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Decision-making in non-cooperative games with conflicting self-objectives

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Abstract

This paper concerns multicriteria decision making by players in a conflict situation. The considered situation is modelled as a non-cooperative game. Moreover, the player has a conflict not only with the opponent, but also she is faced with her own conflicting objectives. Hence, selecting a strategy is dependent on both objective preferences and the inherent uncertainty about the opponent selection. In contrast to most studies on such multiobjective games (MOGs), which employed a scalarization approach, it has recently been suggested to pose such games as MOGs with undecided objective preferences. As in Pareto optimization, the novel solution method to the considered MOGs reveals tradeoff information, which is commonly hindered when using a classical scalarization approach. To complement and highlight the significance of the aforementioned approach, two decision analysis techniques are suggested. The proposed techniques are compared with weighted-sum utility-based analysis, as well as with the Pareto-optimal security strategy approach. The comparison is done using a MOG with two competing travelling salesperson. It is concluded that the suggested techniques allow incorporating tradeoff information to justify the selection of a strategy.

KEYWORDS

competing TSP, game theory, multipayoff game

1 | INTRODUCTION

In conventional game theory, the usual assumption is that decision makers (players) make their decisions based on a scalar payoff. But in many practical problems in the fields of economics and engineering, decision makers must cope with multiple self-objectives. Furthermore, these objectives are often conflicting. When a payoff vector is considered, the game is commonly referred to as a multiobjective game (MOG) or a multipayoff game.

Most studies on MOGs employ a utility function approach (e.g., Chen & Larbani, 2006; Cook, 1976; Corley, 1985; Hannan, 1982; Li, 1999; Lozovanu, Solomon, & Zelikovsky, 2005; Smolyakov, 1994; Somasundaram & Baras, 2008; Somasundaram & Baras, 2009; H. Yu & Liu, 2012; J. Yu & Yuan, 1998; Zeleny, 1975). However, from a decision support viewpoint, utility-based approaches to MOGs lack the insight that may be gained by posing a non-cooperative MOG as a game with undecided objective preferences (Eisenstadt & Moshaiov, 2017; Eisenstadt, Moshaiov, & Avigad, 2016). In such a game, the player is undecided about her own preference of objectives and has no knowledge about the objective preferences of the opponent.

In Eisenstadt and Moshaiov (2017) and Eisenstadt, Moshaiov, and Avigad (2016), the lack of knowledge about the opponents' preferences has been approached using a solution concept that rules out strategies on the basis of Pareto optimality and worst-case considerations. The employed solution concept results with a set of rationalizable strategies (SRS) for each of the players. Each rationalizable strategy is associated with a set of payoff vectors, which has been termed as the anti-optimal front of the strategy. It accounts for all the worst-case interactions of the strategy with the strategies of the opponent.

Studies such as in Eisenstadt and Moshaiov (2017) and Eisenstadt, Moshaiov, & Avigad, (2016) focus on finding the SRS for each of the players. Once found, the SRS can be explored using Multi Criteria Decision Analysis (MCDA) techniques to select a strategy. Yet existing studies on MOGs with undecided objective preferences do not concentrate on the posteriori stage of strategy selection.

In traditional MCDA techniques, each alternative is commonly evaluated by an associated performance vector. In contrast, here, each alternative (strategy) is associated with a set of payoff vectors. Each of these vectors results from an interaction with a specific strategy of the opponent. Hence, evaluating the rationalizable strategies must be based on a comparison between sets of vectors. Moreover, the comparison must take into account that the actual performance of a strategy depends on the interaction with the selected strategy of the opponent.

In contrast to previous studies on MOGs with undecided objective preferences, this work concentrates on the a posteriori stage. It aims to provide a comprehensive discussion and demonstrations on how the results of solving MOGs with undecided objective preferences can be utilized to make a decision on a strategy. The proposed decision analysis techniques employ the revealed tradeoffs, which result from the uncertainty considerations regarding the opponent action. To highlight the significance of the proposed techniques, they are compared with weighted-sum utility-based analysis, as well as with the Pareto-optimal security strategy (POSS) approach (Ghose & Prasad, 1989a, 1989b).

The rest of this paper is organized as follows. Section 2 provides an outline of the MOG problem and its solution concept. Section 3 includes a description of the proposed MCDA methods. Next, Section 4 describes a particular MOG involving competing travelling salespersons. In Section 5, this particular MOG is used for the demonstrations of the suggested MCDA techniques. Finally, Section 6 outlines the conclusions of this study.

2 | SOLVING MOGS WITH UNDECIDED OBJECTIVE PREFERENCES

The considered game is of a normal form, which is also known as a strategic form. In such a form, the players act simultaneously and receive their payoffs, as specified for the combination of their actions (Maschler, Solan, & Zamir, 2013). The game is non-cooperative, namely, the players cannot make any agreements (Friedman, 1971). It is also zero-sum with respect to each component of the payoff vector. This means that the gain of each player in any of the objectives is the exact loss of the other player in the same objective.

In general, a game is said to be of incomplete information if the players, or at least some of them, lack full information about the basic mathematical structure of the game (Harsanyi, 1995). In the considered game, the players do not know what the opponent's objective preferences are. Therefore, the game should be categorized as of incomplete information.

In the considered MOG, each player has a finite set of pure strategies. Let the two players be denoted as P_{min} and P_{max}. Following Eisenstadt, Moshaiov, Avigad, and Branke (2016), let S_{min} and S_{max} be the sets of all possible I and J strategies for P_{min} and P_{max}, respectively, such that

$$\begin{aligned} S_{min} &= \left\{s^1_{min}, \ldots, s^i_{min}, \ldots, s^I_{min}\right\} \\ S_{max} &= \left\{s^1_{max}, \ldots, s^j_{max}, \ldots, s^J_{max}\right\} \end{aligned}. \tag{1}$$

The interaction between the ith and the jth strategies played by P_{min} and P_{max}, respectively, results in the following payoff vector:

$$\bar{f}_{i,j} = \left[f_{i,j^{(1)}}, f_{i,j^{(2)}}, \ldots, f_{i,j^{(k)}}, \ldots, f_{i,j^{(K)}}\right]^T \in \mathbb{R}^K, \tag{2}$$

where K is the number of objectives (payoffs) considered by both players. In the current example, K = 2.

The set of all interactions between a strategy s^i_{min} of player P_{min} and all of the available strategies of the second player P_{max} is the set of payoff vectors that are associated with strategy s^i_{min}:

$$F_{s^i_{min}} = \left\{\bar{f}_{i,1}, \ldots, \bar{f}_{i,j}, \ldots, \bar{f}_{i,J}\right\}. \tag{3}$$

In the same way, the set of the associated performances of strategy s^j_{max} of the maximizer is

$$F_{s^j_{max}} = \left\{\bar{f}_{1,j}, \ldots, \bar{f}_{i,j}, \ldots, \bar{f}_{I,j}\right\}. \tag{4}$$

As suggested in Eisenstadt, Moshaiov, Avigad, and Branke (2016), the associated performances can be analysed to find all rationalizable strategies, from which a strategy can be selected.

Each player takes a worst-case approach. Namely, she takes into account the best alternatives of the opponent. Hence, the problem could be stated as a MiniMax problem for P_{min} and as a MaxiMin problem for P_{max}. In other words:

The problem of P_{min}:

$$\min_{s^i_{min} \in S_{min}} \max_{s^j_{max} \in S_{max}} \bar{f}_{i,j}. \tag{5}$$

The problem of P_{max}:

$$\max_{s^j_{max} \in S_{max}} \min_{s^i_{min} \in S_{min}} \bar{f}_{i,j}. \tag{6}$$

However, the above are ill-defined. When applied on a vector, the operators (max and min) require an interpretation. Based on Eisenstadt, Moshaiov, Avigad, and Branke (2016), the meaning of these operators, for the case of a MOG under undecided objective preference, is provided in this section.

