School of Mathematics
Faculty of Science
National University of Engineering

Test 5

Topic: Weak LLN
Subject: Probability Theory **Period: 2016.1**

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1. [10 pts.] Present a probabilistic proof of the Weierstraß's approximation theorem.

2. [10 pts.] [Bernstein-Chernov bound] Let $n \in \mathbb{N}$ and $p_1, \ldots, p_n \in [0,1]$. Let $X_1, \ldots, X_n$ be independent random variables with $X_i \sim \text{Ber}_{p_i}$ for any $i = 1, \ldots, n$. Define $S_n := X_1 + \ldots + X_n$ and $m := \mathbf{E}[S_n]$. Show that for any $\delta > 0$:

$$\mathbf{P}[S_n \geq (1+\delta)m] \leq \left(\frac{e^{\delta}}{(1+\delta)^{1+\delta}} \right)^m$$

and

$$\mathbf{P}[S_n \leq (1-\delta)m] \leq e^{-\frac{\delta^2 m}{2}}.$$

Thursday, July 7