Network Science (VU) (706.703)

Function of Networks

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Outline

Structure and function of networks

- One of the main goals of studying networks is to understand the behavior of the systems we represent with the networks
- E.g. we study online social network to understand how information and ideas spread through these networks
- We study technological networks to analyze the protocols and improve them
- We study metabolic networks to understand chemical process in the cell
- We study information networks to understand the user behavior

Structure and function of networks

- Studying the structure is just the first step towards understanding
- Another important step is to make the connection between the structure and the function
- Once when we have measured and quantified the structure the question is then how can we turn these results into prediction of the system's behavior?
- There are some areas where some results have been achieved
- Resilience, spreading processes, and dynamical systems on networks

Structure and function of networks

- Resilience
  - Attack the networks and see how resilient are they: percolation
- Spreading processes
  - Disease spreading, computer viruses, ideas, innovation: threshold and dynamics
- Dynamical systems
  - Diffusion, opinion dynamics, activity: dynamics

Percolation

- Imagine removing some fraction of nodes, along with its links from a network
- This process is called percolation (site percolation)
- This simple process can be used as a model of node failures
- E.g. router failures on the Internet
- E.g. vaccination of individuals against the disease

Percolation

- Sometimes links in a network fail
- E.g. communication lines on the Internet fail
- We can model this by removing some fraction of links from the network (bond percolation)
- In general, there are many ways how nodes and links can be removed from the network
- The simplest model is to remove them uniformly at random
- Another possibility is to remove them according to their degrees

Uniform random removal of nodes

- The percolation process is parametrized by probability ϕ, which is the probability that a node is present in the network
- If present the node is said to be *occupied*
- ϕ is called occupation probability
- Thus, if $\phi = 1$ none of the nodes has been removed
- If $\phi = 0$ all nodes have been removed

Uniform random removal of nodes

- When ϕ is close to one, the nodes tend to be connected and form a giant component
- When ϕ is close to zero, there are only few nodes that are typically disconnected
- When we start with a large ϕ and decrease it there comes a point where the giant component breaks up
- When we start with a small ϕ and increase it, there comes a point where the small and disconnected components are connected to form a giant component
- This point is called percolation threshold

Non-uniform random removal of nodes

- Typically, in this model ϕ depends on the degree of a node
- E.g. it might be proportional to the degree and this is biased towards removal of the hubs
- Or it can be inversely proportional to the degree, i.e. we tend to remove low degree nodes first
- The basic process and formation of the giant component still take place
- But, the percolation threshold might change

Percolation in real-world networks

- Typically, we study percolation by simulating node removal
- We are interested in the size of the largest component as the function of ϕ
- We should repeat simulation many times, i.e. thousand times to see what is the typical behavior
- We are also interested in variation around that typical behavior

Percolation in real-world networks

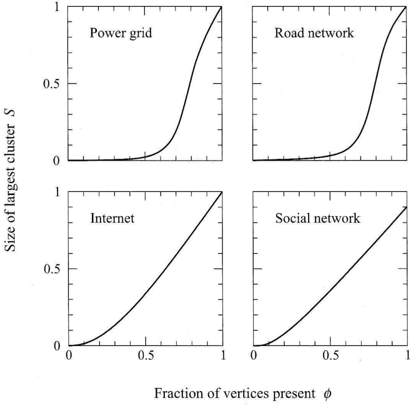

Figure: Uniform removal

Percolation in real-world networks

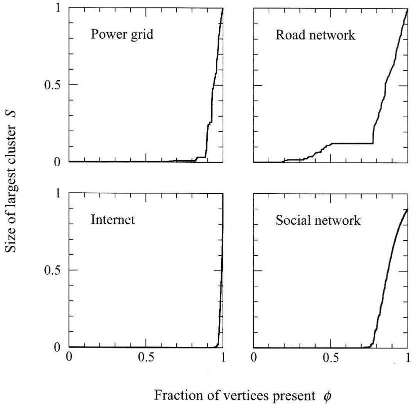

Figure: Non-uniform (targeted) removal

Spread over networks

- Connection of social networks with the spread of disease
- Diseases spread over networks of contacts between individuals
- Similarly, the computer viruses spread over e.g. the Web, Facebook, e-mail communication, etc.
- The mechanics are similar
- Also, information diffusion, gossip, spread of ideas have similar underlying (abstract) dynamics
- Similar models can be used to describe all of these processes