The MiniMax method for single objective games is a decision rule for minimizing the maximal loss. However, in the considered MOG, with undecided objective preferences, there is no one way to determine which strategy minimizes the maximal loss. One strategy may be the one that minimizes the maximal loss in a certain combination of player's preferences, whereas in a different combination of preferences, another strategy will be the solution. Therefore, in such a MOG, there is no single optimal strategy. Instead, it is possible to eliminate all strategies that cannot, under any circumstances, be the preferred solution of the MOG.

The above leads to the solution concept of rationalizability. Namely, the players view as irrational the strategies which are never a best response to any strategy of the opponent. Here, the strategies are compared using a domination relation among sets of payoff vectors. The comparison allows filtering out irrational strategies.

When evaluating the ith strategy of P_{min}, there are J possible interactions with the strategies of P_{max} to consider. Given that P_{max} has not decided what are his objective preferences, then there is a set of nondominated payoff vectors (in a maximization problem), which corresponds to all the best responses of P_{max} to the ith strategy of P_{min}.

The anti-optimal front of strategy $s^i_{\min}$ of the minimizer is the set of all performances due to the best responses of the maximizer to that strategy. It is defined as

$$F^{-*}_{s^i_{min}} = \left\{ \bar{f}_{i,j} \in F_{s^i_{min}} \left| \neg \exists \bar{f}_{i,j'} \in F_{s^i_{min}} : \ \bar{f}_{i,j'} \succ^{max} \bar{f}_{i,j} \right\} \right. \quad \forall j, j', \tag{7}$$

where $\bar{f} \succ^{max} \bar{h}$ (or $\bar{f} \succ^{min} \bar{h}$) means vector $\bar{f}$ dominates vector $\bar{h}$ in a maximization (or minimization) problem.

The set of all the anti-optimal fronts of P_{min} is a set of sets, as defined below.

$$F^{-*}_{min} = \left\{ F^{-*}_{s^1_{min}}, \ldots, F^{-*}_{s^i_{min}}, \ldots, F^{-*}_{s^I_{min}} \right\}. \tag{8}$$

Sorting all the member sets of F^{-*}_{min} and selecting the dominating sets in the maximization problem will result in the set of the minimizer's *irrational* strategies S^{irr}_{min}:

$$S^{irr}_{min} := \left\{ s^i_{min} \in S_{min} \left| \exists s^{i'}_{min} \in S_{min} \ F^{-*}_{s^i_{min}} \succ^{max} F^{-*}_{s^{i'}_{min}} \right. \quad \forall i, i' \in \{1, \ldots, I\} \right\}, \tag{9}$$

where $F \succ^{max} H$ (or $F \succ^{min} H$) means that set F dominates set H in a maximization (or minimization) problem, according to the definitions of set domination in Zitzler, Thiele, Laumanns, Fonseca, and Da Fonseca (2003), which states $F \succ^{max} H := \left\{ \forall \bar{h} \in H \exists \bar{f} \in F | \bar{f} \succ^{max} \bar{h} \right\}$.

The SRS of P_{min} is the relative complement of S_{min} and S^{irr}_{min}, namely:

$$S^R_{min} = S_{min} \ S^{irr}_{min} \tag{10}$$

The set S^R_{min} includes all strategies of the minimizer that are associated with a nondominating anti-optimal front in the maximization problem where the cardinality of S^R_{min} is $\left| S^R_{max} \right| = I'$ and $1 \le I' \le I$. It is noted that this set is nonempty, which results from the fact that the considered game involves finite sets of strategies. Each of the rationalizable strategies is represented in the objective space by its related anti-optimal front. The set of all the anti-optimal fronts which are associated with all the rationalizable strategies form a set of sets in the objective space, which is termed as the rationalizable layer.

Note that the above equations are given for P_{min}. Parallel definitions from the perspective of P_{max} are provided in the following. The anti-optimal front of the jth strategy of P_{max} is

$$F^{-*}_{s^j_{max}} = \left\{ \bar{f}_{i,j} \in F_{s^j_{max}} \left| \neg \exists \bar{f}_{i',j} \in F_{s^j_{max}} : \ \bar{f}_{i',j} \succ^{min} \bar{f}_{i,j} \right\} \right. \quad \forall i, i'. \tag{11}$$

The set of irrational strategies of P_{max} is

$$S^{irr}_{max} := \left\{ s^j_{max} \in S_{max} \left| \exists s^{j'}_{max} \in S_{max} \ F^{-*}_{s^j_{max}} \succ^{min} F^{-*}_{s^{j'}_{max}} \right. \quad \forall j, j' \in \{1, \ldots, J\} \right\}. \tag{12}$$

The SRS of P_{max} is

$$S^R_{max} = S_{max} \ S^{irr}_{max}. \tag{13}$$

Finally, it should be renoted that the considered MOG involves finite sets of strategies. Yet the proposed approach can also be used with infinite sets, provided that conditions of existence are met. Such conditions are yet to be investigated. With this respect, it is noted that a review on studies concerning the existence of Pareto solutions can be found in Miettinen (1998).

It is further noted that many practical MOGs are expected to have no analytical solutions. In such a case, numerical optimization methods should be used. For MOGs involving infinite or unmanageable size of strategy sets, finding the theoretical SRS by a numerical technique cannot be guaranteed in general. Yet the procedures that are presented here are expected to help finding a satisficing SRSs for such cases.

3 | STRATEGY SELECTION TECHNIQUES

MCDA deals with making a decision under the presence of more than one criterion. It concerns issues and dilemmas such as conflicting criteria, incomparable units, and methods for expressing preferences (Greco, Figueira, & Ehrgott, 2005).

The current paper focuses on multicriteria decision making on a strategy to be selected out of the SRS. As emphasized in Section 1, in contrast to the traditional multiobjective problems, in the concerned MOG, each solution (a strategy) is to be evaluated by a *set* of vectors (anti-optimal front). Therefore, methods as listed in Greco et al. (2005) cannot be used here. This means that special selection methods should be devised.

To support the selection of a strategy out of the SRS, the rationalizable strategies should be ordered or partially ordered. A major aspect of the proposed selection approach should be that the Decision-Maker (DM) will be able to consider the tradeoffs which are associated with the selection.

In the following, two partial ordering techniques are suggested. Both of them are based on nondomination ranking in an auxiliary objective space. The auxiliary objectives take into an account performance information on the strategies that is revealed by their associated anti-optimal fronts. The proposed techniques aim at reducing the SRS into a set of preferred strategies (SPS). The SPS of each player is taken as the Pareto-optimal set of strategies based on the auxiliary-objective space. Next, each player may select a strategy out of the SPS, using her auxiliary objective preferences.

The first technique, which is presented in Section 3.1, involves measures of sensitivity and of distance from a reference point. The second technique, which is presented in Section 3.2, involves a weighted-sum grade and a set deviation from an aspired constraint. The former technique is hereby termed as the Sensitivity and Distance (SD) method, whereas the latter one is termed as the Weighted-sum and Aspired Constraint (WAC) method.

It should be noted that there could possibly be other ordering methods. The suggested methods appear to be good examples of using the information that is obtained by the assumption that the players are undecided about their preference of objectives. In particular, the proposed methods consider the worst of the worst when taking into account the information in the anti-optimal fronts.

