Word Problems solutions

Graham Middle School Math Olympiad Team

PROBLEMS 1-4

1. The lengths of the sides of a triangle are in the ratio $4 : 3 : 5$. Find the lengths of the longest side if the perimeter is 18 inches.

PROBLEMS 1-4

1. The lengths of the sides of a triangle are in the ratio $4:3:5$. Find the lengths of the longest side if the perimeter is 18 inches.

Let sides of triangle will be $4x$, $3x$ and $5x$.

$$4x + 3x + 5x = 12x = 18.$$

PROBLEMS 1-4

1. The lengths of the sides of a triangle are in the ratio $4:3:5$. Find the lengths of the longest side if the perimeter is 18 inches.

Let sides of triangle will be $4x$, $3x$ and $5x$.

$$4x + 3x + 5x = 12x = 18.$$

So $x = \frac{18}{12} = \frac{3}{2}$. So the longest side is

$5x = 5 \cdot \frac{3}{2} = \frac{15}{2} = \boxed{7.5}$ inches.

PROBLEMS 1-4

1. The lengths of the sides of a triangle are in the ratio $4:3:5$. Find the lengths of the longest side if the perimeter is 18 inches.

Let sides of triangle will be $4x$, $3x$ and $5x$.

$$4x + 3x + 5x = 12x = 18.$$

So $x = \frac{18}{12} = \frac{3}{2}$. So the longest side is

$5x = 5 \cdot \frac{3}{2} = \frac{15}{2} = \boxed{7.5}$ inches.

2. There are 40 pigs and chickens in a farmyard. Joseph counted 100 legs in all. How many pigs and how many chickens are there?

PROBLEMS 1-4

1. The lengths of the sides of a triangle are in the ratio $4:3:5$. Find the lengths of the longest side if the perimeter is 18 inches.

Let sides of triangle will be $4x$, $3x$ and $5x$.

$$4x + 3x + 5x = 12x = 18.$$

So $x = \frac{18}{12} = \frac{3}{2}$. So the longest side is

$5x = 5 \cdot \frac{3}{2} = \frac{15}{2} = \boxed{7.5}$ inches.

2. There are 40 pigs and chickens in a farmyard. Joseph counted 100 legs in all. How many pigs and how many chickens are there?

Let it be p pigs and c chicken. We got:

$$p + c = 40, \quad 4p + 2c = 100.$$

PROBLEMS 1-4

1. The lengths of the sides of a triangle are in the ratio $4:3:5$. Find the lengths of the longest side if the perimeter is 18 inches.

Let sides of triangle will be $4x$, $3x$ and $5x$.

$$4x + 3x + 5x = 12x = 18.$$

So $x = \frac{18}{12} = \frac{3}{2}$. So the longest side is

$5x = 5 \cdot \frac{3}{2} = \frac{15}{2} = \boxed{7.5}$ inches.

2. There are 40 pigs and chickens in a farmyard. Joseph counted 100 legs in all. How many pigs and how many chickens are there?

Let it be p pigs and c chicken. We got:

$$p + c = 40, \quad 4p + 2c = 100.$$

$p = 40 - c$, so $4(40 - c) + 2c = 160 - 4c + 2c =$
$= 160 - 2c = 100$. $2c = 60$ and $c = \boxed{30}$
chickens. $p = 40 - c = 40 - 30 = \boxed{10}$ pigs.

PROBLEMS 1-4

1. The lengths of the sides of a triangle are in the ratio $4:3:5$. Find the lengths of the longest side if the perimeter is 18 inches.

Let sides of triangle will be $4x$, $3x$ and $5x$.

$$4x+3x+5x=12x=18.$$

So $x=\frac{18}{12}=\frac{3}{2}$. So the longest side is

$5x=5\cdot\frac{3}{2}=\frac{15}{2}=\boxed{7.5}$ inches.

2. There are 40 pigs and chickens in a farmyard. Joseph counted 100 legs in all. How many pigs and how many chickens are there?

Let it be p pigs and c chicken. We got:

$$p+c=40,\quad 4p+2c=100.$$

$p=40-c$, so $4(40-c)+2c=160-4c+2c=$
$=160-2c=100$. $2c=60$ and $c=\boxed{30}$
chickens. $p=40-c=40-30=\boxed{10}$ pigs.

3. The cost of gas rises by 2 cents a liter. Last week a man bought 20 liters at the old price. This week he bought 10 liters at the new price. Altogether, the gas costs $\$9.20$. What was the old price for 1 liter?

PROBLEMS 1-4

1. The lengths of the sides of a triangle are in the ratio $4:3:5$. Find the lengths of the longest side if the perimeter is 18 inches.

