

Example: Counting Change Problem Def'n: ■ Cashier has collection of 'coins' of various denominations ■ Goal is to return a specified sum using the smallest number of coins Example: Counting Change Mathematical Def'n: ■ *n* coins: $P = \{p_1, p_2, p_3, ..., p_n\}$ with value $D = \{d_1, d_2, d_3, ..., d_n\}$ - can have repetition (two dimes, three pennies) - S is a subset of P $S \subseteq P$, such that $s_i = 1$ if $p_i \in S$, $s_i = 0$ if $p_i \notin S$ ■ A: sum to be returned ■ Goal: minimize Σs_i , such that $\Sigma d_i = A$ Brute-force Approach ■ Try all subsets of P - since there are n coins, there are 2^n possible subsets - enumerate all possible subsets - check if a subset equals A · called 'feasible solution' set • O(n) pick subset that minimizes Σs_i · called 'objective function'

Brute-force Approach ■ Best Case $-\Omega(n2^n)$ ■ Worst Case $-O(n2^{n})$ Greedy Approach ■ Go from largest to smallest denomination – Return largest coin p_i from P, such that $d_i \le A$ $-A = A - d_i$ - Find next largest coin ... if money is sorted (by value), then algorithm is O(n)Does Greedy Always Work? Consider A = 20and $D = \{1, 1, 1, 1, 1, 10, 10, 15\}$ Greedy returns 6 coins Optimal is 2 coins

Text Processing

- Brute-force Pattern Matching
- Improved Pattern Matching
 - Boyer-Moore Algorithm
 - not really brute-force
 - not really greedy either

Pattern Matching

- T: text string of length *n*
- P: pattern string of length *m*
- Question: Is P a substring of T?
- Answer: starting index of match or indication that P ∉ T

Pattern Matching: Pseudocode

```
Algorithm BruteForceMatch (T,P)

Input: character string T of length n and character string P of length m

Output: integer -1 if P & T, integer i (start location of P in T) if P & T for i <- 0 to n-m do

j <- 0

while (j < m and T[i+j] = P[j]) do

j <- j + 1

if j = m then

return i

return -1
```

Pattern Matching: Complexity

```
for i <- 0 to n-m do
   j <- 0
                                            0()
   while (j < m \text{ and } T[i+j] = P[j]) do
                                            0()
      j <- j + 1
                                              0()
   if j = m then
                                            0()
     return i
                                              0()
return -1
                                          0()
Worst case complexity:
                              0(
Best case complexity:
                              Ω(
                                       )
```

Better Pattern Matching: Boyer Moore Algorithm

Two Improvements:

- Looking Glass Heuristic
 - When testing P against T, begin at P[m-1]
- Character Jump Heuristic
 - Mismatch T[i] = c with P[j]
 - if c ∉ P, then shift P past T[i]
 - else if last(c) to left of P[j] then
 - shift P to align last(c) with T[i]
 - else shift P to right by one

Summary: Brute & Greedy

- Brute-force:
 - solve problem in simplest way
 - generate entire solution set, pick best
 - will give optimal solution with (typically) poor efficiency
- Greedy:
 - make local, best decision, and don't look back
 - may give optimal solution with (typically) 'better' efficiency
 - depends upon 'greedy-choice property'
 - global optimum found by series of local optimum choices