Statistical Network Analysis

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Lecture 06
Degree-based Ensembles

November 23, 2022

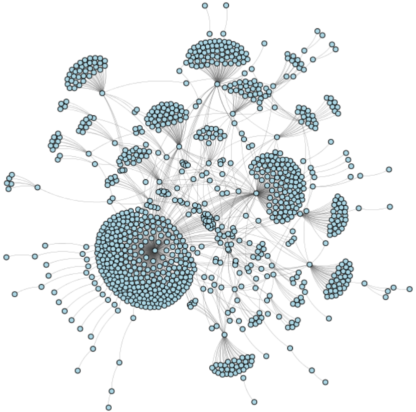
Notes:

- **Lecture L06:** Degree-based Ensembles 23.11.2022
- We introduce stochastic models that can be used to analyze expected properties of random networks with known degree sequence or distribution. We introduce the friendship paradox and reproduce it in random networks with different degree distributions.
  - Ensembles with fixed degrees
  - The Molloy-Reed configuration model
  - The friendship paradox

- **Exercise 04:** Ensembles with fixed degree distribution due 30.11.2022

Motivation

- **random graph models**
  - preserve number of nodes and (expected) number of links
  - basis to analyze **expected properties of random networks**
- random link formation leads to **Binomial degree distribution** (Normal/Poisson limit)
- empirical networks often have **broad degree distributions**
- which properties of the network are due to the **degrees of nodes**?



example: KDE network

collaboration network of developers of the KDE Open Source Software project

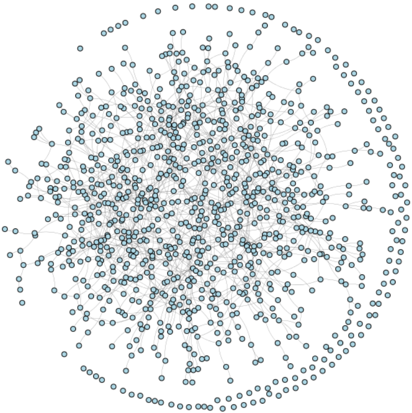
Notes:

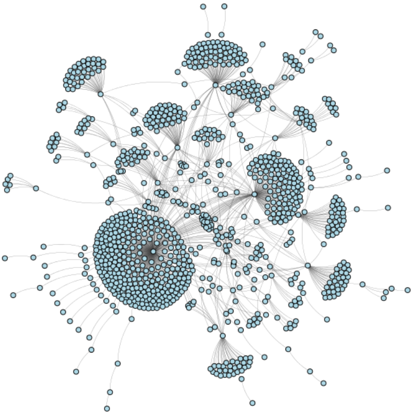
- Last week, we analysed expected properties of microstates generated by random graph models. We have considered two variants of the Erdős-Rényi model for random graphs which preserve ...

  1. the number of nodes n and links m ($G(n, m)$ model)
  2. the number of nodes n and expected number of links m ($G(n, p)$ model)

оценить
- We often use such models as **null models** to assess which properties of a system we can expect at random. Hence, we can use them to **distinguish patterns in networks from noise**. We specifically used the simple independent random link formation in the $G(n, p)$ model to analyse the expected degree distribution of the microstates, which follows a Binomial distribution. Depending on the parameter p, in the limit of infinite networks we can approximate this by a Normal or a Poisson distribution.
- If you think about how we would like to apply the $G(n, p)$ model in terms of a null model for real networks, it is not very satisfying that the number of nodes and (expected) number of edges are the only aggregate properties of a network that we can preserve. For instance, many networks have non-Binomial degree distributions, which is not reproduced by the null models we know until now.
- Considering the degree distribution of a real network is important, because degrees are a source of heterogeneity that is not tied to the topology of the links (i.e. which node is connected to which other node). We often want to incorporate this heterogeneity into our null model, rather than assuming that every node is exactly equal. In the following lectures, we analytically study expected properties of networks beyond simple random graph models, generalizing results for random graphs.

