Kvantik problems, December 2021

PROBLEM 16

On an island, every person is either a knight (always tells the truth) or a liar (always lies). Person A told the story:
"I met B and C. The first one said:
'We are both liars.'
And the second nods: 'This is true.'"
For whom from A, B, and C can you tell for sure whether he is a knight of a liar?

Suppose A is a knight.
Then the first from B and C should be a liar since a knight can't say he is a liar. But that means that the second one should be a knight; otherwise, the first one told the truth. But a knight can't agree with 'We are both liars.'
We got a contradiction. So A **is a liar**. And since he is a liar, we **can't tell for sure anything about** B **and** C.

PROBLEM 17

Solve a cryptarithm:

$$OAK + OAK + OAK + OAK + OAK = SOAK.$$

(Find all answers and prove that there are no other answers. The same letters denote the same digit; the different letters denote different digits. No number has zero as the first digit.)

Substract OAK from both sides, we got:

$$4 \times OAK = S \times 1{,}000$$

or

$$OAK = S \times 250.$$

S can get any value from 1 till 3, otherwise $S \times 250$ would be 4-digit number. $S = 2$ don't work, since $A = K = 0$. For both $S = 1$ and $S = 3$ we got **two solutions**:

$$250 + 250 + 250 + 250 + 250 = 1250,$$

$$750 + 750 + 750 + 750 + 750 = 3750.$$

PROBLEM 18

When Robinson Crusoe had got to an uninhabited island, he had 200 rifle shots. To conserve them, he decided that each following day he should not use more than 5% of the shots he had that morning. At some point, Robinson can't shoot according to this rule. How many shots has he used by this time?

Firstly, Robinson can't use more than 10 shots in one day since he can't get more than 200 shots. Secondly, once Robinson has from 20 to 39 shots, he can't use only one shot per day. So that means that someday Robinson got into that range since it is wider than the number of shots he can use in one day.

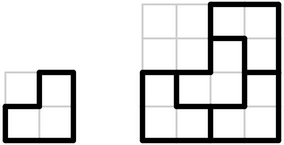
When Robinson was in rage from 20 till 39, he would use one shot daily, until on the last day, when he had 20 shots, he would use a shot and leave with 19 shots. From this day, he couldn't use shots anymore.

So total, **he used 181 shots**.

PROBLEM 19

For which N, a big corner, created from three squares $N \times N$, can be cut by grid lines into smaller three-celled corners?

It is easy to cut a corner for $N = 1$ and $N = 2$.

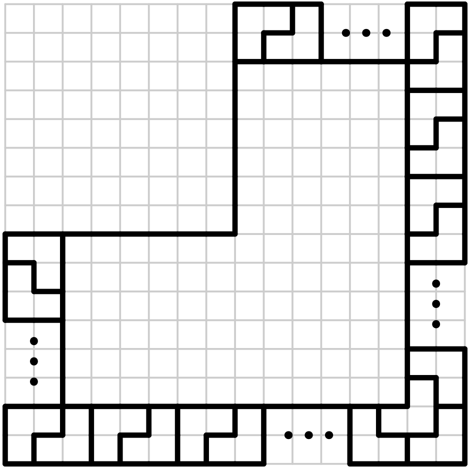


$N = 1$ $N = 2$

Now let's suppose we can cut a corner of some size value K. Let's show how to cut a corner of size $K + 2$.
If we can demonstrate it, that means we can cut corners of sizes $N = 1, 3, 5, 7$ and so on, and corners of sizes $N = 2, 4, 6$ and so on.

This kind of proof is called **proof by induction**.

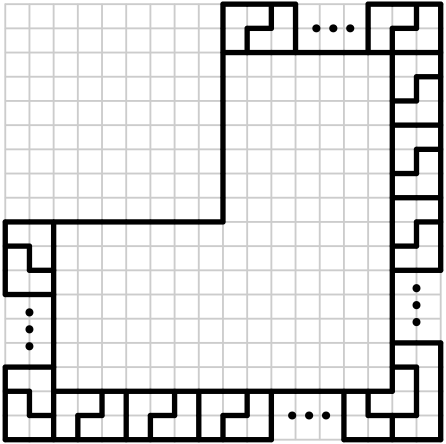
Firstly, let's $K = 3m$ for some counting m.



Here instead of dots we put 2×3 rectangles $m - 1$ or $2m - 2$ times as needed.

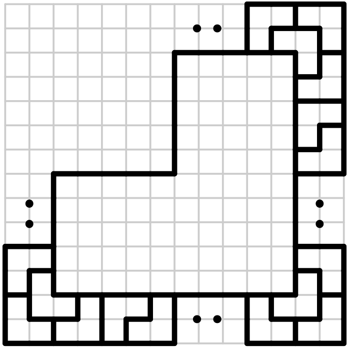
PROBLEM 19, CONTINUED

Secondly, let's $K = 3m + 1$ for some counting m.



Here instead of dots we put 2×3 rectangles $m - 1$ or $2m - 3$ times as needed. If $K = 1$ we remove one or two 2×3 rectangles from sides.

Finally, let's $K = 3m + 2$ for some counting m.



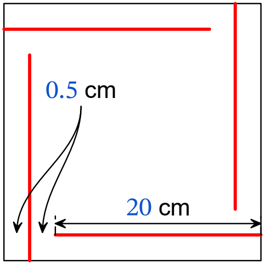
Here instead of dots we put 2×3 rectangles m or $2m - 1$ times as needed. If $K = 2$ we remove two 2×3 rectangles from long sides.

So, for any N it is possible to cut a big corner into small corners.

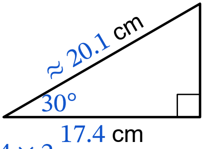
PROBLEM 20

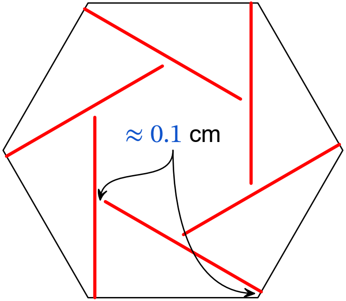
a) Masha made a cake, a square with a side of 21 cm. Then she chose a point on a cake side and made a cut of length 20 cm from this point perpendicular to the chosen side. Masha did the same for each of the 4 sides. Is it necessary for a piece of cake to be cut off?
b) Solve the same problem when Masha made a cake in the form of a regular hexagon with a diameter of 35 cm and made a 20 cm cut perpendicular to each side.

a) If the cut performed 0.5 cm from the edge, as shown, it is possible to keep a cake in one piece. In this case distance between cuts will be 0.5 cm.



b) Let's take a look at this $30-60-90$ triangle. If it longer leg has a length of 17.4 cm, it's hypotenuse has a length (by Pythagorean theorem) of $\dfrac{17.4\times 2}{\sqrt{3}} \approx 20.092$ cm.





This gives us gap of about 0.1cm between cuts like at this picture. So it is also possible to keep a cake in one piece.