Friction Model:

Static friction Coulomb frictiction Viscous friction Asymmetri
Stribeck effect. Position dependence.

$f(\dot{q}) = \gamma_1 (\tanh(\gamma_2 \dot{q}) - \tanh(\gamma_3 \dot{q})) + \gamma_4 \tanh(\gamma_5 \dot{q}) + \gamma_6 \dot{q}$

$\gamma_i \in R \quad \forall i = 1 \cdots 6 > 0$

$q \Rightarrow l \quad \dot{q} \Rightarrow v \quad \ddot{q} \Rightarrow a$

① friction model is symmetric about the origin

③ The static Coefficient of friction can be approximated by $\gamma_1 + \gamma_4$

② The term $\tanh(\underset{\sim}{\gamma_2 \dot{q}}) - \tanh(\underset{\sim}{\gamma_3 \dot{q}})$ captures the Stribeck effect

$\Downarrow$

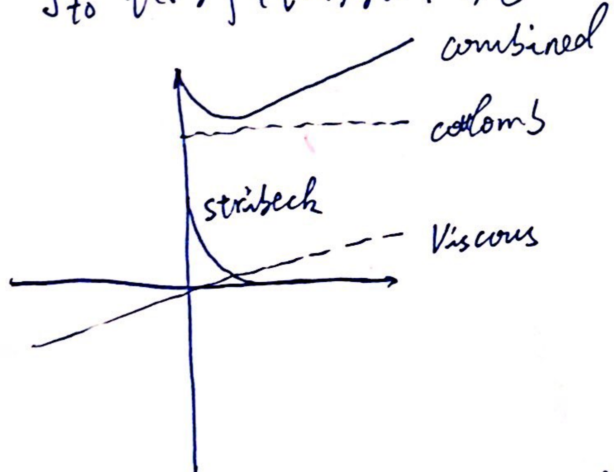
slip velocity.

④ Viscous dissipation : $\gamma_6 \dot{q}$

⑤ Coulombic friction coefficient $\gamma_4 \tanh(\gamma_5 \dot{q})$

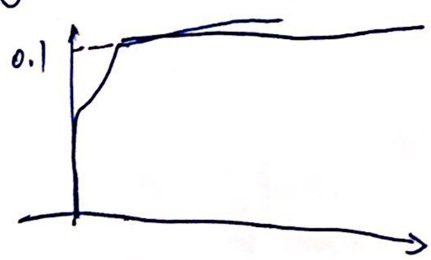
⑥ $f(\cdot) \rightarrow \dot{q}(t) \Rightarrow V$

$\int_{t_0}^{t} \dot{q}(\tau) f(\dot{q}(\tau)) d\tau \geq c^2$ boundary function.



M : mass V_b: Velocity of block V_p : Velocity of plate

Only Coulomb friction set: $\gamma_1 = 0 \quad \gamma_2 = 0 \quad \gamma_3 = 0 \quad \underset{\mu_s?}{\underline{\gamma_4 = 0.1}} \quad \underset{Mg?}{\underline{\gamma_5 = 100}}$

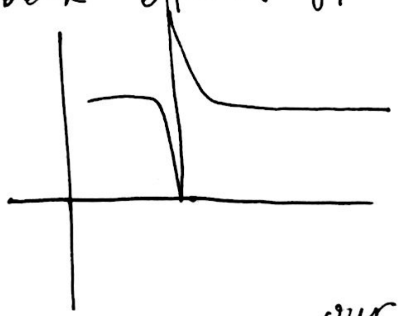


Only Viscous: $\gamma_1=0$ $\gamma_2=0$ $\gamma_3=0$ $\gamma_4=0$ $\gamma_5=0$ $\gamma_6=0.01$

Negative ~~parameter~~ coeffecient friction ?

