

Machine Learning for Complex Networks SoSe 2023

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Exercise Sheet 02

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Please upload your solutions to WueCampus as a scanned document (image format or pdf), a typesetted PDF document, and/or as a jupyter notebook.

1. Molloy-Reed model

(a) Given a random microstate generated based on the configuration model with degree distribution P(k), consider a random node v and follow a random edge to a neighbor of w of v. What is the probability that node w has degree k?

1P

(b) Using the expression obtained above compute the expected degree of the neighbors a random node v. What do we see when we calculate the difference between the expected degree of a random node and the expected degree of a random neighbor of such a node?

2P

(c) Often, rather than the degree of a node at the end of an edge, we are interested in the number of edges attached to the node *other* than the one we arrived through. This number is called the *excess degree* of a node. What is the probability that the node at which you arrive has *excess degree k*?

1P

(d) Consider a Molloy-Reed model with no self-loops and where we allow for the creation of multiple edges between a single pair of nodes. What is the probability that two nodes v and w with degrees d_v and d_w are connected?

2P

2. Inference and Statistical Ensembles

(a) Consider the G(n,p) model for undirected random graphs with no self-loops and no multi-edges. Show that, for a given network G_e with n nodes and m links, a maximum likelihood estimate of parameter p is given as:

2P

 $\hat{p} = \frac{m}{\binom{n}{2}}$

(b) Consider the microstates G_1 and G_2 with n=100 nodes and $m_1=300$ and $m_2=350$ edges, respectively. What is the probability of these microstates within

2P

• a G(n,p) model with n=100 and $p=\frac{5}{99}$? What is the expected number of edges in this model?

• a G(n,m) model with n=100 and m=300?