Overview

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II Problem Solving

3. Solving Problems by Searching

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4. Search in Compex Environments

- Local Search and Optimiziation Problems
- Local Search in Continuous Spaces
- Search with Nondeterministic Agents
- Search in Partially Observable Environments
- Online Search Agents and Unknown Environments

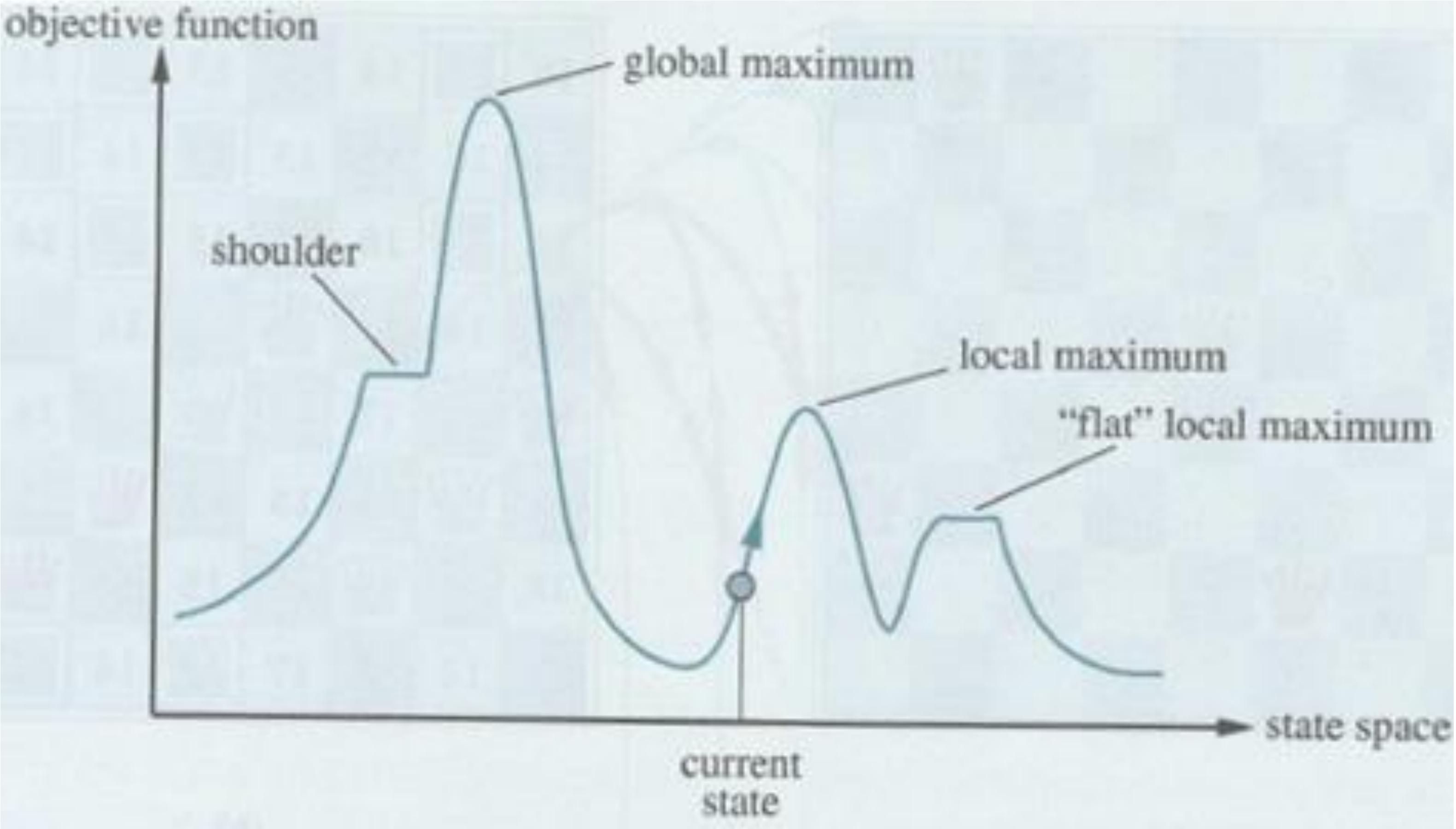
Local Search algorithms

- Local search algorithms are useful, if only the solution is relevant, not the path to the solution.
  - Example: 8-queens-problem
  - Other Examples: VLSI-Layout, factory floor layout, job shop scheduling, automatic programming, telecommunication network optimization, crop planning, portfolio managment
- Idea: Start with a **complete-state formulation** and **improve it by stepwise variations**
- Algorithms:
  - **Hill-Climbing**
  - **Simulated Annealing**
  - **Local Beam Search**
  - **Genetic Algorithms**

