

A	B	$A \Rightarrow B$	$A \wedge B$
T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	F

General difference:

$A \wedge B$ two facts (A and B) but
 $A \Rightarrow B$ only one fact.

To be discussed:

Implication within Implication

Everyone who loves all animals is loved by someone.

If valid: „Wenn y ein Tier ist, wird es von x geliebt“, then there is somebody (z), who loves x.

$\forall x [\forall y \text{ Animal } (y) \Rightarrow \text{Loves } (x,y)] \Rightarrow [\exists z \text{ Loves } (z,x)]$ versus

$\forall x [\forall y \text{ Animal } (y) \wedge \text{Loves } (x,y)] \Rightarrow [\exists z \text{ Loves } (z,x)]$

If all things (all y) are animals and x loves y, then there is somebody (z), who loves x.



$$\forall x [\forall y \text{ Animal } y) \wedge \text{Loves } (x,y)] \Rightarrow [\exists y \text{ Loves } (y,x)]$$

Eliminate implication

$$\forall x \neg [\forall y \text{ Animal } y) \wedge \text{Loves } (x,y)] \vee [\exists y \text{ Loves } (y,x)]$$

Move \neg inwards

$$\forall x [\exists y \neg \text{Animal } (y) \vee \neg \text{Loves } (x,y)] \vee [\exists y \text{ Loves } (y,x)]$$

Standardize variables

$$\forall x [\exists y \neg \text{Animal } (y) \vee \neg \text{Loves } (x,y)] \vee [\exists z \text{ Loves } (z,x)]$$

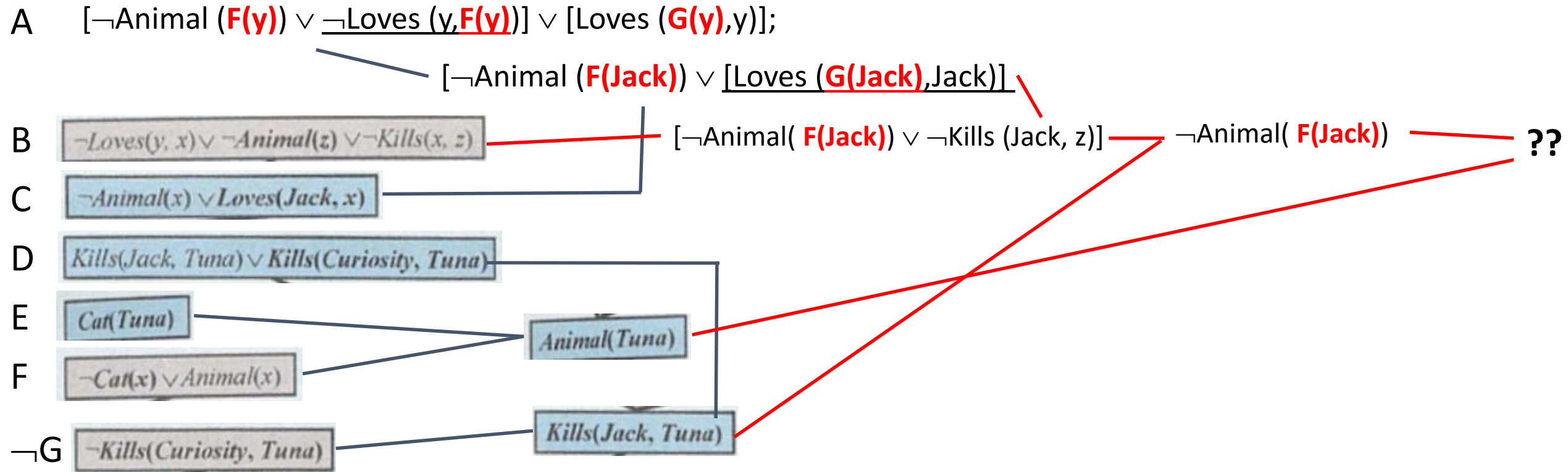
Skolemize (2 x)

$$\forall x [\neg \text{Animal } (\mathbf{F(x)}) \vee \neg \text{Loves } (x,\mathbf{F(x)})] \vee [\text{Loves } (\mathbf{G(x)},x)]$$

Drop universal quantifier

$$[\neg \text{Animal } (F(x)) \vee \neg \text{Loves } (x,F(x))] \vee [\text{Loves } (G(x),x)]$$





Resolutions:

$[\neg \text{Animal}(\mathbf{F(y)}) \vee \neg \text{Loves}(y, \mathbf{F(y)})] \vee [\text{Loves}(\mathbf{G(y)}, y)];$ $\neg \text{Animal}(x) \vee \text{Loves}(\text{Jack}, x) = \text{Jack}; x = \mathbf{F(Jack)}$

$[\neg \text{Animal}(\mathbf{F(Jack)}) \vee \neg \text{Animal}(\mathbf{F(Jack)}) \vee [\text{Loves}(\mathbf{G(Jack)}, \text{Jack})]$

$[\neg \text{Animal}(\mathbf{F(Jack)}) \vee \text{Loves}(\mathbf{G(Jack)}, \text{Jack})];$ $\neg \text{Loves}(y, x) \vee \neg \text{Animal}(z) \vee \neg \text{Kills}(x, z) = \text{G(Jack)}, x = \text{Jack}$

$[\neg \text{Animal}(\mathbf{F(Jack)}) \vee \neg \text{Animal}(z) \vee \neg \text{Kills}(\text{Jack}, z)]$

$[\neg \text{Animal}(\mathbf{F(Jack)}) \vee \neg \text{Kills}(\text{Jack}, z)]$