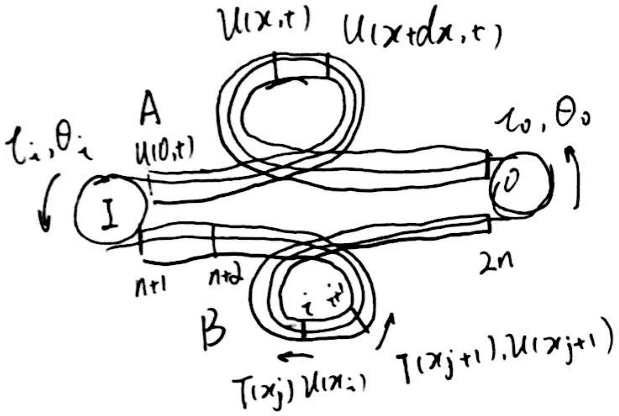
$v \downarrow$ friction $\uparrow$

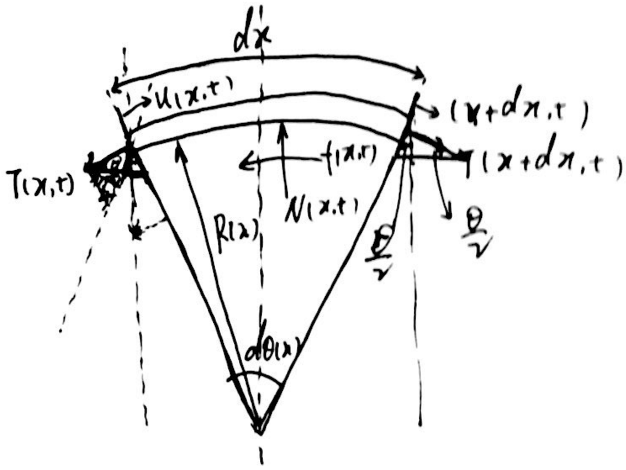
Only Stribeck effect: $\gamma_1=0.25$ $\gamma_2=100$ $\gamma_3=10$ $\gamma_4=0$ $\gamma_5=0$ $\gamma_6=0$



our ~~fr~~ SMT Model Maybe domained by Stribeck effect. the friction Model result has the same Curve.

Cable transmission.





Assumption:

① Inertia effects in the cable can be neglected.

② The cable is restricted to move along Conduit

③ Interaction bwt cable & conduit is Coulomb friction and normal force.

④ Conduit behavior is assumed to be elastic defined by Hooke's law

Our sheath?

$u(x,t)$ axis displacement.

$T(x,t)$ corresponding axial tension of cable.

$$\frac{dT(x,t)}{dx} = T'(x,t) \qquad \frac{dT(x,t)}{dt} = \dot{T}(x,t)$$

Consider Infinitesimal cable segment $[x, x+dx]$ ⇒ our SMT line.

$N(x,t)$ normal force bwt cable & conduit (~~Coulomb friction~~)

$f(x,t)$ frictional force acting on the cable.

$$dx = R(x)\, d\theta(x)$$

force balancing equation, on the two ends of Infinitesimal cable element.

$$N(x,t) = T(x,t)\sin\left(\frac{d\theta(x)}{2}\right) + T(x+dx,t)\sin\left(\frac{d\theta(x)}{2}\right)$$

$$\approx \underline{T(x,t)\, d\theta(x)}$$

Coulomb friction

$$|f(x,t)| \le \mu N(x,t) = \mu T(x,t)\, d\theta(x) = \mu T(x,t)\frac{dx}{R(x)}$$

In order to move

$$T(x+dx,t) - T(x,t) = T'(x,t)dx < \mu T(x,t)\, dx/R(x) \Rightarrow |T'(x,t)| < \mu T(x,t)/R(x)$$

$\dot{u}(x,t) = 0$ static $\Rightarrow$ friction $f(x,t) = T'(x,t)\,dx$.

$\dot{u}(x,t) \neq 0$ friction will be max $f(x,t) = (\mu T(x,t)/R(x))\, \text{sign}(\dot{u}(x,t))\,dx$.

$$T'(x,t)\,dx = \frac{\mu T(x,t)}{R(x)} \text{sign}(\dot{u}(x,t))\,dx \quad (4)$$

To calculate cable strain Assume Hook's law of elasticity can be used to model.

