



Kvantik problems, February 2022



$$\sqrt{x} = 3,14$$
$$3 \times 3 = 9$$



PROBLEM 26

Wise men A and B each received a counting number. The difference between these numbers is 1 . "I don't know whether you know my number," A said to B . Which number has A received?

A has received 2 .

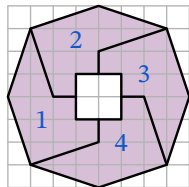
The only case when B may know a number received by A when B got 1 . In any other case B has two options for a number A got.

If A has received a number bigger than 2 , in this case B received number bigger than 1 and B can't know number A got. If A has received 1 , B has to receive 2 and again don't know a number A got. If A has received 2 , B may receive 1 , in this case he knows the number A got, B may also receive 2 , in this case he don't know the number A got.

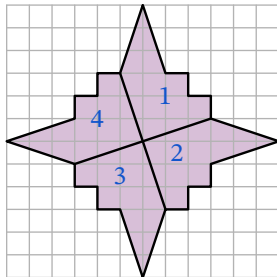
PROBLEM 27

Cut a ring with a hole (pic. 1) into four equal pieces and put them together to make a snowflake (pic. 2).

Here is an example of possible cuts.



Pic. 1



Pic. 2

PROBLEM 28

In the IV round, we asked to solve a cryptarithm $OAK \times 5 = SOAK$, and it has two solutions.

a) Replace the 5 with another digit, so the new cryptarithm has a solution.

b) Prove that there is only one such digit.

c) Prove that the new cryptarithm has only one solution.

(As usual, the same letters denote the same digit; the different letters denote different digits. No number has zero as the first digit.)

Let's n replace 5. Then equation may be written as $OAK \times n = SOAK$ or

$$OAK \times (n - 1) = S \times 1000.$$

The same is

$$OAK = S \times \frac{1000}{n - 1}.$$

For A and K to be different digits, $\frac{1000}{n - 1}$ should not be multiple of 100. So $n - 1$ should be divisible by 4 or 25.

Since $n - 1 < 9$ (n is a digit), $n - 1$ should be multiple of 4, so $n - 1 = 4$ or $n - 1 = 8$.

$n = 5$ can't be solution of the problem, so $n = 9$ is the only solution. Solving

$$OAK = S \times 125$$

for different values of S we got following values for $SOAK$: 1125, 2250, 3375, 4500, 5625, 6750, 7875.

Only 6750 has different digits. So solution for our cryptarithm would be

$$750 \times 8 = 6750.$$

PROBLEM 29

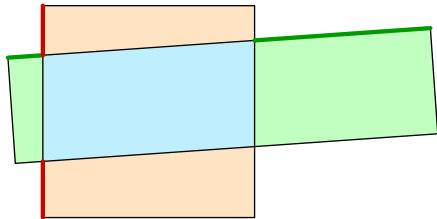
Some cells of a white rectangular board are painted blue. In any row, the number of painted cells is different; and in any column, the number of painted cells is different. Prove if the number of rows and columns is different, the board has an equal number of white and blue cells.

Let our board would be $m \times n$, where $m > n$. And without lost of generality we may say that n is the number of columns. In this case the possible number of colored cells in each row can be $0, 1, 2, \dots, n$: for a total of at most $n + 1$ different values. And since number of rows are greater than number of columns, we got $m = n + 1$.

So board should be $(n + 1) \times n$ and each rows has number of colored cells equal $0, 1, \dots, n$, so total number of colored cells is $\frac{n \cdot (n + 1)}{2}$, which is the half total number of cells in the board.

PROBLEM 30

The square 6×6 and rectangle 3×12 intersect as shown in the picture. Prove that the two green segments sum is twice that of the two red segments.



The area of the square is equal 36 and it is an equal to the area of the rectangle. Let area of the blue area (the intersect of the square and the rectangle) is equal x .

The area of the rectangle formed by two orange trapezoids inside the square and outside of the blue area is

$$6 \times (\text{sum of the two red segments}) = 36 - x.$$

The area of the rectangle formed by two green trapezoids inside the rectangle and outside of the blue area is

$$3 \times (\text{sum of the two red segments}) = 36 - x.$$

$$\text{So } 6 \times (\text{sum of the two red segments}) = 3 \times (\text{sum of the two red segments})$$

$$2 \times (\text{sum of the two red segments}) = x$$

which is we need to prove.