Right Triangles solutions

Graham Middle School Math Olympiad Team

EXERCISES 1-4

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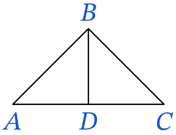

Since $\triangle ABC$ is isosceles, BD is a median and a height. By the Pythagorean theorem $BD = \sqrt{29^2 - 21^2} = 20$, and $S(\triangle ABC) = \dfrac{42 \cdot 20}{20} = \boxed{420}$.

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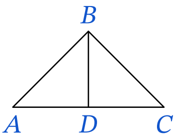

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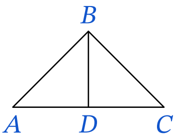

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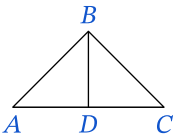

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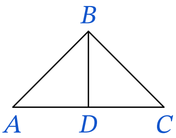

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By Heron's formula the area of the triangle is $S = \sqrt{21 \cdot (21-13) \cdot (21-14) \cdot (21-15)} = \sqrt{21 \cdot 8 \cdot 7 \cdot 6} = 84$. From other side $S = \dfrac{14 \cdot h_{14}}{2} = 84$. So $h_{14} = \boxed{12}$.

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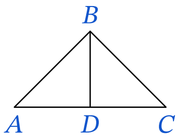

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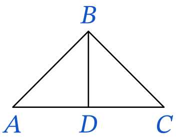

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E4. The altitude of an equilateral triangle is $\sqrt{6}$ units. What is the area of the triangle, in square units?

Let the side of the triangle is a, so $a^2 = \dfrac{a^2}{4} + 6$, and $\dfrac{3}{4}a^2 = 6$. $a^2 = 8$, so $a = 2\sqrt{2}$.

$S = \dfrac{2\sqrt{2} \cdot \sqrt{6}}{2} = \boxed{2\sqrt{3}}$.

EXERCISES 5-6

E5. In pentagon $ABCDE$, $\angle E$ and $\angle C$ are right angles and $m\angle D = 120°$. If $AB = 12$, $AE = BC = 18$ and $ED = DC$, what is ED?

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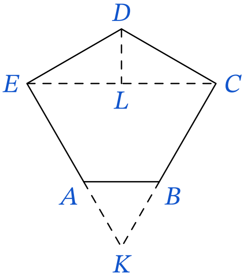

When we continue EA and CB until the intersection in K, we will get equilateral $\triangle ABK$ with the side 12, so equilateral $\triangle ECK$ has a side 30. ED is hypotenuse in the $30-60-90$ triangle with the long side 15. So

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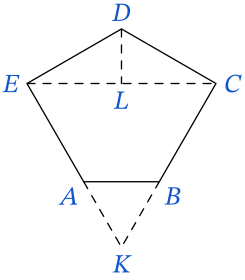

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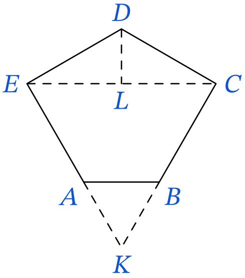

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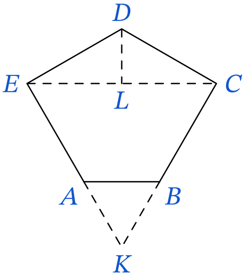

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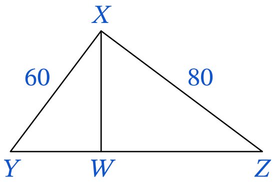

$\triangle XYZ$ is similar to $3-4-5$ triangle, so $\triangle XWZ$ is similar to $3-4-5$ triangle. So $WZ : XZ = 4 : 5$ and

$$WZ = XZ \cdot 4/5 = 80 \cdot 4/5 = \boxed{64}.$$

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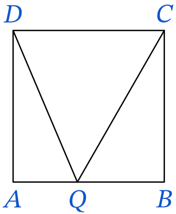

Let the side of the square is a and $AQ = b$. So $a^2 + b^2 = 10$ and $a^2 + (a-b)^2 = 13$.
$(a-b)^2 - b^2 = a^2 - 2ab = 3$,
$b = \frac{a^2-3}{2a}$.
Plugin back into the first equation, we got:

$$a^2 + \left(\frac{a^2-3}{2a}\right)^2 = 10,$$

$$a^2 + \frac{a^4 - 6a^2 + 9}{4a^2} = 10,$$

$$4a^4 + a^4 - 6a^2 + 9 = 40a^2,$$

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$$(a^2 - 9)(5a^2 - 1) = 0, \quad \Rightarrow \quad a^2 = \boxed{9}.$$

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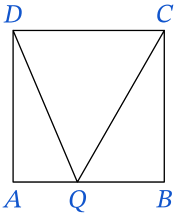

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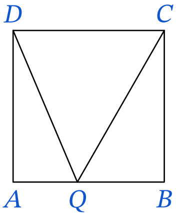

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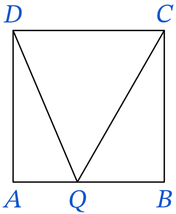

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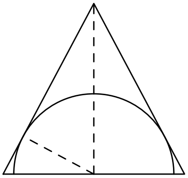

The radius of the semicircle is an altitude in the half of the triangle. The area of the half of the triangle is $\frac{8 \cdot 15}{2} = \boxed{60}$. The hypotenuse of the half of the triangle is $\sqrt{8^2 + 15^2} = 17$. Since $\frac{17 \cdot r}{2} = 60$,
$r = 60 \cdot 2/17 = \boxed{120/17}$.

