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CHAPTER-5

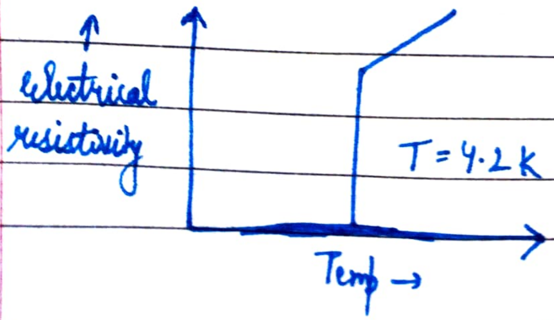
SUPERCONDUCTIVITY

- The phenomena in which electrical resistivity of a material suddenly drops to 0 on cooling it below a certain temp. known as critical temp. is superconductivity

  The materials which show this property are superconductors.

- Critical temp. is the temp. below which material will be superconductor and above which material will be normal conductor.

For Ex:- Critical Temp. of Mercury (Tc) is 4.2 K.



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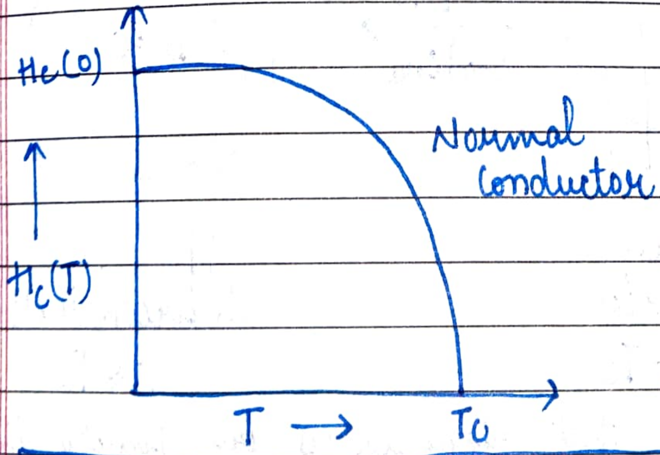
- T_c for

Materials	T_c (K)
Gallium	1.1
Alluminium	1.2
Tin	3.7
Lead	7.2
Niobium	9.3

★ Critical Magnetic field H_c (T) →
It is the minimum magnetic field required to destroy the superconductivity means below H_c materials will be superconductor and above (H_c) it will be normal.



$$\boxed{H_c(T) = H_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]}$$

★ Critical Current (I_c) → The maximum current that can flow through the superconductors without destroying its superconductivity is

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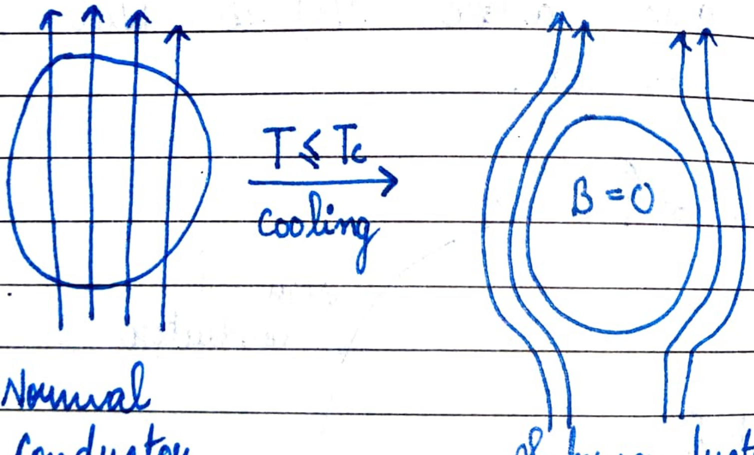
called critical current (I_c).

$$\boxed{I_c = 2\pi r\, H_c}$$

Silsbee's rule — radius of wire — critical field

☆ Meisner Effect (Flux Expulsion) →
The expulsion of magnetic lines of force from the specimen on cooling it below certain temp. known as critical Temp. (T_c) is known as Meisner effect