Example: KDE collaboration network





example: random network 1

random $G(n, m)$ microstate with **same number of nodes and links** as empirical network

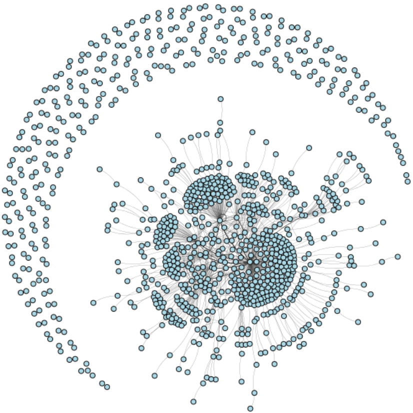
example: KDE network

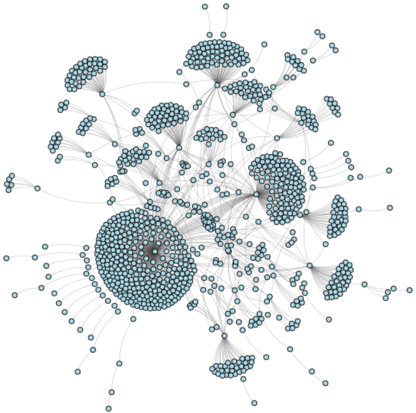
collaboration network of developers of the KDE Open Source Software project

Notes:

- We illustrate the need for null models that are not simple random graphs in the example above. The network on the right shows the collaboration network of the KDE Open Source Software community. How much of what we see (in terms of a topological "pattern") can we expect to be generated "at random"? To address this question, we could generate a random microstate with the $G(n, m)$ model, where we set n and m to the number of nodes and links in the empirical network. Apart from the number of nodes and links, the resulting microstates (see example realization on the left) have almost no similarity to our empirical network. This is not surprising because we know that the degree distributions of those microstates are Binomial, while the empirical network has a broad degree distribution, where some nodes have degrees that are much higher than the average degree.
- So, on the one hand we could conclude that the topology of the network is not "random", i.e. that the observed network cannot be explained by a simple null model that only preserves the number of nodes and links. But what we have actually found is even stronger, because even the **degree distribution (which is independent of the details of the topology)** can not be explained by a random graph model.
- Moreover, this discrepancy between the Binomial degree distribution of Erdős-Rényi random graphs and the broad degree distribution of the empirical network leaves open the question whether the topology (i.e. who is connected to whom) of the empirical network can simply be explained based on the degree distribution. In other words: is there an interesting pattern in the node degrees or is there an additional pattern in how the nodes with those degrees are connected to each other?

Example: KDE collaboration network





example: random network 2

random network with **same node degrees** as empirical network

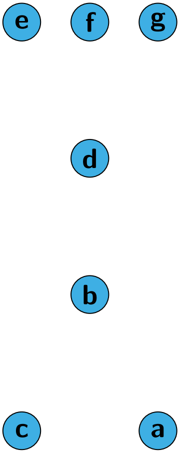
example: KDE network

collaboration network of developers of the KDE Open Source Software project

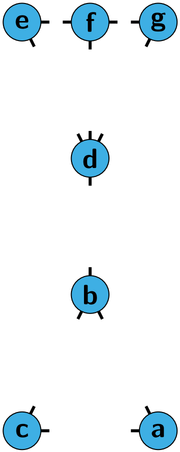
Notes:

- To separate patterns that are due to the degrees of nodes from patterns that are due to the topology of the links, we can **generate a random network which preserves the degrees of nodes** in the empirical network. Such a random realization is shown on the left side. We make the following observations:
  - Compared to the empirical network, the random microstate shows a larger number of isolated pairs, i.e. pairs of connected nodes that have a degree one.
  - The largest connected component of the network on the right looks more similar to the real system than that of the $G(n, m)$ random network. In particular, we have a few high degree nodes that are in the "center" of the network.
  - In the empirical network, nodes with degree one are rather connected to high-degree nodes (giving rise to a negative degree assortativity of $r \approx -0.34$). At random, these nodes are more likely to be connected to each other (giving rise to a close to zero degree assortativity and resulting in the higher number of disconnected pairs of nodes).
- This simple example shows that studying random microstates with a given degree sequence or distribution helps us to understand **which of a system's properties are due to the level of connectivity of nodes** (i.e. *how many* connections they have) rather than due to the topology of these connections (i.e. to *whom* these nodes are connected).