Let sides of triangle will be $4x$, $3x$ and $5x$.

$$4x+3x+5x=12x=18.$$

So $x=\frac{18}{12}=\frac{3}{2}$. So the longest side is

$5x=5\cdot\frac{3}{2}=\frac{15}{2}=\boxed{7.5}$ inches.

2. There are 40 pigs and chickens in a farmyard. Joseph counted 100 legs in all. How many pigs and how many chickens are there?

Let it be p pigs and c chicken. We got:

$$p+c=40,\quad 4p+2c=100.$$

$p=40-c$, so $4(40-c)+2c=160-4c+2c=$
$=160-2c=100$. $2c=60$ and $c=\boxed{30}$
chickens. $p=40-c=40-30=\boxed{10}$ pigs.

3. The cost of gas rises by 2 cents a liter. Last week a man bought 20 liters at the old price. This week he bought 10 liters at the new price. Altogether, the gas costs \$$9.20$. What was the old price for 1 liter?

Let x is the old price of a liter in cents.

$$20x+10(x+2)=920.$$

PROBLEMS 1-4

1. The lengths of the sides of a triangle are in the ratio $4:3:5$. Find the lengths of the longest side if the perimeter is 18 inches.

Let sides of triangle will be $4x$, $3x$ and $5x$.

$$4x+3x+5x=12x=18.$$

So $x=\frac{18}{12}=\frac{3}{2}$. So the longest side is

$5x=5\cdot\frac{3}{2}=\frac{15}{2}=\boxed{7.5}$ inches.

2. There are 40 pigs and chickens in a farmyard. Joseph counted 100 legs in all. How many pigs and how many chickens are there?

Let it be p pigs and c chicken. We got:

$$p+c=40,\quad 4p+2c=100.$$

$p=40-c$, so $4(40-c)+2c=160-4c+2c=$
$=160-2c=100$. $2c=60$ and $c=\boxed{30}$
chickens. $p=40-c=40-30=\boxed{10}$ pigs.

3. The cost of gas rises by 2 cents a liter. Last week a man bought 20 liters at the old price. This week he bought 10 liters at the new price. Altogether, the gas costs $\$9.20$. What was the old price for 1 liter?

Let x is the old price of a liter in cents.

$$20x+10(x+2)=920.$$

$20x+10x+20=30x+20=920$. $30x=900$
and $x=\boxed{30}$ cents.

PROBLEMS 1-4

1. The lengths of the sides of a triangle are in the ratio $4:3:5$. Find the lengths of the longest side if the perimeter is 18 inches.

Let sides of triangle will be $4x$, $3x$ and $5x$.

$$4x+3x+5x=12x=18.$$

So $x=\frac{18}{12}=\frac{3}{2}$. So the longest side is

$5x=5\cdot\frac{3}{2}=\frac{15}{2}=\boxed{7.5}$ inches.

2. There are 40 pigs and chickens in a farmyard. Joseph counted 100 legs in all. How many pigs and how many chickens are there?

Let it be p pigs and c chicken. We got:

$$p+c=40,\quad 4p+2c=100.$$

$p=40-c$, so $4(40-c)+2c=160-4c+2c=$
$=160-2c=100$. $2c=60$ and $c=\boxed{30}$
chickens. $p=40-c=40-30=\boxed{10}$ pigs.

3. The cost of gas rises by 2 cents a liter. Last week a man bought 20 liters at the old price. This week he bought 10 liters at the new price. Altogether, the gas costs \$$9.20$. What was the old price for 1 liter?

Let x is the old price of a liter in cents.

$$20x+10(x+2)=920.$$

$20x+10x+20=30x+20=920$. $30x=900$
and $x=\boxed{30}$ cents.

4. A can do a work in 14 days and working together A and B can do the same work in 10 days. In what time can B alone do the work?

PROBLEMS 1-4

1. The lengths of the sides of a triangle are in the ratio $4:3:5$. Find the lengths of the longest side if the perimeter is 18 inches.

Let sides of triangle will be $4x$, $3x$ and $5x$.

$$4x+3x+5x=12x=18.$$

So $x=\frac{18}{12}=\frac{3}{2}$. So the longest side is

$5x=5\cdot\frac{3}{2}=\frac{15}{2}=\boxed{7.5}$ inches.

2. There are 40 pigs and chickens in a farmyard. Joseph counted 100 legs in all. How many pigs and how many chickens are there?

Let it be p pigs and c chicken. We got:

$$p+c=40,\quad 4p+2c=100.$$

$p=40-c$, so $4(40-c)+2c=160-4c+2c=$
$=160-2c=100$. $2c=60$ and $c=\boxed{30}$
chickens. $p=40-c=40-30=\boxed{10}$ pigs.

3. The cost of gas rises by 2 cents a liter. Last week a man bought 20 liters at the old price. This week he bought 10 liters at the new price. Altogether, the gas costs \$$9.20$. What was the old price for 1 liter?