Hill Climbing Search

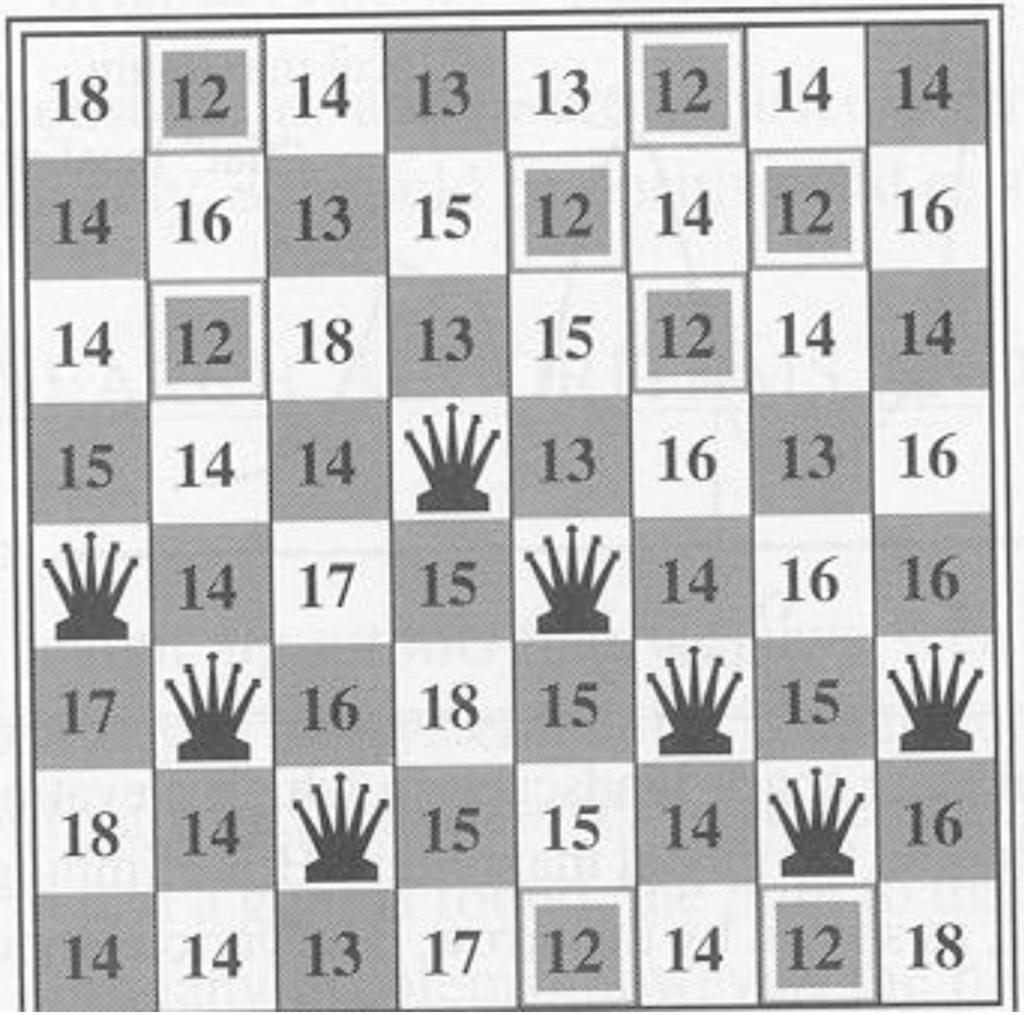
- Choose alway the highest-value sucessor node and return the current node, if no improvement is possible
    - No search tree, no backtracking → very efficient
    - Problems: Local maximas, plateaus, shoulders, flat local maximas

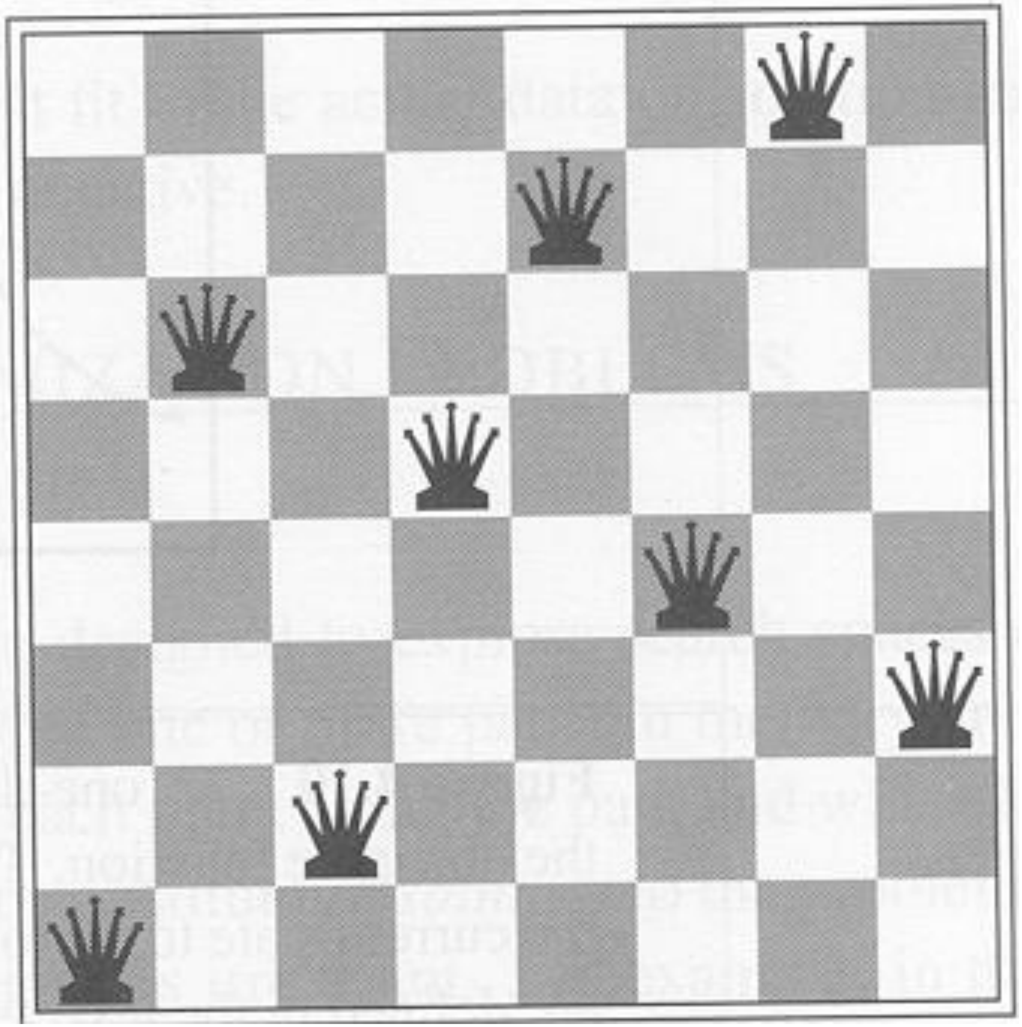
```
function HILL-CLIMBING(problem) returns a state that is a local maximum
  current ← problem.INITIAL
  while true do
      neighbor ← a highest-valued successor state of current
      if VALUE(neighbor) ≤ VALUE(current) then return current
      current ← neighbor
```



Hill-Climing: Example from 8-queens problem

- Left: In each step, a queen is moved to reduce the total number of conflicts (e.g. to 12)
- Right: Local minima, where hill-climbing gets stuck





Improvements for Hill-Climbing

- **Sideways move**: Limited; to pass shoulders but to avoid infinite loops in e.g. flat maxima
- **Stochastic hill-climbing**: Random selection of all improvements
  - **First-choice hill-climbing**: Generate sucessor-nodes and take the first improvement
- **Random-restart hill-climbing**: Repeat hill-climbing with randomly generated initial states