3.1 | The SD method

The proposed SD method involves two auxiliary criteria that are based on $\mathcal{M}_{sens}$ and $\mathcal{M}_{dist}$ measures, as illustrated in Figure 1 and described below. The first measure $\mathcal{M}_{sens}$ aims to evaluate the sensitivity of a strategy to the actions of the opponent. The second measure $\mathcal{M}_{dist}$ aims to assess the optimality of a strategy. It is noted that the rationale of the SD method is described following the description of the measures.

To compute the sensitivity measure of a strategy $\mathcal{M}_{sens}$, it is necessary to find the K extreme payoff vectors of the anti-optimal front of the strategy. Such vectors are denoted by white filled circles in Figure 1 for a biobjective game. For a strategy of the minimizer, the kth extreme payoff vector is the vector that maximizes the kth objective:

$$\bar{f}^{k}_{ext}\left(s^{i}_{min}\right) = \operatorname{argmax}_{\bar{f}_{i,j} \in F^{-*}_{s^{i}_{min}}} \left\{ f_{i,j^{(k)}} \right\}. \tag{14}$$

The sensitivity of strategy s^{i}_{min} is defined as the maximal Euclidean distance between all the extreme vectors of s^{i}_{min}:

$$\mathcal{M}_{sens}\left(s^{i}_{min}\right) = \max_{k,k' \in [1,K]} \left\| \bar{f}^{k}_{ext}\left(s^{i}_{min}\right) - \bar{f}^{k'}_{ext}\left(s^{i}_{min}\right) \right\|_{2}. \tag{15}$$

In the same manner, the extreme vectors of a strategy of the maximizer are

$$\bar{f}^{k}_{ext}\left(s^{j}_{max}\right) = \operatorname{argmin}_{\bar{f}_{i,j} \in F^{-*}_{s^{j}_{max}}} \left\{ f_{i,j^{(k)}} \right\}, \tag{16}$$

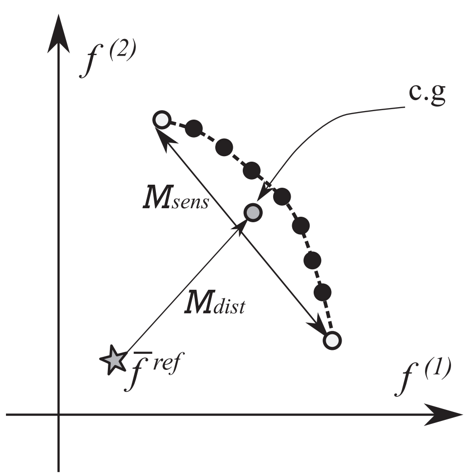

FIGURE 1 Illustration for the SD method for the minimizer. The vectors of the anti-optimal front are denoted by black dots, whereas the white dotes are the extreme vectors of the front. The grey star marks the reference vector. The grey dot is the centre of gravity of the front

and the sensitivity of strategy s^{j}_{max} of the maximizer will be the maximal Euclidean distance between all the extreme vectors of s^{j}_{max}:

$$\mathcal{M}_{sens}\left(s^{j}_{max}\right) = \max_{k,k' \in [1,K]} \left\| \bar{f}^{k}_{ext}\left(s^{j}_{max}\right) - \bar{f}^{k'}_{ext}\left(s^{j}_{max}\right) \right\|_{2}. \tag{17}$$

The second metric $\mathcal{M}_{dist}$ is defined below. It is based on a distance between two points as illustrated in Figure 1. For the minimizer, the first point is the centre of gravity of the anti-optimal front $F^{-*}_{s^{i}_{min}}$ of strategy s^{i}_{min}, which is

$$\bar{f}_{c.g}\left(s^{i}_{min}\right) = \frac{1}{\left|F^{-*}_{s^{i}_{min}}\right|} \sum_{\bar{f}_{i,j} \in F^{-*}_{s^{i}_{min}}} \left\{ \bar{f}_{i,j} \right\}, \tag{18}$$

where $\left|F^{-*}_{s^{i}_{min}}\right|$ is the cardinality of the anti-optimal front. The second point is a reference point, $\bar{f}^{ref}_{min}$, which is to be selected such that $\bar{f}^{ref}_{min} \succ^{min} \bar{f}_{i,j}, \forall i \; and \forall \bar{f}_{i,j} \in F^{-*}_{s^{i}_{min}}$.

The distance measure of a strategy s^{i}_{min} is

$$\mathcal{M}_{dist}\left(s^{i}_{min}\right) = \left\| \bar{f}_{c.g}\left(s^{i}_{min}\right) - \bar{f}^{ref}_{min} \right\|_{2}. \tag{19}$$

In the same manner, the centre of gravity of the anti-optimal front $F^{-*}_{s^{j}_{max}}$ of strategy s^{j}_{max} of the maximizer is

$$\bar{f}_{c.g}\left(s^{j}_{max}\right) = \frac{1}{\left|F^{-*}_{s^{j}_{max}}\right|} \sum_{\bar{f}_{i,j} \in F^{-*}_{s^{j}_{max}}} \left\{ \bar{f}_{i,j} \right\}, \tag{20}$$

and the distance measure of strategy s^{i}_{min} is

$$\mathcal{M}_{dist}\left(s^{j}_{max}\right) = \left\| \bar{f}_{c.g}\left(s^{j}_{max}\right) - \bar{f}^{ref}_{max} \right\|_{2}, \tag{21}$$

where $\bar{f}^{ref}_{max}$ is to be selected such that $\bar{f}^{ref}_{max} \succ^{max} \bar{f}_{i,j}, \forall i \; and \forall \bar{f}_{i,j} \in F^{-*}_{s^{j}_{max}}$.

To select a strategy, each player may define a bi-objective SD auxiliary problem, which is a min–min problem with the objectives $\mathcal{M}_{sens}$ and $\mathcal{M}_{dist}$ to be minimized.

The rationale of the SD method is that a player has to take into account the uncertainty associated with the opponent action. In a game situation, the DM (player) cannot control the payoff when choosing a strategy. This is due to the fact that the payoff is also a result of the strategy selection by the opponent. Moreover, in the considered game, none of the players has information about the preference of the opponent. When selecting a strategy, a player is left with the performance uncertainty, which is reflected in the associated anti-optimal front.

Given that the player has no information on the preferences of the opponent, it appears logical to assume the same probability for any payoff vector that belongs to the anti-optimal front. In such a case, $\bar{f}_{c.g}$ reflects an average of the worst performance vectors, and its distance $\mathcal{M}_{dist}$ from a reference goal provides a measure of optimality under the given uncertainties. A larger $\mathcal{M}_{dist}$ means that the strategy is less optimal with respect to a reference goal. On the other hand, $\mathcal{M}_{sens}$ reflects the range of possible performances. A larger range means that the strategy is more sensitive to the uncertainties of the opponent decision. It appears logical that both players will prefer a less sensitive strategy and, at the same time, a strategy with a smaller distance from the reference goal.

3.2 | The WAC Method

The proposed WAC method involves two auxiliary criteria that are based on $\mathcal{M}_{ws}$ and $\mathcal{M}_{ac}$ measures, which are described below. The first measure, $\mathcal{M}_{ws}$, aims to evaluate the worst weighted-sum utility of the strategy, given a preference of objectives. The second measure, $\mathcal{M}_{ac}$, aims to assess the deviation of the worst possible performance of the strategy from an aspired constraint, in a selected objective.