$T(x,t) = K u'(x,t)$ when $u'(x,t) > 0$

$T(x,t) = 0$ when $u'(x,t) \leq 0$

where $\frac{1}{K} = \frac{1}{K_{cable}} + \frac{1}{K_{conduit}}$.

when $u'(x,t) > 0$ $T(x,t) = K u'(x,t)$

i) $\cancel{K} u''(x,t) \cancel{dx} = \frac{\mu \cancel{K} u'(x,t)}{R(x)} \text{sign}(\dot{u}(x,t)) \cancel{dx}$

$$u'' = \frac{\mu}{R(x)} u'(x,t)\, \text{sign}(\dot{u}(x,t))$$

If $|T'(x,t)| = \frac{\mu}{R(x)} T(x,t)$

ii) $\dot{u}(x,t) = 0$ otherwise

when $u'(x,t \leq 0$

iii) $T(x,t) = 0$

Initial state.

$$u(x,0) = u_0(x)$$

$T(x,0) = K u'(x,0) = K u_0'(x)$

$T(x,0) = T_0$

Discretized Model

$\sum_{i=1}^{n} \Delta x_i = L$

$T(x_i, t_j)$ $u(x_i, t_j)$ $R(x_i)$ (neglect small variation)

Case 1: moving: $\dot{u}(x,t) \neq 0$

$$\int_{x_i}^{x} \frac{T'(x,t)}{T(x,t)} dx = \int_{x_i}^{x} \frac{\mu}{R(x_i)} \operatorname{sign}(\dot{u}(x_i,t)) dx$$

$$\int_{T(x_i)}^{T(x)} \frac{1}{y} dy = \int_{x_i}^{x} \frac{\mu}{R(x_i)} \operatorname{sign}(\dot{u}(x_i,t)) dx$$

$$\ln y \Big|_{T(x_i)}^{T(x)} =$$

$$\ln \frac{T(x)}{T(x_i)} = \frac{\mu(x-x_i)}{R(x_i)} \operatorname{sign}(\dot{u}(x_i,t)) dx.$$

$$\underline{T(x,t)} = T(x_i,t) \exp\left(\frac{\mu(x-x_i)}{R(x_i)} \operatorname{sign}(\dot{u}(x_i,t))\right)$$

$$\int_{x_i}^{x} K u'(x,t) = \int_{x_i}^{x} T(x_i,t) \exp\left(\frac{\mu(x-x_i)}{R(x_i)} \operatorname{sign}(\dot{u}(x_i,t))\right) dx$$

$$\int_{x_i}^{x} u'(x,t) dx = \frac{1}{K} \int_{x_i}^{x} \cdots\cdots$$

$$\int_{x_i}^{x} 1 \, \underline{du(x,t)}$$

$$u(x,t) - u(x_i,t) = \frac{T(x_i,t)}{K} \int_{x_i}^{x} \exp\left(\frac{\mu(x-x_i)}{R(x_i)} \operatorname{sign}(\dot{u}(x_i,t))\right) dx.$$

$$= \frac{T(x_i,t)}{K} \cdot \frac{R(x_i)}{\mu} \int_{0}^{x-x_i} \exp\left(\frac{\mu(x-x_i)}{R(x_i)} \operatorname{sign}(\dot{u}(x_i,t))\right) d\frac{\mu(x-x_i)}{R(x_i)}$$

$$= \frac{T(x_i,t) R(x_i)}{\mu K} \operatorname{sign}(\dot{u}(x_i,t)) \times \left[\exp\left(\frac{\mu(x-x_i)}{R(x_i)} \operatorname{sign}(\dot{u}(x_i,t))\right) - 1\right]$$

$$T_{i+1}^{j} = T_i^j \exp\left(\frac{\mu(x_{i+1}-x_i)}{R(x_i)} \operatorname{sign}_i^j\right)$$

$$u_{i+1}^{j} - u_i^j = S_i^j \frac{R(x_i)}{K\mu} T_i^j \left[\exp\left(\frac{\mu(x_{i+1}-x_i)}{R(x_i)} S_i^j\right) - 1\right]$$