- **Simulated Annealing:** Allow worsening with a low probability
- **Local beam search:** Simultaneous search an several paths

Simulated Annealing

- Idea: Allow „down-hill“ steps to overcome local maxima
  - Model: Gradually cooling of hot material (e.g. metallurgy), freezing of water etc.
- Problem: Control of down-hill steps
- Solution:
  - Random factor for choosing steps: if it improves the solution, choose it with 100%, if not, choose it with a probability depending on the degree of worsening
  - Terminate, by lowering the probability of down-hill steps continuously

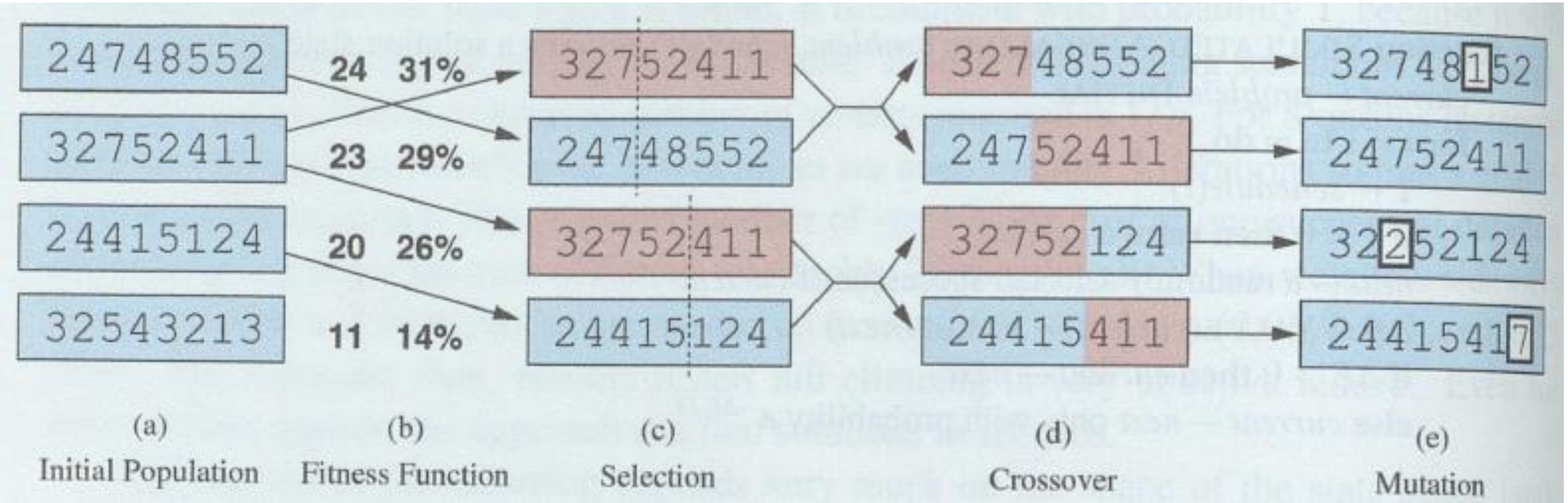
```
function SIMULATED-ANNEALING(problem, schedule) returns a solution state
   current ← problem.INITIAL
   for t = 1 to ∞ do
      T ← schedule(t)
      if T = 0 then return current
      next ← a randomly selected successor of current
      ΔE ← VALUE(current) – VALUE(next)
      if ΔE > 0 then current ← next
      else current ← next only with probability e^(−ΔE/T)
```

Local Beam Search

- **Idea:** Keep a set of k nodes („beam“) instead of one node. From one iteration to the next, the k best sucessors are chosen.
- **Difference to random restart hill-climbing**: Passing of information among parallel searches and concentration of search in promising regions.
- **Problem:** Concentration in a small region of the space (e.g. around a high, but local maxima).
- Improvement: **Stochastic generation of sucessors** nodes similar to stochastic hill-climbing for generation of diversity.

Evolutionary Algorithms

- Variant of stochastic beam search, which can **combine useful solution-parts** (blocks)
- Main steps (data structure is pool of solutions called „**population**“)
  - Repeat until **termination criterium**
    1. **Select:** Select two solutions in population according to fitness
    2. **Recombine:** Generate new solution from both parents
    3. **Repair** (optionally): Repair (improve) new solution
    4. **Mutate:** Modify new solution randomly by mutations
  - **Choose best solutions** (from new population)



Variations of Evolutionary Algorithms

- **Size of population**
- **Representation of individuals**
  - Genetic algorithms: String over an alphabet (in biology DNA over alphabet ACGT)
  - Evolution strategies: Indiviual is a sequence of real numbers
  - Genetic programming: Individual is a computer program
- Mixing p parents: p = 1: Stochastic beam search; p = 2: Standard case; p > 2: Possible
- **Selection process**: Different functions usually depending on fitness
- **Recombination procedure**: Standard is to (randomly) select a crossover point
- **Mutation rate:** Determines, how often an offstring have random mutations
- **Makeup of next generation**: Keep top-scoring parents? Discard individuals below threshhold?

Examples from 8-queens and Assignment

N1 to A1
N2 to A2
N3 to A3
N4 to A4
-5

N1 to A2
N2 to A1
N3 to A4
N4 to A3
-3

N1 to A2
N2 to A3
N3 to A1
N4 to A4
-2

Parent Generation

N1 to A1
N2 to A2
N3 to A4
N4 to A3
-3

N1 to A2
N2 to A3
N3 to A4
N4 to A3

Repair

N1 to A2
N2 to A3
N3 to A4
N4 to A1
0

Child Generation

Genetic Algorithm

```
function GENETIC-ALGORITHM(population, fitness) returns an individual
  repeat
      weights ← WEIGHTED-BY(population, fitness)
      population2 ← empty list
      for i = 1 to SIZE(population) do
          parent1, parent2 ← WEIGHTED-RANDOM-CHOICES(population, weights, 2)
          child ← REPRODUCE(parent1, parent2)
          if (small random probability) then child ← MUTATE(child)
          add child to population2
      population ← population2
  until some individual is fit enough, or enough time has elapsed
  return the best individual in population, according to fitness

function REPRODUCE(parent1, parent2) returns an individual
  n ← LENGTH(parent1)
  c ← random number from 1 to n
  return APPEND(SUBSTRING(parent1, 1, c), SUBSTRING(parent2, c + 1, n))
```