Normal Conductor
$T > T_c$

Superconductor
$T \leq T_c$

⊙ Proove that Superconductors are perfectly diamagnetic

As $B = \mu_0 (H + I)$ — ①

for Superconductor we know
$B = 0$

eqn ① will be

$\mu_0 (H + I) = 0$

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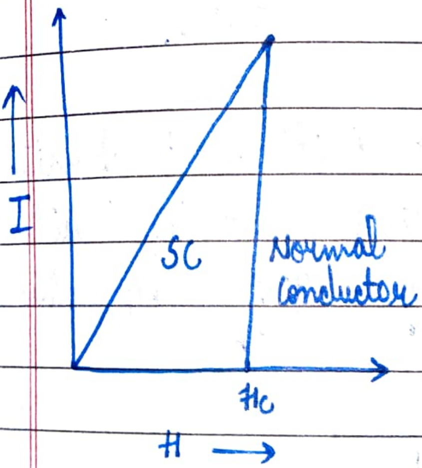
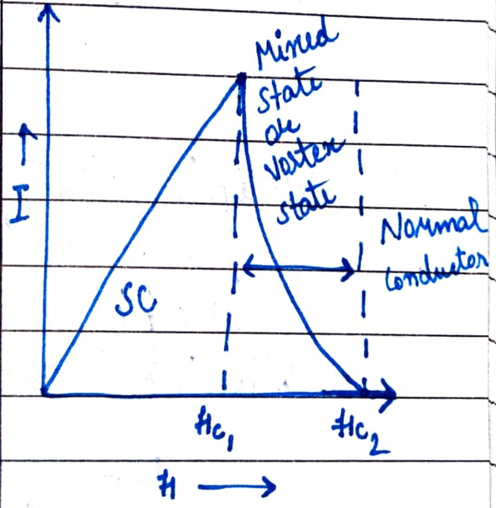
$$H + I = 0$$
$$H = -I$$
$$\frac{I}{H} = -1$$

$$\boxed{\chi = -1}$$

$\because$ χ is -ve, it means superconductors are perfectly diamagnetic.

★ Difference between Type 1 and Type 2 superconductors.

Type → 1 SC	Type → 2 SC
① 	
② They are soft superconductors	They are hard superconductors.
③ They show complete Meissner effect.	③ They show incomplete Meissner effect.

4	Below H_c i.e critical field, material is superconductor and above H_c material is normal conductor	Below H_{c_1} material is superconductor above H_{c_2} material is normal conductor and in b/w H_{c_1} and H_{c_2} material is in mixed or vortex state.
5	Only 1 critical field is required to change the state of the material.	Two critical field i.e lower critical field (H_{c_1}) and upper critical field (H_{c_2}) are required to change the state of the material.

☆ **Isotopic Effect** → Critical temperature of superconductors varies with isotopic mass given by the following relation ⇾

$$\boxed{T_c \propto \frac{1}{\sqrt{M}}}$$

☆ **Entropy (S)** → It measures the degree of randomness.

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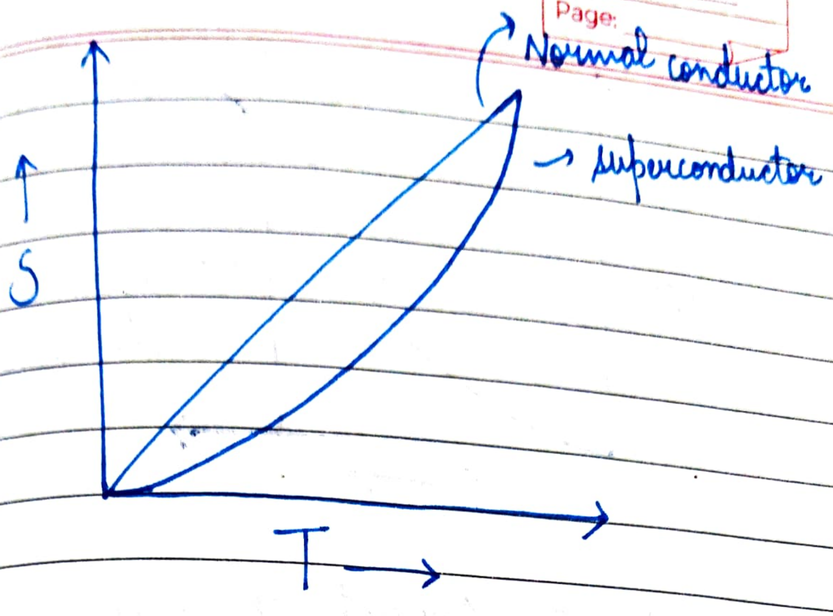

- Graph shows that with increase in temp. entropy (S) will increase. But on comparing entropies of Normal and Super conductor. we observe that Entropy (S) of Superconductor is less than Normal Conductor. means, Super Conductors are more ordered than normal Conductors.