Let x is the old price of a liter in cents.

$$20x+10(x+2)=920.$$

$20x+10x+20=30x+20=920$. $30x=900$
and $x=\boxed{30}$ cents.

4. A can do a work in 14 days and working together A and B can do the same work in 10 days. In what time can B alone do the work?

Let S amount of work and x days are needed B to do the work. S/days is the speed.

$$\frac{S}{10}=\frac{S}{14}+\frac{S}{x}.$$

PROBLEMS 1-4

1. The lengths of the sides of a triangle are in the ratio $4:3:5$. Find the lengths of the longest side if the perimeter is 18 inches.

Let sides of triangle will be $4x$, $3x$ and $5x$.

$$4x+3x+5x=12x=18.$$

So $x=\frac{18}{12}=\frac{3}{2}$. So the longest side is

$5x=5\cdot\frac{3}{2}=\frac{15}{2}=\boxed{7.5}$ inches.

2. There are 40 pigs and chickens in a farmyard. Joseph counted 100 legs in all. How many pigs and how many chickens are there?

Let it be p pigs and c chicken. We got:

$$p+c=40,\quad 4p+2c=100.$$

$p=40-c$, so $4(40-c)+2c=160-4c+2c=$
$=160-2c=100$. $2c=60$ and $c=\boxed{30}$
chickens. $p=40-c=40-30=\boxed{10}$ pigs.

3. The cost of gas rises by 2 cents a liter. Last week a man bought 20 liters at the old price. This week he bought 10 liters at the new price. Altogether, the gas costs \$$9.20$. What was the old price for 1 liter?

Let x is the old price of a liter in cents.

$$20x+10(x+2)=920.$$

$20x+10x+20=30x+20=920$. $30x=900$
and $x=\boxed{30}$ cents.

4. A can do a work in 14 days and working together A and B can do the same work in 10 days. In what time can B alone do the work?

Let S amount of work and x days are needed B to do the work. S/days is the speed.

$$\frac{S}{10}=\frac{S}{14}+\frac{S}{x}.$$

$\frac{1}{10}=\frac{1}{14}+\frac{1}{x}$, $\frac{140x}{10}=\frac{140x}{14}+\frac{140x}{x}$, so
$14x=10x+140$ and $4x=140$, so $x=\boxed{35}$ days.

PROBLEMS 5-8

5. The sum of the first and last of four consecutive odd integers is 52. What are the four integers?

PROBLEMS 5-8

5. The sum of the first and last of four consecutive odd integers is 52. What are the four integers?

Let x is the first odd integer.

$$x + (x + 6) = 52.$$

PROBLEMS 5-8

5. The sum of the first and last of four consecutive odd integers is 52. What are the four integers?

Let x is the first odd integer.

$$x + (x + 6) = 52.$$

$2x = 46$, and $x = 23$. Answer is $\boxed{23, 25, 27, 29}$.

PROBLEMS 5-8

5. The sum of the first and last of four consecutive odd integers is 52. What are the four integers?

Let x is the first odd integer.

$$x + (x + 6) = 52.$$

$2x = 46$, and $x = 23$. Answer is $\boxed{23, 25, 27, 29}$.

6. Annie and Bonnie are running laps around a 400-meter oval track. They started together, but Annie has pulled ahead because she runs 25% faster than Bonnie. How many laps will Annie have run when she first passes Bonnie?

PROBLEMS 5-8

5. The sum of the first and last of four consecutive odd integers is 52. What are the four integers?

Let x is the first odd integer.

$$x + (x + 6) = 52.$$

$2x = 46$, and $x = 23$. Answer is $\boxed{23, 25, 27, 29}$.

6. Annie and Bonnie are running laps around a 400-meter oval track. They started together, but Annie has pulled ahead because she runs 25% faster than Bonnie. How many laps will Annie have run when she first passes Bonnie?

Let x is the speed of Bonnie, $1.25x$ is the speed of Annie. t is the time when Annie passed Bonnie.

$$x \cdot t + 400 = 1.25x \cdot t.$$

5. The sum of the first and last of four consecutive odd integers is 52. What are the four integers?

Let x is the first odd integer.

$$x + (x + 6) = 52.$$

$2x = 46$, and $x = 23$. Answer is $\boxed{23,\ 25,\ 27,\ 29}$.

6. Annie and Bonnie are running laps around a 400-meter oval track. They started together, but Annie has pulled ahead because she runs 25% faster than Bonnie. How many laps will Annie have run when she first passes Bonnie?

Let x is the speed of Bonnie, $1.25x$ is the speed of Annie. t is the time when Annie passed Bonnie.

$$x \cdot t + 400 = 1.25x \cdot t.$$

$0.25x \cdot t = 400$, so $1.25x \cdot t = 2000$. So Annie ran 2000 meters before passing Bonnie, which gives us $\boxed{5}$ laps.

