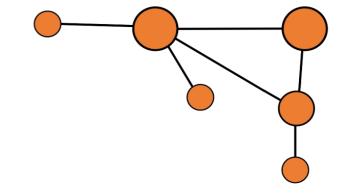


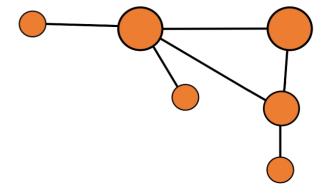
Agenda

- 1 What is Graph Representation Learning?
- 2 The Emergence of Diffusion Models
- 3 Graphs vs. Images
- 4 The Architecture of Directed Diffusion Models
- 5 Benchmarks
- 6 Conclusion

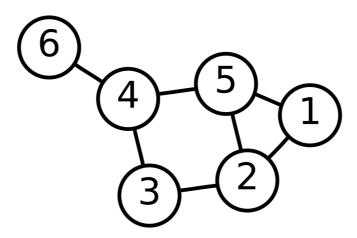


1 What is Graph Representation Learning?

What is a graph?

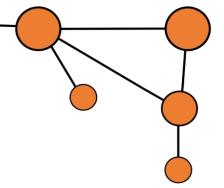


- ightharpoonup G = (V, E), vertices V and edges E
- Graphs can represent anything from molecules to road networks or social networks

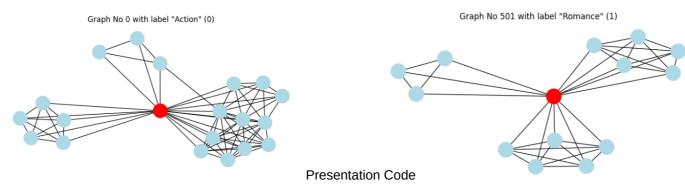


User: AzaToth, Public domain, via Wikimedia Commons



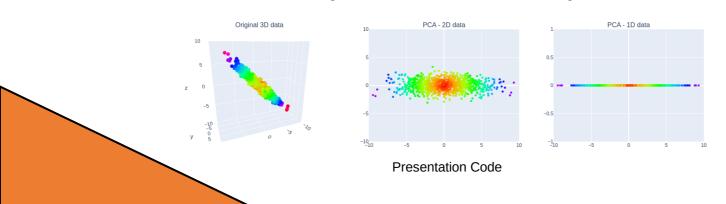


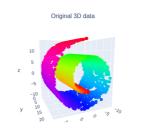
- Graph Learning on Graph Datasets enables the use of AI on those data structures
- ► IMDB-B contains ego-networks of actors from Action or Romance movies (Yanardag et al., 2015)
- ► From only the knowledge of which actors have co-starred, models can determine the genre with an accuracy of up to 95% (Nguyen et al., 2019)

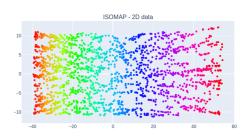


Representation Learning

- ► RL is an important part of Machine Learning that converts data to a form that can be worked with more easily
- ► Here, there is also a difference between unsupervised and supervised learning
- Dimensionality Reduction is a prominent subfield

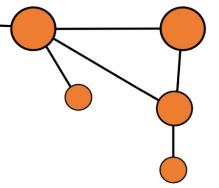




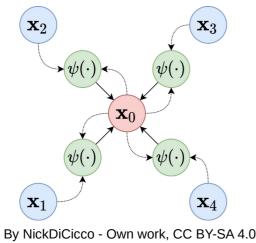


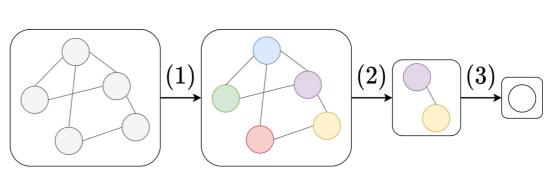
Presentation Code

Graph Representation Learning - GNNs

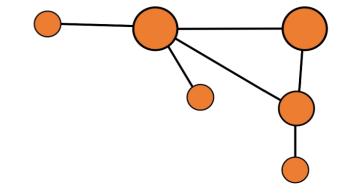


- For RL on Graphs, Graph Neural Networks are often used
- ► They work similar to Convolutional Neural Networks (CNNs)
- ► Instead of using neighboring pixels, the adjacency matrix, which contains the edges, is used



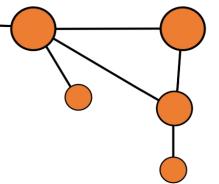


By NickDiCicco - Own work, CC BY-SA 4.0



2 The Successful Diffusion Model

Denoising Probabilistic Diffusion Models



► Introduced to Machine Learning only recently (Ho et al., 2020), but has become the standard for image generation beating former SOTA technology (Dhariwal et al., 2021)

