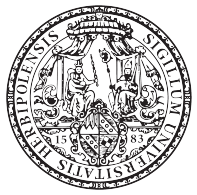

Machine Learning for Complex Networks
SoSe 2023

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Exercise Sheet 01

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Due: May 11, 2023
Total points: 10

Please upload your solutions to WueCampus as a scanned document (image format or pdf), a typesetted PDF document, and/or as a `jupyter` notebook.

1. **Computing connected components in graphs**

   (a) Implement Tarjan's algorithm to compute the (strongly) connected components in a (directed) network. Apply your algorithm to a directed example network that has multiple strongly connected components. Explain how the `low_link` and `dfs_num` counters in your example are used to assign nodes to strongly connected components. [3P]

   (b) For a Laplacian matrix $\mathcal{L}$ of an undirected graph G with n nodes consider the sequence [3P]

   $$\lambda_1 = 0 \leq \lambda_2 \leq \ldots \leq \lambda_n$$

   of eigenvalues in ascending order. Generate undirected networks with $n = 20$ nodes and different numbers of connected components from one to 50. Calculate the eigenvalue sequences of the corresponding Laplacian. What do you observe? Can you explain your observation?

   (c) Repeat the experiment above using the sorted sequence of eigenvalues of the adjacency matrix. [1P]

   (d) Use your finding from the previous tasks to implement a `python` function that uses the Laplacian matrix to calculate the number of connected components in an undirected graph. Test your function in an example network. What is the computational complexity of your function? [3P]