997;155-161.
70. Smola AJ, Scholkopf B. A tutorial on support vector regression. *Stat Comput*. 2004;14(3):199-222.
71. Awad M, Khanna R. Support vector regression. *Efficient Learning Machines*. Springer; 2015.
72. Hastie T, Tibshirani R, Wainwright M. *Statistical Learning with Sparsity: The Lasso and Generalizations*. CRC Press; 2015.
73. Harrington P. *Machine Learning in Action*. Manning Publications Co; 2012.
74. Sutton RS, Barto AG. *Reinforcement Learning: An Introduction*. MIT Press; 2018.
75. Abadi M, Barham P, Chen J, et al. Tensorflow: a system for large-scale machine learning. Paper presented at: 12th USENIX Symposium on Operating Systems Design and Implementation, Savannah, GA; 2016.
76. Pedregosa F, Varoquaux G, Gramfort A, et al. Scikit-learn: machine learning in python. *J Mach Learn Res*. 2011;12:2825-2830.
77. Zaharia M, Xin RS, Wendell PDT, et al. Apache spark: a unified engine for big data processing. *Commun ACM*. 2016;59:56-65.
78. Jia Y, Shelhamer E, Donahue J, et al. Caffe: convolutional architecture for fast feature embedding. Paper presented at: Proceedings of the 22nd ACM International Conference on Multimedia; Orlando, FL; 2014:675-678.
79. Seide F, Agarwal A. CNTK: Microsoft's open-source deep-learning toolkit. Paper presented at: Proceedings of the 22nd ACM SIGKDD International Conference on Knowledge Discovery and Data Mining; San Francisco, CA; 2016:2135-2135.
80. Chang CC, Lin CJ. LIBSVM: a library for support vector machines. *ACM Trans Intell Syst Technol*. 2011;2(3).
81. Hall M, Frank E, Holmes G, Pfahringer B, Reutemann P, Witten IH. The WEKA data mining software: an update. *ACM SIGKDD Explor*. 2009;1(1).
82. Li Z, Kermode JR, De Vita A. Molecular dynamics with on-the-fly machine learning of quantum-mechanical forces. *Phys Rev Lett*. 2015;114(9).
83. Baldi P, Brunak S, Bach F. *Bioinformatics: the Machine Learning Approach*. MIT Press; 2001.
84. Heaton J, Polson N, Witte JH. Deep learning for finance: deep portfolios. *Appl Stoch Model Bus Ind*. 2017;33(1):3-12.
85. Gunduz D, de Kerret P, Sidiropoulos ND, Gesbert D, Murthy C, van der Schaar M. Machine learning in the air. *arxiv Preprint*. 2019;1904:12385.
86. He D, Liu C, Quek TQS, Wang H. Transmit antenna selection in MIMO wiretap channels: a machine learning approach. *IEEE Wirel Commun Lett*. 2018;7(4):634-637.
87. Zhang C, Patras P, Haddadi H. Deep learning in mobile and wireless networking: a survey. *IEEE Commun Surv Tutor*. 2019;21:2224-2287.
88. Bkassiny M, Li Y, Jayaweera SK. A survey on machine-learning techniques in cognitive radios. *IEEE Commun Surv Tutor*. 2012;15(3):1136-1159.
89. Alsheikh MA, Lin S, Niyato D, Tan HP. Machine learning in wireless sensor networks: algorithms, strategies, and applications. *IEEE Commun Surv Tutor*. 2014;16(4):1996-2018.
90. Xin Y, Kong L, Liu Z, et al. Machine learning and deep learning methods for cybersecurity. *IEEE Access*. 2018;6:35365-35381.
91. Samek W, Stanczak S, Wiegand T. The convergence of machine learning and communications. *Arxiv Preprint*;2017, 1708.08299.
92. Thakare VV, Singhal P. Microstrip antenna design using artificial neural networks. *Int J RF Microw Comput Aid Eng*. 2010;20:76-86.
93. Xiao LY, Shao W, Jin FL, Wang BZ. Multiparameter modeling with ANN for antenna design. *IEEE Trans Antenna Propag*. 2018;66:3718-3723.