- **Population:** Ordered list of individuals
- **Weight:** List of fitness values for each individual
- **Fitness:** Function to compute weights

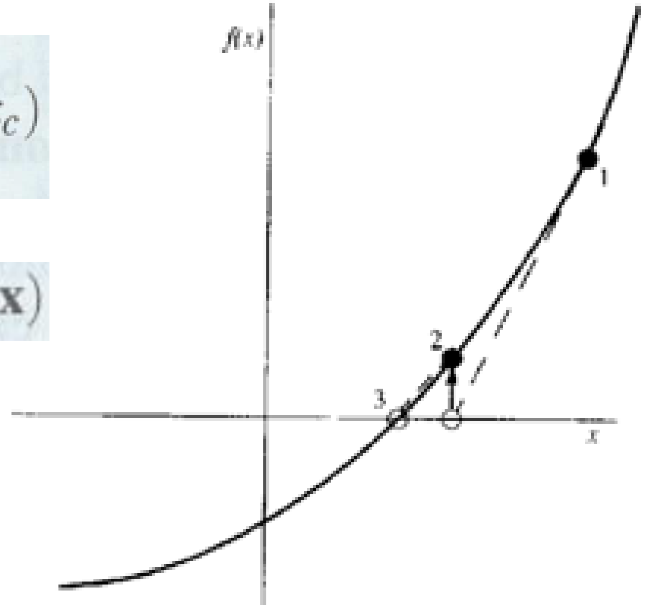
Local Search in Continuous Spaces

- Example problem: Place 3 airports in Romania with minimum square distance to the cities
  - Input: Coordinates of cities C_i (maybe with population weights), whose next airput is i
  - Output: Coordinate (x,y) of the three airports (x_1, y_1; x_2, y_2; x_3 y_3)
  - Optimization criteria: $$f(\mathbf{x}) = f(x_1,y_1,x_2,y_2,x_3,y_3) = \sum_{i=1} \sum_{c \in C_i} (x_i - x_c)^2 + (y_i - y_c)^2$$

Solution Approaches

- **Discretization of neighborhoods**
  - Moving an airport in x or y-direction with a constant d
  - With 6 variables 12 sucessors per state
  - Application of any search algorithms
- **Local gradient search**
  - Derivation of goal function for each variable $$\frac{\partial f}{\partial x_1} = 2 \sum_{c \in C_1} (x_1 - x_c)$$
  - Gradient of goal function is a vector indicating the size and direction of the steepest slope (α = step seize) $$\mathbf{x} \leftarrow \mathbf{x} + \alpha \nabla f(\mathbf{x})$$
  - Newton-Raphson algorithm often effective, also for matrices
- **Linear programming**
  - Constraints for goal function must be linear (e.g. airports should not be placed in mountains)
  - Goal function must be linear too (e.g. sum of distances)

Newton-Raphson for solving equations of the form g(x) = 0
$x \leftarrow x - g(x)/g'(x)$