Spread over networks

- The biological process that leads to infection is complicated
- E.g. multiplication of viruses, bacteria
- The interplay with the immune system
- Previous diseases
- The outcome of the diseases, etc.
- If we want to fully understand this process we need to understand the biology behind it

Spread over networks

- Similarly, the process of how ideas or even information spreads and gets adopted is complicated
- Intrinsic personal tendency to accept new ideas
- Based on education, previous knowledge, previous experience, etc.
- Attractiveness of the idea itself
- The interplay with the social context, etc.
- If we want to fully understand this process we need to understand the psychology, sociology, group psychology, etc.

Models of the spread

- A standard scientific approach is to start with a simplified model of reality
- There are many different approaches to modeling
- A possible categorization:
  1. Threshold models
  2. Agent-based models
  3. Dynamics

Models of the spread

- Another categorization:
  1. Deterministic models
  2. Probabilistic models
- Also we can distinguish between:
  1. Discrete models
  2. Continuous models

Threshold model

- E.g. a threshold model such as linear threshold model by Granovetter
- Each person has an individual threshold (some number)
- If a certain value goes over threshold the person engages in a behavior
- E.g. adopts an idea, or information

Threshold model

- The threshold depends on social context, education, age, personality, etc.
- It abstracts all of these factors with a single number
- Threshold model is a standard deterministic model
- I.e. whenever we pass the threshold the behavior is triggered

Threshold model

- Alternatively, we can also have a probabilistic threshold model
- I.e. a probabilistic model
- We define a probability that a person engages in behavior and draw random numbers
- The probability is based on the same variables as threshold
- However, the emerging behavior is probabilistic

Agent-based models

- Each node is an agent that acts autonomously
- I.e. each agent has a certain behavior
- Agents engage in interactions with other agents
- E.g. neighbors with neighbors or sometimes with arbitrary other agents
- The system evolution is defined by the rules of the interactions

Naming game

- How ideas are spread and adopted in a network
- How does consensus emerge?
- We have a number of people in a social network that are discussing some problems and are trying to reach a consensus
- We model people and their relationships as nodes and links in a social network
- Each person carries a list of words, e.g. names, solutions for the issue that is being discussed

Naming game

- The rules of the game:
  1. We select a random person (speaker) from the network
  2. Then we select a random neighbor (listener) of that person
  3. The the speaker selects a random word from her name list
  4. If the listener has this word in her list they both agree on this word, keep it and throw all other words from their lists
  5. If the listener does not possess the word then she includes it in her list
  6. We repeat the game many times
- The Naming Game in Social Networks: Community Formation and Consensus Engineering: http://arxiv.org/abs/1008.1096

Naming game

- Demo
- Some results:
    1. The network structure is crucial for the process dynamics
    2. E.g. community structure
    3. It is possible to manipulate (engineer) the dynamics by e.g. committed agents
    4. Danger: manipulation of public opinion, mass media, etc.

Dynamical systems

- A general concept of dynamical systems
- We represent the state of a dynamical system by some set of quantitative variables
- Each node has a variable $x_i(t)$
- E.g. the probability of being infected, the number of people waiting on the bus station

Dynamical systems

- A *dynamical system* is any system whose *state changes over time*
- According to some given rules or equations
- E.g. equations of change of probabilities of being infected in the SI model
- E.g. equations of how people arrive and leave bus stations

Dynamical systems

- Many other processes (or simplified versions of these processes) can be represented as dynamical systems
- The spread of news or information between friends
- The movement of money through economy
- The flow of traffic on roads
- Data over the Internet, etc.

Dynamical systems

- For the rest of our discussion on the function of complex networks we will concentrate on such models
- But for this we yet again need some mathematical basics and background
- Differential equations