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Q1 Given : Random Sample $(x_1 \ldots x_n)$

$$L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-u}{\sigma}\right)^2}$$

Taking ln on both sides.

$$\ln(L(\theta_1, \theta_2)) = \sum_{i=1}^{n} \left(-\frac{1}{2}\left(\frac{x-u}{\sigma}\right)^2 - \frac{1}{2}\left(\ln 2\pi\sigma^2\right) \right)$$

To find MLE, diff. log likelihood wrt θ_1, θ_2

for θ_1 ⇒ $$\frac{\partial}{\partial\theta_1} \ln L(\theta_1, \theta_2) = \sum_{i=1} \left(\frac{x_i - u}{\sigma^2}\right) = 0.$$

$$\Rightarrow \sum_{i=1} x_i - nu = 0.$$

$$\theta_1 = \frac{1}{n} \sum_{i=1}^{n} x_i \rightarrow \underline{\underline{\text{Mean}}}$$

for θ_2 ⇒

$$\frac{\partial}{\partial\theta_2} \ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left(-\frac{(x_i - \theta_1)^2}{2\theta_2^2} - \frac{1}{2\theta_2} \right) = 0$$

$$\Rightarrow \sum_{i=1}^{n} \left(\frac{(x_i - \theta_1)^2}{\theta_2}\right) - \frac{n}{\theta_2} = 0.$$

$$\frac{\theta_2^2}{\theta_2} = \frac{1}{n} \sum_{i=1}^{n} (x_i - \theta_1)^2 \Rightarrow \theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

Variance

Q ②

Find MLE of θ for a binomial distribution $B(m, \theta)$ where m is a +ve integer

Sol.

$$L(\theta) = \prod_{i=1}^{n} {}^{m}C_{x_i} \; \theta^{x_i} \; (1-\theta)^{m-x_i}$$

Taking ln

$$\ln(L(\theta)) = \sum_{i=1}^{n} \left(\ln\left({}^{m}C_{x_i}\right) + x_i \ln(\theta) + (m-x_i)\ln(1-\theta) \right)$$

$$\frac{\partial}{\partial \theta} \ln(L(\theta)) = \sum_{i=1}^{n} \left(\frac{x_i}{\theta} - \frac{m-x_i}{1-\theta} \right) = 0$$

Solve for θ

$$\sum_{i=1}^{n} \frac{x_i}{\theta} = \sum_{i=1}^{n} \frac{m-x_i}{1-\theta}$$

$$\theta \sum_{i=1}^{n} x_i = m \sum_{i=1}^{n} \theta$$

$$\theta = \frac{1}{m} \sum_{i=1}^{n} x_i$$

MLE of θ is sample mean of observation