Algorithm 1 Training

1: repeat

- 2: $\mathbf{x}_0 \sim q(\mathbf{x}_0)$
- 3: $t \sim \text{Uniform}(\{1, \dots, T\})$
- 4: $\epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$
- 5: Take gradient descent step on

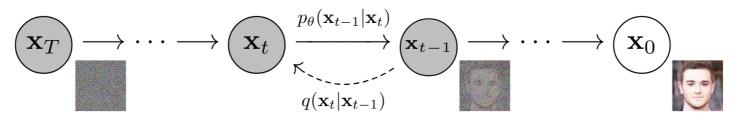
$$\nabla_{\theta} \left\| \boldsymbol{\epsilon} - \boldsymbol{\epsilon}_{\theta} (\sqrt{\bar{\alpha}_t} \mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_t} \boldsymbol{\epsilon}, t) \right\|^2$$

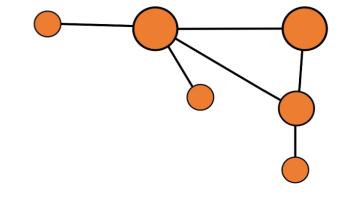
6: until converged

Algorithm 2 Sampling

- 1: $\mathbf{x}_T \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$
- 2: **for** t = T, ..., 1 **do**
- 3: $\mathbf{z} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$ if t > 1, else $\mathbf{z} = \mathbf{0}$
- 4: $\mathbf{x}_{t-1} = \frac{1}{\sqrt{\alpha_t}} \left(\mathbf{x}_t \frac{1-\alpha_t}{\sqrt{1-\bar{\alpha}_t}} \boldsymbol{\epsilon}_{\theta}(\mathbf{x}_t, t) \right) + \sigma_t \mathbf{z}$
- 5: end for
- 6: **return** \mathbf{x}_0

Ho et al., 2020

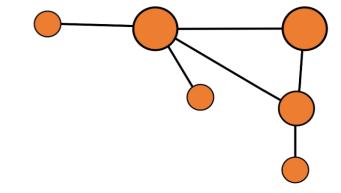




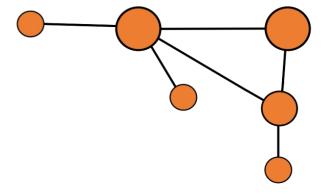
The DM's forward step

- ► To gradually add noise to the images, Diffusion Models add Gaussian noise in each step
- ► Thus, all data is asymptotically converted to a standard Gaussian distribution (Ho et al., 2020)

$$q(\mathbf{x}_{1:T}|\mathbf{x}_0) \coloneqq \prod_{t=1}^{T} q(\mathbf{x}_t|\mathbf{x}_{t-1}) \quad q(\mathbf{x}_t|\mathbf{x}_{t-1}) \coloneqq \mathcal{N}(\mathbf{x}_t; \sqrt{1-\beta_t}\mathbf{x}_{t-1}, \beta_t \mathbf{I})$$

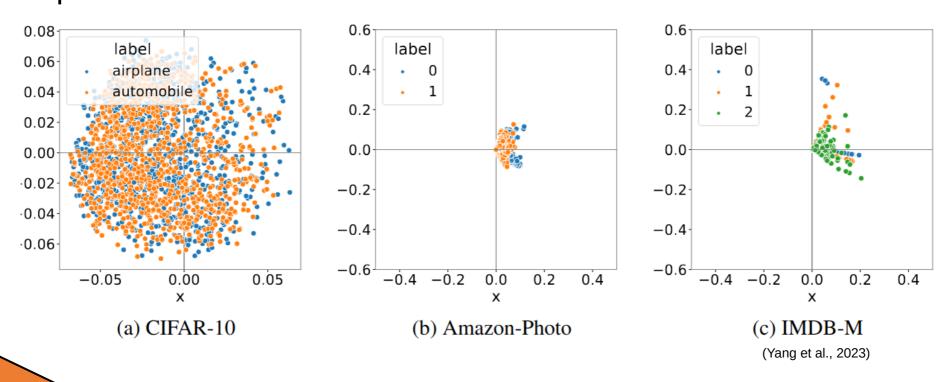


3 Graphs vs. Images



The Anisotropy of Graphs

► While images are naturally isotropic and euclidean, Graphs are anisotropic



se

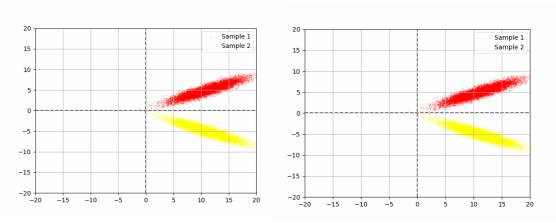
White Noise vs. Directed Noise

- ► The information density of a directed Gaussian declines quickly if White Noise is applied
- ► Hence, Yang et al. introduce "Directional Noise":

