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Exercise Sheet 06

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Due: December 14, 2022
Total points: 10

1. **k-regular Random Graphs**

   (a) Consider the statistical ensemble of random k-regular graphs, i.e. a degree-based ensemble of random microstates where all nodes have exactly degree k. Write down the generating function G_0 of the degree distribution for a k-regular random graph with given $k \in \mathbb{N}$. Use G_0 to calculate the first and second raw moment of the degree distribution. 2P

   (b) Use the Molloy-Reed criterion to derive the critical point for the parameter k above which we expect a random k-regular network to exhibit a giant connected component in the limit of $n \to \infty$. 1P

   (c) Implement a `python` function that generates random microstates from the ensemble above for variable parameters k. Confirm your analytical result by calculating the average largest connected component size in microstates generated with different values of k. 1P

   (d) Compare your finding from 1a to the critical threshold for Erdös-Rényi random graphs derived in lecture L08. What does this result tell you about the influence of the heterogeneity of node degrees on connectivity in random networks? 1P

2. **Epidemic Spreading in Complex Networks**

   For the following questions, consider the so-called susceptible-infected-susceptible (SIS) model, a simple model for the spreading of epidemics in social networks. In this model, nodes can either be susceptible (node is in state S) for a disease or a node is currently infected (node is in state I). In each time step t, (i) each susceptible node is infected with probability λ if at least one of their neighbors are infected ($S \to I$), and (ii) infected nodes recover and become susceptible again ($I \to S$). The only parameter λ in this model controls the spreading rate of the disease, i.e. how transmissible it is.

   (a) Implement this model in python and calculate the average fraction of infected nodes in the limit of large times t for Erdös-Renyi Networks with $n = 1000$ and $p = 3/1000$ and different values for the disease transmissability $\lambda \in (0, 1)$. You can start the simulation with a small number of 10 nodes that are initially infected at $t = 0$. 3P

   (b) Repeat your experiment from 2a for a fixed transmissability $\lambda = \frac{1}{3}$ and different Erös-Rényi random networks with $n = 1000$ and values of p such that $np \in [1, 5]$. How does the average fraction of infected nodes at large times change as we change p. 1P

   (c) Can you explain your results from 2a and 2b based on the theoretical results from the lecture? Which possible implications could those results have in light of the current CoViD-19 situation (keeping in mind the severe limitations of the simple model)? 1P
   <u>Hint:</u> Consider the following article: M Boguna, R Pastor-Satorras, A Vespignani: *Absence of epidemic threshold in scale-free networks with connectivity correlations*, Phys. Rev. Lett., 90, 2003