

## Comparison of $\land$ and $\Rightarrow$



A	В	$A \Rightarrow B$	A ∧ B
Т	Т	Т	Т
Т	F	F	F
F	Т	Т	F
F	F	Т	F

General difference:

 $A \wedge B$  two facts (A and B) but  $A \Rightarrow B$  only one fact.

To be discussed: Implication within Implication

## Everyone who loves all animals is loved by someone.

If valid: "Wenn y ein Tier ist, wird es von x geliebt", then there is somebody (z), who loves x.

 $\forall$  x [ $\forall$  y Animal (y)  $\Rightarrow$  Loves (x,y)]  $\Rightarrow$  [ $\exists$  z Loves (z,x)] versus

 $\forall$  x [ $\forall$  y Animal (y)  $\land$  Loves (x,y)]  $\Rightarrow$  [ $\exists$  z Loves (z,x)]

If all things (all y) are animals and x loves y, then there is somebody (z), who loves x.





## Example 2 for Conversion in CNF with $\land$ instead of $\Rightarrow$



```
\forall x [\forall y Animal y) \land Loves (x,y)] \Rightarrow [\exists y Loves (y,x)]
```

$$\forall x \neg [\forall y \text{ Animal } y) \land \text{Loves } (x,y)] \lor [\exists y \text{ Loves } (y,x)]$$

$$\forall$$
 x [ $\exists$  y  $\neg$ Animal (y)  $\vee$   $\neg$ Loves (x,y)]  $\vee$  [ $\exists$  y Loves (y,x)]

$$\forall$$
 x [ $\exists$  y  $\neg$ Animal (y)  $\vee$   $\neg$ Loves (x,y)]  $\vee$  [ $\exists$  z Loves (z,x)]

$$\forall$$
 x [ $\neg$ Animal ( $F(x)$ )  $\lor \neg$ Loves ( $x,F(x)$ )]  $\lor$  [Loves ( $G(x),x$ )]

$$[\neg Animal (F(x)) \lor \neg Loves (x,F(x))] \lor [Loves (G(x),x)]$$

Eliminate implication

Move *¬* inwards

Standardize variables

Skolemize (2 x)

Drop universal quantifier





## Adapted Resolution (with $\land$ instead of $\Rightarrow$ )



```
[\neg Animal (F(y)) \lor \neg Loves (y,F(y))] \lor [Loves (G(y),y)];
Α
                                              [\neg Animal (F(Jack)) \lor [Loves (G(Jack), Jack)]
                                                                       [\neg Animal(F(Jack)) \lor \neg Kills(Jack, z)]
                                                                                                                              ¬Animal( F(Jack))
       \neg Loves(y, x) \lor \neg Animal(z) \lor \neg Kills(x, z)
В
        \neg Animal(x) \lor Loves(Jack, x)
D
      Kills(Jack, Tuna) V Kills(Curiosity, Tuna)
Ε
       Cat(Tuna)
                                                         Animal(Tuna)
         Cat(x) \lor Animal(x)
                                                          Kills(Jack, Tuna)
\neg G
         Kills(Curiosity, Tuna)
                                         Resolutions:
                                          [\neg Animal (F(y)) \lor \neg Loves (y,F(y))] \lor [Loves (G(y),y)];
                                                                                                                    \neg Animal(x) \lor Loves(Jack, x) = Jack; x=F(Jack)
                                          [\neg Animal (F(Jack)) \lor \neg Animal (F(Jack)) \lor [Loves (G(Jack), Jack)]
                                          [—Animal (F(Jack)) ∨ <u>[Loves (G(Jack),Jack)]</u> ;
                                                                                                     \neg Loves(y, x) \lor \neg Animal(z) \lor \neg Kills(x, z) = G(Jack), x = Jack
                                          [\neg Animal (F(Jack)) \lor \neg Animal (z) \lor \neg Kills (Jack, z)]
                                          \negAnimal( F(Jack)) \lor \negKills (Jack, z)]
```