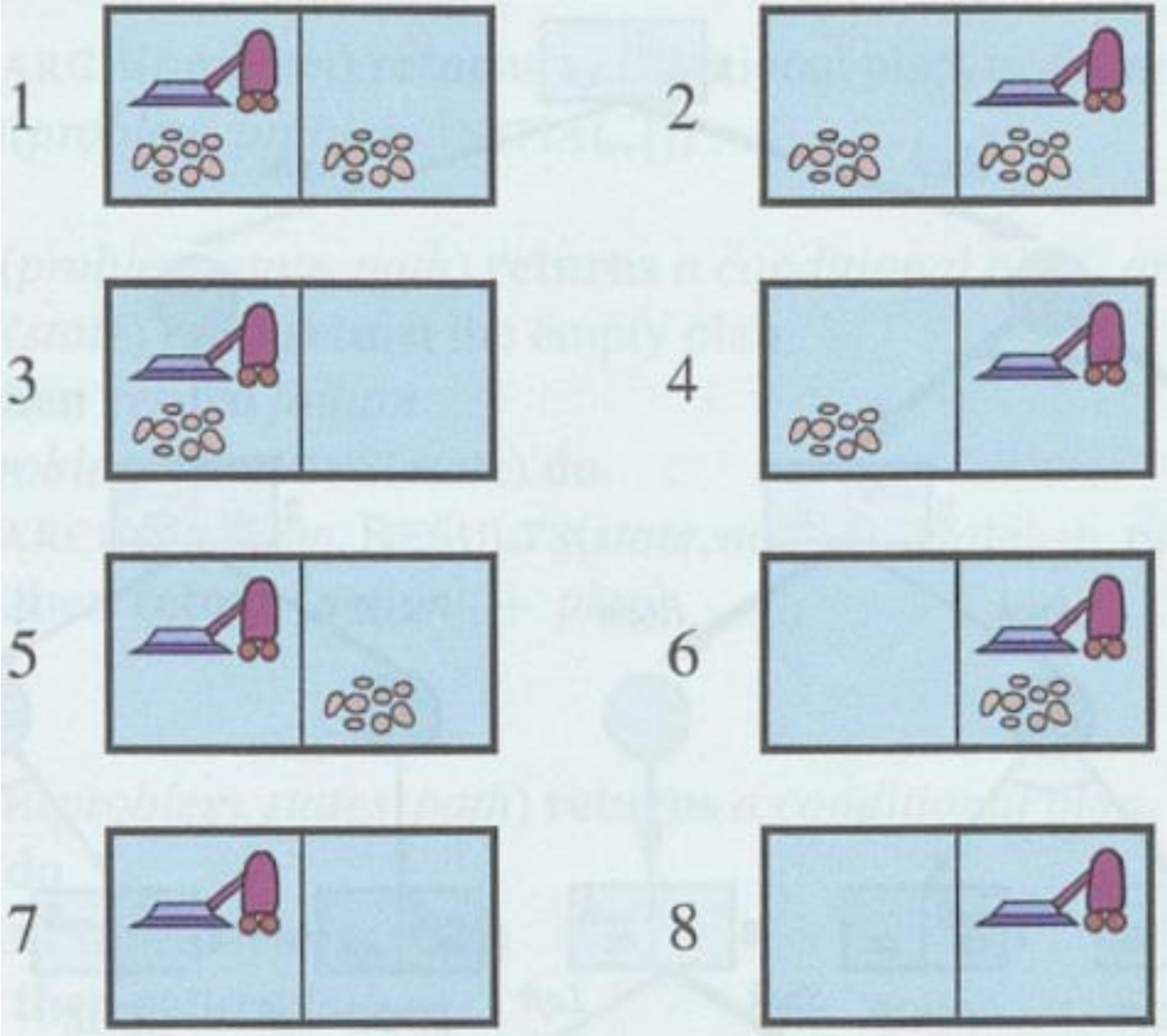


Search with Nondeterministic Actions

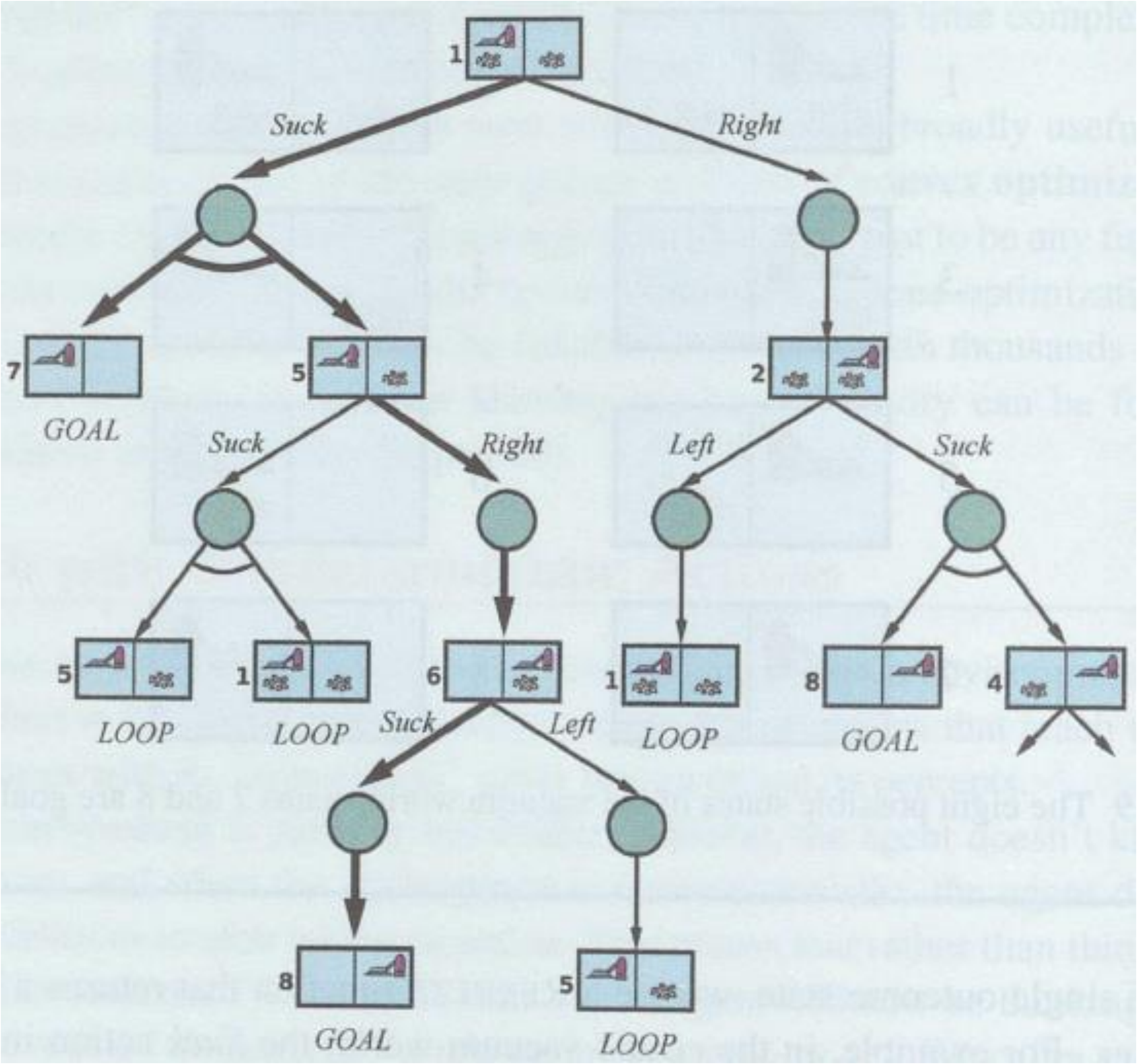
- Example problem:
  - Vacuum world with erratic actions:
    - Suck of a dirty square makes it clean and sometimes cleans the adjacent square too
    - Suck of a clean square sometimes deposits dirt on it
- Solution in state 1
  - Deterministic: No plan available
  - Conditional plan:
    1. Suck
    2. if state = 5

       then [Right, Suck]

       else [ ]



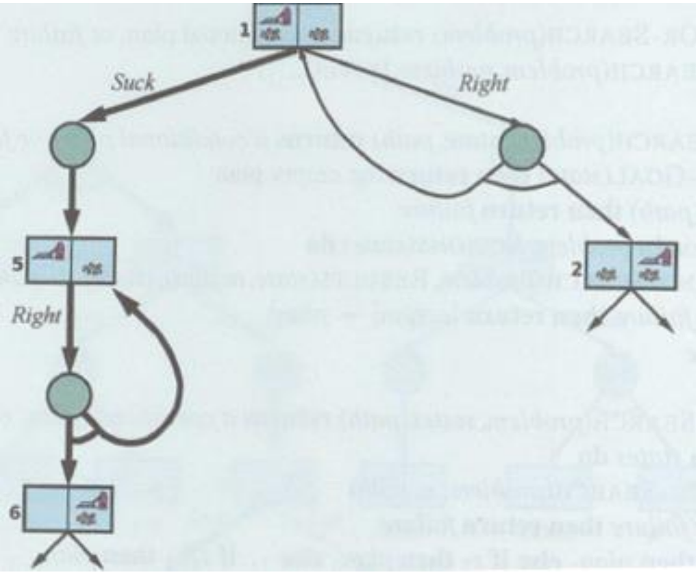
Solution: And-Or-Search Tree



- OR-Nodes: rectangles
- AND-Nodes: circles
  - All results must be handled
  - The state must be checked
- Solution is a subtree of the complete search tree that has …
  - a goal node at every leaf
  - specifies one action for each of its OR-nodes
  - includes every outcome branch at each of its AND-nodes
- Bold arrows: Solution

