Angles and Triangles solutions

Graham Middle School Math Olympiad Team

PROBLEMS 1-3

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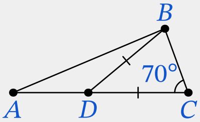

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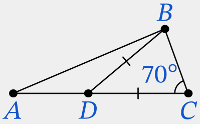

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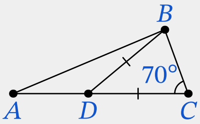

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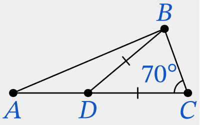

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Let's look at the longest side of a triangle. Then iterate on the length of the middle side.

Case 0: 3 or smaller. The only option $3-2-1$ is degenerated triangle.

Case 1: 4, the only one scalene triangle possible: $4-3-2$.

Case 2: 5: $5-4-2$ and $5-4-3$.

Case 3: 6: $6-4-3$, $6-5-2$, and $6-5-3$.

Case 4: 7: the sum two other sides will be at least 8.

As a result we have $\boxed{6}$ scalene triangles.

PROBLEMS 4-6

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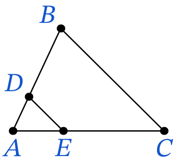

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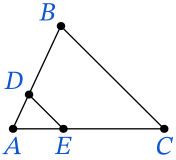

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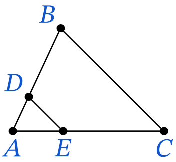

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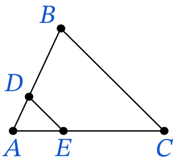

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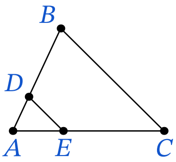

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The total sum of angles in the polygon is $180 \cdot (N-2)$ and from another side, the sum of angles is $150 \cdot q + 80 \cdot (N-q)$.

$$180(N-2) = 150q + 80(N-q),$$
$$180N - 360 = 150q + 80N - 80q,$$
$$100N - 360 = 70q,$$
$$100N = 360 + 70q,$$
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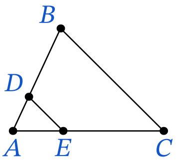

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So $7q$ should be ended up with 4. This is possible when $q = 2, 12, 22$ etc. For $q = 2$, $\boxed{N = 5}$, for $q = 12$, $\boxed{N = 12}$, but for each $q > 12$, $N < q$, so the there are no more solutions.

PROBLEMS 7-8

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Let's look at the longest side:
Case 1: The longest side is 5. The sum of two other sides should be at least 6, so the only options are $2-4-5$ and $3-4-5$.
Case 2: The longest side is 4. The sum of two other sides should be at least 5, so the only option is $2-3-4$.
Case 3: The longest side is 3. The rest two sides are 1 and 2 and we can got only degenerated triangle.
So we have only three cases $2-4-5$, $3-4-5$, and $2-3-4$.

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We let the measures be $5x$ and $4x$ giving us the ratio of $5 : 4$. We know $90-4x > 90-5x$ since this inequality gives $x > 0$, which is true since the measures of angles are never negative. We also know the bigger complement is twice the smaller, so

$$90-4x = 2(90-5x),$$
$$90-4x = 180-10x,$$
$$6x = 90,$$
$$x = 15.$$

Therefore, the angles are 75 and 60, which sum to $\boxed{135}$.

CHALLENGE PROBLEMS 1-2

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The triangle inequality generalizes to all polygons, so $x < 3 + 7 + 15$ and $15 < x + 3 + 7$ yields $5 < x < 25$. Now, we know that there are 19 numbers between 5 and 25 exclusive, but we must subtract 2 to account for the 2 lengths already used that are between those numbers, which gives $19 - 2 = \boxed{17}$.

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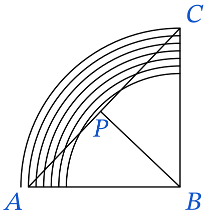

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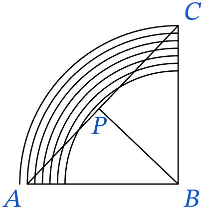

It follows that we can draw circles of radii $15, 16, 17, 18, 19,$ and $20,$ that each contribute two integer lengths (since these circles intersect the hypotenuse twice) from B to $\overline{AC}$ and one circle of radius 21 that contributes only one such segment. Our answer is then $6 \cdot 2 + 1 = \boxed{13}$.