Statistical ensembles



statistical ensemble with
fixed number of n nodes and m links



statistical ensemble with
fixed degree sequence
$S = (2, 3, 2, 4, 2, 3, 2)$

Notes:

- In the previous lectures we introduced statistical ensembles used in statistical network analysis. We consider probability spaces containing all possible networks (microstates) that are consistent with a given macrostate. We can then use the ensemble to calculate the expected properties (or distribution of properties) of such microstates, which gives us a baseline for the analysis of a real network.
- In the motivation of this perspective, we mentioned a statistical ensemble that contains all networks with a given degree sequence (see on the right). But we have not introduced a model that would allow us to construct and analyze the corresponding microstates. This is what we will do in the first part of today's lecture. Moreover we will introduce a framework that we can use to analytically calculate expected network properties.

Ensembles with fixed degree sequence

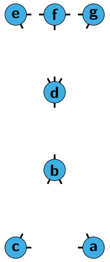
- **macrostate:** degree sequence
  $S = (d_i)_{i \in V}$
- **microstates:** all networks with degree sequence S
- **configuration model** generates equiprobable microstates G with

$$P(G) = \frac{1}{Z(S)}$$

where $Z(S)$ is the number of networks with degree sequence S

- we call this the **Molloy-Reed** model

→ M Molloy & B Reed, 1995



ensemble with **macrostate**
$S = (2, 3, 2, 4, 2, 3, 2)$

Michael Molloy
born ???

Bruce Reed
born 1962

image credit: left: University of Toronto Scarborough, fair use / right: David Eppstein, Wikimedia Commons, CC-BY-SA

Notes:

- We start with the statistical ensemble of networks with a given degree sequence, i.e. the **macrostate** fixes the degree of each of the n nodes, which can be described by a vector S of n integer numbers. The microstates are all possible networks that correspond to the given degree sequence.
- We further assume a **generative model that assigns equal probabilities to each of those microstates**, i.e. we have a discrete uniform probability mass function with a normalization constant $Z(S)$ that corresponds to the number of networks with a given degree sequence. The number of such networks was calculated in $\longrightarrow$ EA Bender and ER Canfield, 1978 .
- An algorithmic formulation of this so-called **configuration model** for networks with a fixed degree sequence was presented in $\longrightarrow$ M Molloy and B Reed, 1995 . For this reason, the configuration model is also often called the **Molloy-Reed model**. Just like for the $G(n, m)$ model, each microstate in this ensemble has the same probability, i.e. $P(G) = \frac{1}{Z(S)}$ where $Z(S)$ is the number of graphs with n nodes and degree sequence S.

Ensembles with fixed degree distribution

- relax constraint that microstates exhibit exact **degree sequence**
- **macrostate:** degree distribution $P(k)$ and number of nodes n
- **microstates:** networks with n nodes and **any** degree sequence
- all microstates with same degree sequence are equiprobable
- how can we generate microstates of such **degree-based ensembles**?

configuration algorithm A

```
def configuration_A(S):
    """configuration model with degree sequence S"""
    stubs = []
    for i in range(len(S)):
      for j in range(S[i]):
        stubs.append(i)

    n := empty network with len(S) nodes
    while len(stubs)>0:
      sample v, w from stubs
      n.add_edge(v,w)
      remove v,w from stubs

    return n
```

configuration algorithm B

```
def configuration_B(n, P):
    """configuration model with degree dist.  P"""
    S := [1]
    while not is_graphic(S):
      S := sample n degrees from P

    return configuration_A(S)
```

Notes:

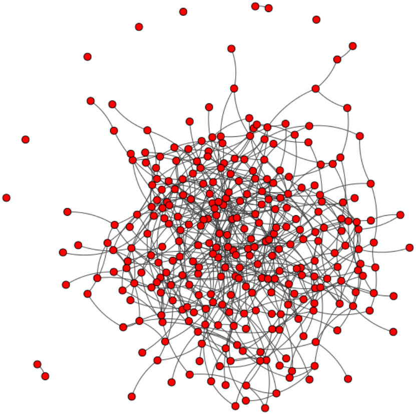
- Just like the $G(n, m)$ and the $G(n, p)$ models for random graphs, the configuration model comes in two variants: In the $G(n, p)$ model, we have relaxed the constraint that all microstates must have a fixed number of links, instead fixing the expected number of links in the network. This allows us to analyze the generated networks based on the simple assumption of independent link probabilities.
- For degree-based ensembles we use the same idea: rather than fixing the degree sequence (i.e. the number of links for each node in the microstates), we can **fix the degree distribution**, i.e. the probability that a node with degree k is generated. The degree sequence can vary across microstates. This model is easier to treat analytically.
- How can we implement both models (i.e. the model generating networks with fixed degree sequence and the one generated networks with fixed degree distribution)? For the model generating networks with fixed degree sequence we start from a network where we add link stubs to each node. We then add those stubs to a list from which we draw pairs uniformly at random without replacement. This resembles the construction procedure of the $G(n, m)$ model.
- We can implement the model for networks with fixed degree distribution as follows: we first draw a degree sequence of length n from the desired degree distribution. We then apply the previous algorithm to the sequence if the sequence is graphic. Like in the $G(n, p)$ model, the resulting ensemble includes all microstates with n nodes and any number of links (and any degree sequence). Depending on their degree sequence, some of these microstates are more likely than others. Microstates with the same degree sequence have the same microstate probability.