94. Mishra R, Patnaik A. Neural network-based CAD model for the design of square-patch antennas. *IEEE Trans Antenna Propag*. 1998;46:1890-1891.
95. Delgado HJ, Thursby MH, Ham FM. A novel neural network for the synthesis of antennas and microwave devices. *IEEE Trans Neural Netw*. 2005;16:1590-1600.
96. Angiulli G, Cacciola M, Versaci M. Microwave devices and antennas modelling by support vector regression machines. *IEEE Trans Magn*. 2007;43:1589-1592.
97. Tokan NT, Gunes F. Support vector characterisation of the microstrip antennas based on measurements. *Progr Electromagn Res*. 2008;5:49-61.

98. De Villiers JP, Jacobs JP. Gaussian process modeling of CPW-fed slot antennas. *Progr Electromagn Res*. 2009;98:233-249.
99. Wu Q, Wang H, Hong W. Multi-stage collaborative machine learning and its application to antenna modeling and optimization. *IEEE Trans Antenna Propag*. 2020.
100. Koziel S, Ogurtsov S, Couckuyt I, Dhaene T. Efficient simulation-driven design optimization of antennas using co-kriging. Paper presented at: IEEE International Symposium on Antennas and Propagation; Chicago, IL; 2012.
101. Ulaganathan S, Koziel S, Bekasiewicz A, Couckuyt I, Laermans E, Dhaene T. Cost-efficient modeling of antenna structures using gradient-enhanced Kriging. Paper presented at: Loughborough Antennas & Propagation Conference (LAPC); Loughborough, England; 2015.
102. Burkov A. *The Hundred-Page Machine Learning Book*; 2019.
103. Baldi P. Gradient descent learning algorithm overview: a general dynamical systems perspective. *IEEE Trans Neural Netw*. 1995;6(1):182-195.
104. Tibshirani R. Regression shrinkage and selection via the lasso. *J R Stat Soc B Methodol*. 1996;58(1):267-288.
105. Shalev-Shwartz S, Ben-David S. *Understanding Machine Learning: From Theory to Algorithms*. Cambridge University Press; 2014.
106. Grohs P, Perekrestenko D, Elbrachter D, Bolcskei H. Deep neural network approximation theory. *Arxiv Preprint*. 2019;1901:02220.
107. Haykin S. *Neural Networks and Learning Machines*. Prentice Hall; 2008.
108. Goodfellow I, Bengio Y, Courville A. *Deep Learning*. MIT Press; 2016.
109. Chauvin Y, Rumelhart DE. *Backpropagation: Theory, Architectures, and Applications*. Psychology Press; 2013.
110. Cherkassky V, Mulier FM. *Learning from Data: Concepts, Theory, and Methods*. John Wiley & Sons; 2007.
111. Mohri M, Rostamizadeh A, Talwalkar A. *Foundations of Machine Learning*. The MIT Press; 2012.
112. Santner TJ, Williams BJ, Notz W, Williams BJ. *The Design and Analysis of Computer Experiments*. Vol 1. Springer; 2003.
113. Hengl T, Heuvelink GB, Rossiter DG. About regression-kriging: from equations to case studies. *Comput Geosci*. 2007;33(10):1301-1315.
114. Klein S, Pluim JP, Staring M, Viergever MA. Adaptive stochastic gradient descent optimisation for image registration. *Int J Comput Vis*. 2009;81(3):227-239.
115. Ruder S. An overview of gradient descent optimization algorithms. *Arxiv Preprint*. 2016;1609.04747.
116. Kingma DP, Ba J. Adam: a method for stochastic optimization. *Arxiv Preprint*. 2014;1412.6980.
117. Moré JJ. The Levenberg-Marquardt algorithm: implementation and theory. *Numer Anal*. 1978;105-116.
118. Hagan MT, Menhaj MB. Training feedforward networks with the Marquardt algorithm. *IEEE Trans Neural Netw*. 1994;5:989-993.
119. Burden F, Winkler D. Artificial neural networks. *Introduction to Artificial Neural Systems*. Springer; 2008.
120. Back T, Fogel DB, Michalewicz Z. *Handbook of Evolutionary Computation*. CRC Press; 1997.
121. Weile DS, Michielssen E. Genetic algorithm optimization applied to electromagnetics: a review. *IEEE Trans Antenna Propag*. 1997;45(3):343-353.