CHALLENGE PROBLEM 3

C3. A square with side length x is inscribed in a right triangle with sides of length 3, 4, and 5 so that one vertex of the square coincides with the right-angle vertex of the triangle. A square with side length y is inscribed in another right triangle with sides of length 3, 4, and 5 so that one side of the square lies on the hypotenuse of the triangle. What is $\frac{x}{y}$?

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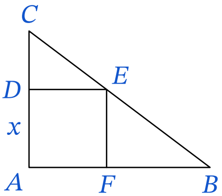

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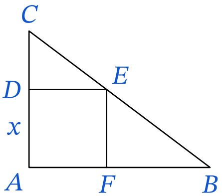

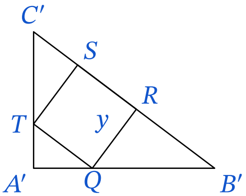

Analyze the first right triangle. Note that $\triangle ABC$ and $\triangle FBE$ are similar, so $\frac{BF}{FE} = \dfrac{AB}{AC}$. This can be written as $\dfrac{4-x}{x} = \dfrac{4}{3}$. Solving, $x = \dfrac{12}{7}$.

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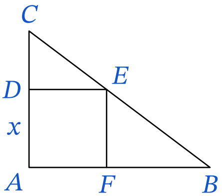

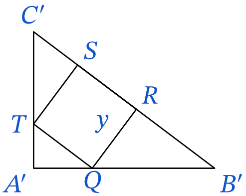

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Now we analyze the second triangle.

Similarly, $\triangle A'B'C'$ and $\triangle RB'Q$ are similar, so $RB' = \frac{4}{3}y$, and $C'S = \frac{3}{4}y$. Thus,

$$C'B' = C'S + SR + RB' = \frac{4}{3}y + y + \frac{3}{4}y = 5.$$

Solving for y, we get $y = \frac{60}{37}$. Thus, $\frac{x}{y} = \boxed{\frac{37}{35}}$.

CHALLENGE PROBLEM 4

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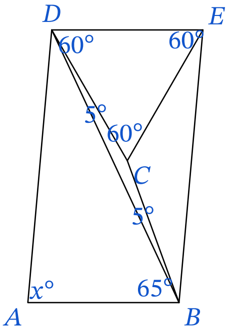

First, connect the diagonal DB, then, draw line DE such that it is congruent to DC and is parallel to AB. Because triangle DCB is isosceles and angle DCB is 170°, the angles CDB and CBD are both $\frac{180-170}{2} = 5^\circ$. Because angle ABC is 70°, we get angle ABD is 65°. Next, noticing parallel lines AB and DE and transversal DB, we see that angle BDE is also 65°, and subtracting off angle CDB gives that angle EDC is 60°.

Now, because we drew $ED = DC$, triangle DEC is equilateral. We can also conclude that $EC = DC = CB$ meaning that triangle ECB is isosceles, and angles CBE and CEB are equal. Finally, we can set up our equation. Denote angle BAD as x°. Then, because $ABED$ is a parallelogram, the angle DEB is also x°. Then, CEB is $(x-60)^\circ$. Again because $ABED$ is a parallelogram, angle ABE is $(180-x)^\circ$. Subtracting angle ABC gives that angle CBE equals $(110-x)^\circ$. Because angle CBE equals angle CEB, we get

$$x - 60 = 110 - x,$$

solving into $x = \boxed{85^\circ}$.

TITLE

TEAM ATTACK 4 SOLUTIONS, PROBLEMS 1-2

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Since she walked 1 km to school and 1 km back home, her total distance is $1 + 1 = 2$ km.
Since she spent 30 minutes walking to school and 10 minutes walking back home, her total time is $30 + 10 = 40$ minutes = $\frac{40}{60} = \frac{2}{3}$ hours.

Therefore her average speed in km/hr is $\frac{2}{\frac{2}{3}} = \boxed{3}$.