☆ Specific Heat C (T) ⇒

It is the heat required to raise the temp. of 1 gram of substance to 1°C it is given by,

$$\boxed{C(T) = \gamma (T) + \beta (T^3)}$$

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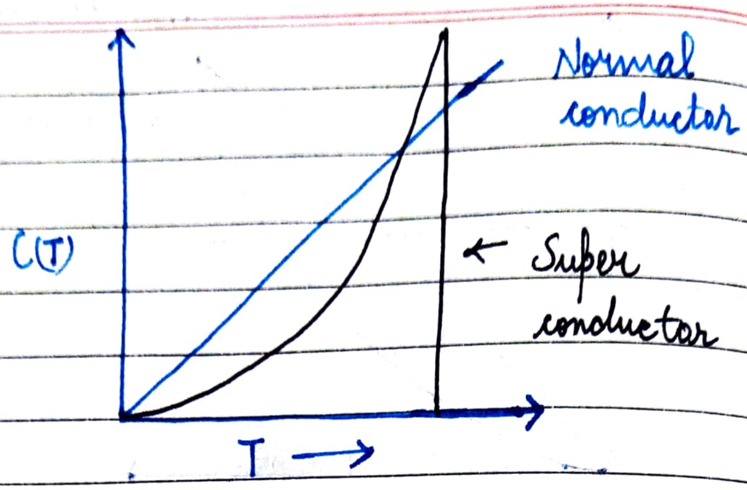

☆ Persistent current → When a superconductor is placed in external magnetic field and electric current is set-up in a superconductor that can persist for a long time known as persistant current.

Ques→ At what Temp. do we get $H_c = 0.1\, H_c(0)$ for lead having $T_c = 7.2$ K

Solⁿ→

$$H_c(T) = H_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

$$0.1\, \cancel{H_c(0)} = \cancel{H_c(0)}\left[1 - \frac{T^2}{(7.2)^2}\right]$$

$$(7.2)^2 (0.1) = (7.2)^2 - T^2$$

$$5.184 = 51.84 - T^2$$

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$$T^2 = 51.84 - 5.184$$
$$T^2 = 46.656$$
$$T = \sqrt{46.656}$$
$$\boxed{T = 6.83\ K}$$

Ans

Ques → The transition temp. for Lead is 7.2 K. However at 5K it loose its superconducting property when subjected to Magnetic field of 3.3×10^4 Amp/m. Find the value of Magnetic field that will allow the metal to retain its superconductivity at 0 K.

Sol → $T_c = 7.2\ K$, $H_c(T) = 3.3 \times 10^4$ Amp/m
$T = 5\ K$

$$H_c(T) = H_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

$$3.3 \times 10^4 = H_c(0)\left[1 - \frac{25}{51.84}\right]$$

$$3.3 \times 10^4 = H_c(0) \times [0.5178]$$

$$H_c(0) = \frac{3.3 \times 10^4}{0.5178}$$

$$H_c(0) = 6.373 \times 10^4 \text{ Amp/m}$$

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Ques → Calculate critical current for a wire of lead having diameter of 1mm at 4.2 K. The critical Temp for lead is 7.18 K and $H_0 = 6.5 \times 10^4$ Amp/m

Solⁿ → $T = 4.2K$; $T_c = 7.18K$
$H_0 = 6.5 \times 10^4$

$$H_c(T) = H_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

$$H_c(T) = 6.5 \times 10^4 \left[1 - \left(\frac{4.2}{7.18}\right)^2\right]$$

$$H_c(T) = 6.5 \times 10^4 \left[1 - \frac{17.64}{51.55}\right]$$

$$H_c(T) = 6.5 \times 10^4 \times 0.658$$

$$H_c = 4.277 \times 10^4$$

$$I_c = 2\pi r H_c$$

$$I_c = 2 \times 3.14 \times 0.5 \times 4.277 \times 10^4 \times 10^{-3}$$

$$\boxed{I_c = 134.29 \text{ Ampere}}$$

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Ques → Lead in the superconducting state has critical Temperature of 6.2 k at 0 Magnetic field and critical field of 0.064 Amp/m at 0 k. Find the critical field at 4 k.