$T_i^j \triangleq T(x_i, t_j)$ $u_i^j \triangleq u(x_i, t_j)$ $S_i^j \triangleq \operatorname{sign}(u(x_i, t_j) - u(x_i, t_{j-1}))$

Case2: All segment is stationary.

$$T_i^j = T_i^{j-1} \qquad T_{i+1}^j = T_{i+1}^{j-1}$$

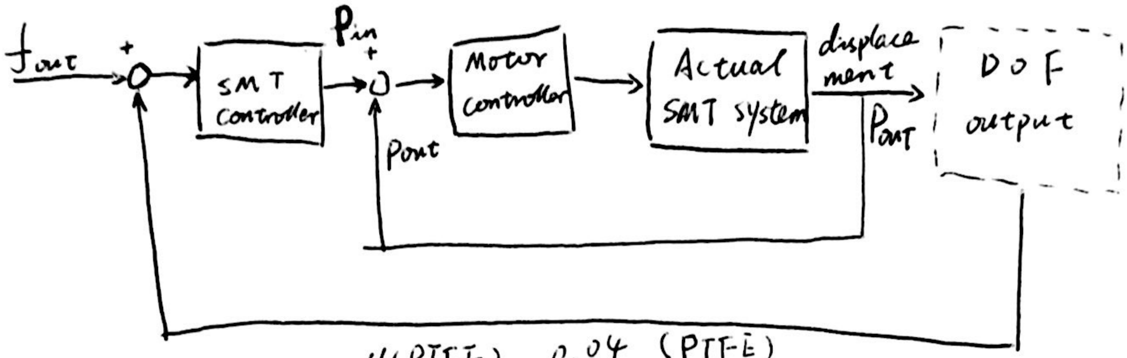
$$u_i^j = u_i^{j-1} \qquad u_{i+1}^j = u_{i+1}^{j-1}$$

Case3: part of the cable is moving part of it is stationary.

Assume node i is moving node $i+1$ is stationary.

Assume $u'(x,t) \approx (1/k)((T_i^j + T_{i+1}^j)/2)$, $\forall x \in (x_i, x_{i+1}]$.

$$u_i^j = u_{i+1}^j - \frac{T_i^j + T_{i+1}^j}{2k}(x_{i+1} - x_i).$$



μ(PTFE) 0.04 (PTFE)
0.05-0.2 (steel)

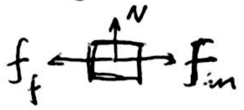
 one segment mass m total mass M

length l total linear length L

$u(x,t)$ displacement
t.

case (1) static.

Only Coulombic friction.



$N = mg$

$\because$ static

$\therefore f_{in} \leq |f_f| = \mu mg$ while $\dot{u}(x,t) = 0$

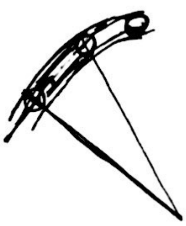
f_{out}

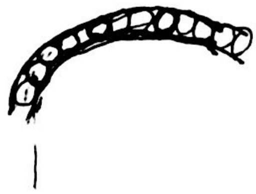
while $\dot{u}(x,t) \neq 0$

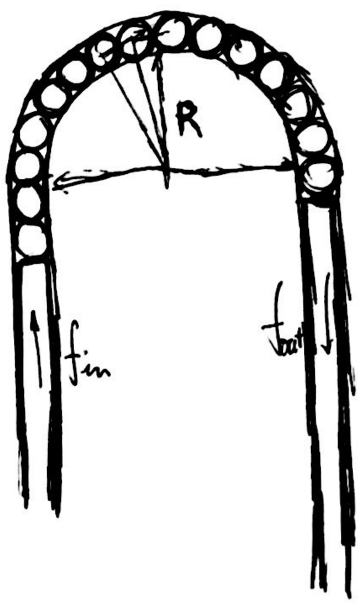
$f_{in} > \mu mg$

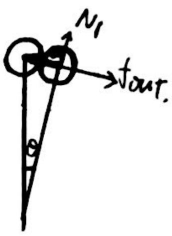
$f_{out} = f_{in} - \mu mg$

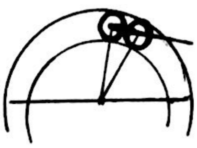
case (2)









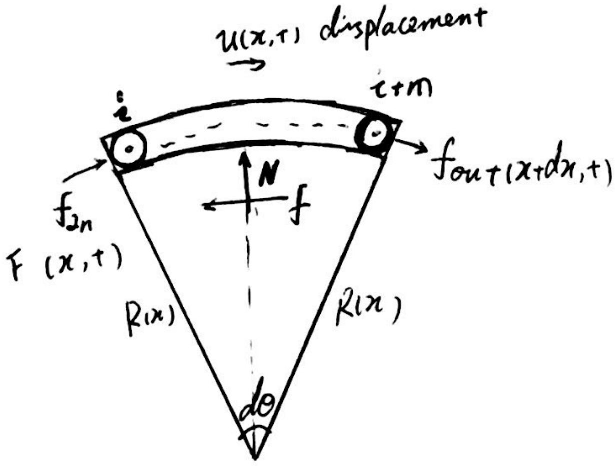


$\cos\theta f_{in} = f_{out}$

$\sin\theta f_{in} = N_1$

$N_2 = mg$

$F_{out} = f_{out} - \mu(mg + N_1)$



balance equation $\dot{u}(x,t)=0$

$$F_0 = f_f$$

$\dot{u}(x,t) \neq 0$

$$f_f = f_{fmax} = \mu \cdot F_0 \cdot \cos\alpha^{m+1} \cdot \sin\alpha \cdot sign(\dot{u}(x,t))$$

$$F_0 - \mu \cdot F_0 \cdot \cos\alpha^{m+1} \cdot \sin\alpha \cdot sign(\dot{u}(x,t)) > 0$$

$$F_0 (1 - \mu \cdot \cos\alpha^{m+1} \cdot \sin\alpha \cdot sign(\dot{u}(x,t)) > 0$$

$$(1 - \mu \cdot \cos\alpha^{m+1} \cdot \sin\alpha \cdot sign(\dot{u}(x,t)) > 0$$

$$\alpha \approx \frac{r}{R(x)}$$

$$(1 - \mu \cdot \cos\frac{r}{R(x)}^{m+1} \cdot \frac{r}{R(x)} \cdot sign(\dot{u}(x,t))) > 0$$

$$(1 - \mu \cdot (1 - (\frac{r}{R(x)})^2)^{m+1} \cdot \frac{r}{R(x)} \cdot sign(\dot{u}(x,t))) > 0$$

$$\cancel{(1 - \mu (1 - (\frac{dx}{2mR(x)})^2)^{m+1} \cdot \frac{dx}{2mR(x)} \cdot sign(\dot{u}(x,t))) > 0}$$

$$(1 - \mu \cdot (1 - \frac{r^2}{R^2})^{m+1} \cdot \frac{r}{R} \cdot sign(\dot{u}(x,t))) > 0$$

$$\frac{r^2}{R^2} \approx 0$$

$$(1 - \mu \cdot \frac{r}{R} \cdot sign(\dot{u}(x,t))) > 0$$

For sigle segment

$$f_{out_i} = f_{2n_i} \cdot \cos\alpha$$

$$f_{N_i} = f_{2n_i} \cdot \sin\alpha.$$

$$f_{f_i} = \mu \cdot f_{2n_i} \cdot \sin\alpha.$$

$$f_f = f_{f_i} + f_{f_{i+1}} + \cdots + f_{f_{i+m}}$$

$$= \mu (f_{2n_i} + f_{2n_{i+1}} + \cdots + f_{2n_{i+m}}) \sin\alpha.$$

$$f_{2n_i} = F_0 \cdot \cos\alpha$$

$$f_{2n_{i+1}} = F_0 \cdot \cos\alpha \cdot \cos\alpha.$$

......

$$f_{2n_{i+m}} = F_0 \cdot \cos\alpha^{m+1}$$

$$|f_f| \leq \mu F_0 \cdot \cos\alpha^{m+1} \cdot \sin\alpha.$$