And-Or Graph Search and Cycles

- Different search algorithms possible, e.g. depth-first, breadth-first, best-first
- Next slide: Recursive depth-first algorithm
- Key aspect: dealing with cycles (arising often in nondetermistic problems)
  - Finding a plan avoiding cycles
  - Keep trying an indetermistic action until the desired outcome occurs
    - Example in a slippery vacuum world, where movements may fail, i.e. the agent may stay in the same location:
      - [Suck,   while State = 5 do Right,   Suck]
      - [Suck, L_1: Right, **if** State = unchanged **then** L_1 **else** Suck]
    - Might result in infinite loop
    - Try a limited number of repetitions (like inserting an electronic card)



Conditional And-Or-Search Algorithm

Solution is a conditional plan considering every nonterministic outcome

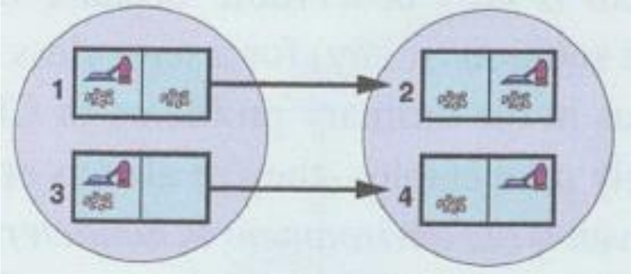
```
function AND-OR-SEARCH(problem) returns a conditional plan, or failure
   return OR-SEARCH(problem, problem.INITIAL, [])

function OR-SEARCH(problem, state, path) returns a conditional plan, or failure
   if problem.IS-GOAL(state) then return the empty plan
   if IS-CYCLE(path) then return failure
   for each action in problem.ACTIONS(state) do
      plan ← AND-SEARCH(problem, RESULTS(state, action), [state] + path])
      if plan ≠ failure then return [action] + plan]
   return failure

function AND-SEARCH(problem, states, path) returns a conditional plan, or failure
   for each s_i in states do
      plan_i ← OR-SEARCH(problem, s_i, path)
      if plan_i = failure then return failure
   return [if s_1 then plan_1 else if s_2 then plan_2 else ...if s_{n-1} then plan_{n-1} else plan_n]
```

Search in Partial Observable Environments

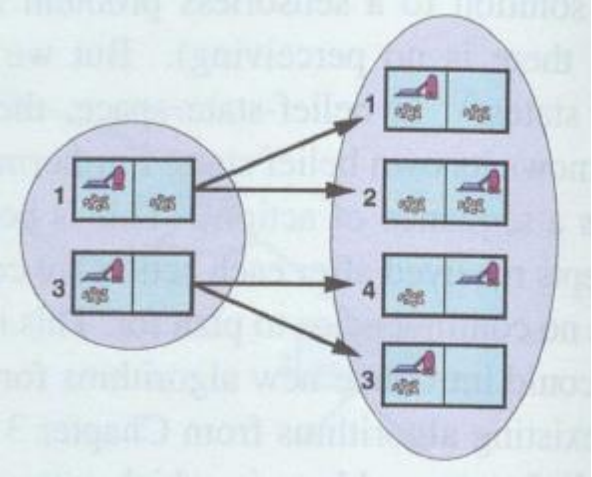
- Example problem: Vacuum world without or with partial sensor information
- Solution approaches: **Search in belief states** and update of belief states
  - **Belief state:** All possible states compatible with the current information
    - Should include the physical state

Search without Observation

- Sensorless (conformant) problem
- Advantages: Sensors are expensive and often not reliable
- Examples:
  - Producing a base state for machines (e.g. restart a computer)
  - Infections: Prescribing broadband antibiotics
  - Sensorless vacuum world:
    - Effect of the deterministic action „right“ in a belief state:



    - Effect of the indeterministic action „right“ in a belief state:

