CSE848: Evolutionary Computation
Solution to HA1
Spring 2021

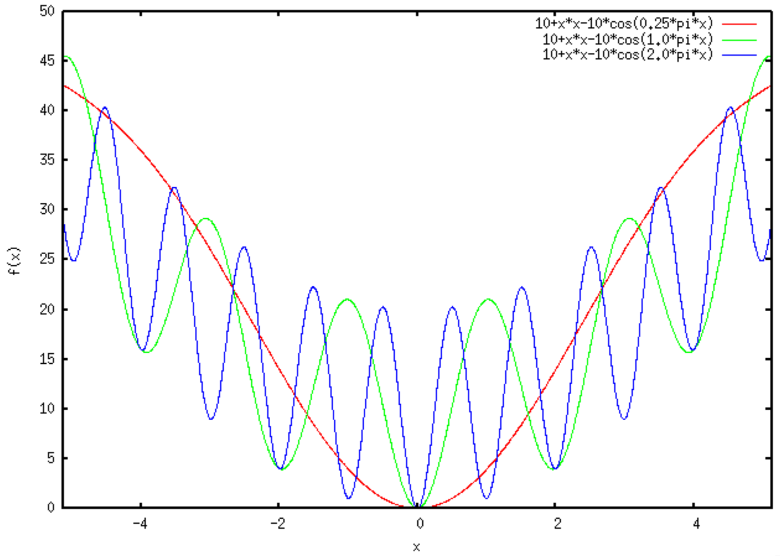
Q1. A Matlab code is written with fminsearch() routine:

```
% HA1Q1.m
clear all;
n = 5; % five variables
xl = -5.12*ones(1,n); % lower bound
xu = 5.12*ones(1,n);  % upper bound
nruns = 100; % 100 runs
options = optimset('MaxFunEvals',5000,'MaxIter',2000);
alpha = [0.25, 1.0, 2.0];
%
fprintf('alpha\t  x(1)\t x(2)\t  f\n');
for k=1:length(alpha)
    xbest = []; fbest = 1e8;
    for i=1:nruns
        x0 = xl + (xu-xl) .* rand(1,n); % random initial point
        fun = @(x) 10*n + x*x' - 10*cos(alpha(k) * pi * x) *
ones(n,1);
        [xv,fv] = fminsearch(fun,x0,options);
        if (fv < fbest)  % store the best-ever solution
            fbest = fv;
            xbest = xv;
        end
    end
    % Print best solution and obj value for each alpha
    fprintf('%5.2f %7.3f %7.3f
%7.3f\n',alpha(k),xbest(1),xbest(2),fbest);
end
```

Results obtained from the code is shown below. The optimal solution is the zero vector with a function value of zero.

```
>> HA1Q1new
alpha    x(1)    x(2)      f
 0.25    0.000  -0.000    0.000
 1.00   -1.960   1.960   11.761
 2.00    0.995   0.000    1.990
```

The results indicate that $\alpha = 0.25$ finds the best solution (f=0). This is because with $\alpha = 0.25$, the function has a single minimum in the range $x_i \in [-5.12, 5.12]$, as shown below. For unimodal problems, fminsearch() finds the minimum every time. With $\alpha = 1.00$, there are five minima for each x_i, making a total of $5^5 = 3{,}124$ local minima and one global minimum. Interestingly, the complexity of the problem does not scale up with an increase in α, despite the increase in number of local minima. The overall parabolic and periodic structure of the Rastrigin function makes a random solution to have smaller function value with increase in α.