5. The sum of the first and last of four consecutive odd integers is 52. What are the four integers?

Let x is the first odd integer.

$$x + (x + 6) = 52.$$

$2x = 46$, and $x = 23$. Answer is $\boxed{23, 25, 27, 29}$.

6. Annie and Bonnie are running laps around a 400-meter oval track. They started together, but Annie has pulled ahead because she runs 25% faster than Bonnie. How many laps will Annie have run when she first passes Bonnie?

Let x is the speed of Bonnie, $1.25x$ is the speed of Annie. t is the time when Annie passed Bonnie.

$$x \cdot t + 400 = 1.25x \cdot t.$$

$0.25x \cdot t = 400$, so $1.25x \cdot t = 2000$. So Annie ran 2000 meters before passing Bonnie, which gives us $\boxed{5}$ laps.

7. A washer costs 25% more than a dryer. If the store clerk gave a 10% discount for the dryer and a 20% discount for the washer, how much is the washer before the discount if you paid 1900 dollars?

5. The sum of the first and last of four consecutive odd integers is 52. What are the four integers?

Let x is the first odd integer.

$$x + (x + 6) = 52.$$

$2x = 46$, and $x = 23$. Answer is $\boxed{23, 25, 27, 29}$.

6. Annie and Bonnie are running laps around a 400-meter oval track. They started together, but Annie has pulled ahead because she runs 25% faster than Bonnie. How many laps will Annie have run when she first passes Bonnie?

Let x is the speed of Bonnie, $1.25x$ is the speed of Annie. t is the time when Annie passed Bonnie.

$$x \cdot t + 400 = 1.25x \cdot t.$$

$0.25x \cdot t = 400$, so $1.25x \cdot t = 2000$. So Annie ran 2000 meters before passing Bonnie, which gives us $\boxed{5}$ laps.

7. A washer costs 25% more than a dryer. If the store clerk gave a 10% discount for the dryer and a 20% discount for the washer, how much is the washer before the discount if you paid 1900 dollars?

Let x is the washer price, $x/1.25$ — dryer.

$$0.80x + 0.90 \cdot \frac{x}{1.25} = 1900.$$

5. The sum of the first and last of four consecutive odd integers is 52. What are the four integers?

Let x is the first odd integer.

$$x + (x + 6) = 52.$$

$2x = 46$, and $x = 23$. Answer is $\boxed{23, 25, 27, 29}$.

6. Annie and Bonnie are running laps around a 400-meter oval track. They started together, but Annie has pulled ahead because she runs 25% faster than Bonnie. How many laps will Annie have run when she first passes Bonnie?

Let x is the speed of Bonnie, $1.25x$ is the speed of Annie. t is the time when Annie passed Bonnie.

$$x \cdot t + 400 = 1.25x \cdot t.$$

$0.25x \cdot t = 400$, so $1.25x \cdot t = 2000$. So Annie ran 2000 meters before passing Bonnie, which gives us $\boxed{5}$ laps.

7. A washer costs 25% more than a dryer. If the store clerk gave a 10% discount for the dryer and a 20% discount for the washer, how much is the washer before the discount if you paid 1900 dollars?

Let x is the washer price, $x/1.25$ — dryer.

$$0.80x + 0.90 \cdot \frac{x}{1.25} = 1900.$$

$0.8x \cdot 1.25 + 0.9x = 1900 \cdot 1.25$, $1x + 0.9x = 1.9x = 1900 \cdot 1.25$, $x = 1000 \cdot 1.25 = \boxed{\$1250}$.

5. The sum of the first and last of four consecutive odd integers is 52. What are the four integers?

Let x is the first odd integer.

$$x + (x + 6) = 52.$$

$2x = 46$, and $x = 23$. Answer is $\boxed{23,\ 25,\ 27,\ 29}$.

6. Annie and Bonnie are running laps around a 400-meter oval track. They started together, but Annie has pulled ahead because she runs 25% faster than Bonnie. How many laps will Annie have run when she first passes Bonnie?

Let x is the speed of Bonnie, $1.25x$ is the speed of Annie. t is the time when Annie passed Bonnie.

$$x \cdot t + 400 = 1.25x \cdot t.$$

$0.25x \cdot t = 400$, so $1.25x \cdot t = 2000$. So Annie ran 2000 meters before passing Bonnie, which gives us $\boxed{5}$ laps.

7. A washer costs 25% more than a dryer. If the store clerk gave a 10% discount for the dryer and a 20% discount for the washer, how much is the washer before the discount if you paid 1900 dollars?