122. Johnson JM, Rahmat-Samii V. Genetic algorithms in engineering electromagnetics. *IEEE Antennas Propag Mag*. 1997;39(4):7-21.
123. Hoorfar A. Evolutionary programming in electromagnetic optimization: a review. *IEEE Trans Antenna Propag*. 2007;55(3):523-537.
124. Naser-Moghaddas M. A heuristic artificial neural network for analyzing and synthesizing rectangular mcrostrip antenna. *Int J Comput Sci Netw Secur*. 2007;7:278-281.
125. Turker N, Gunes F, Yildirim T. Artificial neural design of microstrip antennas. *Turk J Electr Eng Comp Sci*. 2007;14:445-453.
126. Tokan NT, Gunes F. Support vector design of the microstrip antenna. Paper presented at: IEEE 16th Signal Processing, Communication and Applications Conference; Aydin, Turkey; 2008.
127. Zheng Z, Chen X, Huang K. Application of support vector machines to the antenna design. *Int J RF Microw Comput Aid Eng*. 2011;21:85-90.
128. Ulker S. Support vector regression analysis for the design of feed in a rectangular patch antenna. Paper presented at: International Symposium on Multidisciplinary Studies and Innovative Technologies (ISMSIT); Ankara, Turkey; 2019.
129. Khan T, Roy C. Prediction of slot-position and slot-size of a microstrip antenna using support vector regression. *Int J RF Microw Comput Aid Eng*. 2019;29:e21623.
130. Khan T, De A, Uddin M. Prediction of slot-size and inserted air-gap for improving the performance of rectangular microstrip antennas using artificial neural networks. *IEEE Antennas Wirel Propag Lett*. 2013;12:1367-1371.
131. Singh BK. Design of rectangular microstrip patch antenna based on artificial neural network algorithm. Paper presented at: 2nd International Conference on Signal Processing and Integrated Networks (SPIN); Noida, India; 2015.
132. Vilovic I, Burum N, Brailo M. Microstrip antenna design using neural networks optimized by PSO. Paper presented at: ICECom; Dubrovnik, Croatia; 2013.
133. Neog DK, Pattnaik SS, Panda DC, Devi S, Khuntia B, Dutta M. Design of a wideband microstrip antenna and the use of artificial neural networks in parameter calculation. *IEEE Antennas Propag Mag*. 2005;47:60-65.
134. Yildiz C, Gultekin S, Guney K, Sagiroglu S. Neural models for the resonant frequency of electrically thin and thick circular microstrip antennas and the characteristic parameters of asymmetric coplanar waveguides backed with a conductor. *AEU-Int J Electron Commun*. 2002;56:396-406.
135. Vilovic I, Burum N. Design and feed position estimation for circular microstrip antenna based on neural network model. Paper presented at: European Conference on Antennas and Propagation (EUCAP); Prague, Czech Republic; 2012.
136. Malathi P, Kumar R. On the design of multilayer circular microstrip antenna using artificial neural networks. *Int J Recent Trends Eng*. 2009.
137. Mishra A, Janvale G, Pawar B, Patil A. The design of circular microstrip patch antenna by using Conjugate Gradient algorithm of ANN. Paper presented at: IEEE Applied Electromagnetics Conference (AEMC); Kolkata, India; 2011.
138. Sivia JS, Pharwaha APS, Kamal TS. Neurocomputational models for parameter estimation of circular microstrip patch antennas. *Proc Comput Sci*. 2016;85:393-400.

139. Arora P, Dhaliwal BS. Parameter estimation of dual band elliptical fractal patch antenna using ANN. Paper presented at: International Conference on Devices and Communications (ICDeCom); Mesra, India; 2011.
140. Suganthi S, Raghavan S. ANN based pattern generation, design and simulation of broadband fractal antenna for wireless applications. Paper presented at: International Conference on Emerging Trends in Engineering, Technology and Science (ICETETS); Pudukkottai, India; 2016.