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TA2. Members of the Rockham Soccer League buy socks and T-shirts. Socks cost \$4 per pair and each T-shirt costs \$5 more than a pair of socks. Each member needs one pair of socks and a shirt for home games and another pair of socks and a shirt for away games. If the total cost is \$2366, how many members are in the League?

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Since T-shirts cost 5 dollars more than a pair of socks, T-shirts cost $5 + 4 = 9$ dollars.
Since each member needs 2 pairs of socks and 2 T-shirts, the total cost for 1 member is $2(4 + 9) = 26$ dollars.
Since 2366 dollars was the cost for the club, and 26 was the cost per member, the number of members in the League is $2366 \div 26 = \boxed{91}$.

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By the triangle inequality, no side may have a length greater than the semiperimeter, which is $\frac{1}{2} \cdot 7 = 3.5$.
Since all sides must be integers, the largest possible length of a side is 3. Therefore, all such triangles must have all sides of length 1, 2, or 3. Since $2 + 2 + 2 = 6 < 7$, at least one side must have a length of 3. Thus, the remaining two sides have a combined length of $7 - 3 = 4$. So, the remaining sides must be either 3 and 1 or 2 and 2. Therefore, the number of triangles is $\boxed{2}$.

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Notice that
$1 \cdot 60 = 2 \cdot 30 = 3 \cdot 20 = 4 \cdot 15 = 5 \cdot 12 = 6 \cdot 10 = 60$.
Hence, 60 has 12 factors, of which 6 are less than 7. Thus, the answer is $\frac{6}{12} = \boxed{\frac{1}{2}}$.

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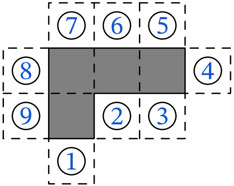
TA6. The sum of the two 5-digit numbers $AMC10$ and $AMC12$ is 123422. What is $A+M+C$?

We know that $AMC12$ is 2 more than $AMC10$. We set up $AMC10 = x$ and $AMC12 = x+2$. We have $x+x+2 = 123422$. Solving for x, we get $x = 61710$. Therefore, the sum $A+M+C = \boxed{14}$.

TA5. The polygon enclosed by the solid lines in the figure consists of 4 congruent squares joined edge-to-edge. One more congruent square is attached to an edge at one of the nine positions indicated. How many of the nine resulting polygons can be folded to form a cube with one face missing?

TA5. The polygon enclosed by the solid lines in the figure consists of 4 congruent squares joined edge-to-edge. One more congruent square is attached to an edge at one of the nine positions indicated. How many of the nine resulting polygons can be folded to form a cube with one face missing?



A cube missing one face has 5 of its 6 faces. Since the shape has 4 faces already, we need another face. The only way to add another face is if the added square does not overlap any of the others. 1, 2, and 3 overlap, while squares 4 to 9 do not. The answer is $\boxed{6}$.

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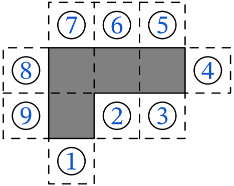

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TA7. A point (x, y) is randomly picked from inside the rectangle with vertices $(0, 0)$, $(4, 0)$, $(4, 1)$, and $(0, 1)$. What is the probability that $x < y$?

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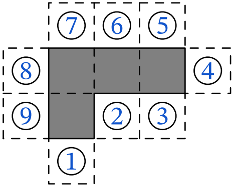

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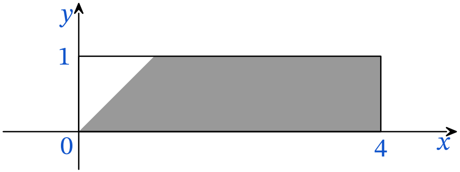

The area of this rectangle is $4 \cdot 1 = 4$.
The line $x = y$ intersects the rectangle at $(0,0)$ and $(1,1)$.

The area of this triangle is $\frac{1}{2} \cdot 1^2 = \frac{1}{2}$

Therefore, the probability that $x < y$ is $\frac{\frac{1}{2}}{4} = \boxed{\frac{1}{8}}$.