Let x is the washer price, $x/1.25$ — dryer.

$$0.80x + 0.90 \cdot \frac{x}{1.25} = 1900.$$

$0.8x \cdot 1.25 + 0.9x = 1900 \cdot 1.25$, $1x + 0.9x =$
$= 1.9x = 1900 \cdot 1.25$, $x = 1000 \cdot 1.25 = \boxed{\$1250}$.

8. In the right triangle ABC, $AC = 12$, $BC = 5$, and angle C is a right angle. A semicircle is inscribed in the triangle as shown. What is the radius of the semicircle?

5. The sum of the first and last of four consecutive odd integers is 52. What are the four integers?

Let x is the first odd integer.

$$x + (x + 6) = 52.$$

$2x = 46$, and $x = 23$. Answer is $\boxed{23,\ 25,\ 27,\ 29}$.

6. Annie and Bonnie are running laps around a 400-meter oval track. They started together, but Annie has pulled ahead because she runs 25% faster than Bonnie. How many laps will Annie have run when she first passes Bonnie?

Let x is the speed of Bonnie, $1.25x$ is the speed of Annie. t is the time when Annie passed Bonnie.

$$x \cdot t + 400 = 1.25x \cdot t.$$

$0.25x \cdot t = 400$, so $1.25x \cdot t = 2000$. So Annie ran 2000 meters before passing Bonnie, which gives us $\boxed{5}$ laps.

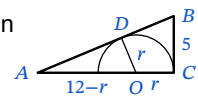
7. A washer costs 25% more than a dryer. If the store clerk gave a 10% discount for the dryer and a 20% discount for the washer, how much is the washer before the discount if you paid 1900 dollars?

Let x is the washer price, $x/1.25$ — dryer.

$$0.80x + 0.90 \cdot \frac{x}{1.25} = 1900.$$

$0.8x \cdot 1.25 + 0.9x = 1900 \cdot 1.25$, $1x + 0.9x =$
$= 1.9x = 1900 \cdot 1.25$, $x = 1000 \cdot 1.25 = \boxed{\$1250}$.

8. In the right triangle ABC, $AC = 12$, $BC = 5$, and angle C is a right angle. A semicircle is inscribed in the triangle as shown. What is the radius of the semicircle?



$\triangle ADO \sim \triangle ACB$,

5. The sum of the first and last of four consecutive odd integers is 52. What are the four integers?

Let x is the first odd integer.

$$x + (x + 6) = 52.$$

$2x = 46$, and $x = 23$. Answer is $\boxed{23,\ 25,\ 27,\ 29}$.

6. Annie and Bonnie are running laps around a 400-meter oval track. They started together, but Annie has pulled ahead because she runs 25% faster than Bonnie. How many laps will Annie have run when she first passes Bonnie?

Let x is the speed of Bonnie, $1.25x$ is the speed of Annie. t is the time when Annie passed Bonnie.

$$x \cdot t + 400 = 1.25x \cdot t.$$

$0.25x \cdot t = 400$, so $1.25x \cdot t = 2000$. So Annie ran 2000 meters before passing Bonnie, which gives us $\boxed{5}$ laps.

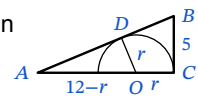
7. A washer costs 25% more than a dryer. If the store clerk gave a 10% discount for the dryer and a 20% discount for the washer, how much is the washer before the discount if you paid 1900 dollars?

Let x is the washer price, $x/1.25$ — dryer.

$$0.80x + 0.90 \cdot \frac{x}{1.25} = 1900.$$

$0.8x \cdot 1.25 + 0.9x = 1900 \cdot 1.25$, $1x + 0.9x =$
$= 1.9x = 1900 \cdot 1.25$, $x = 1000 \cdot 1.25 = \boxed{\$1250}$.

8. In the right triangle ABC, $AC = 12$, $BC = 5$, and angle C is a right angle. A semicircle is inscribed in the triangle as shown. What is the radius of the semicircle?



$\triangle ADO \sim \triangle ACB$, $\dfrac{12-r}{r} = \dfrac{13}{5}$,

5. The sum of the first and last of four consecutive odd integers is 52. What are the four integers?

Let x is the first odd integer.

$$x + (x + 6) = 52.$$

$2x = 46$, and $x = 23$. Answer is $\boxed{23,\ 25,\ 27,\ 29}$.

6. Annie and Bonnie are running laps around a 400-meter oval track. They started together, but Annie has pulled ahead because she runs 25% faster than Bonnie. How many laps will Annie have run when she first passes Bonnie?

Let x is the speed of Bonnie, $1.25x$ is the speed of Annie. t is the time when Annie passed Bonnie.

$$x \cdot t + 400 = 1.25x \cdot t.$$

$0.25x \cdot t = 400$, so $1.25x \cdot t = 2000$. So Annie ran 2000 meters before passing Bonnie, which gives us $\boxed{5}$ laps.

