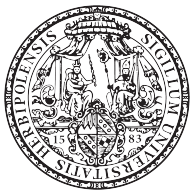

Statistical Network Analysis
WiSe 2022/2023

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Exercise Sheet 04

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Total points: 10

1. Molloy-Reed model

   (a) Given a random microstate generated based on the configuration model with degree distribution $P(k)$, consider a random node v and follow a random edge to a neighbor of w of v. What is the probability that node w has degree k? [1P]

   (b) Using the expression obtained above compute the expected degree of the neighbors a random node v. What do we see when we calculate the difference between the expected degree of a random node and the expected degree of a random neighbor of such a node? [2P]

   (c) Often rather than the degree of a node at the end of an edge we are interested in the number of edges attached to the node *other* than the one we arrived through. This number is called the *excess degree* of a node and will play an important role in the coming lectures. What is the probability that the node at which you arrive has *excess degree* k? [2P]

2. Friendship Paradox and Generating Functions

   (a) Consider a random network with a given log-normal degree distribution with parameters μ and σ. Use the Molloy-Reed model to generate microstates from this statistical ensemble and calculate the difference between the mean degree $\langle k \rangle$ and the mean neighbour degree $\langle k_n \rangle$. How does a change of parameters μ and σ influence $\langle k_n \rangle$ compared to $\langle k \rangle$? [2P]

   (b) Consider a number sequence $\{a_k\}_{k=0}^{\infty}$. We call a power series [3P]

   $$F_0(x) := \sum_{k=0}^{\infty} a_k x^k$$

   a generating function of the sequence $\{a_k\}_{k=0}^{\infty}$. Consider the sequence $\{b_k\}_{k=0}^{\infty}$ with

   $$\{b_k\}_{k=0}^{\infty} := \{a_0, 0, a_2, 0, a_4, 0, a_6, \ldots\}$$

   Specify a function $F_1(x)$ that generates $\{b_k\}_{k=0}^{\infty}$, where F_1 is an expression in terms of $F_0(x)$. Hint: Consider the sequence generated by $F_0(-x)$.