$$x_{t,i} = \sqrt{\bar{\alpha}_t} x_{0,i} + \sqrt{1 - \bar{\alpha}_t} \epsilon',$$

$$\epsilon' = \operatorname{sgn}(x_{0,i}) \odot |\bar{\epsilon}|,$$

$$\bar{\epsilon} = \mu + \sigma \odot \epsilon \quad \text{where } \epsilon \sim \mathcal{N}(0, \mathbf{I})$$

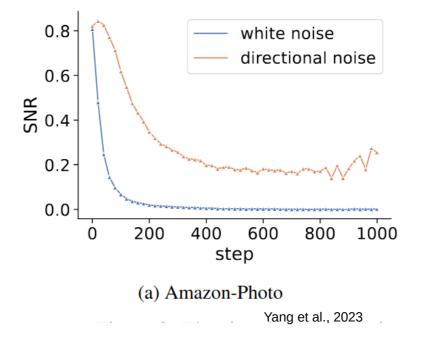


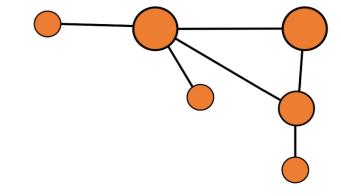
Presentation Code

The effect on the Signal-To-Noise-Ratio



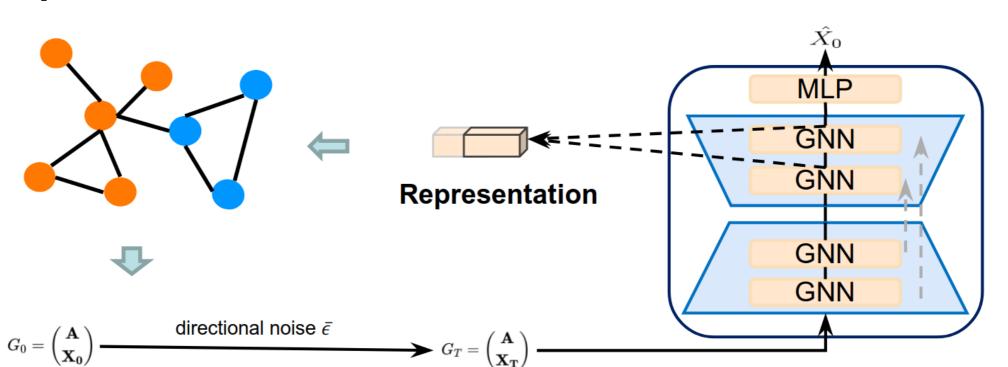
- ► The Signal-To-Noise-Ratio is fundamental for the learning process of Diffusion Models
- ► The application of directional noise has a vital effect on the SNR



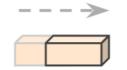


4 Directed Diffusion Models - Architecture

Components of the Model



Training stepEmbedding step

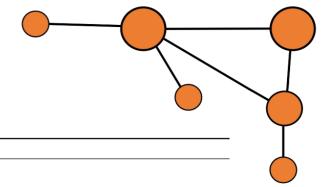


Skip connection Representation vector

Yang et al., 2023

The Algorithm

- ► The two algorithms work similar to Ho's algorithm
- ► Instead of generating an image, a representation is generated



```
Algorithm 1 The training algorithm.
```

Input: A batch of graphs $\mathcal{G} = \{G_1, \dots G_B\}$ **Output:** The denoising network f_{θ}

```
1: Initialize: the denoising network f_{\theta}
2: Compute \mu, the mean of node features across batch \mathcal{G}
3: Compute \sigma, the standard deviation of node features across batch \mathcal{G}
4: while not convergence do
5: for G_i in \mathcal{G} do
6: for t = 1, \dots, T do
7: Sample directional noise \epsilon' using equation (2)
8: Take gradient descent step on \nabla_{\theta} \| \mathbf{X}_0 - f_{\theta}(\sqrt{\bar{\alpha}_t}\mathbf{X_i} + \sqrt{1 - \bar{\alpha}_t}\epsilon', \mathbf{A}, t) \|
9: end for
10: end for
11: end while
```

Algorithm 2 Extracting representations.