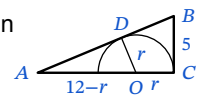
7. A washer costs 25% more than a dryer. If the store clerk gave a 10% discount for the dryer and a 20% discount for the washer, how much is the washer before the discount if you paid 1900 dollars?

Let x is the washer price, $x/1.25$ — dryer.

$$0.80x + 0.90 \cdot \frac{x}{1.25} = 1900.$$

$0.8x \cdot 1.25 + 0.9x = 1900 \cdot 1.25$, $1x + 0.9x =$
$= 1.9x = 1900 \cdot 1.25$, $x = 1000 \cdot 1.25 = \boxed{\$1250}$.

8. In the right triangle ABC, $AC = 12$, $BC = 5$, and angle C is a right angle. A semicircle is inscribed in the triangle as shown. What is the radius of the semicircle?



$\triangle ADO \sim \triangle ACB$, $\dfrac{12-r}{r} = \dfrac{13}{5}$,
$5(12-r) = 13r$, $60 - 5r = 13r$,
$18r = 60$, $\boxed{\text{r = 10/3}}$.

CHALLENGE PROBLEMS 1-2

C1. A basketball team's players were successful on 50% of their two-point shots and 40% of their three-point shots, which resulted in 54 points. They attempted 50% more two-point shots than three-point shots. How many three-point shots did they attempt?

CHALLENGE PROBLEMS 1-2

C1. A basketball team's players were successful on 50% of their two-point shots and 40% of their three-point shots, which resulted in 54 points. They attempted 50% more two-point shots than three-point shots. How many three-point shots did they attempt?

Let x is the number of two-points shots and y is the number of three-points shots.

$$x = 1.5 \cdot y,$$
$$0.5 \cdot 2 \cdot x + 0.4 \cdot 3 \cdot y = 54.$$

CHALLENGE PROBLEMS 1-2

C1. A basketball team's players were successful on 50% of their two-point shots and 40% of their three-point shots, which resulted in 54 points. They attempted 50% more two-point shots than three-point shots. How many three-point shots did they attempt?

Let x is the number of two-points shots and y is the number of three-points shots.

$$x = 1.5 \cdot y,$$

$$0.5 \cdot 2 \cdot x + 0.4 \cdot 3 \cdot y = 54.$$

$x + 1.2y = 54$, $x = 54 - 1.2y$, $54 - 1.2y = 1.5y$,

$2.7y = 54$, $y = \boxed{20}$ shots.

CHALLENGE PROBLEMS 1-2

C1. A basketball team's players were successful on 50% of their two-point shots and 40% of their three-point shots, which resulted in 54 points. They attempted 50% more two-point shots than three-point shots. How many three-point shots did they attempt?

Let x is the number of two-points shots and y is the number of three-points shots.

$$x = 1.5 \cdot y,$$

$$0.5 \cdot 2 \cdot x + 0.4 \cdot 3 \cdot y = 54.$$

$x + 1.2y = 54$, $x = 54 - 1.2y$, $54 - 1.2y = 1.5y$, $2.7y = 54$, $y = \boxed{20}$ shots.

C2. Two sides of a triangle have lengths 10 and 15. The length of the altitude to the third side is the average of the lengths of the altitudes to the two given sides. How long is the third side?

CHALLENGE PROBLEMS 1-2

C1. A basketball team's players were successful on 50% of their two-point shots and 40% of their three-point shots, which resulted in 54 points. They attempted 50% more two-point shots than three-point shots. How many three-point shots did they attempt?

Let x is the number of two-points shots and y is the number of three-points shots.

$$x = 1.5 \cdot y,$$

$$0.5 \cdot 2 \cdot x + 0.4 \cdot 3 \cdot y = 54.$$

$x + 1.2y = 54$, $x = 54 - 1.2y$, $54 - 1.2y = 1.5y$, $2.7y = 54$, $y = \boxed{20}$ shots.

C2. Two sides of a triangle have lengths 10 and 15. The length of the altitude to the third side is the average of the lengths of the altitudes to the two given sides. How long is the third side?

Let S is the area of the triangle. x is the length the third side and h is the length of altitude to third side. a and b is altitudes to sides 10 and 15.

$$S = \frac{1}{2}xh = \frac{1}{2}10a = \frac{1}{2}15b,$$

$$\frac{a+b}{2} = h.$$

C1. A basketball team's players were successful on 50% of their two-point shots and 40% of their three-point shots, which resulted in 54 points. They attempted 50% more two-point shots than three-point shots. How many three-point shots did they attempt?

Let x is the number of two-points shots and y is the number of three-points shots.

$$x = 1.5 \cdot y,$$

$$0.5 \cdot 2 \cdot x + 0.4 \cdot 3 \cdot y = 54.$$

$x + 1.2y = 54$, $x = 54 - 1.2y$, $54 - 1.2y = 1.5y$, $2.7y = 54$, $y = \boxed{20}$ shots.

