School of Mathematics
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Test 4

Topic: Moments
Subject: Probability Theory **Period: 2016.1**

Throughout this test $(\Omega, \mathcal{A}, \mathbf{P})$ is our abstract probability space and $X, Y, X_1, X_2, \ldots : (\Omega, \mathcal{A}) \to (\mathbb{R}, \mathcal{B}(\mathbb{R}))$ are real random variables.

1. [5 pts.] Suppose that X represents the number of errors per 100 lines of software code and has the following probability distribution:

X	2	3	4	5	6
Probability	0.01	0.25	0.40	0.30	0.04

   (a) Find the variance of X

   (b) Find the mean and variance of $3X - 2$.

2. [5 pts.] Let a particle in the x axis with probability $2/3$ to move one meter to the right and $1/3$ to move one meter to the left. Take a number on $\mathbb{N}_0$ and call it T; suppose that $T \sim \mathrm{Poi}_3$. Then starting at the origin, the particle performs T movements along the axis, say $X_1, \ldots, X_T$. What is the expected final position of this particle?

3. [5 pts.] Suppose that X and Y are independent with probability densities:

$$f_X(x) = \begin{cases} \frac{8}{x^3} & \text{if } x > 2, \\ 0 & \text{elsewhere,} \end{cases}$$

   and

$$f_Y(x) = \begin{cases} \frac{2}{y} & \text{if } 0 < y < 1, \\ 0 & \text{elsewhere,} \end{cases}$$

   Find $\mathbf{E}[XY]$.

4. [5 pts.] Let $X_1, X_2, \ldots \geq 0$ be i.i.d. Prove the following proposition.

$$\limsup_{n\to+\infty} \frac{1}{n} X_n = \begin{cases} 0 \text{ a.s.} & \text{if } \mathbf{E}[X_1] < +\infty, \\ +\infty \text{ a.s.} & \text{if } \mathbf{E}[X_1] = +\infty. \end{cases}$$