Assume that the weight vector $\overline{w}_{min} \in \mathcal{R}^K_{\geq 0}$ expresses the minimizer's objective preferences. In such a case, the weighted-sum utility of a payoff vector $\overline{f}_{i,j}$ for the minimizer is the inner product:

$$u_{min}\left(\overline{f}_{i,j}, \overline{w}_{min}\right) = \left\langle \overline{f}_{i,j}, \overline{w}_{min} \right\rangle. \tag{22}$$

The weighted-sum grade, $\mathcal{M}_{ws}$, of strategy s^i_{min} of the minimizer is the maximal (worst) u_{min} which is

$$\mathcal{M}_{ws}\left(s^i_{min}\right) = \max_{\overline{f}_{i,j} \in F^*_{s^i_{min}}} \left\langle \overline{f}_{i,j}, \overline{w}_{min} \right\rangle. \tag{23}$$

Similarly, the weighted-sum utility of a payoff vector $\overline{f}_{i,j}$ for the maximizer with a weight vector $\overline{w}_{max} \in \mathcal{R}^K_{\geq 0}$ is the inner product:

$$u_{max}\left(\overline{f}_{i,j}, \overline{w}_{max}\right) = \left\langle \overline{f}_{i,j}, \overline{w}_{max} \right\rangle, \tag{24}$$

and the weighted-sum grade, $\mathcal{M}_{ws}$, of strategy s^j_{max} of the maximizer is the minimal (worst) u_{max} such that

$$\mathcal{M}_{ws}\left(s^j_{max}\right) = \min_{\overline{f}_{i,j} \in F^*_{s^j_{max}}} \left\langle \overline{f}_{i,j}, \overline{w}_{max} \right\rangle. \tag{25}$$

The second measure, $\mathcal{M}_{ac}$, is based on an aspired constraint on the performance in a selected objective. The aspired constraint of the minimizer is denoted as $f_{min^{(k)}}$, where k denotes her selected objective. The deviation is defined as $f_{i,j^{(k)}} - f_{min^{(k)}}$. Note that negative deviation means that the minimizer does not violate the constraint and vice versa.

The measure of deviation for the minimizer, which is illustrated in Figure 2, is

$$\mathcal{M}_{ac}\left(s^i_{min}\right) = \max_{\overline{f}_{i,j} \in F^*_{s^i_{min}}} \left\{ f_{i,j^{(k)}} - f_{min^{(k)}} \right\}. \tag{26}$$

The aspired constraint of the maximizer is denoted as $f_{max^{(k)}}$, and the deviation is defined as $f_{max^{(k)}} - f_{i,j^{(k)}}$. Note that, similar to the case of the minimizer, a negative deviation means that the maximizer does not violate the constraint.

The measure of deviation for the maximizer is

$$\mathcal{M}_{ac}\left(s^j_{max}\right) = \max_{\overline{f}_{i,j} \in F^*_{s^j_{max}}} \left\{ f_{max^{(k)}} - f_{i,j^{(k)}} \right\}. \tag{27}$$

To select a strategy, the WAC auxiliary problem is defined, based on the auxiliary objectives $\mathcal{M}_{ws}$ and $\mathcal{M}_{ac}$, as a min–min problem for the minimizer and as a max–min problem for the maximizer.

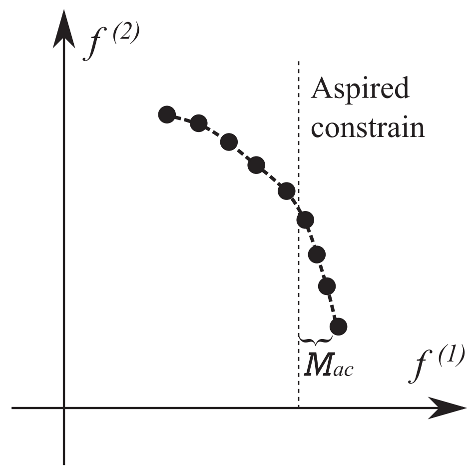

FIGURE 2 Illustrating the aspired constraint for the minimizer. The vectors of the anti-optimal front are denoted by black dots

The rationale of the WAC method is that although expressing a certain preference of objectives via the weights, the player may also be concerned with the uncertainty that is associated with a strategy selection. For the above purpose, the $\mathcal{M}_{ac}$ measure is defined. It measures the worst possible deviation from an aspired constraint under the uncertainty of the opponent action, as revealed by the anti-optimal front of the strategy. If the player has an aspired constraint, then the proposed method allows its incorporation in the decision making. The player may be willing to consider a deviation from the aspired constraint if it yields some improvement in the weighted-sum grade, $\mathcal{M}_{ws}$,and vice versa. Such a tradeoff analysis between the worst-case utility and deviation is supported by the auxiliary space. Finally, it should also be noted that determining the weights (objective preference) prior to the optimization procedure conceals the tradeoff information that is critical for justifying the strategy selection under the uncertainties of the MOG.

4 | THE TRAVELLING SALESMAN PROBLEM AS A MOG

The travelling salesman problem (TSP) is a famous and widely studied combinatorial optimization problem. As apparent from reviews such as in Gutin and Punnen (2006), the classical TSP has been modified over the years into several variations. The variations which are most relevant to the current paper are described below. As in the original TSP, in all the considered versions, the problem is represented as a graph. The graph consists of cities (vertices) with known coordinates and roads (edges), which connect the cities.

4.1 | Multiple objectives in travelling salesperson problems

Treating the problem of TSP as a multiobjective problem is not new. There are two main types of such problems, which have been dealt with. The first type is a version of the classical TSP, which is commonly termed as multiobjective TSP. In this case, the salesperson has to visit each city exactly once. However, in contrast to the classical TSP, which is a single-objective problem, the multiobjective version involves more than one cost for travelling an edge. These are commonly conflicting costs; hence, a Pareto-optimal set is expected. Among such works are Agrawal (2011); Angel, Bampis, and Gourvès (2004); Changdar, Mahapatra, and Kumar Pal (2014); Luo, Liu, Hao, and Liu (2014); Lust (2008); Lust and Teghem (2010); Shim, Tan, Chia, and Chong (2011); Shim, Tan, and Tan (2012); and Zhou, Gao, and Zhang (2013). The second type of multiobjective TSP is the selective TSP (STSP), also known as TSP with profits. In this type of a problem, each city is associated with a profit, and each edge is associated with a cost. Here, the salesperson is free to choose which cities to visit in order to maximize the profit while minimizing the travelling cost. Among such studies are Fekete, Fleischer, Fraenkel, and Schmitt (2004); Filippi and Stevanato (2013); Grimme, Meisel, Trautmann, Rudolph, and Wölck (2015); Keller and Goodchild (1988); Kendall and Li (2012); and Mohtadi and Nogondarian (2014).

4.2 | TSP as a competitive game problem

When there is more than one salesperson, the TSP is posed as a game between salespersons, which may belong to either a competitive game type (e.g. Fekete et al., 2004; Kendall & Li, 2012) or to a cooperative game type (e.g., Mohtadi & Nogondarian, 2014; Qu, Yi, & Tang, 2007). Salespersons often have to compete with those of rival companies. Competing salespersons problems may include two or more competing agents. A competition between two salespersons has been studied in Fekete et al. (2004). Each one of the competitors aims at reaching any of the cities before the opponent does. In Fekete et al. (2004), it is assumed that the first salesperson to arrive a city wins all the customers at this location. A similar problem, which is termed competitive TSP, is described in Kendall and Li (2012). Each salesperson receives a payoff if first to visit a city, while paying a travelling cost proportional to the distance that was travelled to reach that city. In this case, each player aims to be the first to visit as many cities as possible with the minimum travelling distance. This is an extensive form game with perfect information. In such a case, both players move in turns by changing from the current city to the next one.