C2. Two sides of a triangle have lengths 10 and 15. The length of the altitude to the third side is the average of the lengths of the altitudes to the two given sides. How long is the third side?

Let S is the area of the triangle. x is the length the third side and h is the length of altitude to third side. a and b is altitudes to sides 10 and 15.

$$S = \frac{1}{2}xh = \frac{1}{2}10a = \frac{1}{2}15b,$$

$$\frac{a+b}{2} = h.$$

$xh = 10a \Rightarrow a = xh/10$,
$xh = 15b \Rightarrow b = xh/15$,

$$\frac{a+b}{2} = \frac{\frac{xh}{10} + \frac{xh}{15}}{2} = \frac{3xh + 2xh}{60} = \frac{5xh}{60} = \frac{xh}{12} = h.$$

$x = \boxed{12}$.

C3. It takes Clea 60 seconds to walk down an escalator when it is not operating, and only 24 seconds to walk down the escalator when it is operating. How many seconds does it take Clea to ride down the operating escalator when she just stands on it?

C3. It takes Clea 60 seconds to walk down an escalator when it is not operating, and only 24 seconds to walk down the escalator when it is operating. How many seconds does it take Clea to ride down the operating escalator when she just stands on it?

Let S meters be the length of the escalator.
v meters per second is the speed of Clea and
x meters per second is the speed of the escalator.

$$S = 60 \cdot v = 24 \cdot (v + x).$$

C3. It takes Clea 60 seconds to walk down an escalator when it is not operating, and only 24 seconds to walk down the escalator when it is operating. How many seconds does it take Clea to ride down the operating escalator when she just stands on it?

Let S meters be the length of the escalator.
v meters per second is the speed of Clea and
x meters per second is the speed of the escalator.

$$S = 60 \cdot v = 24 \cdot (v + x).$$

$60v = 24v + 24x$, $36v = 24x$, $v = \frac{2}{3}x$.

$S = 60 \cdot v = 60 \cdot \frac{2}{3}x$, $S/x = \boxed{40}$ seconds.

C3. It takes Clea 60 seconds to walk down an escalator when it is not operating, and only 24 seconds to walk down the escalator when it is operating. How many seconds does it take Clea to ride down the operating escalator when she just stands on it?

Let S meters be the length of the escalator.
v meters per second is the speed of Clea and
x meters per second is the speed of the escalator.

$$S = 60 \cdot v = 24 \cdot (v + x).$$

$60v = 24v + 24x$, $36v = 24x$, $v = \frac{2}{3}x$.

$S = 60 \cdot v = 60 \cdot \frac{2}{3}x$, $S/x = \boxed{40}$ seconds.

C4. Paula the painter and her two helpers each paint at constant, but different, rates. They always start at $8:00$ AM, and all three always take the same amount of time to eat lunch. On Monday the three of them painted 50% of a house, quitting at $4:00$ PM. On Tuesday, when Paula wasn't there, the two helpers painted only 24% of the house and quit at $2:12$ PM. On Wednesday Paula worked by herself and finished the house by working until $7:12$ PM How long, in minutes, was each day's lunch break?

C4. Paula the painter and her two helpers each paint at constant, but different, rates. They always start at $8:00$ AM, and all three always take the same amount of time to eat lunch. On Monday the three of them painted 50% of a house, quitting at $4:00$ PM. On Tuesday, when Paula wasn't there, the two helpers painted only 24% of the house and quit at $2:12$ PM. On Wednesday Paula worked by herself and finished the house by working until $7:12$ PM How long, in minutes, was each day's lunch break?

Let x hours be the length of the lunch break.
S total amount of work, v work/hours is the speed of Paula's work and u is the speed of two helpers.

$$0.5S = (v+u)\cdot(8-x), \quad (1)$$

$$0.24S = u\cdot(6.2-x), \quad (2)$$

$$0.26S = v\cdot(11.2-x). \quad (3)$$

C4. Paula the painter and her two helpers each paint at constant, but different, rates. They always start at $8:00$ AM, and all three always take the same amount of time to eat lunch. On Monday the three of them painted 50% of a house, quitting at $4:00$ PM. On Tuesday, when Paula wasn't there, the two helpers painted only 24% of the house and quit at $2:12$ PM. On Wednesday Paula worked by herself and finished the house by working until $7:12$ PM How long, in minutes, was each day's lunch break?

