Q: Prove that $3n+4 = \Theta(n)$:

By def of Θ: $\underbrace{\overbrace{C_1 \cdot g(n) \leq f(n)}^{\Omega(n)} \leq C_2 \cdot g(n)}_{O(n)}$

$f(n) = 3n+4$
$g(n) = n$

$\boxed{1}$ Find the Big-O

$\because f(n) \leq C_2 \cdot g(n), \forall n \geq n_0$

$3n+4 \leq 4n$, "suppose $C_2 = 4$"

- Now, we need to find n_0

so:

$4 \leq 4n - 3n$

$4 \leq n$

$\therefore n_0 = 4$

True: $\forall n \geq 4$

if $n_0 = 4$ & $C_2 = 4$

① Try when $n=4$

$3*4+4 \leq 4*4$

$16 \leq 16$

② Try when $n=6$

$3*6+4 \leq 4*6$

$22 \leq 24$

And so on....

$\boxed{2}$ Find the Big Omega

$f(n) = 3n + 4$
$g(n) = n$

$\because f(n) \geq c_1 \cdot g(n), \forall n \geq n_0$

$3n + 4 \geq 2n$, "suppose $c_1 = 2$"

- Now, we need to find n_0

so:

$4 \geq 2n - 3n$

$4 \geq -n$

$-4 \leq n$

$\therefore n_0 = -4$, but n can't be negitive or zero

So $n_0 = 1$

True: $\forall n \geq 1$

if $n_0 = 1$ & $c_1 = 2$

① Try when $n = 1$

$3 * 1 + 4 \geq 2 * 1$

$7 \geq 2$

② Try when $n = 7$

$3 * 7 + 4 \geq 2 * 7$

$25 \geq 14$

$\because \Omega(n)$ & $O(n)$ for $f(n) = 3n + 4$

$\therefore \Theta(n)$, for $f(n) = 3n + 4$