141. Silva PH, Oliveira EE, d'Assuncao AG. Using a multilayer perceptrons for accurate modeling of quasi-fractal patch antennas. Paper presented at: International Workshop on Antenna Technology (iWAT); Lisbon, Portugal; 2010.
142. Agrawal A, Vakula D, Sarma N. Design of elliptical microstrip patch antenna using ANN. Paper presented at: PIERS proceedings; Suzhou, China; 2011.
143. Badra N, Allam A, El-Rafei A, et al. WiFi antenna design and modeling using artificial neural networks. Paper presented at: International Conference on Innovative Trends in Computer Engineering (ITCE); 2019.
144. Pandit M, Bose T. Application of neural network model for designing circular monopole antenna. Paper presented at: ISDMISCS; Gangtok, Sikkim, India; 2011.
145. Sharma Y, Wu J, Xin H, Zhang HH. Sparse linear regression for optimizing design parameters of double T-shaped monopole antennas. Paper presented at: IEEE International Symposium on Antennas and Propagation & USNC/URSI National Radio Science Meeting; San Diego, CA; 2017.
146. Sharma Y, Zhang HH, Xin H. Machine learning techniques for optimizing design of double T-shaped monopole antenna. *IEEE Trans Antenna Propag*. 2020;1.
147. Gianfagna C, Swaminathan M, Raj PM, Tummala R, Antonini G. Enabling antenna design with nano-magnetic materials using machine learning. Paper presented at: 2015 IEEE Nanotechnology Materials and Devices Conference (NMDC); Anchorage, AK; 2015.
148. Gianfagna C, Yu H, Swaminathan M, Pulugurtha R, Tummala R, Antonini G. Machine-learning approach for design of nanomagnetic-based antennas. *J Electron Mater*. 2017;46(8):4963-4975.
149. Chetioui M, Boudkhil A, Benabdallah N, Benahmed N. Design and optimization of SIW patch antenna for Ku band applications using ANN algorithms. Paper presented at: 4th International Conference on Optimization and Applications (ICOA); Mohammedia, Morocco; 2018.
150. Wu Q, Wang H, Hong W. Broadband millimeter-wave SIW cavity-backed slot antenna for 5G applications using machine-learning-assisted optimization method. Paper presented at: International Workshop on Antenna Technology (iWAT); Miami, FL; 2019.
151. Patnaik A, Anagnostou D, Christodoulou CG, Lyke JC. Neurocomputational analysis of a multiband reconfigurable planar antenna. *IEEE Trans Antenna Propag*. 2005;53:3453-3458.
152. Siakavara K. Artificial neural network employment in the design of multilayered microstrip antenna with specified frequency operation. Paper presented at: PIERS; 2007.
153. Thakare VV, Singhal PK. Bandwidth analysis by introducing slots in microstrip antenna design using ANN. *Progr Electromagn Res*. 2009;9:107-122.
154. Selvan PT, Raghavan S. Neural network model for design of compact CPW—fed monopole antenna for 5.8 GHz RFID application. Paper presented at: Second International conference on Computing, Communication and Networking Technologies; Karur, India; 2010.
155. Bose T, Gupta N. Design of an aperture-coupled microstrip antenna using a hybrid neural network. *IET Microw Antennas Propag*. 2012;6:470-474.
156. Wang Z, Fang S, Wang Q, Liu H. An ANN-based synthesis model for the single-feed circularly-polarized square microstrip antenna with truncated corners. *IEEE Trans Antenna Propag*. 2012;60:5989-5992.
157. Umut Ozkayaa LS. Dimension optimization of microstrip patch antenna in X/Ku band via artificial neural network. *Procedia Soc Behav Sci*. 2015;195:2520-2526.
158. Hammoodi AI, Milanova M, Khaleel H. Design of flexible antenna for UWB wireless applications using ANN. Paper presented at: International Conference on Computational Science and Computational Intelligence (CSCI); Las Vegas, NV; 2017.
159. Tighilt Y, Bouttout F, Khellaf A. Modeling and design of printed antennas using neural networks. *Int J RF Microw Comput Aid Eng*. 2011;21:228-233.
160. Quevedo-Teruel O, Rajo-Iglesias E. Design of short-circuited ring-patch antennas working at TM01 mode based on neural networks. *IEEE Antennas Wirel Propag Lett*. 2006;5:559-562.
