Kvantik problems, April 2022

PROBLEM 36

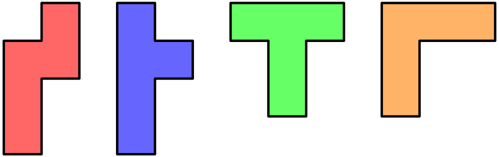
A postman has a pack of envelopes from which he needs precisely 50. While he was meticulously counting them off one by one, his son, who is in fifth grade, said: ”If you only knew how many envelopes there are in total, you could count them twice as fast.” What does son mean, and how many envelopes are in one pack?

The other way to get 50 envelopes is to count some number of envelopes in a pack and get the rest. Since in this case we need to count the half of 50, e.g. 25, the total amount of envelopes in a pack is 75.

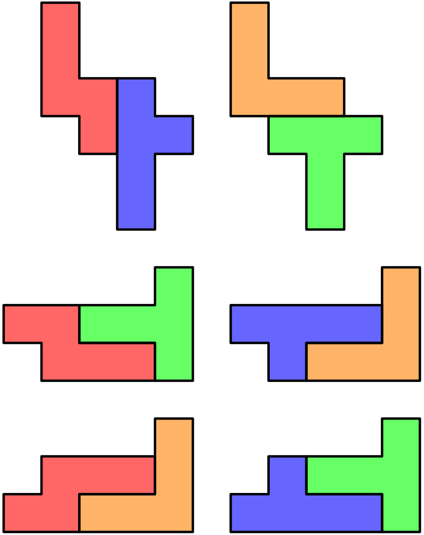
PROBLEM 37

There is a set of four different pentominoes (figures made of 5 connected equal-sized squares). For each possible pairing within the set, the pentominoes in the two pairs can be combined to create two congruent figures. Provide an example of such pentominoes and figures.

We can get following pentominoes:



And combine them in following ways:



PROBLEM 38

Kvantik the Robot permuted numbers in sequence $1,\ 2,\ 3, \dots,\ 100$ in "alphabetical order", so the first come the numbers starting with 1, then starting with 2, and so on (numbers beginning with the same digit are ordered by the second digit). He got the sequence $1,\ 10,\ 100,\ 11,\ 12,$ etc. How many numbers stay in their original place?

1 is obvious on its place. No more single digit number are on their places, since this places are occupied by numbers 10, 100, 11, 12 and so on. 100 is also on its place.

Let's consider two digit numbers: $\overline{ab} = 10a + b$, where a and b are digits. All numbers started from the same digit (except 1) are in groups of 11 numbers:

$$a,\ \overline{a0},\ \overline{a1},\ \overline{a2},\ \overline{a3},\ \overline{a4},\ \overline{a5},\ \overline{a6},\ \overline{a7},\ \overline{a8},\ \overline{a9}.$$

So we can calculate the new position of $\overline{ab}$ which is

$$11(a-1) + b + 3,$$

where $a-1$ because we are starting from 1, not 0, $+2$ because $\overline{a0}$ are on the second place and $+1$ for 100 which is in the beginning. The formula is incorrect only for 10 because of the position of 100.

For other numbers we can find the numbers that keeps their places:

$$11(a-1) + b + 3 = 10a + b.$$

So

$$a = 8,$$

and that means all two digit numbers started from 8 are stay in their original places.

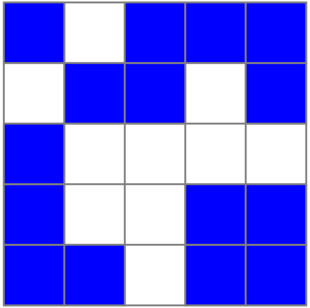
As the result we have 11 numbers to stay in place:

$$1,\ 80,\ 81,\ 82,\ 83,\ 84,\ 85,\ 86,\ 87,\ 88,\ 89.$$

PROBLEM 39

Color some cells of a 5×5 white grid in blue so that all 16 2×2 squares have different colorings (they may not fit together when shifted).

That may be done in the following way:

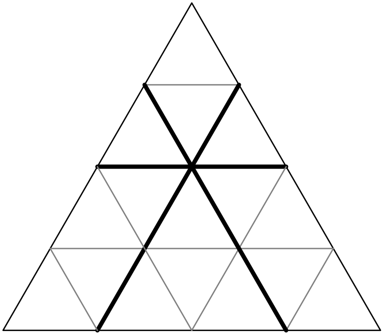


An observation that the total amount of different 2×2 squares is $2^4 = 16$ is equal to the total amount of different 2×2 subsquares of 5×5 squares may help to check whether the solution is correct.

PROBLEM 40

Through a point inside an equilateral triangle, we have drawn three lines parallel to the sides of the triangle. Then we measured the areas of the six pieces we got. Can these areas have exactly three different values?

Lets divide the triangle into 16 smaller equilateral triangles.



And we got:
3 figures with area 4;
1 figure with area 2 and
2 figures with area 1.
So areas have exactly 3 different values.