Probability

Graham Middle School Math Olympiad Team

PROBABILITY

An **outcome** is a possible result of an experiment.

We say "the outcome of throwing the dice was 4." The experiment was throwing the die and its result (outcome) was that the face with 4 pips on it came up.

Probability problems are either based on *countable* outcomes or on outcomes that are not countable. The later case is called *geometric probability* or *continuous probability*.

If the number of outcomes is countable and *if all oucomes have the same probability*, then the **probability** of desired oucome is:

$$P = \frac{\text{(number of desired outcomes)}}{\text{(total number of events)}}.$$

Example: We write each of the names of all 30 students in a math club on a card and we draw a card at random. If there are 14 sixth-graders, 9 seventh-graders, and 7 eighth-graders in this club, what is the probability of the name on the card being that of a sixth-grader?

The total number of *outcomes* is all students in the club. The desired *outcomes* are all sixth-graders. So *probability* is

$$P = \frac{14}{30} = \frac{7}{15}.$$

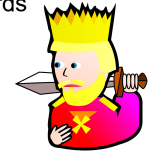
In the *geometric case*, the **probability** of a desired outcome is calculated as the ratio:

$$P = \frac{\text{(desired area)}}{\text{(total area)}}.$$

Probabilities in any case are always between 0 and 1, inclusive.

ADDITION RULE FOR PROBABILITY

What is the probability of choosing *an ace* **or** *a king* from a full deck of 52 cards?



A card that we got from the deck of cards may not be both an ace and a king. So we have 4 *oucomes* of getting an ace and 4 *oucomes* of getting a king. The result number of *desired oucomes* is 8. The *total number of possible outcomes* is 52. The total probability is

$$P = \frac{4+4}{52} = \frac{8}{52} = \frac{2}{13}.$$

We also may count the probability in other way. The *probability* of getting an ace is $4/52$ and the *probability* of getting a king is also $4/52$. Since both cases are good for us, the total probability is

$$P = \frac{4}{52} + \frac{4}{52} = \frac{1}{13} + \frac{1}{13} = \frac{2}{13}.$$

If two events are *oucomes* of one experiment and they *may not happen together*, then *probability* of any of this event is **sum of their probabilities**

$$P(A \text{ or } B) = P(A) + P(B).$$

What is the probability of choosing *a multiple of* 4 or *a multiple of* 5 from a numbers from 1 to 100?

The probability of getting a multiple of 4 is $25/100$, the probability of getting a multiple of 5 is $20/100$. But there are multiples of 20, which we counted twice. To make up for this case, we need to subtract the probability of getting a multiple of 20, which is $5/100$. So total probability would be

$$P = \frac{25}{100} + \frac{20}{100} - \frac{5}{100} = \frac{25+20-5}{100} = \frac{40}{100} = \frac{2}{5}.$$

So the *probability of two events* of one experiment

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).$$

MULTIPLICATION RULE FOR PROBABILITY

If the experiment *"throw a die and toss a coin"* is performed, what is the probability of the event *"a 4 and a tails"* to happen?

Let's first throw a die. If 4 didn't happen, we don't need to toss a coin, the desired event will not happen in any case. So we need to toss a coin only when the die rolls 4. The 4 has a probability of $1/6$ and, in this case only, we toss a coin and with a probability of $1/2$ we will get tails. So the overall probability would be

$$P = \frac{1}{6} \cdot \frac{1}{2} = \frac{1}{12}.$$

If two events are *independent* the **probability** of both of them happen is *a multiple of their probabilities*

$$P(A \text{ and } B) = P(A) \cdot P(B).$$

The **conditional probability** of an event E that depends on another event F to happen is denoted

$$P(E \mid F)$$

and is probability of E to happen if we know that F has already happened.

We used that rule to solve the problem *"throw a die and toss a coin"*.

If two events are *dependent* the **probability** of both of them happen is

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A).$$

What is the probability of choosing *an ace* **and then** *a king* from a full deck of 52 cards?

The probability to choose an ace is $4/52$. Then we selecting a king from the deck without one card, so the probability to choose a king is $4/51$. The total probability is

$$P(A \text{ and then } K) = P(A) \cdot P(K \mid A) = \frac{4}{52} \cdot \frac{4}{51} = \frac{4}{663}.$$

USING THE NEGATIVE CASE

THE BIRTHDAY PROBLEM