



# Angles and Triangles

Graham Middle School Math Olympiad Team



$$\sqrt{x} = 3, 14$$
$$3 \times 3 = 9$$

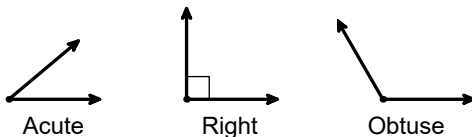


## TYPES OF ANGLES AND THEIR MEASURES

We typically measure angles in degrees (symbol  $^{\circ}$ ), with an entire circle having a measure of  $360^{\circ}$ . A pair of rays intersecting to form a straight line therefore form a  $180^{\circ}$  angle. The number 360 is somewhat arbitrary. It was developed in ancient Babylonia where they used a sexagesimal (base 60) number system and had a 360 day calendar.

Angles are sometimes also measured in radians, where  $2\pi$  radians is equal to 360 degrees.

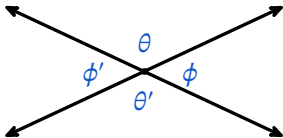
Angles are classified as acute, right, or obtuse depending on whether they measure less than, equal to, or greater than  $90^{\circ}$  respectively.



Pairs of angles whose measures sum to  $90^{\circ}$  are called **complementary** angles.

Pairs of angles that sum to  $180^{\circ}$  are called **supplementary**.

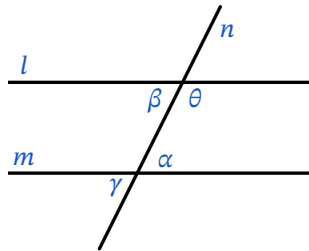
## VERTICAL ANGLES AND ANGLES FORMED BY PARALLEL LINES



Two pairs of vertical angles ( $\theta$  and  $\theta'$ ,  $\phi$  and  $\phi'$ ) are formed by intersecting lines. Since they are angles that make a line,  $\theta$  and  $\phi$  sum to  $180^\circ$ . Likewise,  $\theta'$  and  $\phi$  sum to  $180^\circ$ . Therefore  $\theta = \theta'$  and the angles are said to be congruent. Since  $\phi$  and  $\phi'$  both form lines when combined with  $\theta$ , we also see that  $\phi = \phi'$ .

Intersecting lines form two pairs of congruent **vertical** angles.

In the figure below,  $l$  and  $m$  are parallel lines and line  $n$  is called a **transversal**.



Angles  $\alpha$  and  $\beta$  are called **alternate interior angles** and they are congruent.

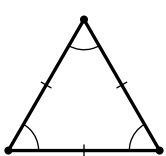
Angles in same relative locations to lines  $l$  and  $m$  respectively are **corresponding angles**, such as  $\gamma$  and  $\beta$ , and they are also congruent.

## TYPES OF TRIANGLES

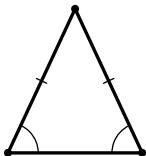
Any 3 points that *don't lie on the same line* can be vertices of a triangle.

The length of any side in a triangle must be **less than the sum of the lengths of the other two sides**.

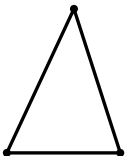
Triangles can be classified by the number of sides of equal length.



Equilateral  
Triangle  
All sides and  
angles are  
equal

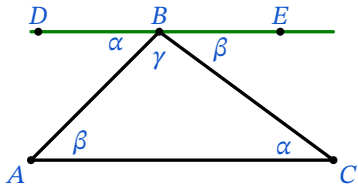


Isosceles  
Triangle  
One pair of  
sides and  
angles equal



Scalene  
No sides or  
angles equal

The sum of the angles in a triangle is always  $180^\circ$ .



Proof: Draw a line parallel to side  $AB$  of triangle  $ABC$  passing through point  $C$ . As alternate interior angles, we have the pairs of angles labeled  $\alpha$  and  $\beta$  in the figure equal. Since they form a line  $ED$ ,  $\angle ACB + \angle ECA + \angle BCD = 180^\circ$ . This means  $\gamma + \alpha + \beta = 180^\circ$ . Q.E.D.