As in Kendall and Li (2012), in the current study, a conflict exists among not only the players, but also each player is faced with her own conflicting objectives.

4.3 | The TSP as a MOG

The considered TSP, which is based on Eisenstadt, Moshaiov, and Avigad (2016), is an amalgamation of selective TSP and competitive TSP types. It concerns a MOG between two salespersons that compete over markets which are located in different cities. Each of the salespersons has multiple conflicting objectives (two in the current example), while choosing a route among her selective set of cities. The basic assumption is that neither of the players knows what the chosen route of the opponent is. The considered game is a game of pure strategy, single-act, non-cooperative, and with imperfect information.

The game arena is presented as a graph. The considered graph contains N vertices (cities), where each vertex represents a city from the set of cities $C = \{c(1), c(2), \ldots, c(n), \ldots, c(N)\}$. Each city $c(n)$ has a value $v(c(n))$. This value represents the profit of the first salesperson that arrives to that city. The arcs of the graph represent the roads between the cities. The arc value is the road length.

The game is between two competing salespersons (players), which are denoted by P_{min} and P_{max}. A strategy of a player, which is a chosen route, is defined as a partial permutation of the cities' set C. Each player may visit a city no more than once and returns to the first city at the end of the path. The routes of the players are described as the ordered sets $Path_{min} = \left\{c_{min^{(1)}}, c_{min^{(2)}}, \ldots, c_{min^{(N_{min})}}, c_{min^{(1)}}\right\}$ and $Path_{max} = \{c_{max^{(1)}}, c_{max^{(2)}}, \ldots, c_{max^{(N_{max})}}, c_{max^{(1)}}\}$, where $1 \leq N_{min}$, N_{max}- N for the first and second player, respectively. Each element $c_{min^{(q)}}$ or $c_{max^{(p)}}$ is associated with a member city in the set C. Namely, $c_{min^{(q)}}$, $c_{max^{(p)}} \in C \forall q, p$. Also, $c_{min^{(q)}} \neq c_{min^{(q')}} \forall q, q,$ and $c_{max^{(p)}} \neq c_{max^{(p')}} \forall p, p'$. It is noted that the subscripts min and max indicate which player visited that city. On the other hand, the superscripts indicate the order by which the player visited it.

Each selected path has a length that is calculated as the sum of the distances between all successive cities of the path. The distance between two successive cities of the P_{min} route is denoted as $d\left(c_{min^{(q)}}, c_{min^{(q+1)}}\right)$. Namely, the route lengths of P_{min} and P_{max} are

$$L_{min} = length(Path_{min}) = \sum_{n=2}^{N_{min}} d\left(c_{min}^{(n-1)}, c_{min}^{(n)}\right) + d\left(c_{min}^{(N_{min})}, c_{min}^{(1)}\right), \tag{28}$$

and

$$L_{max} = length(Path_{max}) = \sum_{n=2}^{N_{max}} d\left(c_{max}^{(n-1)}, c_{max}^{(n)}\right) + d\left(c_{max}^{(N_{max})}, c_{max}^{(1)}\right). \tag{29}$$

When considering a path, each player takes into account not only the path length but also the value of the chosen route.

In the considered game, the value of a city, $v(c(n)) \in [v_{min}, v_{max}]$ is taken proportional to its market size. The salesperson with the shortest route to a city (the first to arrive) earns the city's value. If both salespersons arrive together to city $c(n)$, then each one earns $\frac{1}{2}v(c(n))$. The values of the routes of the players are

$$V_{min} = value(Path_{min}) = \sum_{\substack{c(n) \in Pathe_{min} \\ \wedge l_{min}(c(n)) < l_{max}(c(n))}} v(c(n)) + \sum_{\substack{c(n) \in Pathe_{min} \\ \wedge l_{min}(c(n)) = l_{max}(c(n))}} \frac{1}{2}v(c(n)), \tag{30}$$

and

$$V_{max} = value(Path_{max}) = \sum_{\substack{c(n) \in Pathe_{max} \\ \wedge l_{max}(c(n)) < l_{min}(c(n))}} v(c(n)) + \sum_{\substack{c(n) \in Pathe_{max} \\ \wedge l_{max}(c(n)) = l_{min}(c(n))}} \frac{1}{2}v(c(n)), \tag{31}$$

where $l_{min}(c(n))$ and $l_{max}(c(n))$ are the route lengths from the first city of each player to city $c(n)$. It is noted that when calculating the path value (Equations (30) and (31)), if a city is not in the route of a player, then the route length to this city is taken as infinite, namely, $l(c(n)) = \infty\ \ \forall\ c(n) \notin Path$.

Here, each player is interested not only in shortening her path, while collecting the highest value, but also in causing the opponent the maximal damage. Namely, each player aims at minimizing her own path length while maximizing the opponent path length and collecting the maximal value while causing the opponent to collect the minimal value. The rationale is that each player wants not only to increase her profit but also to cause some damage to the opponent with the hope to eventually cause the rival to avoid the considered markets. Therefore, the payoff vector components are defined as

$$f^{(1)} = L_{min} - L_{max}\ \ f^{(2)} = V_{max} - V_{min}. \tag{32}$$

4.4 | Implementation aspects

The classical TSP is Nondeterministic Polynomial time (NP-hard), let alone the type of TSP presented here. In the considered problem with N cities, the number of possible paths of one salesperson, which starts the path from a given city, is

$$\Omega = \sum_{n=1}^{N} \frac{(N-1)!}{(N-n)!}. \tag{33}$$

As the problem presented here is a game, then the number of all possible interactions is Ω^2. Considering a small problem with $N = 10$, the number of all possible paths for each player is $\Omega = 986409$, and the number of all possible interactions is $\Omega^2 = 9.73 \cdot 10^{11}$.

As presented in Section 5, to demonstrate the proposed method for multiple criteria decision making on a strategy, two different arenas are devised, each with $N = 5$ ($\Omega = 326$ and $\Omega^2 = 106276$). This allows full sorting of all the interactions and finding the exact SRS at a reasonable time. For larger problems, specific search algorithms should be developed (e.g., Harel, Matalon-Eisenstadt, & Moshaiov, 2017; Zitzler et al., 2003).

5 | CASE STUDIES

This section aims to demonstrate the proposed selection techniques. In addition, to highlight the significance of making a selection out of the SRS, the results of the proposed selection techniques are compared with those obtained by strategy selection using a weighted-sum approach and with those obtained by the POSS approach (see Appendix A).

First, Section 5.1 provides a description of two arenas and associated MOGs between competing travelling salespersons. Next, in Sections 5.2 and 5.3, the results are provided of solving the first and second game, respectively.

5.1 | The considered arenas

Two arenas are presented in Figure 3. For the sake of simplicity and given that the graph is complete, the arcs between the cities are

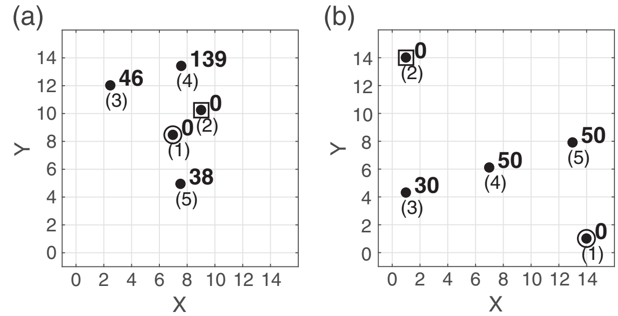

FIGURE 3 Arenas 1 and 2 (left and right, respectively). The five cities are denoted by black dots. The index of each city is presented in parentheses below the city and the other number (top left to each city) presents the city value. The first cities of the minimizer and maximizer are encircled by square and circle, respectively

omitted from the figure. Nevertheless, each distance between two cities may be extracted from the figure as the Euclidean distance.