Example: Poisson degree distribution

example: Poisson degree distribution

- $P(k) = \frac{\lambda^k e^{-\lambda}}{k!}$
- λ is mean degree $\langle k \rangle$
- degree distribution of sparse random networks

- for large n we recover the ensemble generated by $G(n, p)$ model with $\lambda = np$
- Molloy-Reed model can be viewed as a **generalization** of Erdős-Rényi random graphs



microstates drawn from statistical ensemble with **Poisson degree distribution**

Notes:

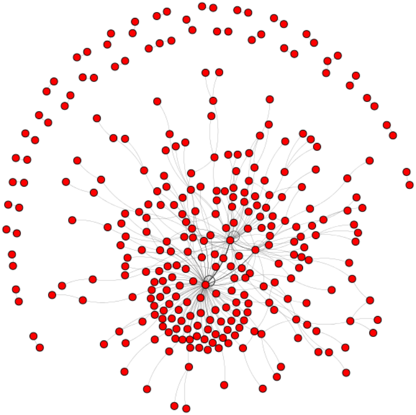
- Let us consider some examples for microstates generated in this way. We first fix the degree distribution to be a Poisson distribution with parameter λ. Using different values for λ enables us to generate networks with different mean degrees, while the degree distribution of the generated microstates is expected to be Poissonian.
- The network on the right is one microstate generated by the Molloy-Reed model and the video shows multiple of these microstates. These microstates will have different degree sequences, however the probability that a randomly chosen node in this network has degree k is fixed by the given degree distribution $P(k)$.
- We know that the $G(n, p)$ model generates networks with a Poisson degree distribution. So by fixing the degree distribution in the Molloy-Reed model to a Binomial/Poisson/Normal distribution, we obtain an alternative formulation of the $G(n, p)$ model. This means that we can actually view the Molloy-Reed model as a generalization of classical random graph models to networks with arbitrary degree distributions (other than e.g. Poisson). The whole set of $G(n, p)$ ensembles is contained in the parameter space of the Molloy-Reed degree-based ensemble if we fix $P(k)$ to correspond to the degree distribution of random graphs.

Example: Zipf degree distribution

example: Zipf degree distribution

- $P(k) = k^{-\gamma} \cdot \zeta(\gamma)^{-1}$
- **power law** with exponent $\gamma \in [2, \infty]$
- ζ is normalizing Riemann Zeta function

- paradigmatic example for a **broad degree distribution**
- for certain exponents γ we obtain **scale-free** networks



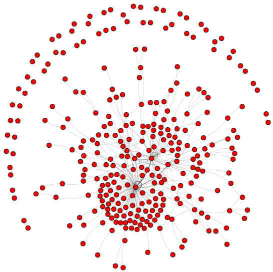
microstates drawn from statistical ensemble with **Zipf degree distribution**

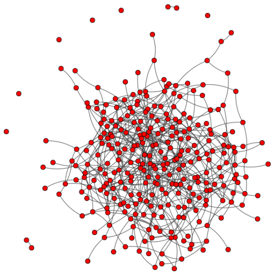
Notes:

- How does this generalization help us to analyze empirical networks? Many of the empirical networks that we have seen do not exhibit a Poisson degree distribution. Empirical degree distributions are often broad, which can not be reproduced by a random graph model. We can now fix this issue: We can use the Molloy-Reed model to generate microstates whose degree distributions are fixed to the degree distribution of an empirical network, which provides us with a more reasonable baseline model that we can use to distinguish patterns from noise.
- The Zipf distribution is a paradigmatic and (in the context of complex networks) particularly well-studied example for such a broad distribution. The example on the right shows a microstate drawn from an ensemble of random graphs with a fixed **Zipf degree distribution**. The video again shows several of these microstates.
- We can now use this distribution as the macrostate of our ensemble, and study which of the observed properties of the empirical network we can expect due to the heterogeneous node degrees.