CHALLENGE PROBLEMS 1-2

CP1. Points $A(11, 9)$ and $B(2, -3)$ are vertices of $\triangle ABC$ with $AB = AC$. The altitude from A meets the opposite side at $D(-1, 3)$. What are the coordinates of point C?

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Without loss of generality, let AB be a horizontal segment of length 10. Now realize that C has to lie on one of the lines parallel to AB and vertically 20 units away from it. But $10 + 20 + 20$ is already 50, and this doesn't form a triangle. So there are no such points, the answer is $\boxed{0}$.

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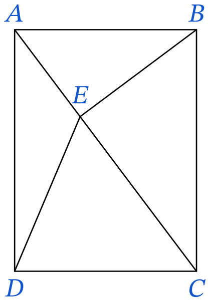

$S(\triangle AED) = S(\triangle AEB)$ since they share the base and altitudes have the same length because of symmetry. $\triangle ABE$ similar to $3 - 4 - 5$ triangle, so $AE : AB = 3 : 5$ and $AE = 9/5$, $EB : AB = 4 : 5$, so $EB = 12/5$.

$$S(\triangle AED) = S(\triangle AEB) = \frac{\frac{9}{5} \cdot \frac{12}{5}}{2} = \frac{12 \cdot 9}{2 \cdot 5 \cdot 5} = \boxed{\frac{54}{25}}.$$

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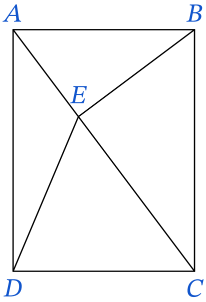

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CP4. In rectangle $PQRS$, $PQ = 8$ and $QR = 6$. Points A and B lie on $\overline{PQ}$, points C and D lie on $\overline{QR}$, points E and F lie on $\overline{RS}$, and points G and H lie on $\overline{SP}$ so that $AP = BQ < 4$ and the convex octagon $ABCDEFGH$ is equilateral. Find the length of a side of this octagon.

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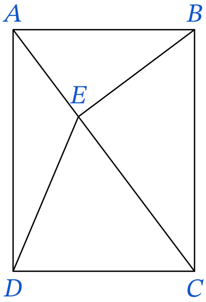

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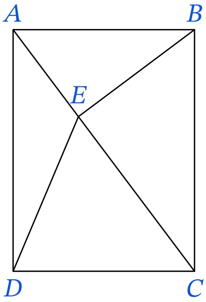

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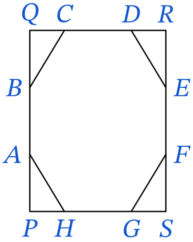

Let $AP = BQ = x$. Then $AB = 8 - 2x$. Now notice that since $CD = 8 - 2x$ we have $QC = DR = x - 1$. Thus by the Pythagorean Theorem we have $x^2 + (x-1)^2 = (8-2x)^2$ which becomes

$$2x^2 - 30x + 63 = 0 \implies x = \frac{15 - 3\sqrt{11}}{2}.$$

Our answer is $8 - (15 - 3\sqrt{11}) = \boxed{3\sqrt{11} - 7}$.

EMPTY

TEAM ATTACK PROBLEMS 1-4

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TEAM ATTACK PROBLEMS 5-6

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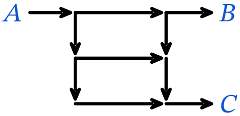

The probability to get point B is $0.7 \cdot 0.7 = 0.49$. So $1 - 0.49 = 0.51 = \boxed{51\%}$ of traffic will exists at C.

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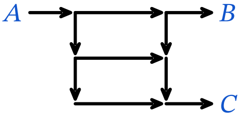

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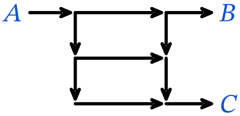

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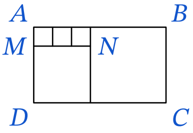

Let $AM = n$ cm, then $AN = 3n$ cm, $AD = 4n$ cm and $AB = 7n$ cm, so $S(ABCD) = 4n \cdot 7n = 28n^2$ sq cm. If $n = 1$, $28 < 100$, if $n = 2$, $28 \cdot 4 = \boxed{112}$ sq cm.

TEAM ATTACK PROBLEM 7

TA7. Two semicircles are inscribed in a square with side 8 meters as shown. Approximate the area of the shaded region to the nearest tenth of a square meter. Use the approximation 3.14 for π.

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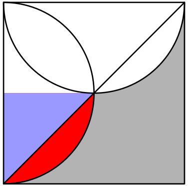

The gray area is a half of the big square minus two red sections.
To calculate area of the red section, we need to get area of quarter of the circle minus area of the blue triangle.

$S(\text{quarter of the circle}) = \dfrac{\pi r^2}{4} = \dfrac{\pi 4^2}{4} = 4\pi \approx 4 \cdot 3.14 = 12.56.$

$S(\text{blue triangle}) = \dfrac{4 \cdot 4}{2} = 8.$

$S(\text{red area}) = 12.56 - 8 = 4.56.$

$S(\text{grey area}) = \dfrac{8 \cdot 8}{2} - 2 \cdot 4.56 = 32 - 9.12 = 22.88 \approx \boxed{22.9}$ sq meters.