The arenas have five cities ($N = 5$) which are marked by black circles. The first cities of P_{max} and P_{min} (Cities 1 and 2) are encircled by a circle and a square, respectively.

5.2 | Results of arena No. 1

5.2.1 | The rationalizable strategies of arena No. 1

In this arena, the maximizer has five rationalizable strategies, and the minimizer has three. The five rationalizable strategies (paths) of the maximizer are shown in Figure 5. These paths are marked by numbers including 1, 3, 5, 10, and 11. Their associated anti-optimal fronts are shown in Figure 4.

It can be seen from Figure 4 that, as defined in Equation (12), none of the anti-optimal fronts of a rationalizable strategy dominates another anti-optimal front.

Considering path No. 1 of the maximizer (see Panel a of Figure 5), it is noted that this strategy reflects the extreme case in which the player prefers minimizing her path length. In contrast, Panel d of Figure 5 (path No. 10) reflects a case in which the collected value is the leading objective. When comparing the associated anti-optimal fronts of these strategies, it can easily be observed that the position

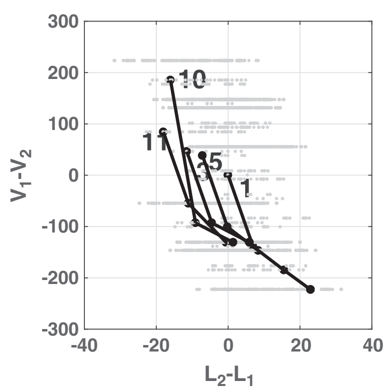

FIGURE 4 The rationalizable layer of P_{max} of arena No. 1. The associated numbers are the path indices. The small grey dots are all the payoff vectors from all the interactions between the minimizer and maximizer

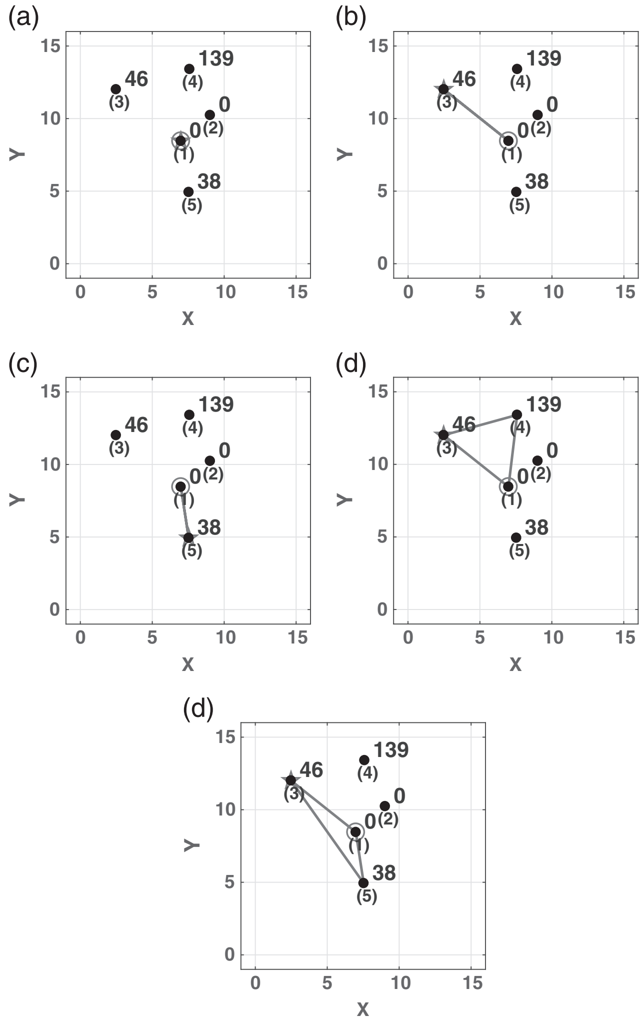

FIGURE 5 The five rationalizable strategies of P_{max} in arena No. 1. The first and second cities of the maximizer are denoted by a circle and a star, respectively. Path No. 1 in panel (a), path No. 3 in panel (b), path No. 5 in panel (c), path No. 10 in panel (d) and path No. 11 in panel (e)

of the front of path No. 1, as compared with that of path No. 10, is in agreement with the associated objective preferences.

The following provides some details and analysis of one specific strategy of the maximizer. Namely, path No. 11, {1, 3, 5, 1}. Figure 6 depicts the anti-optimal front of this path, whereas Figure 7 shows all the associated interactions.

The anti-optimal front of path No. 11 consists of the worst performances (from P_{max} viewpoint), which results from interactions with paths No. 1, 4, 14, and 17 of P_{min}.

For example, the payoff vector (−17.99, 84) is the result of interaction with path No. 1 of the minimizer (see Panel a of Figure 7), in which P_{min} stays in her first city (City 2). In such a case, although V_{min} = 0, the length is L_{min} = 0. This means that P_{min} is willing to earn no value in order to earn the shortest path.

The payoff vector (−11.05, −55) is the result of interaction with path No .4 (see Panel b of Figure 7). Here, P_{min} travels to the nearest city with the highest value (139).

The remaining payoff vector is (−0.54, −131). This is the result of either the interaction with path No .14 or with No .17 of P_{min}. These two paths are the two permutations {2, 4, 5, 2} and {2, 5, 4, 2} (see

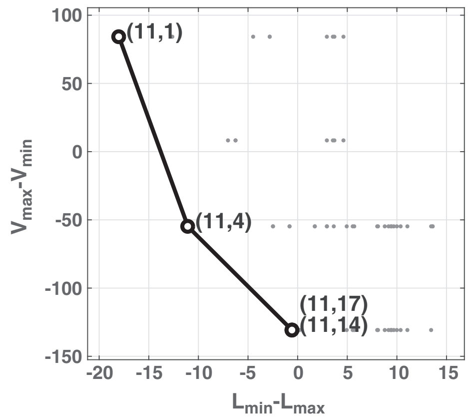

FIGURE 6 The anti-optimal front of path No. 11. The payoff vectors of all the interactions of this strategy are denoted by small grey dots, and the three payoff vectors of the front are marked by white filled circles. The numbers near each payoff vector are the indexes of the interacting strategies that created this vector

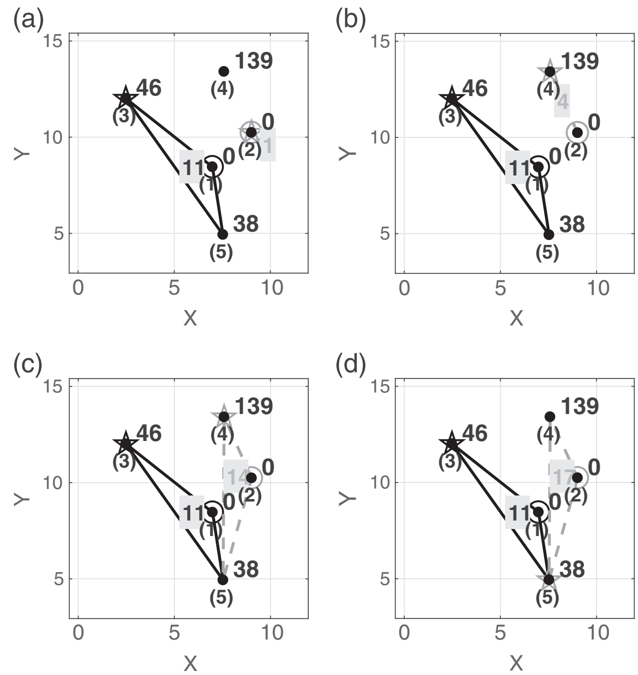

FIGURE 7 The interactions of the rationalizable strategy s_{max}^{11} that form its anti-optimal front. The paths of the maximizer and minimizer are denoted by black and grey dashed lines, respectively. The first city in each path is marked by circle, and the second by star with the corresponding colour. The interaction with the minimizer's path No. 1 in panel (a), path No. 4 in panel (b), path No. 14 in panel (c), and with path No. 17 in panel (d)

Panels c and d of Figure 7, respectively). It is noted that the latter two strategies are irrationalizable strategies of P_{min}. However, from the worst-case approach, it is assumed that they might be selected by the minimizer.