Input: $G = (\mathbf{A}, \mathbf{X})$, forward step set $\{T_0, T_1, \dots, T_K\}$, pre-trained denoising network f_{θ} **Output:** \mathbf{H} , the representation of G

```
1: Compute \mu the mean of node features

2: Compute \sigma the standard deviation of node features

3: for k in \{T_0, T_1, \ldots, T_K\} do

4: Sample directional noise \epsilon' using equation (2)

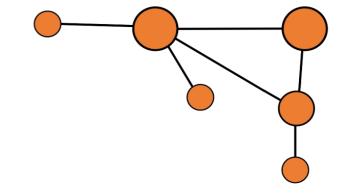
5: \mathbf{X_k} \leftarrow \sqrt{\bar{\alpha}_k} \mathbf{X_0} + \sqrt{1 - \bar{\alpha}_k} \epsilon'

6: \mathbf{H}_k \leftarrow f_{\theta}(\mathbf{X_k}, \mathbf{A}, k)

7: end for

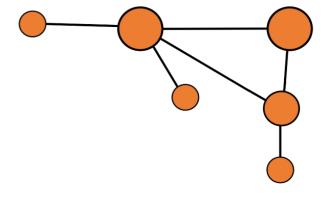
8: Concatenate \mathbf{H} = [\mathbf{H}_{T_0}, \mathbf{H}_{T_1}, \ldots, \mathbf{H}_{T_K}]

9: return \mathbf{H}
```

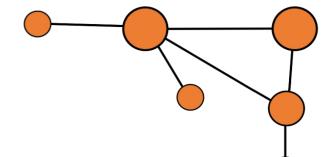


5 Resulting Benchmarks





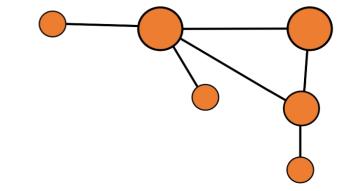
- ► The paper compares multiple State-Of-The-Art models with DDMs
- SVMs are used on the learned representations
- ► While here only graph classification results are presented, the results from node classification are similarly promising



Results

Dataset	IMDB-B	IMDB-M	COLLAB	REDDIT-B	PROTEINS	MUTAG
GIN	75.1±5.1	52.3±2.8	80.2±1.9	92.4±2.5	76.2 ± 2.8	89.4±5.6
DiffPool	72.6±3.9	-	78.9 ± 2.3	92.1±2.6	75.1 ± 2.3	85.0±10.3
Infograph	73.03±0.87	49.69±0.53	70.65±1.13	82.50±1.42	74.44±0.31	89.01±1.13
GraphCL	71.14±0.44	48.58±0.67	71.36±1.15	89.53±0.84	74.39 ± 0.45	86.80 ± 1.34
JOAO	70.21±3.08	49.20±0.77	69.50±0.36	85.29±1.35	74.55±0.41	87.35±1.02
GCC	72	49.4	78.9	89.8	-	-
MVGRL	74.20±0.70	51.20±0.50	-	84.50±0.60	-	89.70±1.10
GraphMAE	75.52±0.66	51.63±0.52	80.32±0.46	88.01±0.19	75.30±0.39	88.19±1.26
DDM	76.40±0.22	52.53±0.31	81.72±0.31	89.15 ±1.3	75.47 ±0.50	91.51 ±1.45

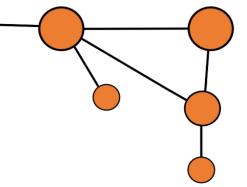
Yang et al., 2023



6 Conclusion

Research Outlook

- ► Yang et al. only introduce the idea, they admit that their hyperparameters are not optimal yet
- One open question is how the optimal set of diffusion steps can be determined
- Variants of DDMs could bring value to areas such as computer vision and natural language processing



What makes this paper special?

- ► As mentioned, the technology introduced holds great potential for the future
- The benchmarks are remarkable
- ► The researchers consider themselves "among the pioneers in the literature" regarding the "exploration of anisotropic structure in graph data"

References & Weblinks

- ► Yang et al. (2023). Directional diffusion models for graph representation learning
- ► Yanardag et al. (2015). Deep Graph Kernels
- ▶ Nguyen et al. (2019). Universal Graph Transformer Self-Attention Networks
- ▶ Ho et al. (2020). Denoising Diffusion Probabilistic Model
- ▶ Dhariwal et al. (2021). Diffusion Models Beat GANs on Image Synthesis
- Presentation Code: https://github.com/JavaLangMarlon/ddm-proseminar-tu-dortmund
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