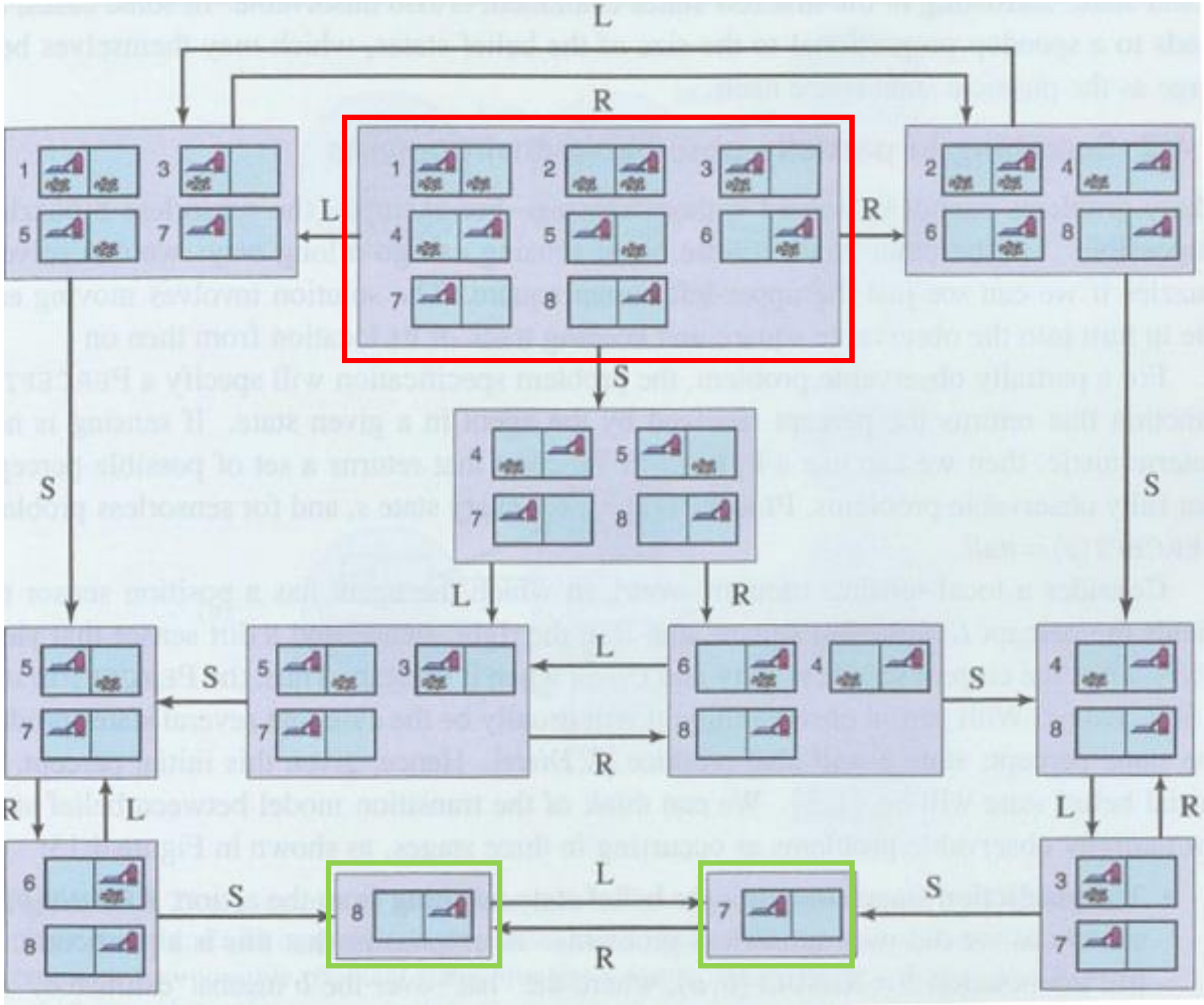


Search State for Sensorlos Planning in Vacuum World

… with deterministic actions

- Start state in red
- goal states in green

- Possible solutions:
  [right, suck, left, suck]
  [left, suck, right, suck]



Construction of Search Space in Belief State (1)

- **Belief state:** Set of all possible physical states
  - With N physical states, there are 2^N belief states
- **Initial state:** Without knowledge all 2^N belief states
- **Actions:**
  - Assumption: Illegal actions have no effect in environment
    - New belief state b' = union of all actions a in every physical state p of the current belief state b

$$\text{ACTIONS}(b) = \bigcup_{s \in b} \text{ACTIONS}_P(s)$$

  - Assumption: Illegal actions might cause (great) damage
    - It is safer to allow only actions, which are legal in all the states (computed by intersection)

Construction of Search Space in Belief State (2)

- **Transition model:**
  - For deterministic actions: the new belief state b has one result state s for each of the possible actions a: set b‘ = Result (b, a) = {s‘ : s‘ = Result$_p$ (s,a) und s $\in$ b}
  - For inderministic action: the new belief state may have several result states for each of the possible actions: set b‘ = Result (b, a) = {s‘ : s‘$\in$ Result$_p$ (s,a) und s $\in$ b} = $\bigcup_{s \in b} \text{RESULTS}_P(s,a)$
  - The size of the result set may decrease for deterministic actions and may increase for interdeterministic actions.
- **Goal test:** All physical states p in belief state b should satisfy the goal test
- **Action costs:** Could be difficult, if the same action has different costs if applied to different states, otherwise straightforward.

Search Algorithms in Sensorless Planning

- Apply **ordinary search algorithms** in belief state representation
  - Special treatment of supersets: if a superset {1, 3, 5, 7} is solved, all subsets, e.g. (e.g. {5,7}) are solved too.
  - The representation of one belief state is already exponential
    - Compacter representation of belief states by using logic instead of enumaration, e.g. after action [right]: „not in left cell“
  - Very large space 2^N instead of N, and even N is often too large for e.g. breadth-first search or A* search.
- **Incremental belief state search**
  - Search a solution for just one physical state in the belief state
  - Check, whether this plan works also for the other physical states
  - If not, search for another plan in the first physical state etc.
  - Is efficient to find out, whether a problem has no solution

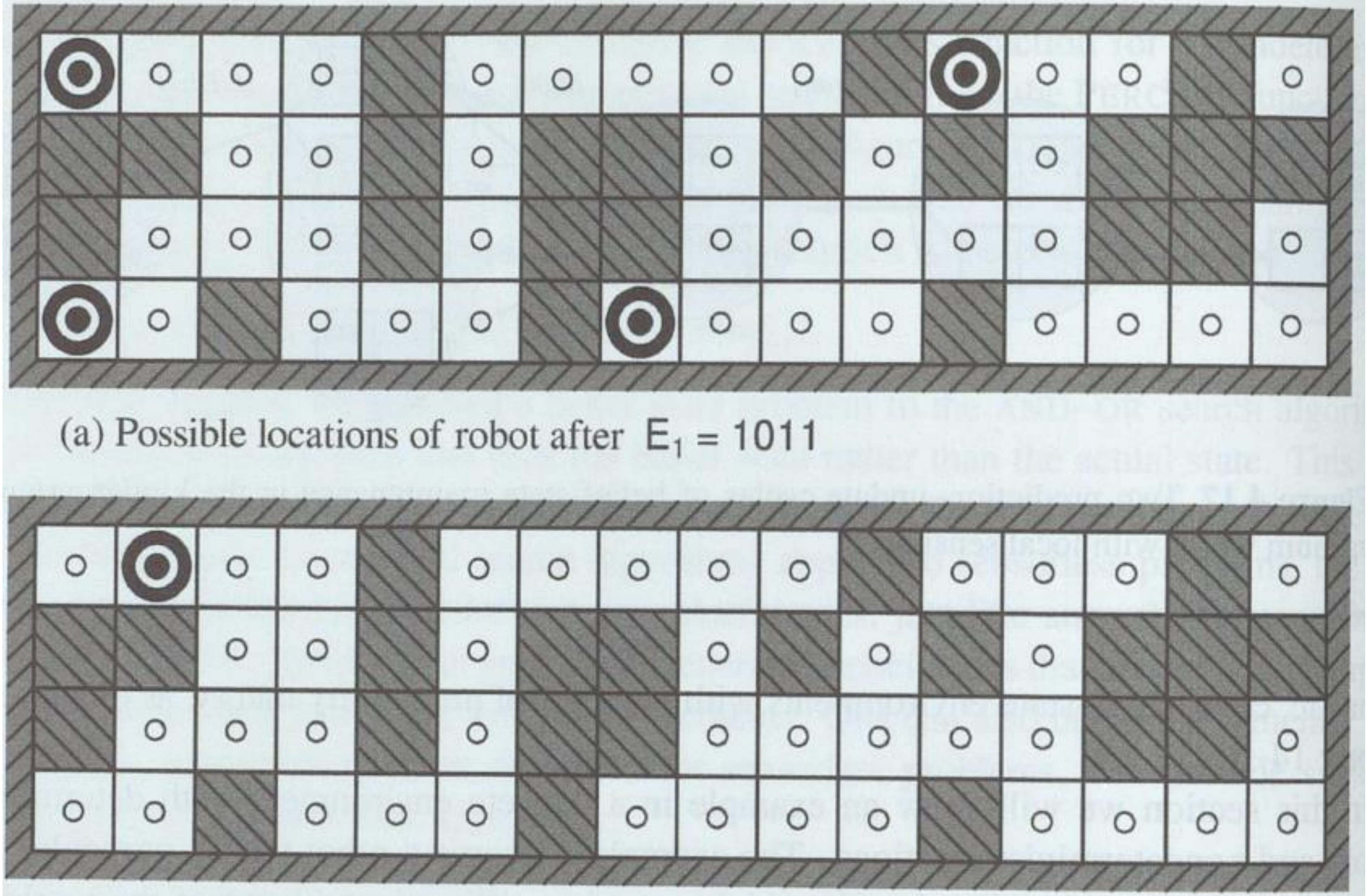
Searching in Partially Observable Environments

- Sensorless planning often impossible (e.g. 8-puzzle)
- A little bit of sensing often useful (e.g. one square in 8-puzzle sufficient)

- Algorithm aspects
  - Prediction step similar to sensorloss planning
    - But results in possible percepts, that could be observed in predicted belief state
  - Update step computes for each possible percept new belief state from percept
    - For deterministic sensing, belief states for different possible percepts are disjoint
  - AND-OR search algorithm applicable
    - Solution is a conditional plan; agent tests condition and execute appropriate branch
    - Agent updates its belief state after each action (and percept)
    - Probabilistic information for indeterministic actions would be useful (treated later)

Example: Robot Navigation with non-deterministic Actions

- A robot has 4 sonar sensors telling him, whether there is a wall, e.g. 1011 means North: Wall; East: No wall; South and West: Wall.
- Robot has 4 action to move in each direction.
    - However, move is non-deterministically, so the robot may end on any adjacent field.



(a) Possible locations of robot after $E_1 = 1011$

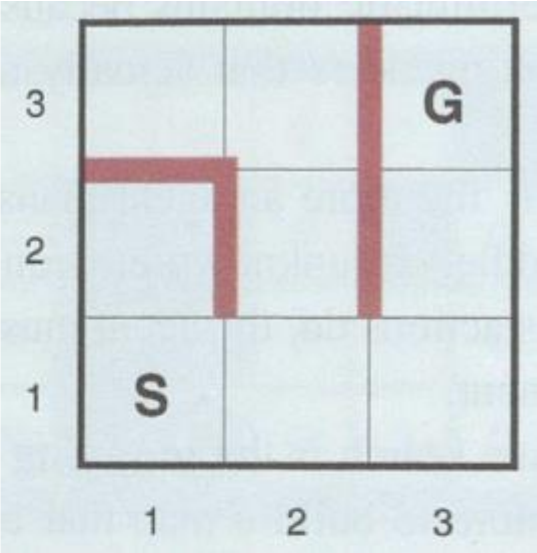
(b) Possible locations of robot after $E_1 = 1011$, $E_2 = 1010$

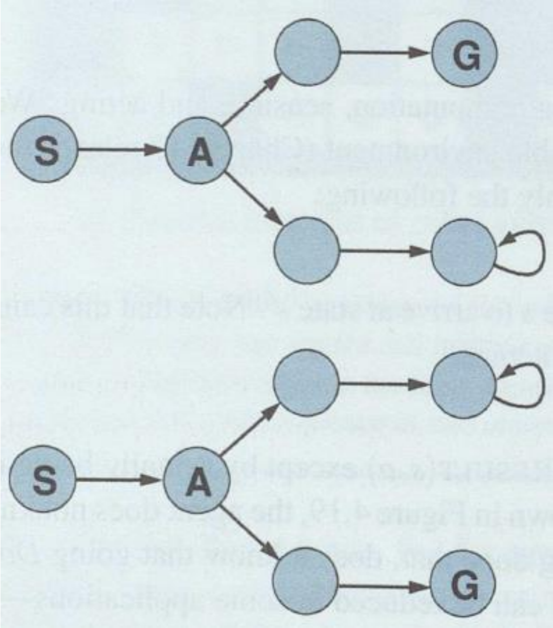
Online-Search

- Offline-Search: Solution is computed before execution
- Online-Search: Computation and Execution overlap
- Necessary in unknown environments (but goal and action with costs are known)
- Problems: Irreversible action leading in dead ends (e.g. robot drops off a cliff, one-way streets)

Solution Approaches for Online-Search (1)

- **Online-Depth-First Search:**
  - In each state, follow the action given by depth-first search
  - If no action possible, go back to last branch (must be stored) and try another action
  - In maze (right), the agent would walk: (1,1) → (1,2) → (1,1) → (2,1) → (2,2) → (2,3) → (1,3) → (2,3) → (2,2) → (2,1) → (3,1) → (3,2) → (3,3)
  - Indefinite paths would prevent the agent from finding the goal (right below)
- Online variant of **iterative deepening depth-first search** would avoid indefinite paths and would find „shallow" goals much faster.





Solution Approaches for Online-Search (2)

- **Hill-climbing search** (does not explore the environment, might get stuck)
- Hill-climbing with random restarts and random walk (very slow)
- Augmenting hill-climbing with memory: For each state, an heuristic cost-estimate to reach the goal is stored and updated with more experience
- **Real-time A* (LRTA*):** Agent builts a map in its result table:
  - The estimated cost to reach a goal through neighbor state s‘ is cost to get to s‘ + estimated cost to goal, i.e.: c(s, a, s‘) + H (s‘).
  - Example: cost increase in red circle to escape plateau
  - Prefer untried states u by H(u) = 0