In each of the cases of interacting with path No. 14 or with No. 17 of the minimizer, both players arrive to a common city (City 5). In the case of path No. 14, P_{min} starts travelling to City 4 and then to City 5, whereas in the case of path No .17, P_{min} first travels to City 5 and then to City 4. In both cases, P_{min} arrives to City 5 before P_{max}. Therefore, P_{max} earns the value of City 5, whereas the opponent does not. This is also the reason for the lowest value for P_{min} (see Figure 6).

5.2.2 | Strategy selection in arena No. 1

Figures 8 and 9, respectively, present the SD and WAC auxiliary performances for the first arena.

The left and right panels of Figure 8 show the SD performances of the minimizer and the maximizer, respectively. The left and right panels of Figure 9 depict the WAC performances of the minimizer and maximizer, respectively.

Based on the results presented in the right panel of Figure 8, the maximizer will choose one of the four paths 3, 5, 10, or 11. These strategies are in the Pareto front of the auxiliary space, and therefore they constitute the SPS of the maximizer in this arena based on the SD strategy selection technique. It is noted that only Path 1 is not included in this front.

Path No. 3 ($\{1,3,1\}$) and No. 5 ($\{1,5,1\}$) are very similar in their auxiliary performances. The anti-optimal fronts of both paths are characterized by a relatively large distance from the reference point and small sensitivity. On the other hand, if the maximizer is interested in the best distance (smaller) she will choose path No. 10 ($\{1,3,4,1\}$). However, there is no free lunch. Improving the distance comes with an increased sensitivity to the action of the opponent.

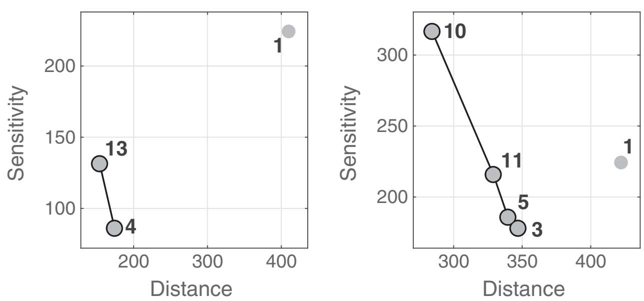

FIGURE 8 The SD auxiliary spaces. The minimizer's and maximizer's results are on the left and right panels, respectively. The auxiliary performance vectors of all the rationalizable strategies are denoted by grey filled circles. The auxiliary performance vectors of the set of preferred strategies are connected by a black line

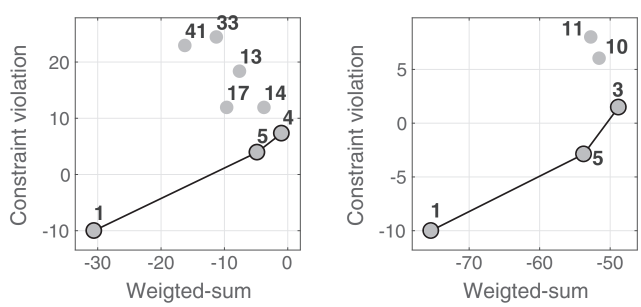

FIGURE 9 The WAC auxiliary spaces. The minimizer's and maximizer's results are on the left and right panels, respectively. The auxiliary performance vectors of all the rationalizable strategies are denoted by grey filled circles. The auxiliary performance vectors of the set of preferred strategies are connected by a black line

The auxiliary WAC performances, which are presented in Figure 9, are based on the weight-vectors $w_{min} = w_{max} = [0.6, 0.4]$ and the aspired constraints $f_{min^{(1)}} = 20$ and $f_{max^{(1)}} = -10$. If the maximizer will use the WAC approach, then she may choose one of the three strategies of the SPS, Paths 1, 3, or 5, as can be observed from the right panel of Figure 9.

In contrast to the above, when considering the weighted-sum results only, the performance of path No. 1 is the worst one. In the WAC auxiliary decision space, path No. 1 is included in the SPS. This is due to the fact that its anti-optimal front is the best with respect to the aspired constraint. In fact, this path is far from violating the aspired constraint. If the maximizer is willing to compromise on the constraint violation grade, she can choose path No. 5. This alternative strategy offers a better weighted-sum grade at the cost of a small violation of the aspired constraint.

In the following, the proposed MCDA approaches are compared with the POSS approach as applied to the first arena. The set of POSS includes Strategies 1, 3, and 5. Namely, the vectors of security levels of these strategies dominate the other vectors in a maximization problem (see the right panel of Figure 10). Note that according to Ghose and Prasad (1989a, 1989b), the selection of a strategy out of the POSS is done by the weighted-sum approach. Note that if the maximizer employs the weight-vector, as in the WAC approach ($w_{max} = [0.6, 0.4]$), then the selected POSS is path No. 3.

5.3 | Results of arena No. 2

In contrast to arena No. 1, in arena No. 2, there is no city with a substantially higher value than the others. Hence, it appears that in arena No. 2, the rationalizable strategies are less intuitive. Table 1 lists all the rationalizable strategies of each player for the current arena.

Figure 11 presents the results, for the second arena, in the DS and auxiliary space. Considering the right panel, if P_{max} is interested in reducing the sensitivity, then she may choose either path No. 4 or 5. Alternatively, the distance can be improved on the expense of the sensitivity. In this case, the maximizer may choose either path No. 41, 14, or 17.

Figure 12 presents, for the second arena, the results in the WAC auxiliary space. As in the case of the first arena, the weight-vectors of

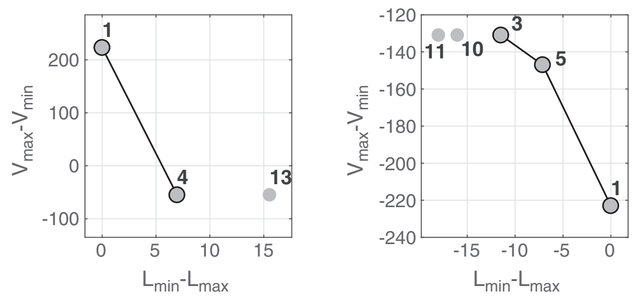

FIGURE 10 The security level vectors of the rationalizable strategies. The minimizer's and maximizer's results are on the left and right panels, respectively. The vectors of security levels of all the rationalizable strategies are denoted by grey filled circles, and each of the auxiliary performance of the Pareto-optimal security strategy is marked by black line

TABLE 1 The rationalizable strategies of the minimizer (two left columns) and maximizer (two right columns) for arena No. 2