161. Guney K, Sarikaya N. A hybrid method based on combining artificial neural network and fuzzy inference system for simultaneous computation of resonant frequencies of rectangular, circular, and triangular microstrip antennas. *IEEE Trans Antenna Propag*. 2007;55:659-668.
162. Caputo D, Pirisi A, Mussetta M, Freni A, Pirinoli P, Zich R. Neural network characterization of microstrip patches for reflectarray optimization. Paper presented at: 3rd European Conference on Antennas and Propagation; Berlin, Germany; 2009.
163. Mussetta M, Pirinoli P, Cong P, Orefice M, Zich R. Characterization of microstrip reflectarray square ring elements by means of an artificial neural network. Paper presented at: Fourth European Conference on Antennas and Propagation; Berlin, Germany; 2010.
164. Robustillo P, Zapata J, Encinar JA, Arrebola M. Design of a contoured-beam reflectarray for a EuTELSAT European coverage using a stacked-patch element characterized by an artificial neural network. *IEEE Antennas Wirel Propag Lett*. 2012; 11:977-980.
165. Nesil S, Gunes F, Kaya G. Analysis and design of X-band reflectarray antenna using 3-D EM-based artificial neural network model. Paper presented at: IEEE International Conference on Ultra-Wideband; Syracuse, NY; 2012.
166. Richard V, Loison R, Gillard R, et al. Spherical mapping of the second-order phoenix cell for unbounded direct reflectarray copolar optimization. *Progr Electromagn Res*. 2019;90: 109-124.
167. Freni A, Mussetta M, Pirinoli P. Neural network characterization of reflectarray antennas. *Int J Antennas Propag*. 2012; 2012:1-10.
168. Robustillo P, Zapata J, Encinar JA, Rubio J. ANN characterization of multi-layer reflectarray elements for contoured-

beam space antennas in the Ku-band. *IEEE Trans Antenna Propag*. 2012;60:3205-3214.
169. Tenuti L, Oliveri G, Bresciani D, Massa A. Advanced learning-based approaches for reflectarrays design. Paper presented at: 11th European Conference on Antennas and Propagation (EUCAP); Paris, France; 2017.
170. Salucci M, Tenuti L, Oliveri G, Massa A. Efficient prediction of the EM response of reflectarray antenna elements by an advanced statistical learning method. *IEEE Trans Antenna Propag*. 2018;66:3995-4007.
171. Prado DR, Fernandez JAL, Arrebola M, Pino MR, Goussetis G. General framework for the efficient optimization of reflectarray antennas for contoured beam space applications. *IEEE Access*. 2018;6:72295-72310.
172. Prado DR, López-Fernández JA, Arrebola M, Goussetis G. Efficient shaped-beam reflectarray design using machine learning techniques. Paper presented at: 15th European Radar Conference (EuRAD); Madrid, Spain; 2018.
173. Prado DR, Lopez-Fernandez JA, MaGG A. Support vector regression to accelerate design and crosspolar optimization of shaped-beam reflectarray antennas for space applications. *IEEE Trans Antenna Propag*. 2018;67.
174. Prado DR, Fernandez JAL, Arrebola M, Rodriguez-Pino M, Goussetis G. Wideband shaped-beam reflectarray design using support vector regression analysis. *IEEE Antennas Wirel Propag Lett*. 2019.
175. Prado DR, Lopez-Fernandez JA, Barquero G, Arrebola M, Las-Heras F. Fast and accurate modeling of dual-polarized reflectarray unit cells using support vector machines. *IEEE Trans Antenna Propag*. 2018;66:1258-1270.
176. Kim Y, Keely S, Ghosh J, Ling H. Application of artificial neural networks to broadband antenna design based on a parametric frequency model. *IEEE Trans Antenna Propag*. 2007;55:669-674.
177. Sarmah K, Sarma KK. Design optimization of loop antenna using competitive learning ANN. Paper presented at: 2nd National Conference on Emerging Trends and Applications in Computer Science; Shillong, India; 2011:1-4.