Sol → $T_c = 6.2K$, $T = 4K$, $H_c(0) = 0.064\,A/m$

$$H_c(T) = H_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

$$H_c(T) = 0.064\left[1 - \left(\frac{4}{6.2}\right)^2\right]$$

$$H_c(T) = 0.064\left[1 - \frac{16}{38.44}\right]$$

$$H_c(T) = 0.064 \times 0.584$$

$$H_c(T) = 0.0373\ A/m$$

☆ **London Equations** → According to Meissner on cooling the specimen below critical temp. (T_c) the magnetic field (B) is 0 inside the specimen. But F. London and H. London repeated the same experiment with thin film materials and observed that

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value of B is not 0 everywhere inside the specimen. It is maximum at the surface and decreases towards the centre of the specimen and becomes minimum at the centre.

In order to explain this observations they derived two eq^n's whose assumptions are given below :-

① In superconductor two types of electrons exists, normal electrons and super electrons.

② Below T_c super electrons will be dominating and above T_c normal electrons will be dominating.

③ In super conductors, total no. of electrons are,

$$n = n_n + n_s$$

n_n = normal e^-

n_s = super e^-

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* Eq^n of motion for Super e^- ↔

$$m\frac{dv_s}{dt} = -eE \quad -①$$

$F = ma$
$F = qE$

$$\frac{dv_s}{dt} = \frac{-eE}{m} \quad -②$$

- current density (J_s)

$$J_s = -n_s e v_s \quad -③$$

- differentiating eq^n ③

$$\frac{dJ_s}{dt} = -n_s e \frac{dv_s}{dt} \quad -④$$

- Put $\frac{dv_s}{dt}$ from eq^n ② in eq^n ④

$$\boxed{\frac{dJ_s}{dt} = \frac{n_s e^2 E}{m}} \quad -⑤$$

- Eq^n 5 represents first London Equation.

Note → If $E = 0$, $\frac{dJ_s}{dt} = 0$

$J_s \to$ constant

- It means no electric field is required for the flow of current through superconductors.

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- Taking curl of eqn ⑤

$$\nabla \times \frac{dJ_s}{dt} = \nabla \times \left(\frac{n_s e^2 E}{m}\right)$$

$$\nabla \times \frac{dJ_s}{dt} = \frac{n_s e^2}{m} (\nabla \times E)$$

$$\frac{d}{dt}(\nabla \times J_s) = \frac{-n_s e^2}{m} \frac{dB}{dt}$$

$$\boxed{\nabla \times J_s = -\frac{n_s e^2 B}{m}} \quad -⑥$$

- Eqn ⑥ represents Second London Eqn.

☆ London penetration depth (λ) ⇒ London Equations are used to calculate London penetration depth (λ).

- Using Maxwell eqn

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J_s} \quad -①$$

- Taking curl

$$\vec{\nabla} \times \vec{\nabla} \times \vec{\nabla} = \mu_0 (\vec{\nabla} \times \vec{J_s})$$

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$$\nabla(\nabla.\vec{B}) - (\nabla.\nabla)B = \frac{-\mu_0 n_s e^2 B}{m}$$ (Using 2^{nd} London eq^{n})

$$+\nabla^2.B = \frac{+\mu_0 n_s e^2 B}{m} \quad -②$$

- eq^n ② represent 2^{nd} Order partial differential eq^n in B.

where, $\frac{\mu_0 n_s e^2}{m} = \frac{1}{\lambda^2}$ (constant)

$$\lambda = \sqrt{\frac{m}{\mu_0 n_s e^2}}$$

$$\boxed{\nabla^2 B = \frac{1}{\lambda^2} B} \quad -③$$

- Solution of eq^n ③ is given by

$$\boxed{B = B_0 e^{-x/\lambda}} \quad -④$$

where, x = Penetration Depth

λ = London penetration Depth

☆ Special case ⇒ (I) when $x=0$ (at Surface)

$\boxed{B = B_0}$ (maximum value)

So, B will be maximum at the surface.