Let x hours be the length of the lunch break.
S total amount of work, v work/hours is the speed of Paula's work and u is the speed of two helpers.

$$0.5S = (v+u)\cdot(8-x), \quad (1)$$
$$0.24S = u\cdot(6.2-x), \quad (2)$$
$$0.26S = v\cdot(11.2-x). \quad (3)$$

Addition of (2) and (3) is equal (1).

$$(v+u)(8-x) = u\cdot(6.2-x) + v\cdot(13.2-x),$$

C4. Paula the painter and her two helpers each paint at constant, but different, rates. They always start at $8:00$ AM, and all three always take the same amount of time to eat lunch. On Monday the three of them painted 50% of a house, quitting at $4:00$ PM. On Tuesday, when Paula wasn't there, the two helpers painted only 24% of the house and quit at $2:12$ PM. On Wednesday Paula worked by herself and finished the house by working until $7:12$ PM How long, in minutes, was each day's lunch break?

Let x hours be the length of the lunch break.
S total amount of work, v work/hours is the speed of Paula's work and u is the speed of two helpers.

$$0.5S = (v+u)\cdot(8-x), \qquad (1)$$
$$0.24S = u\cdot(6.2-x), \qquad (2)$$
$$0.26S = v\cdot(11.2-x). \qquad (3)$$

Addition of (2) and (3) is equal (1).

$$(v+u)(8-x) = u\cdot(6.2-x) + v\cdot(13.2-x),$$
$$8v + 8u - vx - ux = 6.2u - ux + 13.2v - vx,$$
$$8v + 8u = 6.2u + 11.2v,$$
$$1.8u = 3.2v, \quad u = \frac{32}{18}v = \frac{16}{9}v.$$

C4. Paula the painter and her two helpers each paint at constant, but different, rates. They always start at $8:00$ AM, and all three always take the same amount of time to eat lunch. On Monday the three of them painted 50% of a house, quitting at $4:00$ PM. On Tuesday, when Paula wasn't there, the two helpers painted only 24% of the house and quit at $2:12$ PM. On Wednesday Paula worked by herself and finished the house by working until $7:12$ PM How long, in minutes, was each day's lunch break?

Let x hours be the length of the lunch break.
S total amount of work, v work/hours is the speed of Paula's work and u is the speed of two helpers.

$$0.5S = (v+u)\cdot(8-x), \qquad (1)$$
$$0.24S = u\cdot(6.2-x), \qquad (2)$$
$$0.26S = v\cdot(11.2-x). \qquad (3)$$

Addition of (2) and (3) is equal (1).

$$(v+u)(8-x) = u\cdot(6.2-x) + v\cdot(13.2-x),$$
$$8v + 8u - vx - ux = 6.2u - ux + 13.2v - vx,$$
$$8v + 8u = 6.2u + 11.2v,$$
$$1.8u = 3.2v, \quad u = \frac{32}{18}v = \frac{16}{9}v.$$

Now using (2) and (3):

$$S = \frac{u(6.2-x)}{0.24} = \frac{v(11.2-x)}{0.26},$$

C4. Paula the painter and her two helpers each paint at constant, but different, rates. They always start at $8:00$ AM, and all three always take the same amount of time to eat lunch. On Monday the three of them painted 50% of a house, quitting at $4:00$ PM. On Tuesday, when Paula wasn't there, the two helpers painted only 24% of the house and quit at $2:12$ PM. On Wednesday Paula worked by herself and finished the house by working until $7:12$ PM How long, in minutes, was each day's lunch break?

Let x hours be the length of the lunch break.
S total amount of work, v work/hours is the speed of Paula's work and u is the speed of two helpers.

$$0.5S = (v+u)\cdot(8-x), \quad (1)$$
$$0.24S = u\cdot(6.2-x), \quad (2)$$
$$0.26S = v\cdot(11.2-x). \quad (3)$$

Addition of (2) and (3) is equal (1).

$$(v+u)(8-x) = u\cdot(6.2-x) + v\cdot(13.2-x),$$
$$8v+8u-vx-ux = 6.2u-ux+13.2v-vx,$$
$$8v+8u = 6.2u+11.2v,$$
$$1.8u = 3.2v, \quad u = \frac{32}{18}v = \frac{16}{9}v.$$

Now using (2) and (3):

$$S = \frac{u(6.2-x)}{0.24} = \frac{v(11.2-x)}{0.26},$$
$$26\cdot u(6.2-x) = 24\cdot v(11.2-x),$$
$$26\frac{16}{9}v(6.2-x) = 24\cdot v(11.2-x),$$
$$26\cdot 16\cdot(6.2-x) = 24\cdot 9\cdot(11.2-x),$$
$$13\cdot 4\cdot(6.2-x) = 3\cdot 9\cdot(11.2-x),$$
$$52\cdot 6.2 - 52x = 27\cdot 11.2 - 27x,$$
$$25x = 52\cdot 6.2 - 27\cdot 11.2 = 322.4 - 302.4 = 20,$$
$$x = \frac{20}{25} = \frac{4}{5}.$$

The lunch break is $\boxed{48}$ minutes.