MOEMS Contests

Graham Middle School Math Olympiad Team

CONTEST OVERVIEW

Unlike many of our contests, time should not be a factor for MOEMS. You have 30 minutes for 5 problems.
Please take this time to check your work.

In general, the problems are arranged with the easiest first, and the harder ones at the end of the contest. Be sure to read the question carefully to insure that you actually are answering the question that is being asked. Solving for the area of a square when they ask for a side length is a frustrating (and avoidable) way to miss a question.

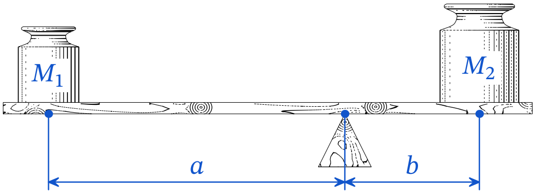
For individual and school awards, what matters is your cumulative score for the 5 contests. This has several important implications:

- If you know that you will be absent the day a contest is given, let the coaches know – it might be possible to change the contest date if it works for the team.
- If a particular contest is not going well for you, don't be discouraged. That extra question you solve could be an important point in your cumulative score.

BALANCE QUESTION

Although this is more of a physics question, MOEMS often asks questions about balancing weights on two sides of a pivot point.

$$M_1 \times a = M_2 \times b$$



The tip of the triangle represents the **pivot point** or **fulcrum**. The beam or board the weights rest on is called the **lever arm**. Unless told otherwise, ignore the mass of the lever arms themselves in these problems.

The system is **balanced** when the *product of the mass times the distance* from the fulcrum on one side is **equal** to the *product of the mass and the distance* from the fulcrum on the other side.
As shown in the illustration on the right, this occurs when $M_1 \cdot a = M_2 \cdot b$.

Even though the blocks M_1 and M_2 have some width, the problems assume the mass is distributed uniformly, so we can simplify the problem so that the mass is acting as if it were all concentrated at this center point of the block.

If there are **multiple** masses on a side, to find the total torque for all masses we sum the products of $M_i\, r_i$ where M_i are the individual masses and r_i are their corresponding distances to the fulcrum.

CRYPTARITHM

A cryptarithm is a popular type of mathematical logic problem, where each digit now corresponds to a number. Read the rules for a particular cryptarithm carefully. Typically a letter can represent only one digit, and every instance of a digit always corresponds to the same letter. In other words, if d corresponds to 7, it cannot also represent 8, and 7 cannot be represented by any letter other than d. If a number has 4 digits, the **leading digit cannot be zero** (it would be a 3-digit number then. Don't forget that zero could appear elsewhere. Ignoring the possibility of zero is often disastrous if the problem is asking something like, "find the smallest possible value for the codeword."

Don't be shy to try using some trial and error techniques to crack the puzzle – you do have 30 minutes after all.

AB, CD, EF, GH, and JK are five 2-digit numbers. Different letters represent different digits. Find the greatest possible value for the fraction

$$\frac{AB + CD + EF}{GH - JK}.$$

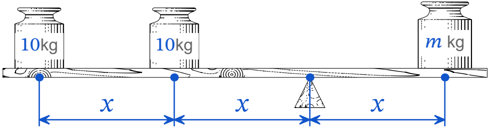
The smallest denominator is 1, which can be obtained using $20 - 19$, $30 - 29$, $40 - 39$, etc. To maximize the numerator, we should choose $20 - 19$. The remaining digits are 3 through 8. To make the numerand as large as possible, use 8, 7, 6 for the tens digits (order doesn't matter), and 3, 4, 5 for the ones digits (again, in any order). The greatest possible value is 222.

EXERCISES

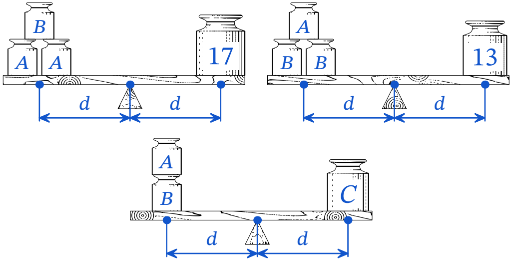
1. AB, CD, EF, GH, and JK are five 2-digit numbers. Different letters represent different digits. Find the smallest possible value for the fraction:

$$(AB + CD + EF)/(GH - JK).$$

2. Both P and $(98 - P)$ are prime numbers. What is the least possible value for P?
3. Line segment AB has endpoints $A(-5,\ 4)$ and $B(7,\ 13)$. Point C lies on AB and is two-thirds of the way from point A to point B. Find the coordinates $(x,\ y)$ of point C.
4. In the diagram below, the 10 kg weights are at a distance of x and $2x$ from the fulcrum. The weight on the right side is also a distance x from the fulcrum. What is the mass on the right side of the balance when the system is in equilibrium?



5. The arithmetic mean of five positive integers is 30. What is the greatest possible value of their median?
6. Find the least value of the fraction a/b such a/b is an improper fraction in lowest terms; and if a/b is divided by either $6/25$ or $8/15$, the quotient is a whole number.
7. Find the greatest prime factor of the sum $5! + 7!$.
8. A, B, and C represent weights in the 3 balanced scales shown below, with lever arms of equal lengths on both sides. Find C.



CHALLENGE PROBLEMS

1. Six girls of differing heights are arranged in 2 rows of 3 girls each. Each girl is taller than the girl in front of her and also taller than the girl to her right. How many distinct arrangements of the six girls are possible?

2. What is the probability that a fraction chosen at random from the list of 49 fractions below will terminate?

$$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots, \frac{1}{48}, \frac{1}{49}, \frac{1}{50}$$

3. 480 identical cubes are placed in the corner of a room and arranged in a rectangular solid that is $6 \times 8 \times 10$ cubes and bounded by the floor and 2 walls. How many of the those cubes have at least one face visible to an observer in the room?

4. In the aluminum can shown below, the height is 8 cm and the circumference is 12 cm. Points A and B lie "opposite" each other on the two rims (they would be connected by a diameter if B were translated vertically to the upper rim). Find the **shortest distance along the surface of the can** from A to B.