Degree-based ensembles

- we can **generate random microstates** with arbitrary degree distributions
- we can reproduce microstates in the $G(n, p)$ ensemble
- we can preserve degree sequence or distribution of **empirical networks**
- how can we reason about **expected properties** of random microstates?





Notes:

- Thanks to the Molloy-Reed model, we can generate random microstates whose degree distribution matches that of an empirical network, just as we have used the $G(n, p)$ model to generate random networks where the numbers of nodes and (expected) links match those of an empirical network. In the last two lectures, we have seen examples of how we can make (analytical) statements about the expected properties of random networks based on the underlying $G(n, p)$ model (cf. our analysis of degree distribution, clustering coefficient, and diameter).
- We want to be able to make make similar analytical statements about the expected properties of microstates drawn from a statistical ensemble with a given degree distribution. One could think that this is a more complicated than what we have done for the simple Erdős-Rényi model, where we used the independent link probabilities p in the $G(n, p)$ model.
- Next week, we will introduce an analytical framework that allows us to generalize this analysis to random networks with arbitrary degree distributions. For now, we can adopt a sampling approach, i.e. we can generate many microstates and then compare the distribution of properties of those microstates with the properties of an empirical network. We will show this approach in the following practice session.

Practice session

- we study necessary conditions for graphic degree sequences
- we implement two configuration models for networks with fixed degree sequence and degree distribution
- we compare properties of Molloy-Reed random networks to those of empirical networks

practice session

see notebook 06-01 – 06-02 in `gitlab` repository at

→ https://gitlab.informatik.uni-wuerzburg.de/ml4nets_notebooks/2022_wise_sna_notebooks

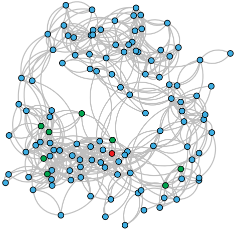
Notes:

- In the first practice session of this week, we use the implementations of random graph models introduced last week to study their expected properties. We further compare those properties to the properties observed in real networks.

From random nodes to random neighbors

- to analyze path lengths, component sizes, connectivity, etc. we must **move beyond randomly chosen nodes**

- what if we consider **random neighbors** of randomly chosen nodes?



thought experiment

1. check **friend count** of each of your Facebook/Instagram/Twitter friends
2. compute **average friend count** of your friends
3. compare this value to **your own friend count**

example

contact network of high school students
$\langle k \rangle = 8.4 < 9.7 = \langle k_n \rangle$

- **friendship paradox**: your friends have – on average – more friends than you

$\longrightarrow$ Scott L Feld, 1991

Scott L. Feld
born 1949

image credit: Purdue University, fair use

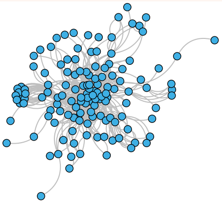
Notes:

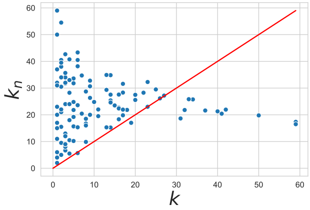
- To motivate the analytical framework that we will introduce next week, let us come back to an issue that we encountered in the approximate analysis of the diameter of random graphs in the previous lecture. FOr our analysis, we considered a node v chosen uniformly at random and we argued that the expected degree of such a node is given by the mean of the degree distribution. We then picked a randomly chosen neighbour w and argued about the number of links of node w. But what degree do we expect for this node?
- You could assume that a neighbour w uniformly chosen at random from the neighbors of v is no different from the node v chosen uniformly at random among all nodes. If that was the case, we could use the mean of the degree distribution to calculate the expected degree of node w. However, this is a fallacy that is often paraphrased as the **friendship paradox** in (social networks).
- We motivate this with a thought experiment that you can actually do at home. For this experiment, you should calculate (e.g. via a script) the average number of friends that each of your Facebook/Twitter/Instagram friend has. If you have many friends and want a quick estimate, you can randomly pick a set of friends and estimate the population average based on this small sample.
- Do your friends have – on average – have more, less or the same number of friends than you? What is your intuitive expectation? We try this in the social network shown above. If we compare the mean degree of that network to the mean degree of the neighbors of random nodes, we find that the neighbors tend to have a larger degree. This seemingly unintuitive finding was first observed (and explained) in → Scott L. Feld, 1991