178. Huang CC, Chu TH. Analysis of wire scatterers with nonlinear or time-harmonic loads in the frequency domain. *IEEE Trans Antenna Propag*. 1993;41:25-30.
179. Xiao LY, Shao W, Ding X, Liu QH, Joines WT. Multigrade artificial neural network for the design of finite periodic arrays. *IEEE Trans Antenna Propag*. 2019;67:3109-3116.
180. Modi AY, Mehta J, Pisharody N. Design of optimum gain L-band pyramidal horn using artificial neural network. Paper presented at: IEEE Applied Electromagnetics Conference (AEMC); Bhubaneswar, India; 2013.
181. Fedi G, Manetti S, Pelosi G, Selleri S. Profiled corrugated circular horns analysis and synthesis via an artificial neural network. *IEEE Trans Antenna Propag*. 2001;49:1597-1602.
182. Watson PM, Gupta KC. EM-ANN models for microstrip vias and interconnects in dataset circuits. *IEEE Trans Microw Theory Tech*. 1996;44:2495-2503.
183. Tak J, Kantemur A, Sharma Y, Xin H. A 3-D-printed W-band slotted waveguide array antenna optimized using machine learning. *IEEE Antennas Wirel Propag Lett*. 2007;17(11):2008-2012.
184. Yuan L, Yang XS, Wang C, Wang BZ. Multibranch artificial neural network modeling for inverse estimation of antenna array directivity. *IEEE Trans Antenna Propag*. 2020;68:4417-4427.
185. Delgado HJ, Thursby MH. A novel neural network combined with FDTD for the synthesis of a printed dipole antenna. *IEEE Trans Antenna Propag*. 2005;53:2231-2236.
186. Silva CR, Martins SR. An adaptive evolutionary algorithm for UWB microstrip antennas optimization using a machine learning technique. *Microw Opt Technol Lett*. 2013;55(8):1864-1868.
187. Martins SR, Lins HW, Silva CR. A self-organizing genetic algorithm for UWB microstrip antenna optimization using a machine learning technique. Paper presented at: International Conference on Intelligent Data Engineering and Automated Learning; Berlin, Heidelberg; 2012.
188. Jain SK. Bandwidth enhancement of patch antennas using neural network dependent modified optimizer. *Int J Microw Wirel Technol*. 2016;8(7):1111-1119.
189. Jain SK, Patnaik A, Sinha SN. Design of custom-made stacked patch antennas: a machine learning approach. *Int J Mach Learn Cybernet*. 2013;4:189-194.
190. Jain S, Patnaik A, Sinha S. Neural network based particle swarm optimizer for design of dual resonance X/Ku band stacked patch antenna. Paper presented at: IEEE International Symposium on Antennas and Propagation (APSURSI); Spokane, WA; 2011.
191. Patnaik A, Sinha S. Design of custom-made fractal multi-band antennas using ANN-PSO. *IEEE Antennas Propag Mag*. 2011;53:94-101.
192. Chen XH, Guo XX, Pei JM, Man WY. A hybrid algorithm of differential evolution and machine learning for electromagnetic structure optimization. Paper presented at: 32nd Youth Academic Annual Conference of Chinese Association of Automation (YAC); Hefei, China; 2017.
193. Liu B, Aliakbarian H, Ma Z, Vandenbosch GAE, Gielen G, Excell P. An efficient method for antenna design optimization based on evolutionary computation and machine learning techniques. *IEEE Trans Antenna Propag*. 2014;62(1):7-18.
194. Koziel S, Ogurtsov S, Jacobs JP. Low-cost design optimization of slot antennas using Bayesian support vector regression and space mapping. Paper presented at: Loughborough Antennas & Propagation Conference (LAPC); Loughborough, England; 2012.
195. Jacobs JP, Koziel S, Ogurtsov S. Computationally efficient multi-fidelity Bayesian support vector regression modeling of planar antenna input characteristics. *IEEE Trans Antenna Propag*. 2012;61:980-984.
196. Jacobs JP, Koziel S, Ogurtsov S. Reduced-cost Bayesian support vector regression modeling and optimization of planar slot antennas. Paper presented at: IEEE International Symposium on Antennas and Propagation; Chicago, IL; 2012.

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