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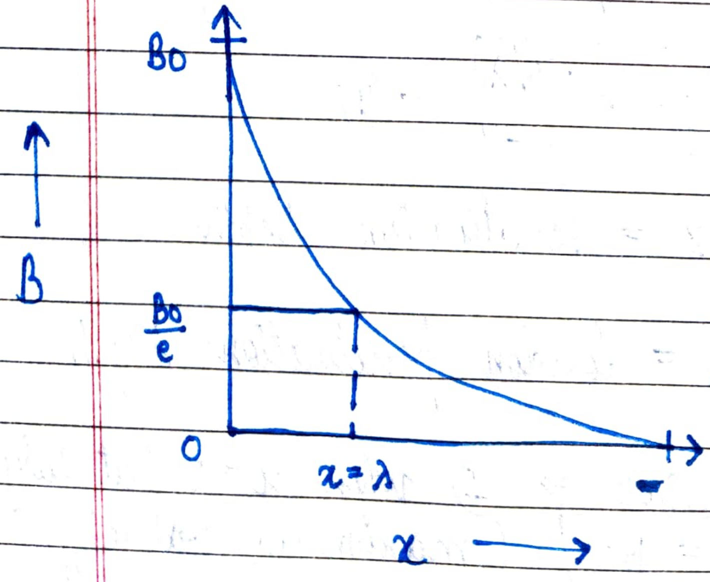
(II) when $x = \lambda$

$$B = \frac{B_0}{e}$$

- λ maybe defined as penetration depth at which magnetic field will reduce to $1/e^{th}$ value of its maximum value at the surface.

- As x increases value of B decreases exponentially and finally it will become negligible at the centre of specimen and this behaviour can be shown in the following graph.



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Ques → A Superconducting Tin has T_c of 3.7 K in 0 magnetic field and a critical field of 0.0306 Amp/m at 0 K. Find critical field at 2 K.

Solⁿ → $T = 2K$; $T_c = 3.7K$, $H_c(0) = 0.0306$

$$H_c(T) = H_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

$$H_c(2) = 0.0306\left[1 - \left(\frac{2}{3.7}\right)^2\right]$$

$$H_c(2) = 0.0306\left[1 - \frac{4}{13.69}\right]$$

$$H_c(2) = 0.0306 \times 0.708$$

$$H_c(2) = 0.0216 \text{ Amp/m}$$

(Ans)

Ques → The critical Temp. for mercury with isotopic mass 199.5 is 4.185 K. Find its critical Temp. when its isotopic Mass changes to 203.4.

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Sol^n: $T_c \propto \frac{1}{\sqrt{M}}$

Let K be a constant

$$T_{c_1} = K \times \frac{1}{\sqrt{M_1}}$$

$$K = T_c \times \sqrt{M}$$

$$K = 4.185 \times \sqrt{199.5}$$
$$K = 59.108$$

$$T_{c_2} = K \times \frac{1}{\sqrt{M_2}}$$

$$T_{c_2} = 59.108 \times 0.070$$

$$\boxed{T_{c_2} = 4.14\ K}$$

☆ BCS theory or Cooper Pair ⇒

Bardeen Cooper Schrieffer explained the behaviour of superconductors according to which there is interaction between electrons via mediating particles known as phonons.

phonons are the packets of lattice vibrations so electron - electron interaction will

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dominate over the coloumb repulsion among the electrons. Hence, an electron pair will be formed known as cooper pair.

These cooper pairs are made by the mediating particles phonons. Each cooper pair consists of two electrons having opposite movements and opposite spins. Thus superconductors are having number of cooper paires so, they show exceptional behaviour or properties.

☆ High temperature Superconductors →

These are oxide materials that behave as superconductors at high temperature.

Examples →

Material	Temperature (T_c)
$YBa_2Cu_3O_7$	92 K
$Bi_2Sr_2CuO_6$	20 K