No.	Path	No.	Path
1	{2}	1	{1}
3	{2, 3, 2}	4	{1, 4, 1}
10	{2, 3, 4, 2}	5	{1, 5, 1}
		13	{1, 4, 3, 1}
		14	{1, 4, 5, 1}
		17	{1, 5, 4, 1}
		33	{1, 4, 3, 5, 1}
		41	{1, 5, 4, 3, 1}

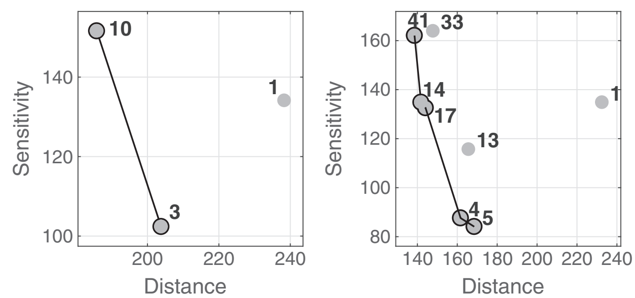

FIGURE 11 The SD auxiliary spaces for arena No. 2. The minimizer's and maximizer's results are on the left and right panels, respectively. The auxiliary performance vectors of all the rationalizable strategies are denoted by grey filled circles. The auxiliary performance vectors of the set of preferred strategies are connected by a black line

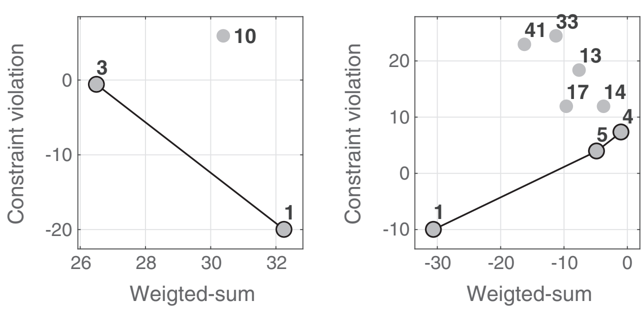

FIGURE 12 Selecting a strategy in arena No. 2 using the WAC auxiliary spaces. The minimizer's and maximizer's results are on the left and right panels, respectively. The auxiliary performance vectors of all the rationalizable strategies are denoted by grey filled circles. The auxiliary performance vectors of the set of preferred strategies are connected by a black line

both players are $w_{min} = w_{max} = [0.6, 0.4]$, and the aspired constraint of the minimizer and the maximizer are $f_{min^{(1)}} = 20$ and $f_{max^{(1)}} = -10$, respectively.

If the maximizer is using the WAC approach (see the right panel of Figure 12), then she may choose path 1, 4, or 5 which constitutes the SPS. When examining the weighted-sum results only, the best path is No. 4, and the worst is No. 1. Yet, if the maximizer does not accept any constraint violation, then she will consider only path No. 1.

The POSS results, for the second arena, are presented in Figure 13. The set of POSS includes Strategies 1 and 5, which is

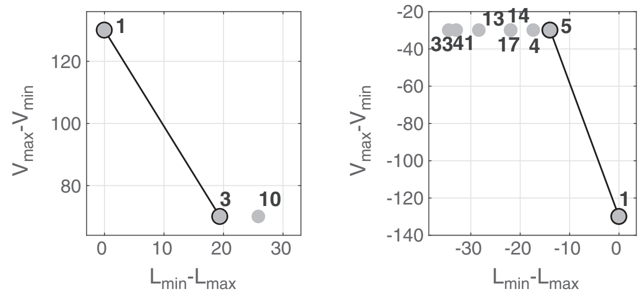

FIGURE 13 Selecting a strategy in arena No. 2 using the Pareto-optimal security strategy approach. The minimizer's and maximizer's results are on the left and right panels, respectively. The vectors of security levels of all the rationalizable strategies are denoted by grey filled circles, and each of the auxiliary performance of the Pareto-optimal security strategy is marked by black line

different from the set that is obtained by the WAC approach. If the maximizer is using the same weights-vector as she used for the WAC approach ($w_{max} = [0.6, 0.4]$), then the selected strategy out of the POSS set is path No. 5.

6 | SUMMARY AND CONCLUSIONS

A novel solution approach has recently been suggested for solving MOGs. In contrast to the traditional scalarization approach to MOGs, the aforementioned approach assumes that the players are undecided about their preference of objectives. The solution amounts to finding a SRS for each player, which can be analysed before a final strategy selection is made. Each such strategy is associated with a set of payoff vectors (anti-optimal front). The anti-optimal fronts of the rationalizable strategies hold tradeoff information, which is unavailable by the traditional scalarization approaches to MOGs. This paper proposes two multicriteria decision analysis techniques, which are based on the utilization of the information inherent to the anti-optimal fronts. In particular, the proposed techniques take into consideration the uncertainty regarding the opponent action, which is revealed by the anti-optimal fronts.

To demonstrate the proposed MCDA techniques, two case studies, which involve competing travelling salespersons, are employed. The demonstration includes a comparison of the results as obtained by the suggested MCDA techniques with those obtained by the classical weighted-sum utility-based selection and by the POSS approach. When considering the results from the two arenas, it can be observed that the proposed techniques may result in different SPSs as compared with the sets obtained by the POSS approach. Also, as should have been expected, each of the obtained sets include more alternatives as compared with the weighted-sum utility-based approach.

The above observations are not a surprised. This is because the proposed MCDA techniques utilize information that is extracted from the anti-optimal fronts, which is different from the information used when employing the POSS approach or the weighted-sum approach. It is concluded from this study that the suggested techniques allow incorporating tradeoff information to justify the selection of a strategy. Moreover, it is concluded that there is no unique way to utilize the information that is inherent to the anti-optimal fronts.

The utilization of the SRSs and their associated anti-optimal fronts may be viewed as an advantage or disadvantage. On the one hand, having the possibility of approaching the selection by different auxiliary measures opens up the way for a comprehensive analysis of the available options. Namely, each possible auxiliary-objective reflects different possible consideration by the decision maker. Yet there is no free lunch. There is a cost associated with finding the SRS and the associated fronts, which is normally larger as compared with the cost involves with the other methods.

As discussed in Section 2, the extension of this study to MOGs with infinite sets of strategies is possible, yet it will require a study on the existence of SRSs. As noted in Section 3, other MCDA techniques, of utilizing the information inherent to the anti-optimal fronts, could be devised in the future. Also, prospective work may include combining the suggested MCDA techniques with evolutionary search algorithms to form an interactive search approach. Another potential topic is to expand this study to the case of mixed strategies.

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APPENDIX A.

Solving multiobjective game with Pareto-optimal security strategy

Ghose and Prasad (1989a, 1989b) introduced the idea of Pareto-optimal security strategy (POSS). A strategy is considered to be a POSS if its worst performances are equal to or better than other strategies' worst performances. The strategy is evaluated by its security level. The kth element of the vector $\bar{J}$ of security level of a strategy s^i_{min} of the minimizer is defined as

$$\bar{J}^{(k)}\left(s^i_{min}\right) = \max_j \bar{f}_{i,j^{(k)}} \tag{A.1}$$

and the k^{th} element of the vector $\underline{J}$ of security level of a strategy s^j_{max}

of the maximizer is defined as

$$\underline{J}^{(k)}\left(s^j_{max}\right) = \min_i \bar{f}^{(k)}_{i,j}. \tag{A.2}$$

A strategy is considered to be a POSS if its vector of security level is nondominated by any other security level vector of all other strategies. Note that the nondomination is with respect to the player's viewpoint.

The POSS approach can be used to solve MOGs in which the players do not have enough information about their opponent preferences. The POSS offers security against the uncertainty about the opponent strategy selection.