Example: friendship paradox

example A

network of character co-occurrences in *The Lord of the Rings*

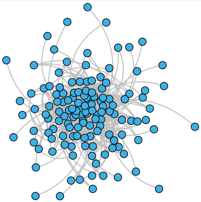


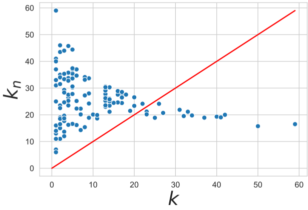


$$\langle k \rangle = 10.4 < 22.35 = \langle k_n \rangle$$

example B

random Molloy-Reed microstate with degree sequence of *The Lord of the Rings* network



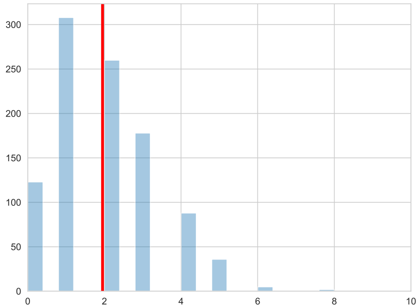


$$\langle k \rangle = 10.4 < 23.34 = \langle k_n \rangle$$

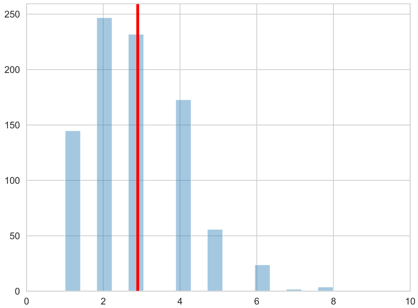
Notes:

- Since we observed the friendship paradox in an actual social network, one could think that this property is due to the special mechanisms by which links are formed in social networks, e.g. a preference of nodes to specifically link to nodes with higher degrees. To study this further, consider a "social network" that is actually not a real social network, but a fictional one. The network on the left shows the contact network of characters in the novel *The Lord of the Rings*.
- For each node v in this network, we can calculate the degree k and the mean degree k_n of all neighbors of v, which we plot below the network. Each point above the red diagonal line corresponds to a node where the mean neighbour degree is larger than its on degree. In this example, almost all characters would find that their neighbors have a higher degree than themselves. We can also calculate $\langle k_n \rangle$ as the mean of all mean neighbour degrees k_n. We find that in this network, $\langle k_n \rangle > \langle k \rangle$ by a large margin.
- You could think that this is simply due to the fact that the network of characters in *The Lord of the Rings* is just carefully and realistically constructed. To rule out this possibility, we generate a random network that has the same degree sequence, i.e. we randomize the topology of links. We find that the friendship paradox is even stronger in this random microstate, i.e. the **friendship paradox is an example for a property that can be explain based on the degree sequence/distribution of the network!** Next week, we we will show that we can use the the Molloy-Reed model to derive an elegant analytical explanation of this (at first glance) surprising result.

Degree vs. neighbor degree distribution 1/2



degree distribution of nodes v chosen uniformly at random



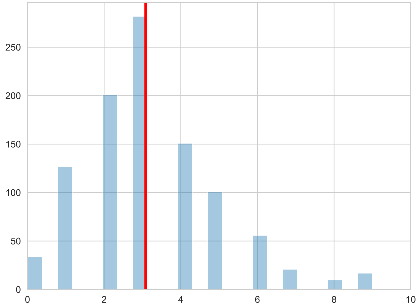
degree distribution of random neighbours w of randomly chosen nodes v

random network generated with $G(n, p)$ model with parameters $n = 1000$ and $p = 0.002$

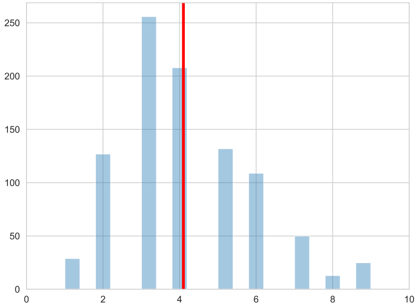
Notes:

- Before we analytically explain the friendship paradox next week, let us explore the difference between the degree distribution of (i) nodes chosen uniformly at random and (ii) nodes that were chosen as a random neighbour of such randomly chosen nodes. We test this for random Erdős-Rényi networks with different mean degrees.
- The left plot shows the **Binomial degree distribution** of such a random network with parameters $n = 1000$ and $p = 2$, which has a mean degree of $\langle k \rangle = np = 2$.
- On the right, we plot the **neighbour degree distribution** of the same network, which has a mean neighbour degree $\langle k_n \rangle = 3$, i.e. the mean neighbour degree is larger than the mean degree. This simple experiment suggests that the friendship paradox also holds in random Erdős-Rényi networks.

Degree vs. neighbor degree distribution 2/2



degree distribution of nodes v chosen uniformly at random



degree distribution of random neighbours w of randomly chosen nodes v

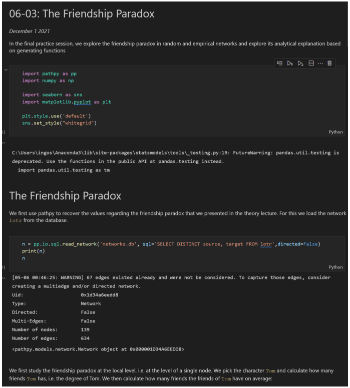
random network generated with $G(n, p)$ model with parameters $n = 1000$ and $p = 0.003$

Notes:

- We repeat this experiment for a network generated using a $G(n, p)$ model with parameters $n = 1000$ and $p = 0.003$, which has a mean degree $\langle k \rangle = 3$. We again find that the mean neighbour degree is larger than the mean degree. Moreover, the difference between the mean neighbour degree and the mean degree is again one.
- Can you explain how this relates to the approximate analysis of the expected diameter of random networks that we have done in the last week? Here we said that we simply ignore the fact that the mean degree of the neighbour of a randomly chosen node is not the same as the mean degree of the network. Doesn't this invalidate the results of our analysis?

Practice session

- we study the friendship paradox in empirical and random networks
- we explore the dependence of the friendship paradox on the degree distribution and the mean degree



practice session

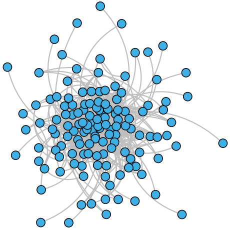
see notebook 06-03 in `gitlab` repository at

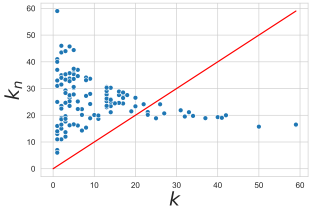
→ https://gitlab.informatik.uni-wuerzburg.de/ml4nets_notebooks/2022_wise_sna_notebooks

Notes:

In summary

- statistical network ensembles with **fixed degree sequence or distribution**
- help us to separate effects of **node degrees** from effects of **link topology**
- we considered **degrees of neighbors** of randomly chosen nodes
- we rediscovered the **friendship paradox in (social) networks**
- can be interpreted as a **sampling bias** that occurs when we sample neighbors of nodes





$$\langle k \rangle = 10.4 < 23.34 = \langle k_n \rangle$$

Notes:

- In summary, we introduced a new class of degree-based statistical ensembles that we can use to study expected properties of networks with a given degree sequence or distribution.
- We have finally motivated the so-called friendship paradox, which states that your friends have - on average - more friends than you. Next week, we will see that we can analytically explain and understand the friendship paradox as a property that follows from the degree distribution of a network.