Q2. The problem is coded in Matlab and solved using fmincon() routine.

```
% HA1Q2.m
x0 = [1,1]; % given
[x,fval,history]=myproblem_fmincon(x0),
% plot history of points
figure(2);
plot(history(:,1),history(:,2),'r-'); hold on;
plot(history(:,1),history(:,2),'bo');
grid on; xlim([-1 3]); ylim([-2 2]);
xlabel('x1'); ylabel('x2');
% plot contour
x = linspace(-1,3);
y = linspace(-2,2);
[X,Y] = meshgrid(x,y);
Z = (X+1).^2+(Y+1).^2;
contour(X,Y,Z);
plot(x,sqrt(5-(x-1).^2)/2,'g-');
plot(x,-sqrt(5-(x-1).^2)/2,'g-');
plot(x,x-1,'g-');
plot(x,1-0.5*x,'g-');
hold off;


% subroutines are here
function [x, fval, history] = myproblem_fmincon(x0)
history = x0;
options = optimset('OutputFcn',@myoutput,...
'Display','Iter','PlotFcns',@optimplotfval);
objfun = @(x) (x(1)+1)^2+(x(2)+1)^2;
[x, fval,exitflag,output,lambda] =
fmincon(objfun,x0,[],[],[],[],[],[],...
    @constraint,options);
figure(1);

    function stop = myoutput(x,optimvalues,state)
        stop = false;
        if isequal(state,'iter')
            history = [history; x];
```

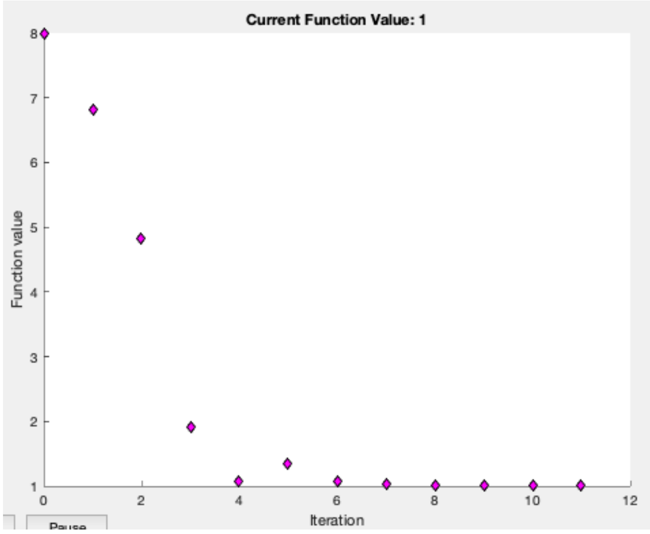
```
            end
        end
    end

    function [c,ceq] = constraint(x)
    c(1) = (x(1)-1.0)^2 + 4 * x(2)^2 - 5.0;
    c(2) = 1.0 - x(1) + x(2);
    c(3) = x(1) + 2*x(2) - 2.0;
    ceq=[];
    end
```

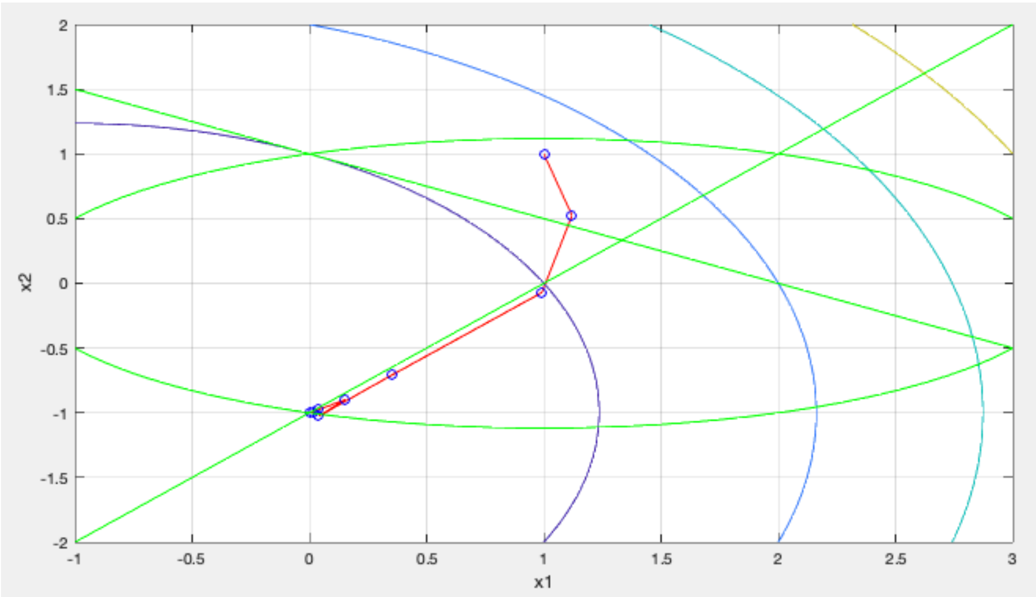
The following output is obtained:

```
>> HA1Q2
                                            First-order      Norm of
 Iter F-count            f(x)  Feasibility   optimality         step
    0       3    8.000000e+00    1.000e+00    5.406e+00
    1       6    6.820038e+00    4.043e-01    4.455e+00    4.904e-01
    2       9    4.829925e+00    0.000e+00    3.264e+00    6.093e-01
    3      13    1.914113e+00    0.000e+00    1.990e+00    9.026e-01
    4      17    1.081783e+00    1.364e-01    6.246e-01    4.462e-01
    5      20    1.336132e+00    0.000e+00    2.653e-01    1.696e-01
    6      23    1.076754e+00    0.000e+00    7.141e-02    1.385e-01
    7      26    1.040257e+00    0.000e+00    2.082e-02    2.309e-02
    8      29    1.001432e+00    0.000e+00    8.875e-03    2.047e-02
    9      32    1.000401e+00    0.000e+00    2.072e-04    6.154e-04
   10      35    1.000004e+00    0.000e+00    5.685e-05    2.120e-04
   11      38    1.000004e+00    0.000e+00    2.000e-06    7.184e-08

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-
decreasing in
feasible directions, to within the value of the optimality
tolerance,
and constraints are satisfied to within the value of the constraint
tolerance.

<stopping criteria details>

x =
    0.0000   -1.0000


fval =
    1.0000


history =
    1.0000    1.0000
    1.0000    1.0000
    1.1203    0.5246
    0.9917   -0.0710
    0.3524   -0.7081
    0.0398   -1.0264
    0.1515   -0.8988
    0.0374   -0.9774
    0.0199   -0.9925
```

```
    0.0007    -0.9996
    0.0002    -0.9999
    0.0000    -1.0000
    0.0000    -1.0000
```

The respective plots are shown below.





Q3. First, we solve the problem using linprog() and then solve using intlinprog():

```
% HA1Q3.m
% Mixed Integer Linear programming
A = [1 5; 3 1; -1 -2];
b = [10; 15; -1];
f = [1 4];
% Linear programming
options=optimoptions('linprog','DISPLAY','Off');
[x,fv] = linprog(-f,A,b,[],[],[],[],[],options);
fprintf("LP:\t x(1) = %6.3f, x(2) = %6.3f, f = %6.3f\n",x(1),x(2),-
fv);
```

```
%
intcon = 1;
options=optimoptions('intlinprog','DISPLAY','Off');
%
% x1 is an integer
[x,fv] = intlinprog(-f,intcon,A,b,[],[],[],[],[],options);
fprintf("MILP:\t x(1) = %6.3f, x(2) = %6.3f, f =
%6.3f\n",x(1),x(2),-fv);
```

Results are shown below:

```
>> HA1Q3
LP:     x(1) =  4.643, x(2) =  1.071, f =  8.929
MILP:   x(1) =  4.000, x(2) =  1.200, f =  8.800
```

It can be noticed that LP solution is the best with obj value of 8.929 (max), followed by mixed-integer LP solution. Variable x(1) moves to the nearest integer (=4) and the respective x(2) gets adjusted. Here is a plot to show the points. Feasible space is marked in blue color.