Proof that $3n+2 = \Theta(n)$;

$*$ By def of Θ; $\underbrace{\overbrace{c_1 \cdot g(n) \leq f(n)}^{\Omega(n)} \leq c_2 \cdot g(n)}_{O(n)}$

$f(n) = 3n+2$
$g(n) = n$

(1) Find the Big-O

$\because f(n) \leq c_2 \cdot g(n), \forall n \geq n_0$

$3n+2 \leq 4n$, "I suppose $c_2=4$"

– we need to find n_0

So,

$2 \leq 4n-3n$

$2 \leq n$

$\therefore n_0 = 2$

True: $\forall n \geq 2$

if $n_0=2$ & $c_2=4$

(1) Try when $n=2$

$3*2+2 \leq 4*2$

$8 \leq 8$

(2) Try when $n=6$

$3*6+2 \leq 4*6$

$20 \leq 24$

And so on

② Find the Big-Omega

$\boxed{\begin{array}{l} f(n) = 3n+2 \\ g(n) = n \end{array}}$

$\because f(n) \geq C_1 \cdot g(n), \forall\, n \geq n_0$

$3n+2 \geq 2n$, "suppose $C_1 = 2$"

- Now, we need to find n_0

So:

$2 \geq 2n - 3n$

$2 \geq -n$

$-2 \leq n$,

$n_0 = -2$, but "n" can't be negitave or zero

so $n_0 = 1$

True: $\forall\, n \geq 1$

if $n_0 = 1$ & $C_1 = 2$

① Try when $n = 1$

$3*1+2 \geq 2*1$

$5 \geq 2$

② Try when $n = 7$

$3*7+2 \geq 2*7$

$23 \geq 14$

$\because \Omega(n)$ & $O(n)$ for $f(n) = 3n+2$

$\therefore \Theta(n)$, for $f(n) = 3n+2$