Statistical Network Analysis
WiSe 2022/2023

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Exercise Sheet 05

Published: November 30, 2022
Due: December 07, 2022
Total points: 10

1. **Generating functions**

   (a) Implement a `python` function that plots the generating function of a given probability mass function in its domain. Write down the generating functions of three random variables corresponding to dice rolls of fair dice with six, ten, and 12 sides respectively. Plot the functions and explain the differences in their shapes. **2P**

   (b) Derive a closed form for a probability generating function G_0 that generates the probability mass function $P(k) = \frac{1}{2^{k+1}}$, for $k \in \mathbb{N}_0$. **2P**

   (c) Consider the following three functions in the domain $x \in [0, 1]$ **2P**

   - $G_1(x) = \mathrm{e}^x$
   - $G_2(x) = x^3 - x^2 + x$
   - $G_3(x) = \mathrm{e}^{x-1}$

   Which of those functions generate a probability mass function $P(X = k)$ for a discrete random variable X assuming values in $\mathbb{N}_0$. Proof your answers and give the probability mass function if possible.

2. **Generating functions and Molloy-Reed Model**

   (a) Consider a network with Poisson degree distribution $P(k)$ with mean degree λ. Write down the corresponding probability generating function of $P(k)$. Simplify the function as much as possible. **1P**

   (b) Consider a random microstate generated by the Molloy-Reed Model with an arbitrary fixed degree distribution $P(k)$. Further consider a probability mass function $Q(k)$ for the excess degree distribution calculated in Task 1(c) of Exercise Sheet 04. Write down the probability generating function corresponding to $Q(k)$. Use the properties of generating functions introduced in Lecture 5 to obtain an expression for the *expected excess degree* in a random microstate in the Molloy-Reed ensemble with given degree distribution $P(k)$. **3P**