Exercise sheet 04

- **fourth exercise sheet** is available on WueCampus
  - study the Molloy-Reed configuration model
  - explore the friendship paradox in random networks
- solutions are due **December 1st** (via WueCampus)
- present your solution to earn bonus points

Statistical Network Analysis
WiSe 2021/2022

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Exercise Sheet 04

Published: November 30, 2021
Due: December 8, 2021
Total points: 10

1. Molloy-Reed model
   (a) Given a random microstate generated based on the configuration model with degree distribution $P(k)$, consider a random node v and follow a random edge to a neighbor of w of v. What is the probability that node w has degree k? [1P]
   (b) Using the expression obtained above compute the expected degree of the neighbors a random node v. What do we see when we calculate the difference between the expected degree of a random node and the expected degree of a random neighbor of such a node? [2P]
   (c) Often rather than the degree of a node at the end of an edge we are interested in the number of edges attached to the node *other* than the one we arrived through. This number is called the *excess degree* of a node and will play an important role in the coming lectures. What is the probability that the node at which you arrive has *excess degree* k? [2P]
2. Friendship Paradox and Generating Functions
   (a) Consider a random network with a given log-normal degree distribution with parameters μ and σ. Use the Molloy-Reed model to generate microstates from this statistical ensemble and calculate the difference between the mean degree $\langle k \rangle$ and the mean neighbour degree $\langle k_n \rangle$. How does a change of parameters μ and σ influence $\langle k_n \rangle$ compared to $\langle k \rangle$? [2P]
   (b) Consider a number sequence $\{a_k\}_{k=0}^{\infty}$. We call a power series [3P]
   $$F_0(x) := \sum_{k=0}^{\infty} a_k x^k$$
   a generating function of the sequence $\{a_k\}_{k=0}^{\infty}$. Consider the sequence $\{b_k\}_{k=0}^{\infty}$ with
   $$\{b_k\}_{k=0}^{\infty} := \{a_0, 0, a_2, 0, a_4, 0, a_6, \dots\}$$
   Specify a function $F_1(x)$ that generates $\{b_k\}_{k=0}^{\infty}$, where F_1 is an expression in terms of $F_0(x)$.
   Hint: Consider the sequence generated by $F_0(-x)$.

Notes:

Self-study Questions

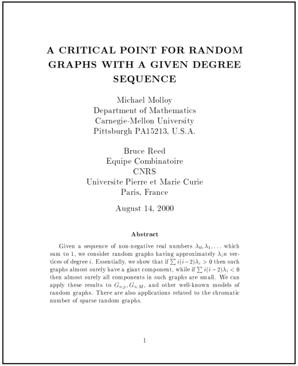
1. Give a necessary condition for a graphic degree sequence.
2. Write down a pseudocode formulation of the Molloy-Reed algorithm generating networks with a fixed degree distribution.
3. Consider the Molloy-Reed model for networks with a fixed degree distribution. Consider two random microstates with the same number of nodes and the same number of links but different degree sequences. How can we compare the probabilities of those two microstates in the Molloy-Reed ensemble?
4. In what sense are random networks with fixed degree sequence better "null models" than Erdős-Rényi random graphs?
5. Explain and test the friendship paradox in a social network of your choice.
6. Can you find a network in which the friendship paradox does not apply?
7. Consider the difference between the degree and neighbour degree distribution in random Erdős-Rényi networks and explain how it affects our analysis of the expected diameter in L05.
8. Can you think of a potential practical application of the friendship paradox, esp. considering the current pandemic.

Notes:

References

reading list

- EA Bender, ER Canfield: **The Asymptotic Number of Labeled Graphs with Given Degree Sequences**, Journal of Combinatorial Theory, 1978
- M Molloy, B Reed: **A critical point for random graphs with a given degree sequence**, Random Structures & Algorithms, 1995
- MEJ Newman, SH Strogatz, DJ Watts: **Random graphs with arbitrary degree distributions and their applications**, Physical Review E, 2001
- SF Feld: **Why Your Friends Have More Friends Than You Do**, American Journal of Sociology 96, 1991
- NA Christakis, JH Fowler: **Social Network Sensors for Early Detection of Contagious Outbreaks**, PLOS ONE. 5, 2010



A CRITICAL POINT FOR RANDOM GRAPHS WITH A GIVEN DEGREE SEQUENCE

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Abstract

Given a sequence of non-negative real numbers $\lambda_0, \lambda_1, \ldots$ which sum to 1, we consider random graphs having approximately $\lambda_i n$ vertices of degree i. Essentially, we show that if $\sum i(i-2)\lambda_i > 0$ then such graphs almost surely have a giant component, while if $\sum i(i-2)\lambda_i < 0$ then almost surely all components in such graphs are small. We can apply these results to $G_{n,p}, G_{n,M}$, and other well-known models of random graphs. There are also applications related to the chromatic number of sparse random graphs.

1

Notes: