Machine Learning for Complex Networks
SoSe 2023

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Exercise Sheet 02

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Total points: 10

Please upload your solutions to WueCampus as a scanned document (image format or pdf), a typesetted PDF document, and/or as a jupyter notebook.

1. **Molloy-Reed model**

   (a) Given a random microstate generated based on the configuration model with degree distribution $P(k)$, consider a random node v and follow a random edge to a neighbor of w of v. What is the probability that node w has degree k? [1P]

   (b) Using the expression obtained above compute the expected degree of the neighbors a random node v. What do we see when we calculate the difference between the expected degree of a random node and the expected degree of a random neighbor of such a node? [2P]

   (c) Often, rather than the degree of a node at the end of an edge, we are interested in the number of edges attached to the node *other* than the one we arrived through. This number is called the *excess degree* of a node. What is the probability that the node at which you arrive has *excess degree* k? [1P]

   (d) Consider a Molloy-Reed model with no self-loops and where we allow for the creation of multiple edges between a single pair of nodes. What is the probability that two nodes v and w with degrees d_v and d_w are connected? [2P]

2. **Inference and Statistical Ensembles**

   (a) Consider the $G(n,p)$ model for undirected random graphs with no self-loops and no multi-edges. Show that, for a given network G_e with n nodes and m links, a maximum likelihood estimate of parameter p is given as: [2P]

$$\hat{p} = \frac{m}{\binom{n}{2}}$$

   (b) Consider the microstates G_1 and G_2 with $n = 100$ nodes and $m_1 = 300$ and $m_2 = 350$ edges, respectively. What is the probability of these microstates within [2P]

   - a $G(n,p)$ model with $n = 100$ and $p = \frac{5}{99}$? What is the expected number of edges in this model?
   - a $G(n,m)$ model with $n = 100$ and $m = 300$?