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Exercise Sheet 01

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Total points: 14

Please upload your solutions to WueCampus as a scanned document (image format or pdf), a typesetted PDF document, and/or as a `jupyter` notebook.

1. **Shortest Paths and Diameter**

   (a) Investigate and explain the Bellman-Ford algorithm[1] to calculate all shortest path between a given node v and all other nodes w in a weighted network. Implement the algorithm in `python` and test your method in the example network from the theory lecture. **3P**

   (b) Develop an algorithm that uses the powers of adjacency matrices to calculate the diameter of a directed network. You can assume that the network is connected, i.e. your algorithm does not need to terminate if the network is disconnected. Implement your algorithm in `python` and test it in a directed network, e.g. using the software package `pathpy`. **3P**

2. **Modularity and Community Structure**

   Answer the following questions about the partition quality measure $Q(G, C)$ that was introduced in lecture L02.

   (a) Consider a fully-connected (i.e. all links exist) and undirected network $G = (V, E)$ with n nodes and no self-loops. Further assume that all nodes are assigned to a single community, i.e. consider a partition $C = \{V\}$. Prove that $Q(G, C) = 0$. **2P**

   (b) Consider an undirected network $G = (V, E)$ that exclusively contains self-loops. Assume that self-loops are represented by a one-entry on the main diagonal of the adjacency matrix. i.e. $\mathbf{A} = \text{diag}(1, \ldots, 1)$. Consider a community partition C, where all nodes are assigned to different communities, i.e. $C = \{\{v_1\}, \{v_2\}, \ldots, \{v_n\}\}$ for $V = \{v_1, \ldots, v_n\}$. Prove that $Q(G, C) \to \frac{1}{2}$ for $n \to \infty$. **2P**

3. **Node centralities**

   (a) Construct a network in which the node with the highest betweenness centrality has the smallest degree centrality. Use `pathpy` to demonstrate the correctness of your example. **1P**

   (b) Construct a network in which exactly one node has the maximum possible closeness centrality. **1P**

   (c) Give an example for a network with 10 nodes where exactly one node has the maximum betweenness centrality possible in a network with that size. Prove that the maximum possible betweenness centrality in a network with n nodes is $n^2 - 2n - n + 2$. **2P**

[1] Richard Bellman: **On a routing problem**, In Quarterly of Applied Mathematics, No. 16, pp. 87–90