دانشگاه صنعتی امیرکبیر
(پلی تکنیک تهران)

Electrical and Electronic Circuits

chapter 12. Transistor

Afarghadan@aut.ac.ir

عظیم فرقدان

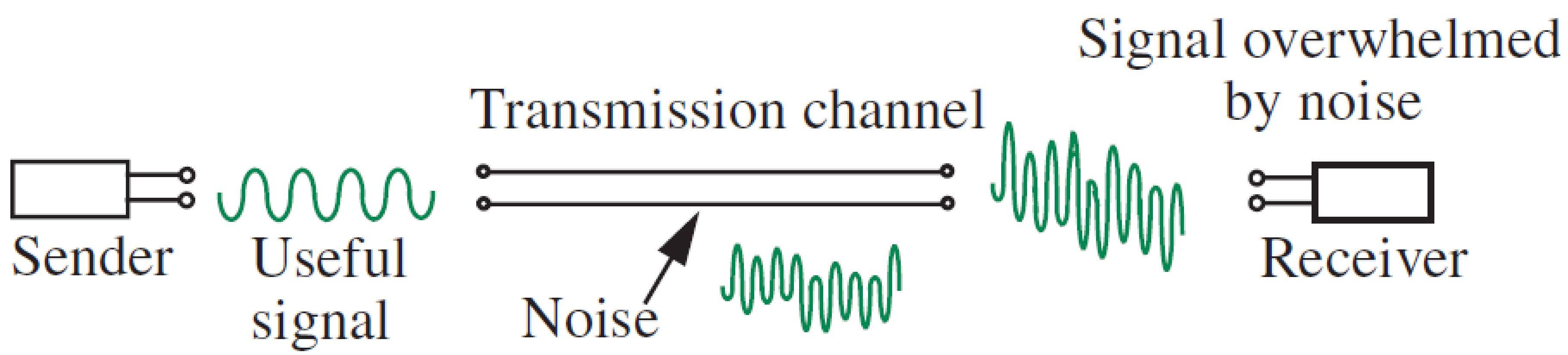
مهر ۱۴۰۳

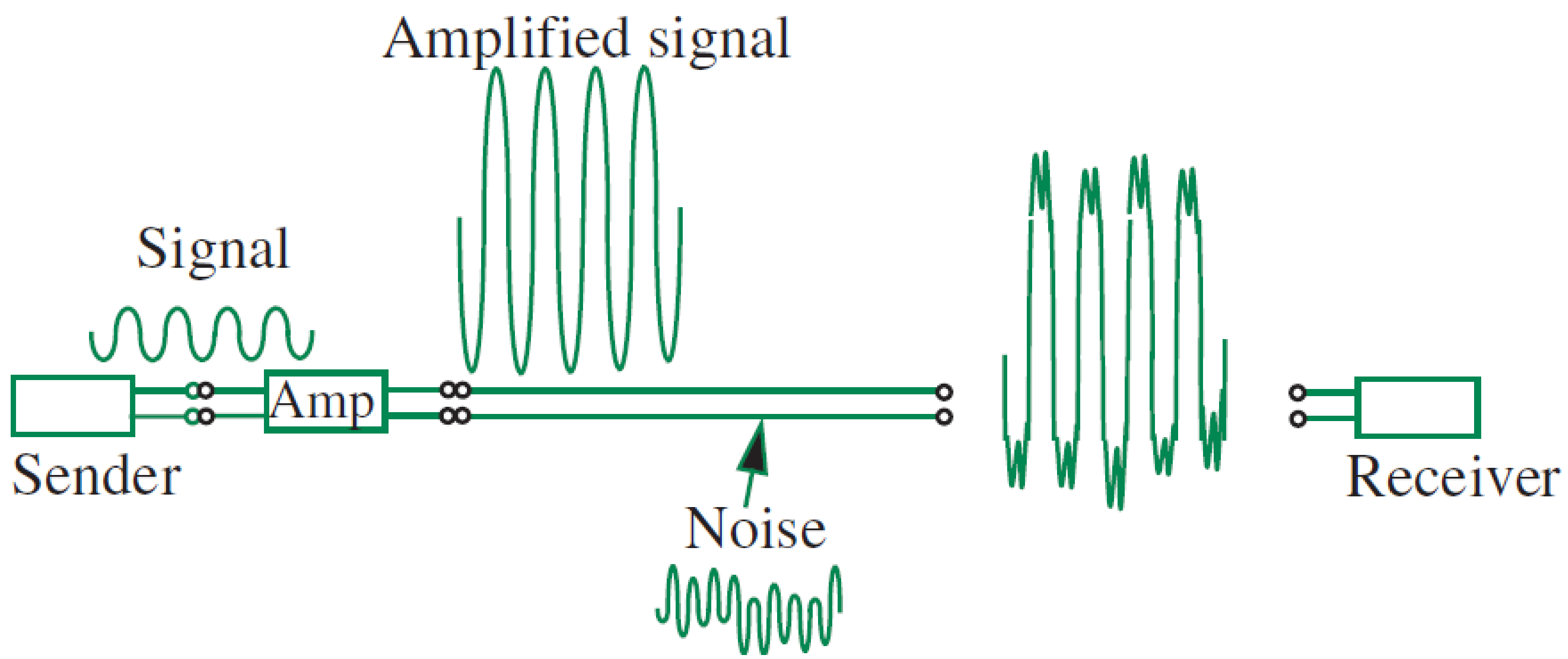
Objectives of the Lecture

- **Transistor Modelling as an Amplifier**
  - Large-Signal Analysis
  - Small-Signal Analysis
- DC and AC Analysis of Transistors
- Various Configurations

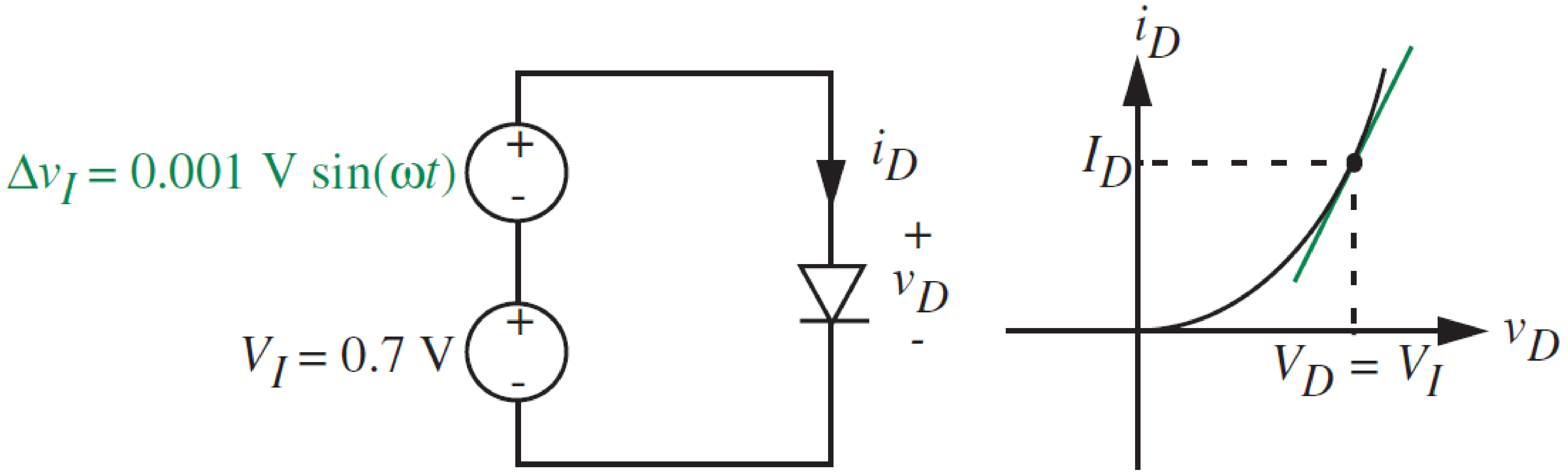
Applications of Transistors

In analog circuits, transistors are utilized as amplifiers.





Nonlinear Behaviour of the Diode



- ✓ Using the Nonlinear Model for Calculating Diode Current, We Have:
- ✓ $\boldsymbol{i_D = I_s[e^{(V_I+\Delta v_I)/V_{TH}} - 1]}$

Using the taylor expansion around V_I, we have:

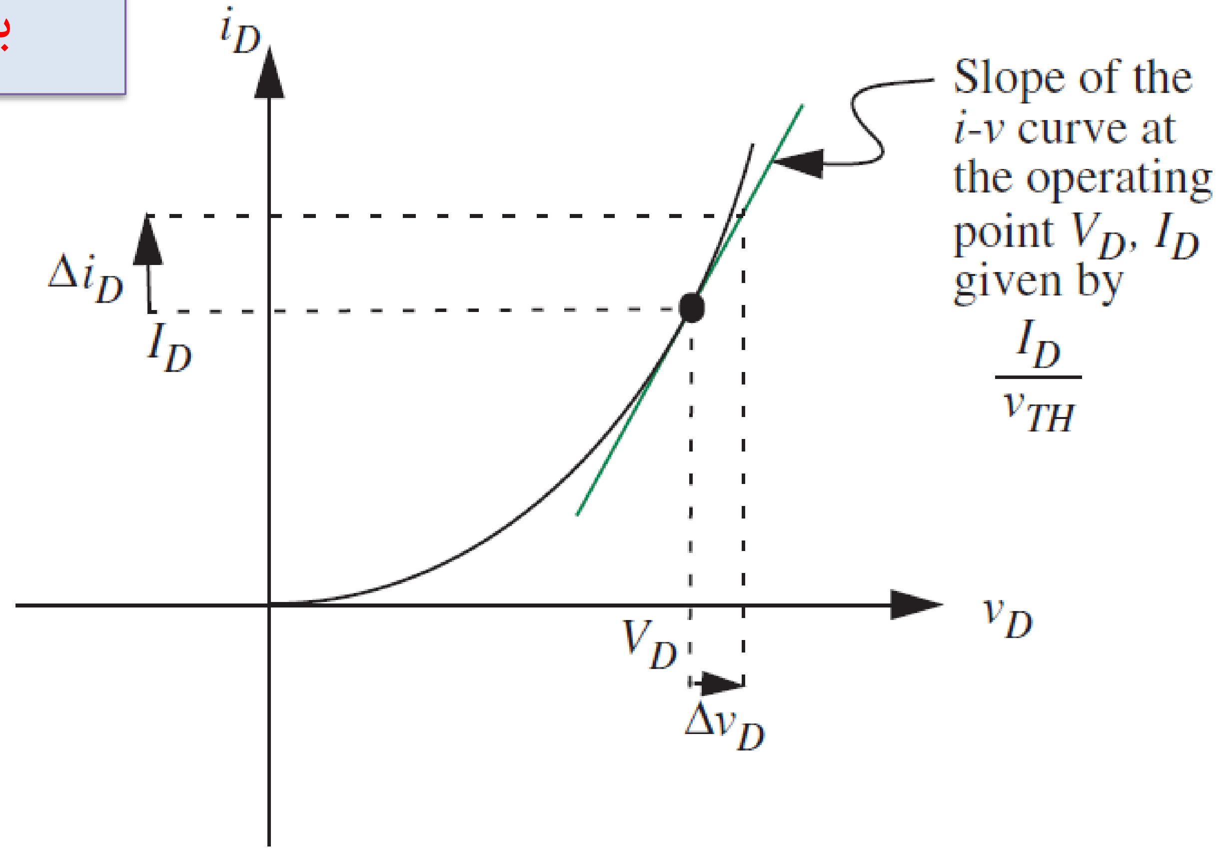
- $f(x) = f(a) + f'(a)(x-a) + \frac{1}{2!}f''(a)(x-a)^2 + \cdots$
  - $i_D = f(x) = I_s[e^{x/V_{TH}} - 1]$
  - $x = V_I + \Delta v_I$
  - $a = V_I$
- $i_D = I_s[e^{(V_I+\Delta v_I)/V_{TH}} - 1]$
- $= I_s(e^{V_I/V_{TH}} - 1) + I_s e^{V_I/V_{TH}}\left[\frac{\Delta v_I}{V_{TH}} + \frac{1}{2}\left(\frac{\Delta v_I}{V_{TH}}\right)^2 + \cdots\right]$
- $\approx I_s(e^{V_I/V_{TH}} - 1) + \frac{I_D}{V_{TH}}\Delta v_I$

Small-Signal and Large-Signal Models

$$i_D \approx I_s(e^{V_I/V_{TH}} - 1) + \frac{I_D}{V_{TH}}\Delta v_I$$

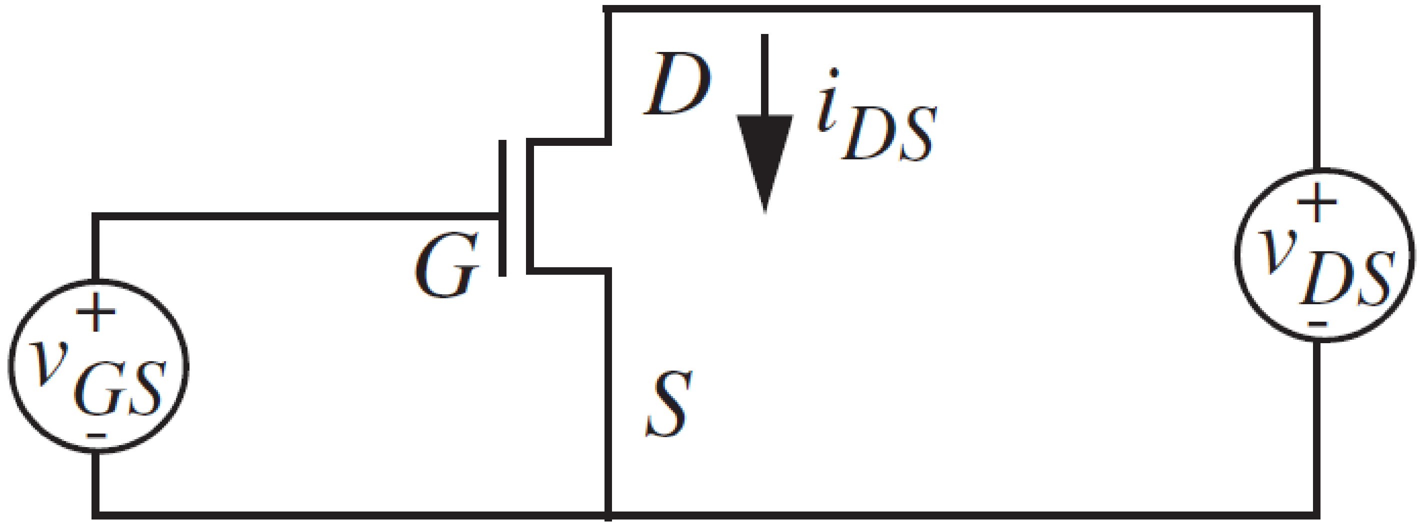
- ✓ بخش DC جریان دیود با بخش DC منبع رابطه غیرخطی دارد.
- ✓ یعنی دیود در برابر یک سیگنال بزرگ رفتاری غیرخطی دارد.
- ✓ به این مدل غیرخطی که قبلا نیز دیده بودیم، مدل سیگنال بزرگ گویند.

- ✓ بخش AC جریان دیود با بخش AC منبع رابطه تقریباً خطی دارد.
- ✓ یعنی دیود در برابر یک سیگنال کوچک رفتاری خطی دارد و مانند یک مقاومت با مقدار $\frac{V_{TH}}{I_D}$ عمل می‌کند.
- ✓ به این مدل خطی، مدل سیگنال کوچک گویند.



Large-Signal Model of MOSFET

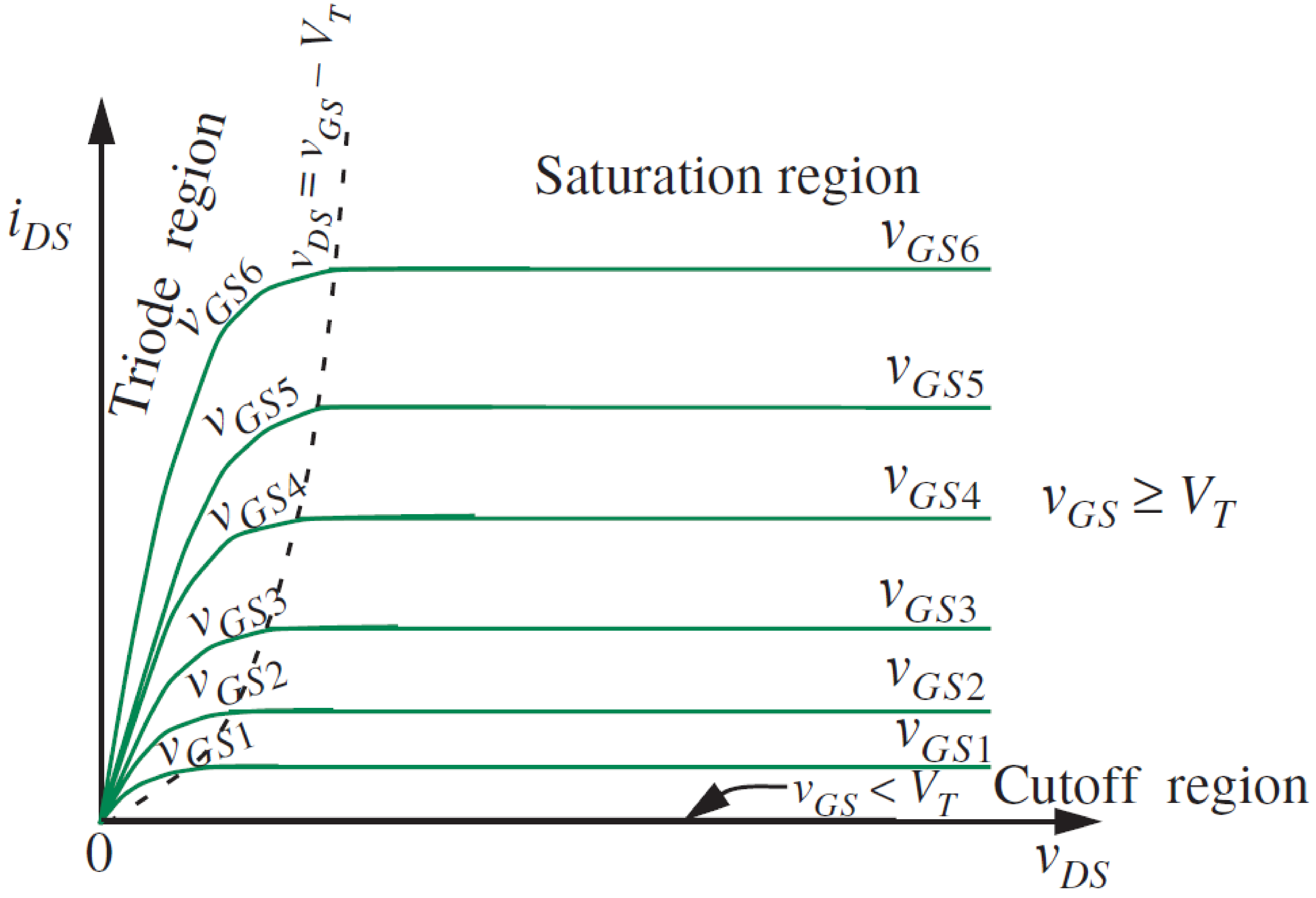
✓ First-Order Shockley model:



$$I_{ds} = \begin{cases} 0 & V_{gs} < V_t & Cut\ off \\ K\left(V_{gs} - V_t - \dfrac{V_{ds}}{2}\right)V_{ds} & V_{ds} < V_{gs} - V_t & Triode \\ \dfrac{K}{2}\left(V_{gs} - V_t\right)^2 & V_{ds} > V_{gs} - V_t & Saturation \end{cases}$$

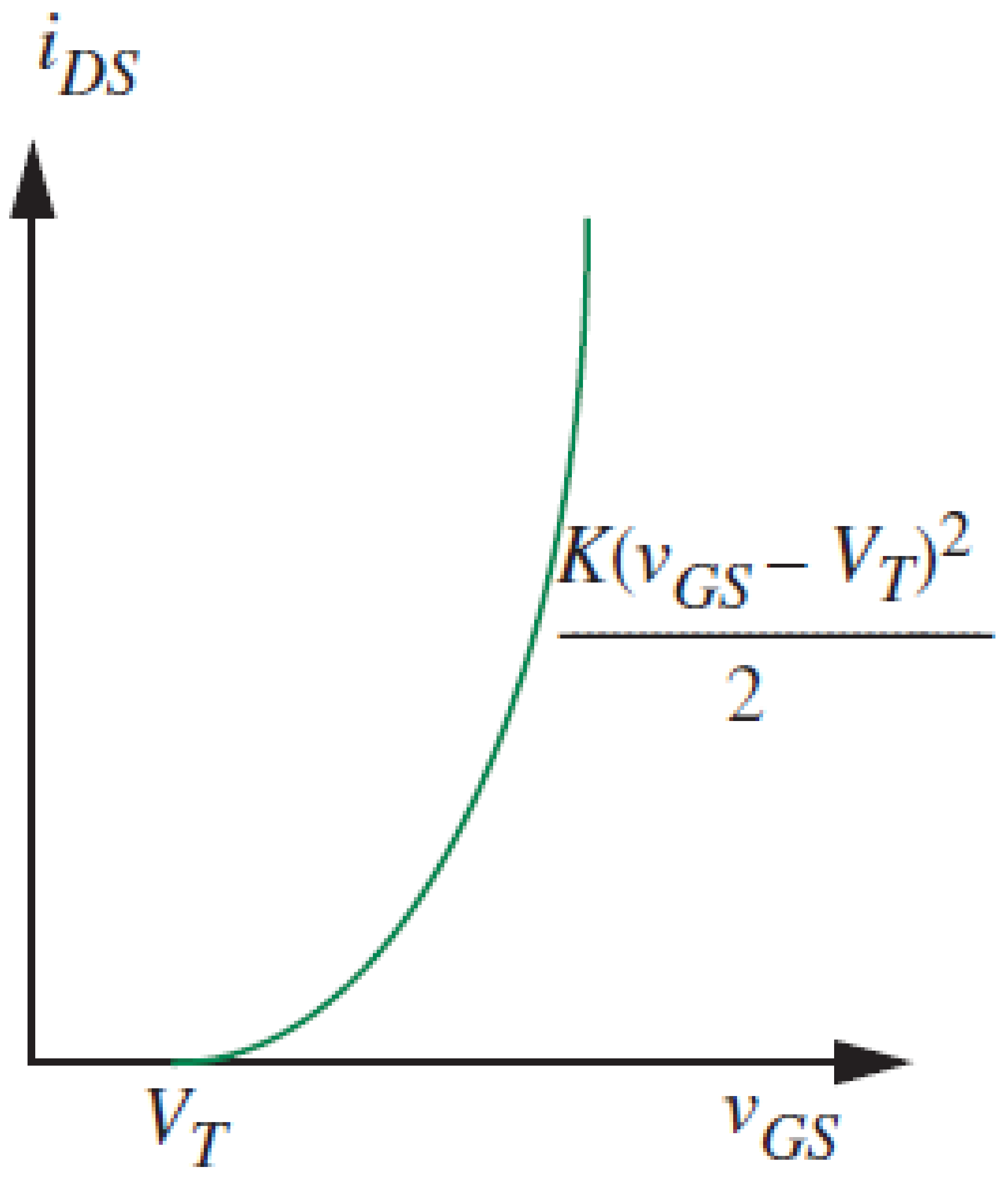
MOSFET Voltage-Current Characteristics

- The relationship between drain current (i_{DS}) and drain-source voltage (v_{DS})



MOSFET Voltage-Current Characteristics

- The relationship between drain current (i_{DS}) and Gate-source voltage (v_{GS})

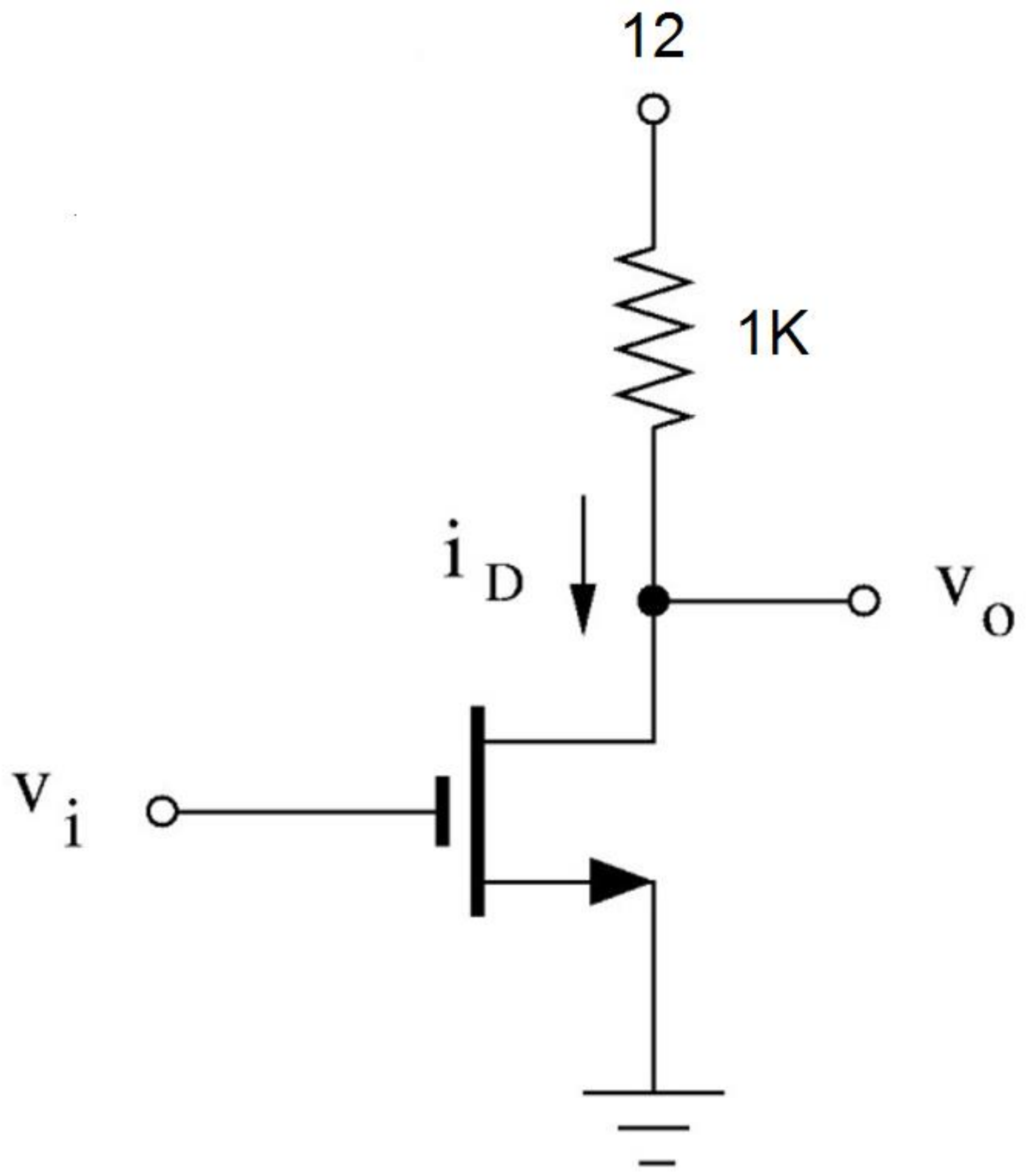


valid when $v_{DS} \geq v_{GS} - V_T$

DC Analysis of MOSFET Transistor

- ✓ روابط KVL دو طرف ترانزیستور را بنویسید.
- ✓ با استفاده از این روابط، ولتاژ V_{gs} را به‌دست آورید. (جریان گیت همیشه صفر است).
- ✓ اگر $V_t < V_{gs}$ باشد، ترانزیستور قطع است: $I_{ds} = 0$
- ✓ اگر $V_t > V_{gs}$ باشد، یا اشباع است یا خطی.
- ✓ فرض می‌کنیم اشباع است: $I_{ds} = \frac{K}{2}\left(V_{gs} - V_t\right)^2$
- ✓ ولتاژ V_{ds} را به‌دست آورده و شرط اشباع بودن را چک می‌کنیم:
  - ✓ $V_{ds} > V_{gs} - V_t$
- ✓ اگر تناقض داشت خطی است: $I_{ds} = K\left(V_{gs} - V_t - \frac{V_{ds}}{2}\right)V_{ds}$

Example



✓ولتاژ V_o را بیابید. $(\mathrm{K} = 0.5\frac{mA}{V^2}, V_t = 2, v_i = 0)$

✓روابط KVL دو طرف:

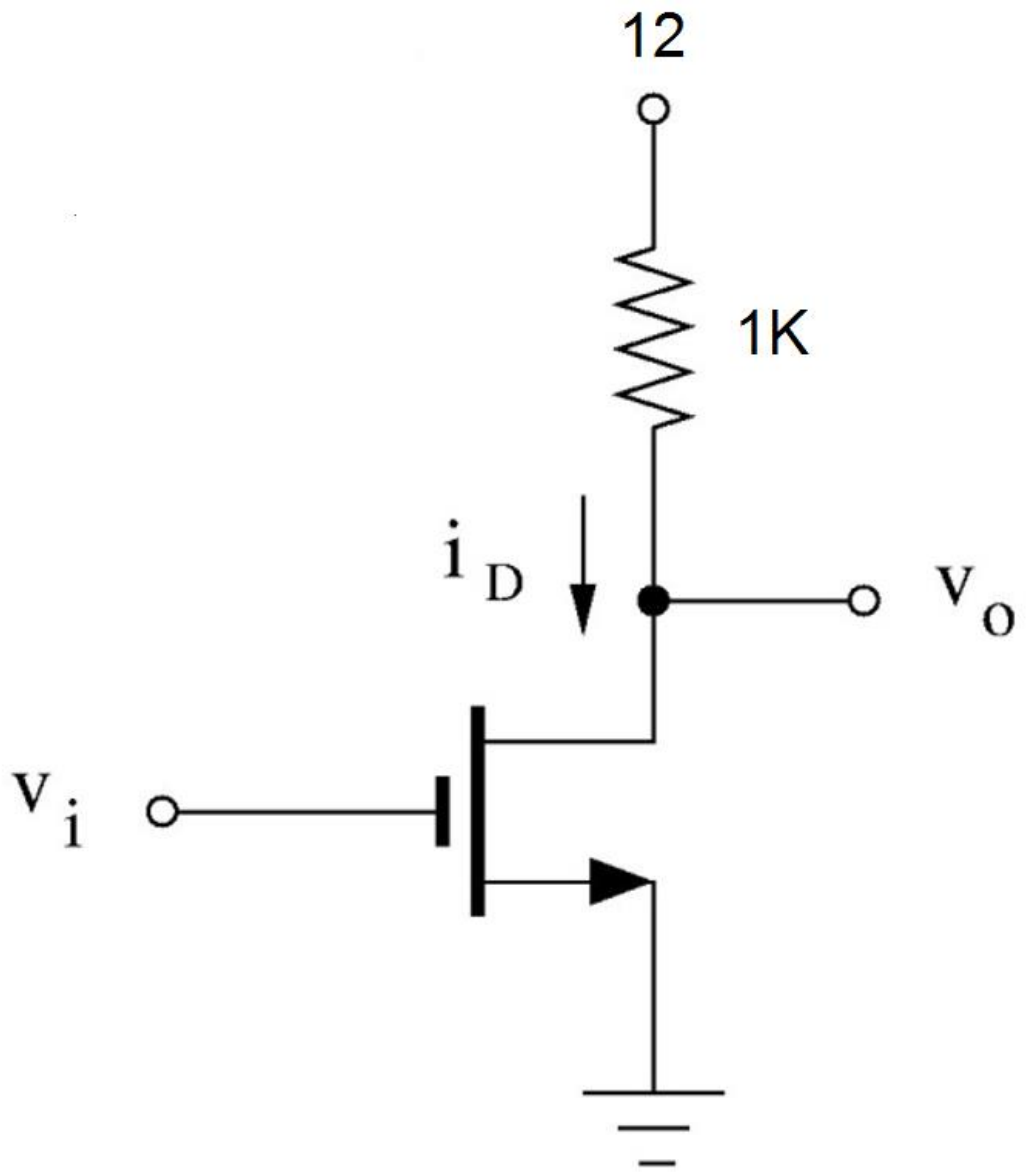
$$v_{gs} = v_i = 0$$ ✓

$$-12 + 1000 i_D + v_{ds} = 0$$ ✓

✓$v_{gs} < v_t$ پس قطع است: $i_D = 0$

✓$v_{ds} = 12$

Example



✓ V_o را بیابید. $\quad (K = 0.5\frac{mA}{V^2}, V_t = 2, v_i = 6)$

✓ روابط KVL دو طرف:

$$v_{gs} = v_i = 6 \;✓$$

$$-12 + 1000 i_D + v_{ds} = 0 \;✓$$

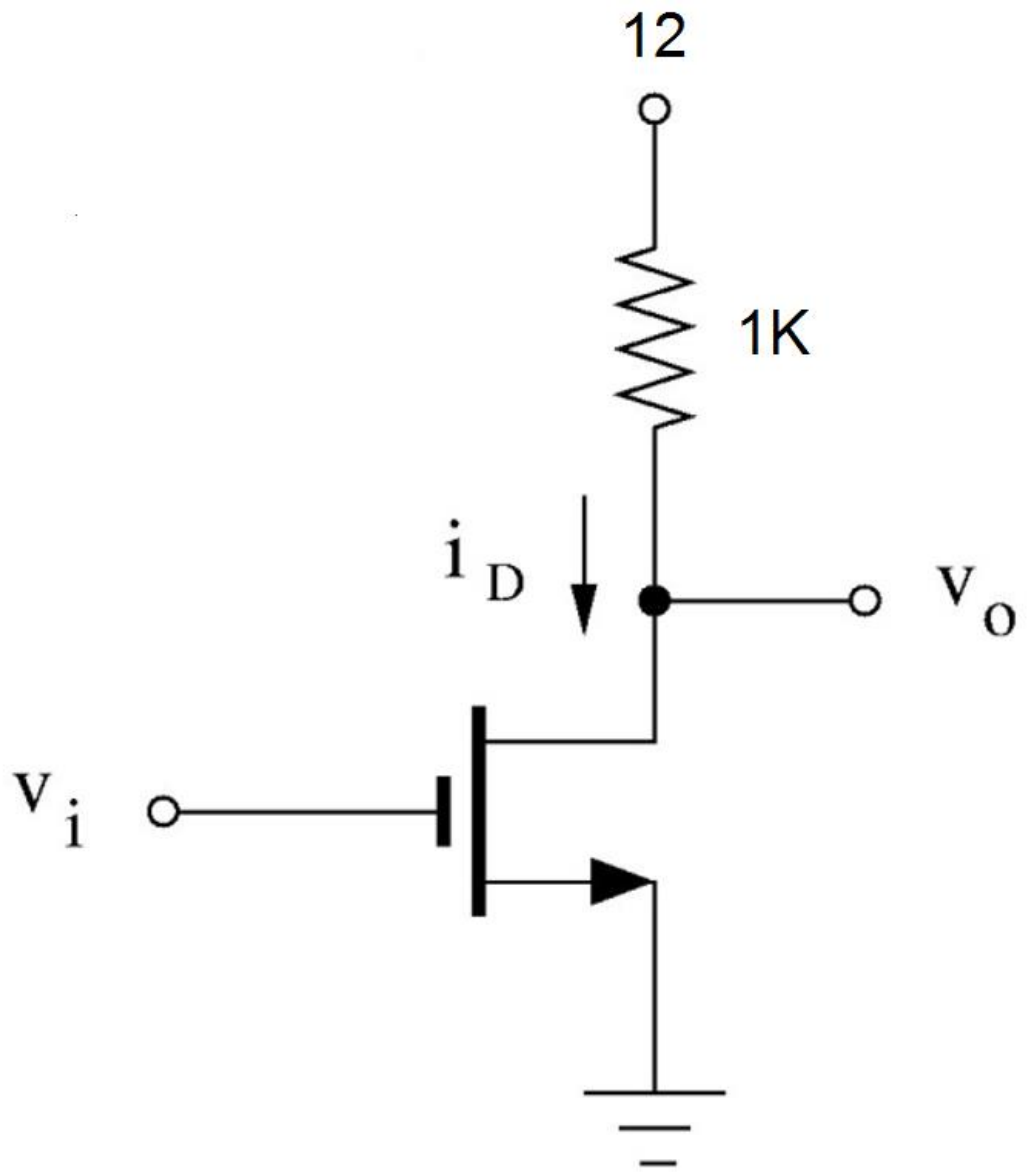
✓ $v_{gs} > V_t$ پس یا خطی است یا اشباع

✓ فرض می‌کنیم اشباع است:

$$i_D = \frac{K}{2}\left(v_{gs} - V_t\right)^2 = 4mA \rightarrow v_{ds} = 12 - 4 = 8 \;✓$$

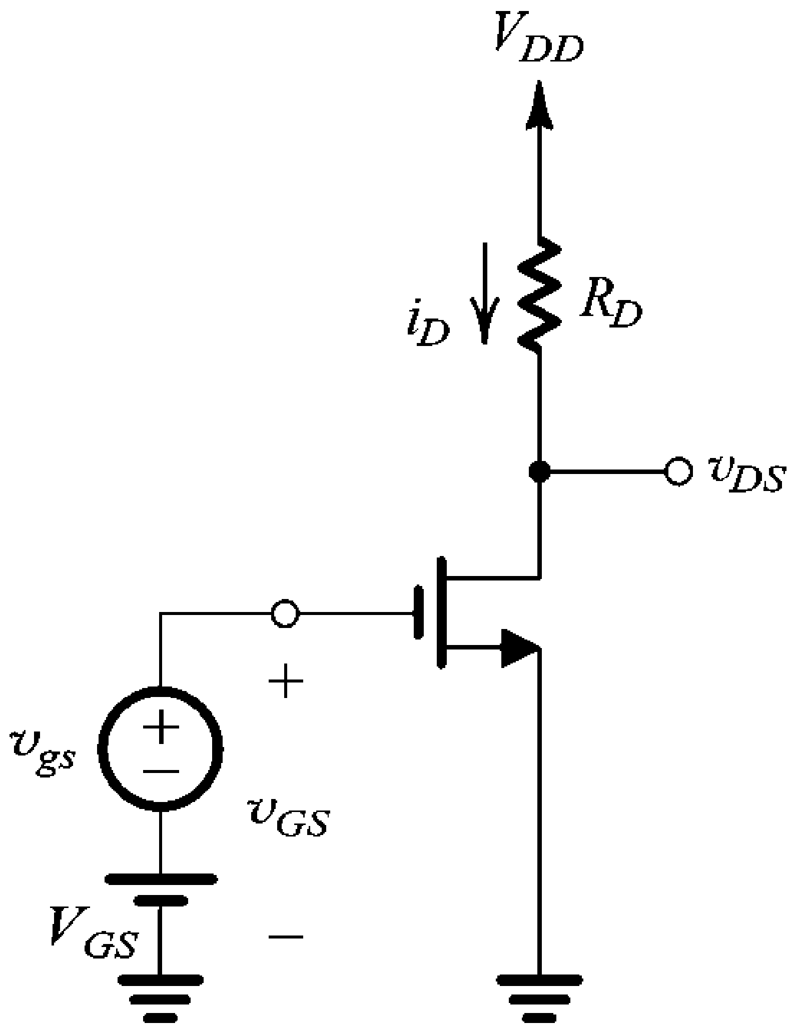
$$v_{ds} > v_{gs} - V_t = 4 \;✓$$

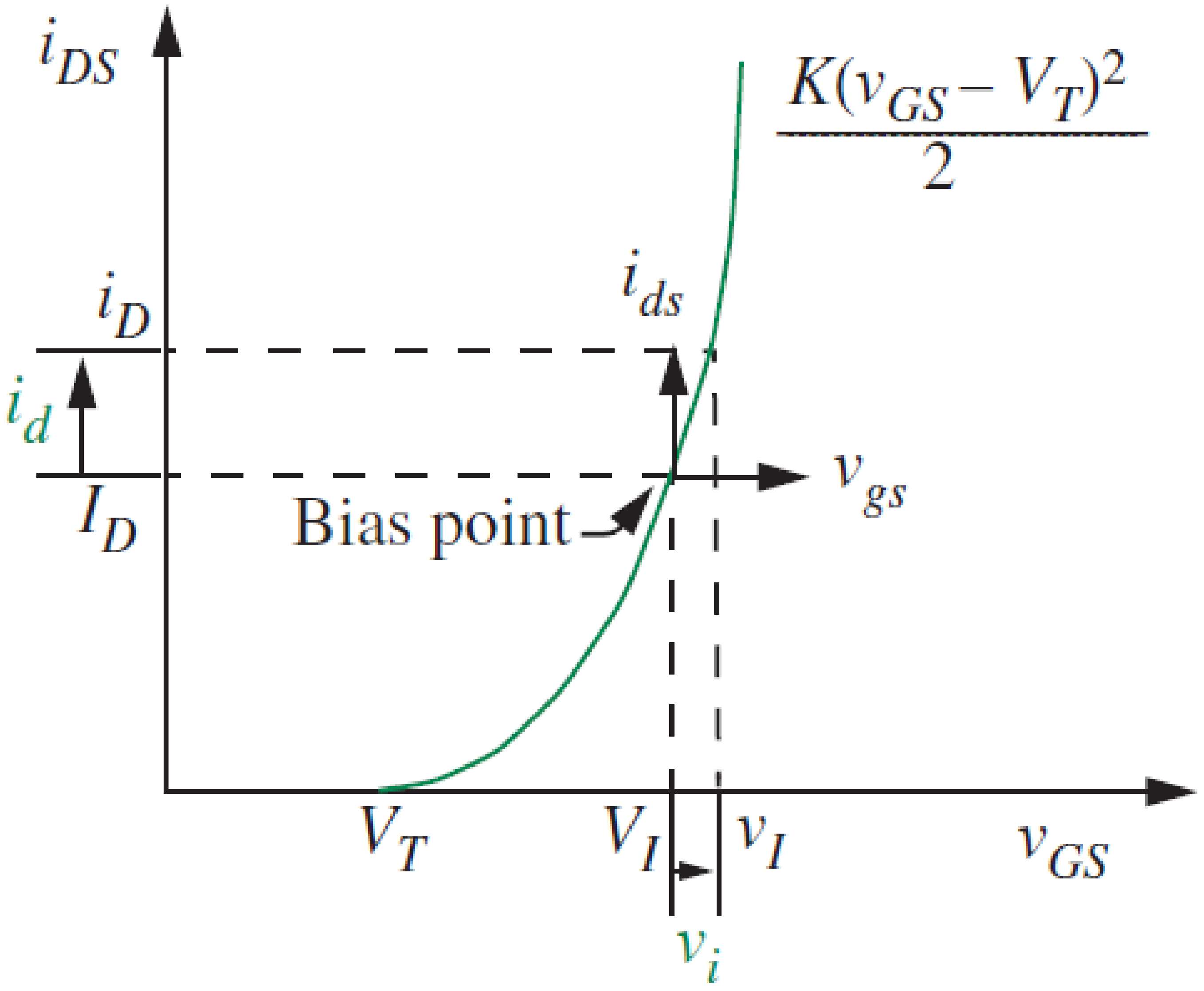
Example



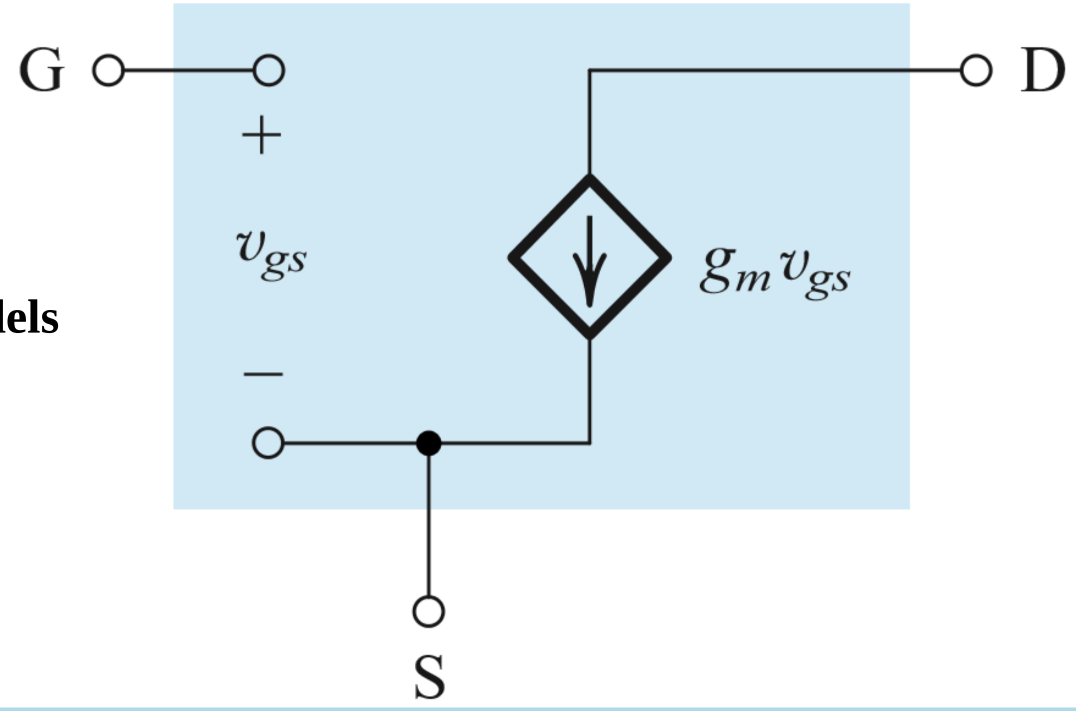
- ✓ V_o را بیابید. $(K = 0.5\frac{mA}{V^2}, V_t = 2, v_i = 12)$
- ✓ روابط KVL دو طرف:
  - ✓ $v_{gs} = v_i = 12$
  - ✓ $-12 + 1000i_D + v_{ds} = 0$
- ✓ $v_{gs} > V_t$ یا اشباع است یا خطی
- ✓ فرض اشباع:
  - ✓ $i_D = \frac{K}{2}\left(v_{gs} - V_t\right)^2 = 25mA \rightarrow v_{ds} = 12 - 25 = -13$
  - ✓ $v_{ds} < v_{gs} - V_t = 10$
- ✓ فرض خطی:
  - ✓ $i_D = K\left(v_{gs} - V_t - \frac{v_{ds}}{2}\right)v_{ds} = 0.25\left(20v_{ds} - v_{ds}^2\right) \xrightarrow{use\ DS\ KVL}$
  - ✓ $v_{ds} = 2.2 < 10$ $\qquad$ $v_{ds} = 21.8\ \times$

Small-Signal Models





Small-Signal Models

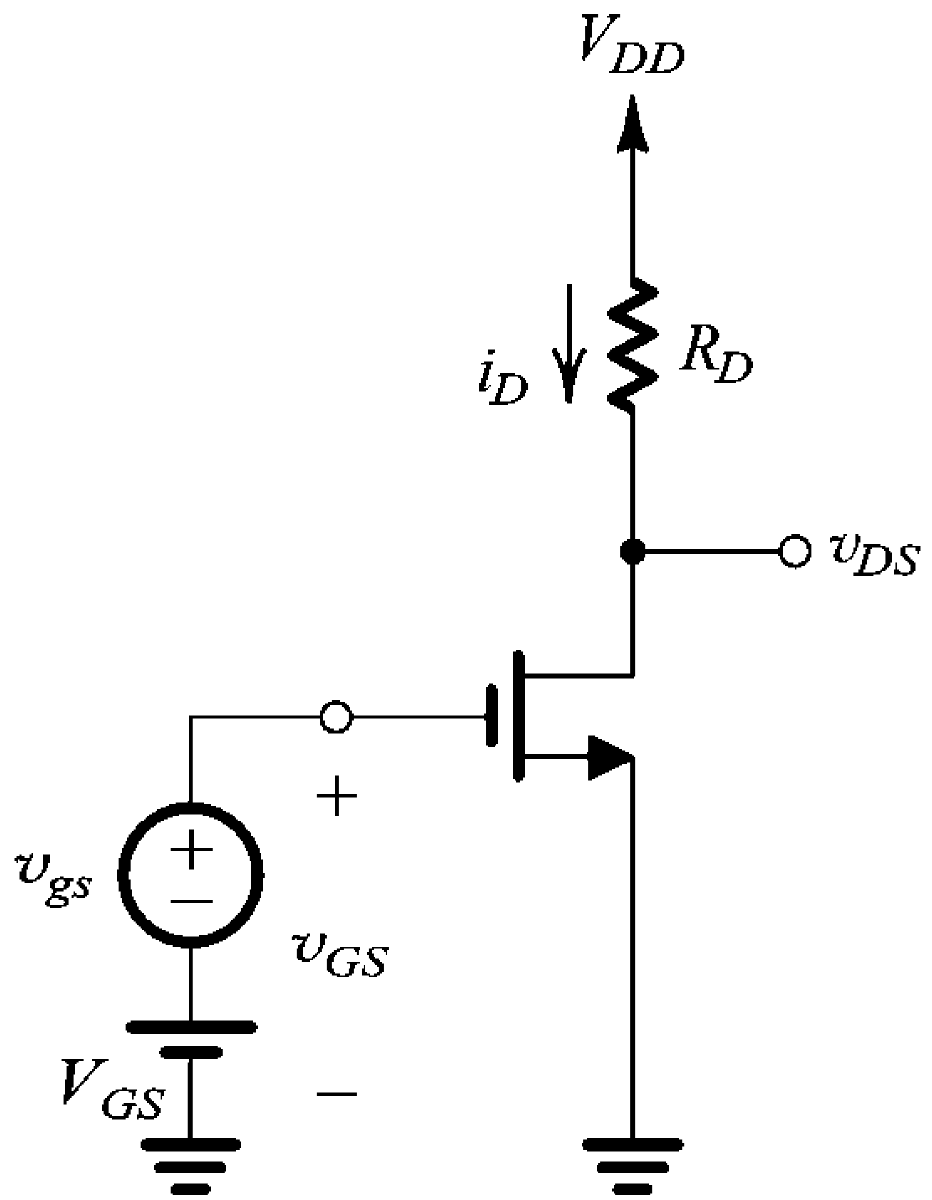


Saturation Region:

$$g_m = \frac{\partial i_D}{\partial v_{GS}}\Big|_{v_{GS}=V_{GS}} = K(V_{GS} - V_t)$$

The separation of DC analysis from AC analysis

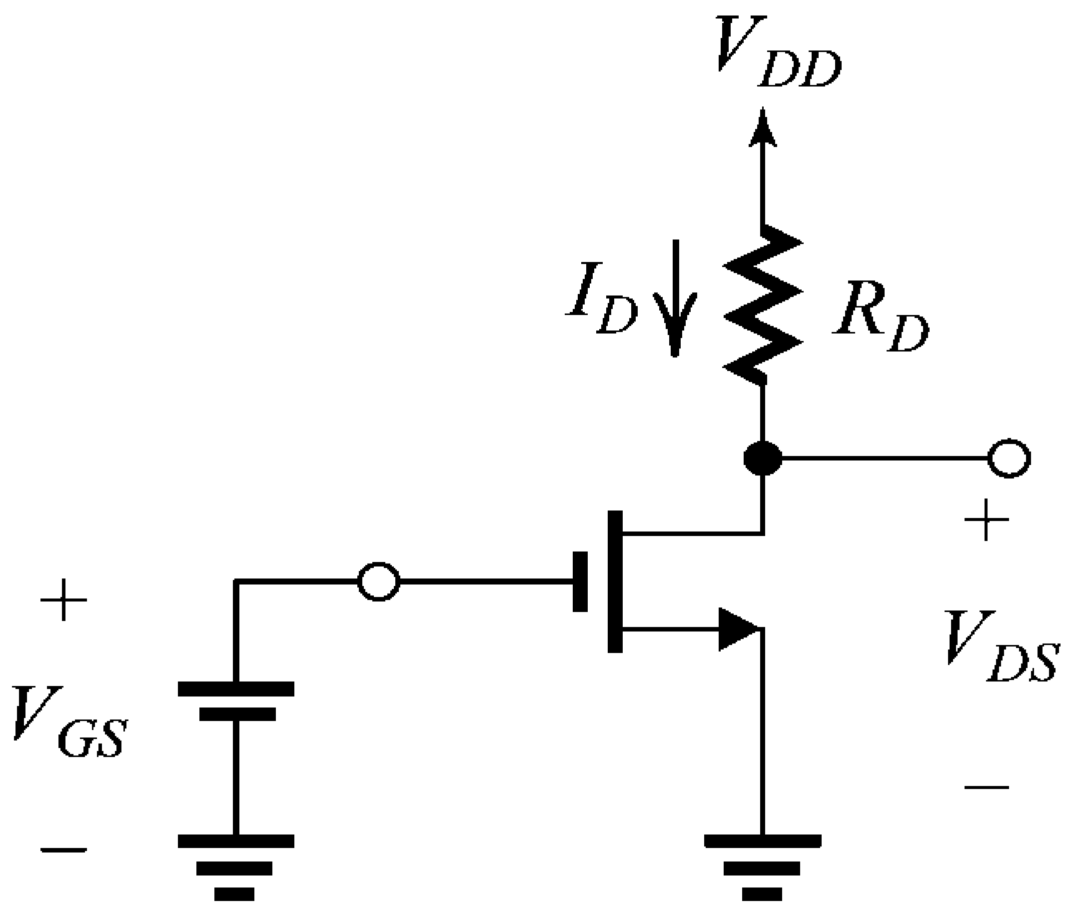
- ✓ $v_{DS} = V_{DD} - i_D R_D$
- ✓ $i_D = \underbrace{I_D}_{\text{DC component}} + \underbrace{i_d}_{\text{AC component}}$
- ✓ $v_{DS} = \underbrace{V_{DS}}_{\text{DC component}} + \underbrace{v_{ds}}_{\text{AC component}}$
- ✓ $V_{DS} = V_{DD} - R_D I_D$
- ✓ $v_{ds} = -R_D i_d$



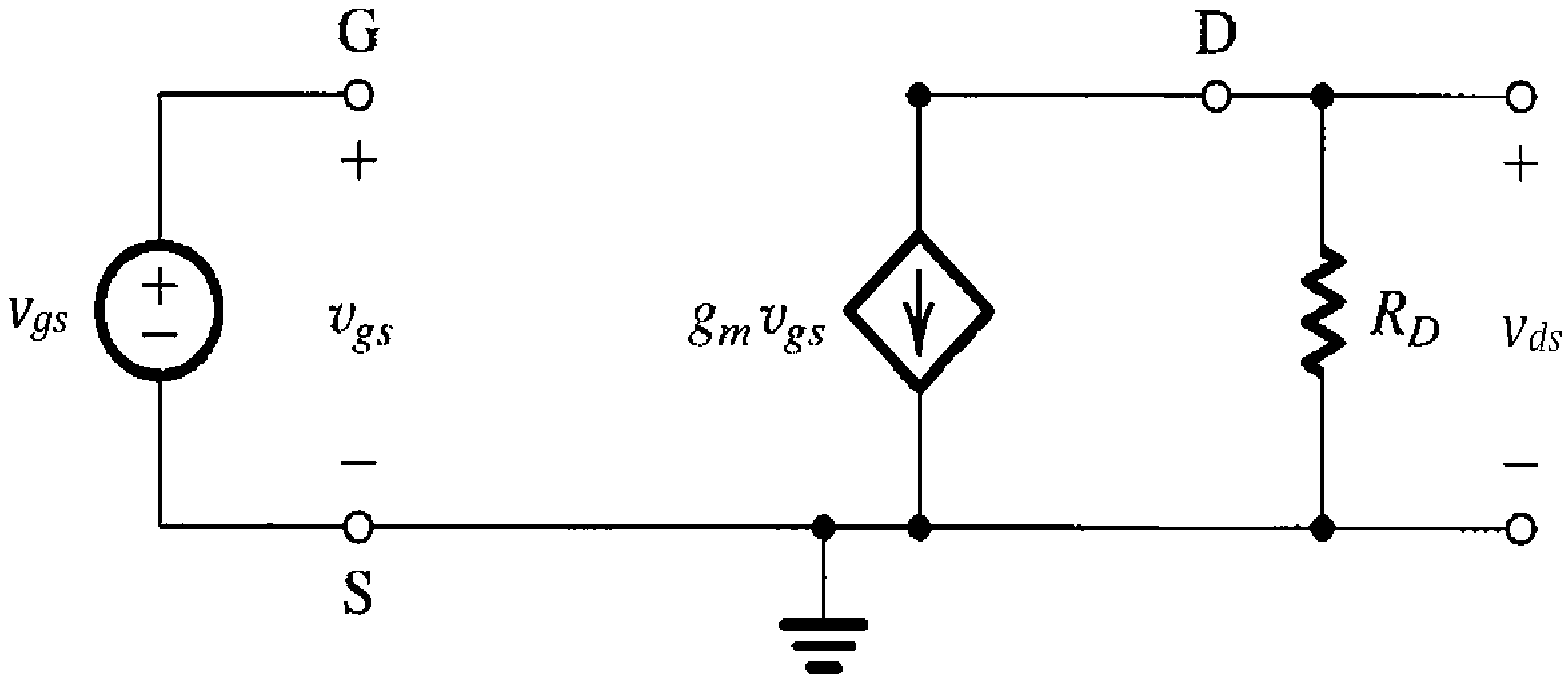
- ✓ The response of the DC component is determined by the DC input
- ✓ The response of the AC component is determined by the AC input.
- ✓ Consequently, DC and AC analyses can be performed independently.

- $v_{ds} = -R_D i_d = -R_D g_m v_{gs}$

- $A_v = \dfrac{v_{ds}}{v_{gs}} = -g_m R_D$ Voltage Gain



DC analysis



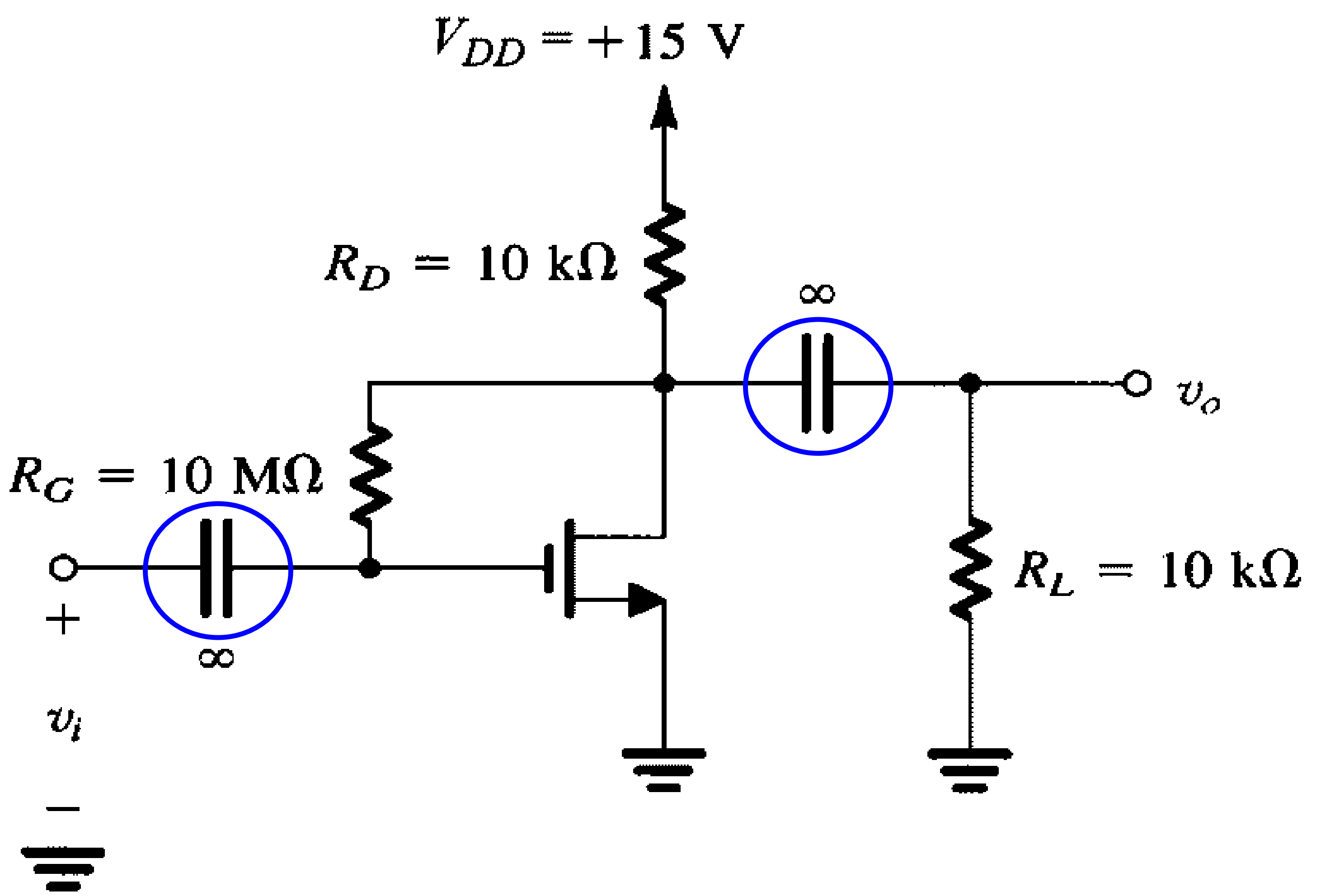
AC analysis

- Why is the terminal of the resistor R_D connected to the ground?

➢ Reason for Using Coupling Capacitors:

Capacitors act as open circuits for DC voltages. Therefore, when multiple stages of these circuits are connected, the DC bias of one stage does not affect the other. This simplifies the design of the amplifier.

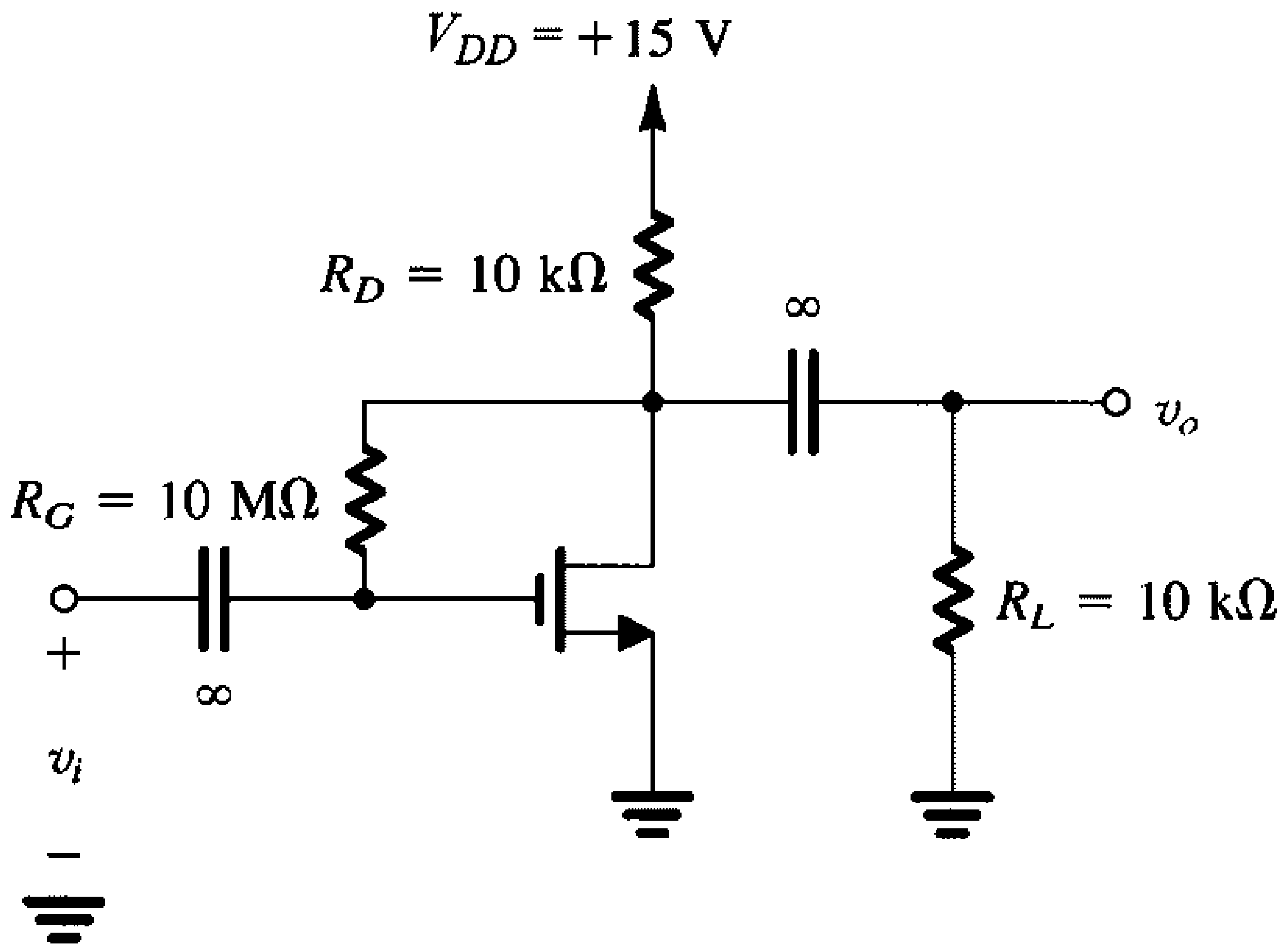
- ✓ A capacitor acts as a short circuit for AC signals ($\frac{1}{cj\omega}\approx 0$ for high frequencies).
- ✓ As a result, AC signals can pass through the capacitor.



Example (Continue)

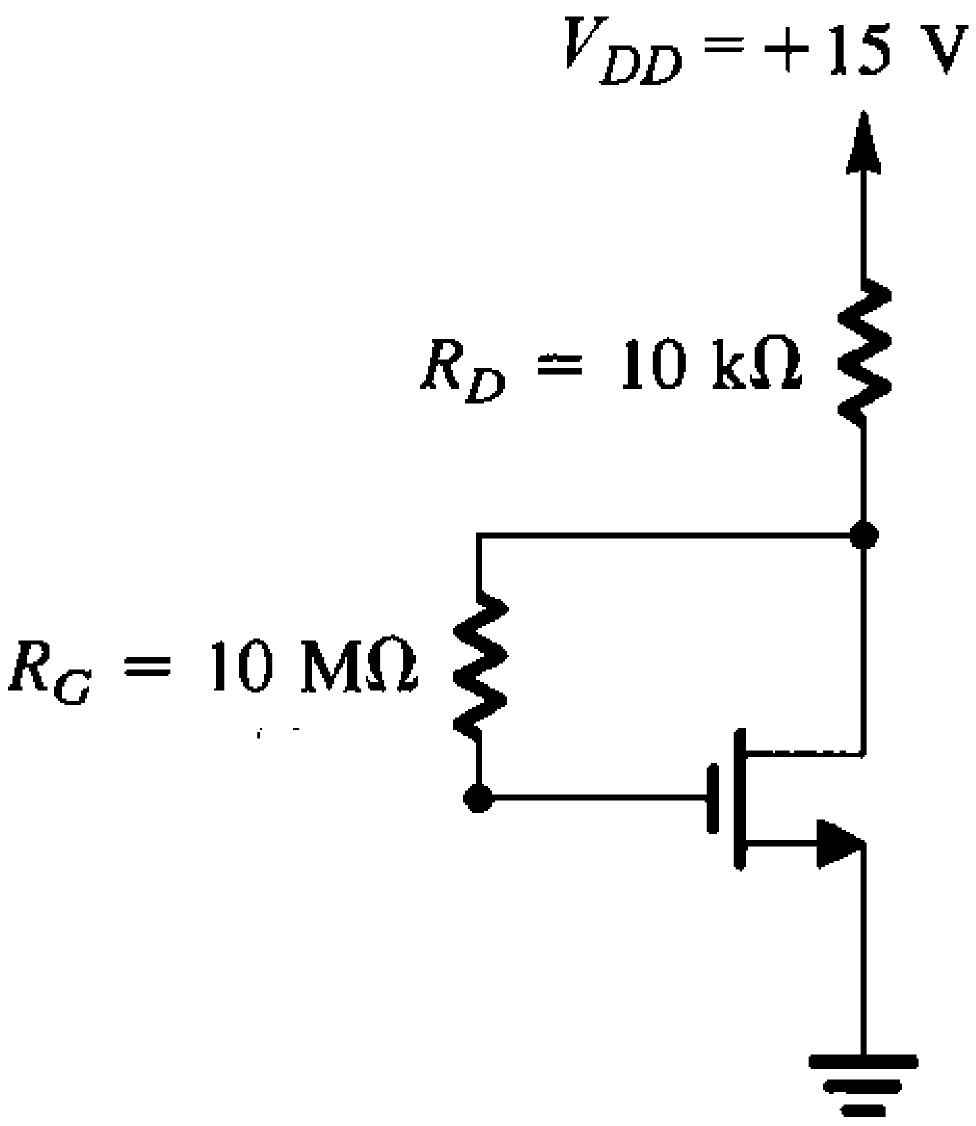
Determine the Voltage Gain A_v:

- **DC Analysis:**
  - ✓ Capacitors are considered open circuits.
  - ✓ Find the biasing conditions and the g_m.
- **AC Analysis:**
  - ✓ Capacitors are treated as short circuits.
  - ✓ Calculate the voltage gain A_v.

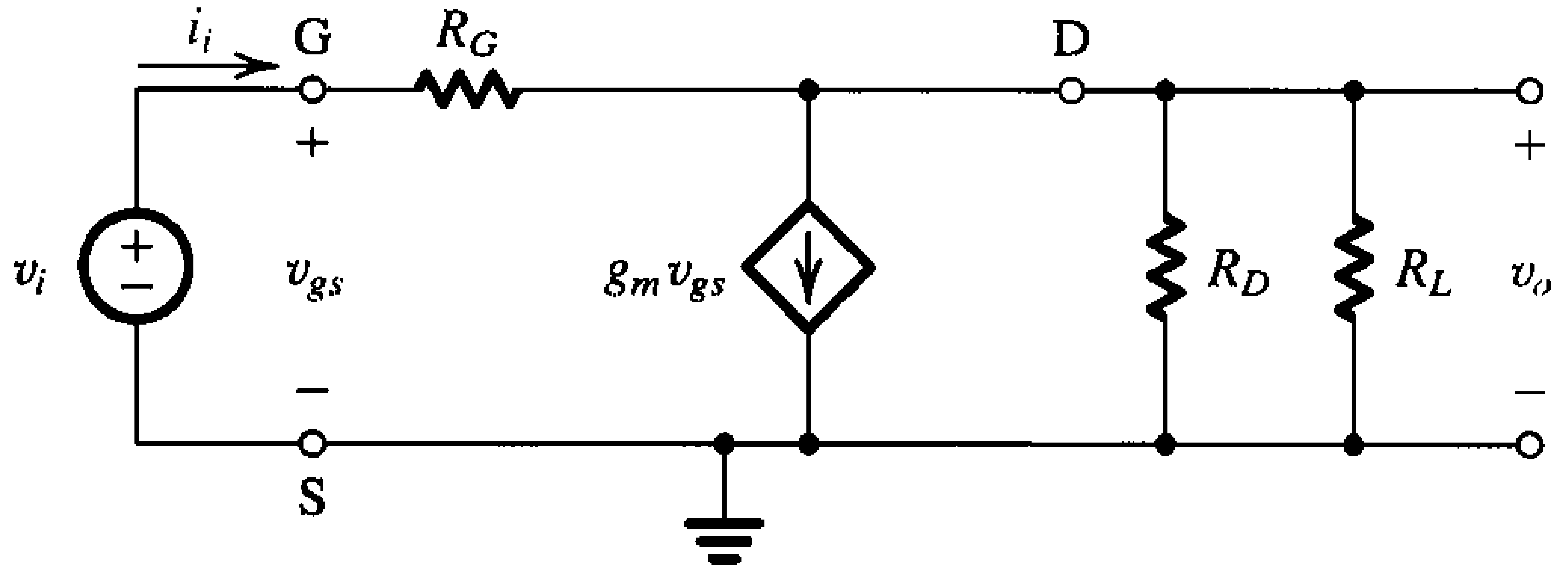


Example (Continue: DC analysis)

- ✓ $(K = 0.25\frac{mA}{V^2}, V_t = 1.5)$
- ✓ $I_G = 0 \rightarrow V_{GS} = V_{DS}$
- ✓ $V_{DS} = 15 - 10000 I_{DS}$
- ✓ Cut off Assumption:
  - ✓ $I_{DS} = 0 \rightarrow V_{DS} = 15$
  - ✓ $V_{GS} = 15 > V_t$ ×
- ✓ Saturation Assumption:
  - ✓ $I_{DS} = 0.125(V_{GS} - 1.5)^2 \rightarrow I_{DS} = 1.06mA$
  - ✓ $V_{GS} = V_{DS} = 4.4V$
  - ✓ $g_m = k(V_{GS} - V_t) = 0.725mA/V$



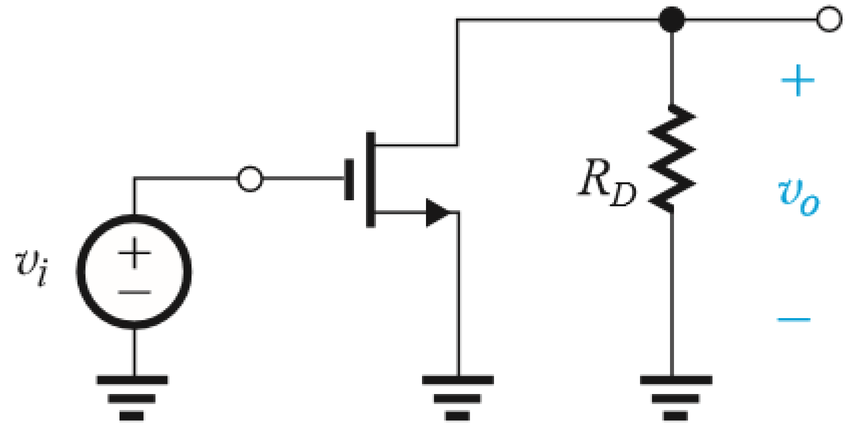
Example (Continue: AC analysis)



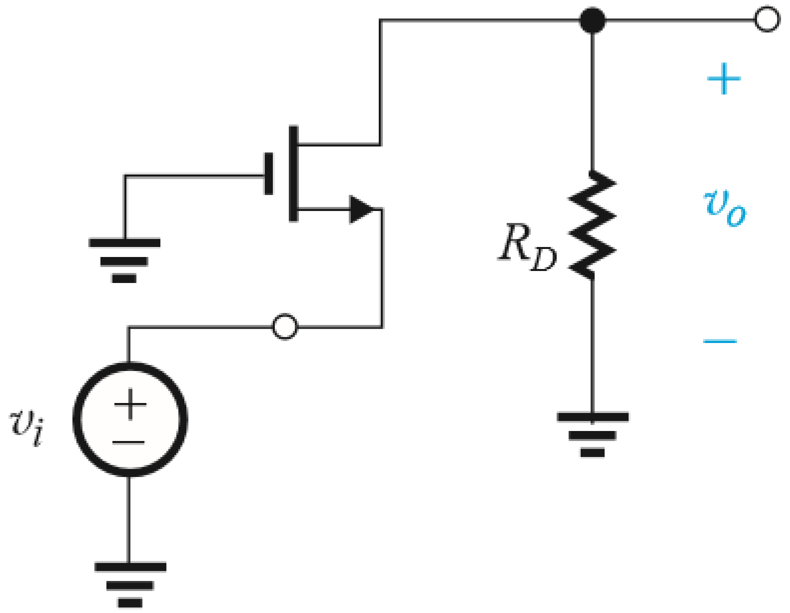
- ✓ $R_L' = R_D || R_L = 5K$
- ✓ $v_o = R_L'(i_i - g_m v_i)$
- ✓ $v_i = R_G i_i + v_o$
- ✓ $\rightarrow A_v = \frac{v_o}{v_i} = \frac{1 - g_m R_G}{1 + R_G / R_L'} \approx -g_m R_L' = -3.625$

Different Configurations of MOSFET

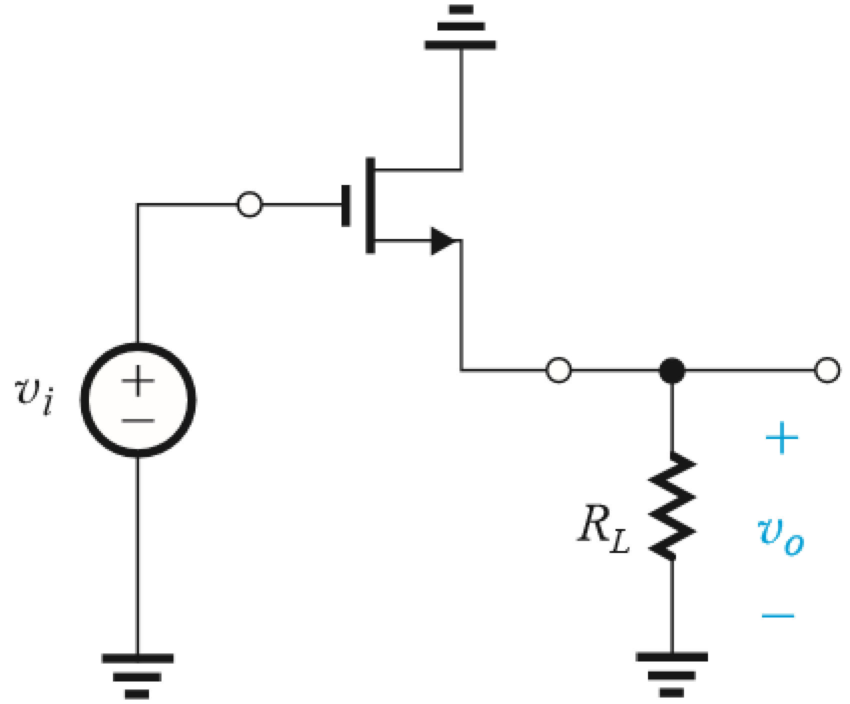
- ✓**Common Source (CS):**
  - ✓Applications: Amplifiers.
- ✓**Common Drain (CD) or voltage Follower:**
  - ✓Applications: Buffer stages in amplifiers.
- ✓**Common Gate (CG):**
  - ✓Applications: High-frequency amplifiers.



(a) Common Source (CS)



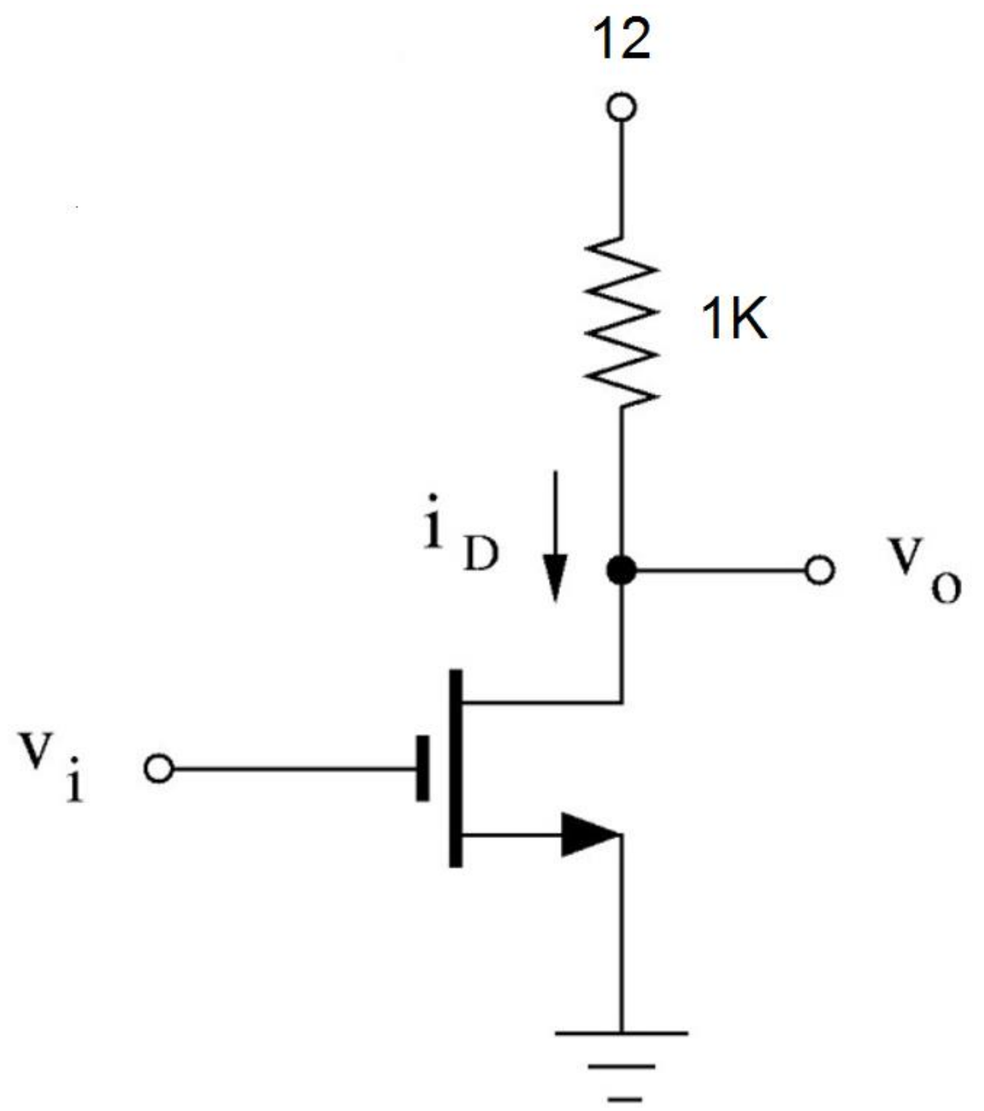
(b) Common Gate (CG)



(c) Common Drain (CD)

In Example 3: Circuit Behavior

- $v_i = 0 \rightarrow v_o = 12$
- $v_i = 12 \rightarrow v_o = 2.2$



MOSFET Operation as a Switch

- For low v_i (v_i is close to 0): The MOSFET is in the cutoff region (OFF state)

  - The output voltage v_o remains high ($v_o \approx 12$).

- For high v_i (v_i is sufficiently large): The MOSFET enters the linear region (ON state)

  - The output voltage v_o drops ($v_o \approx 2.2$).

Inverter Behavior

This circuit acts as an inverter, where the output voltage is the logical inverse of the input voltage:

- When $v_i = 0$, $v_o = 1$ (logical high).

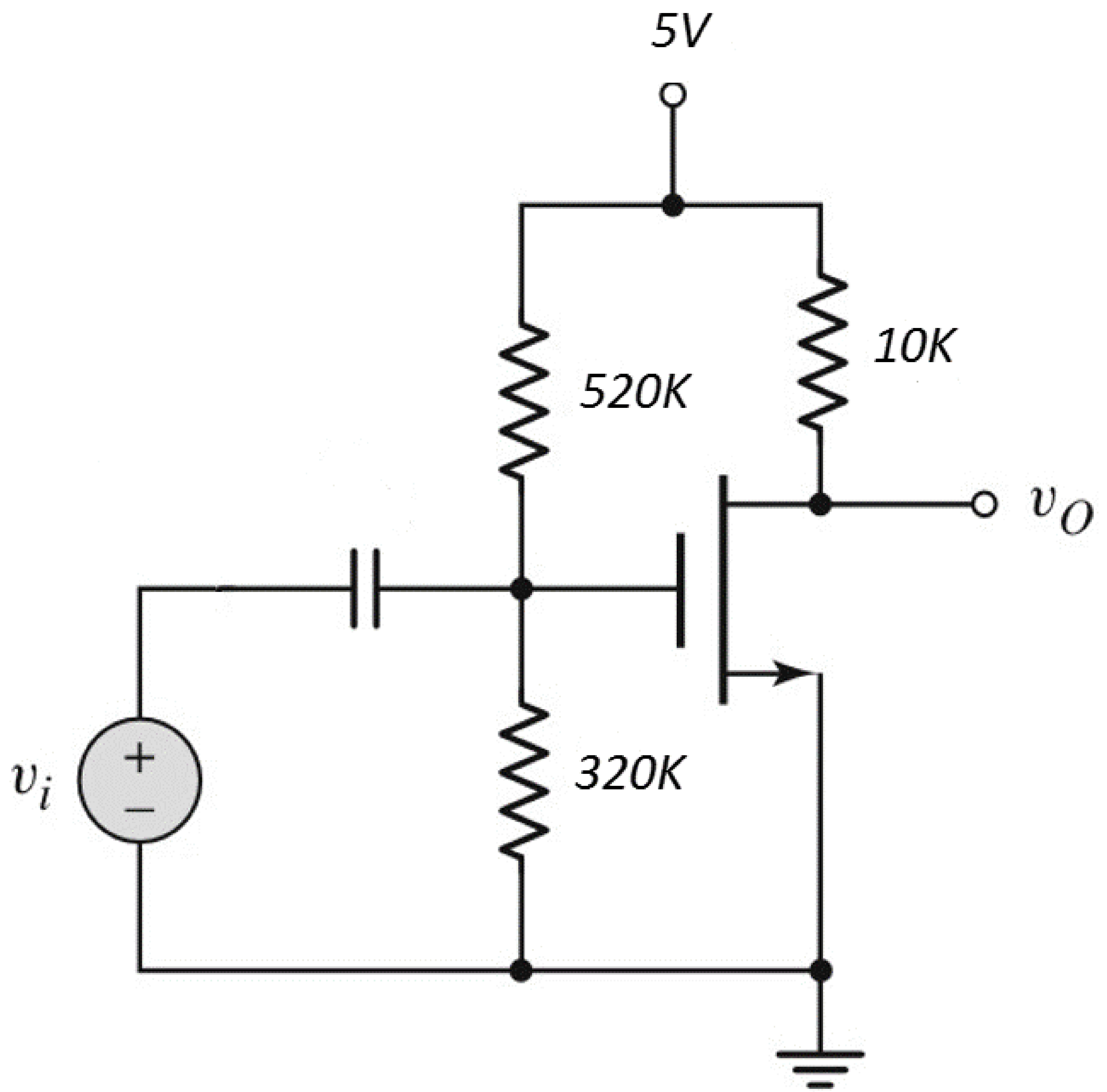
- When $v_i = 1$, $v_o \approx 0$ (logical low).

Class Exercise 1: Common Source

- Determine g_m.

- Calculate the voltage gain ($A_v = \frac{v_o}{v_i}$).

$V_T = 0.8V, K = 0.4mA/V^2$

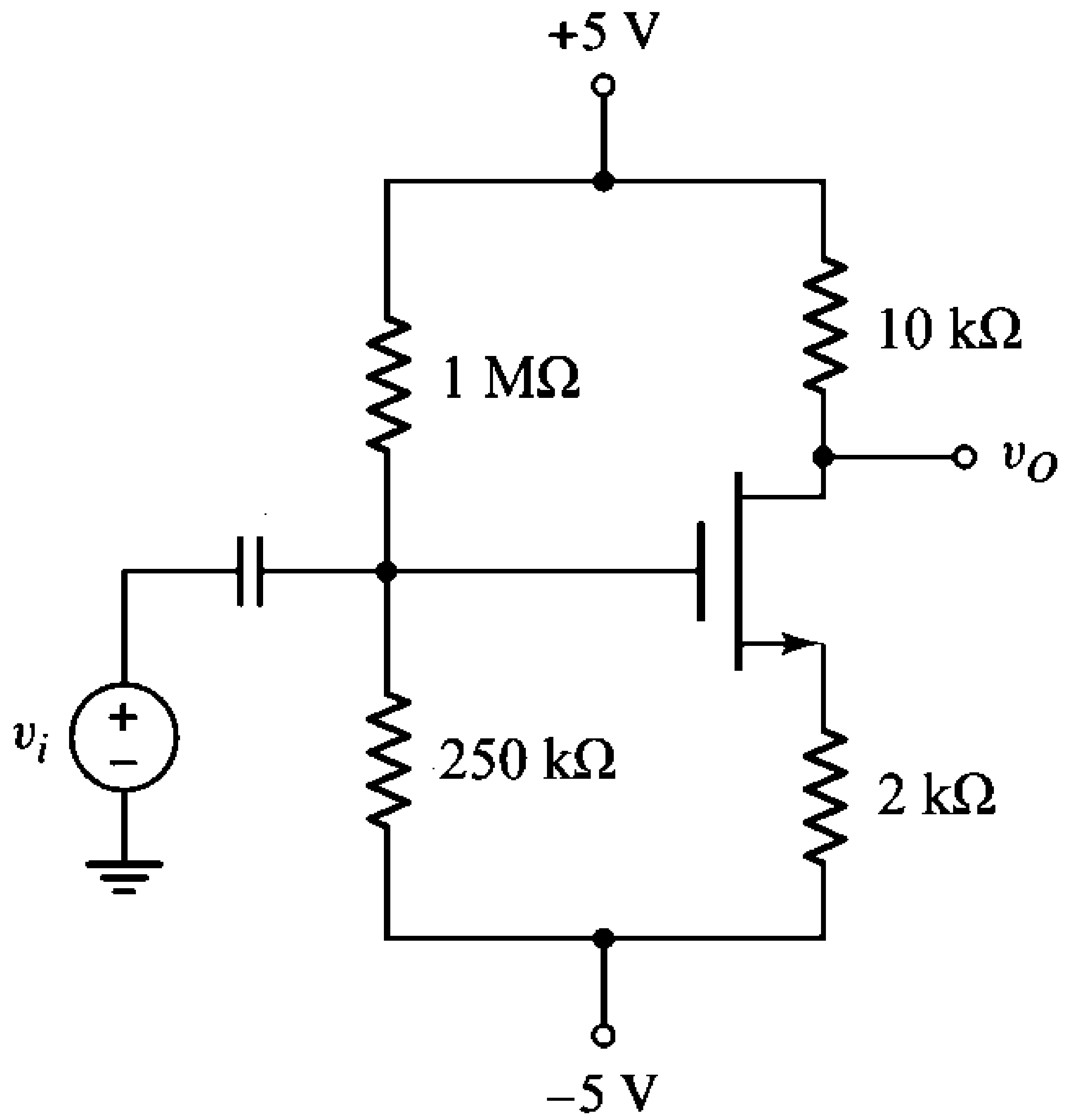


Answer: $g_m = 0.44mA/V$, $A_v = -4.4$

Class Exercise 2: Common Source

- ➢Determine g_m.
- ➢Calculate the voltage gain ($A_v = \frac{v_o}{v_i}$).

✓$V_T = 0.6V, K = 1mA/V^2$

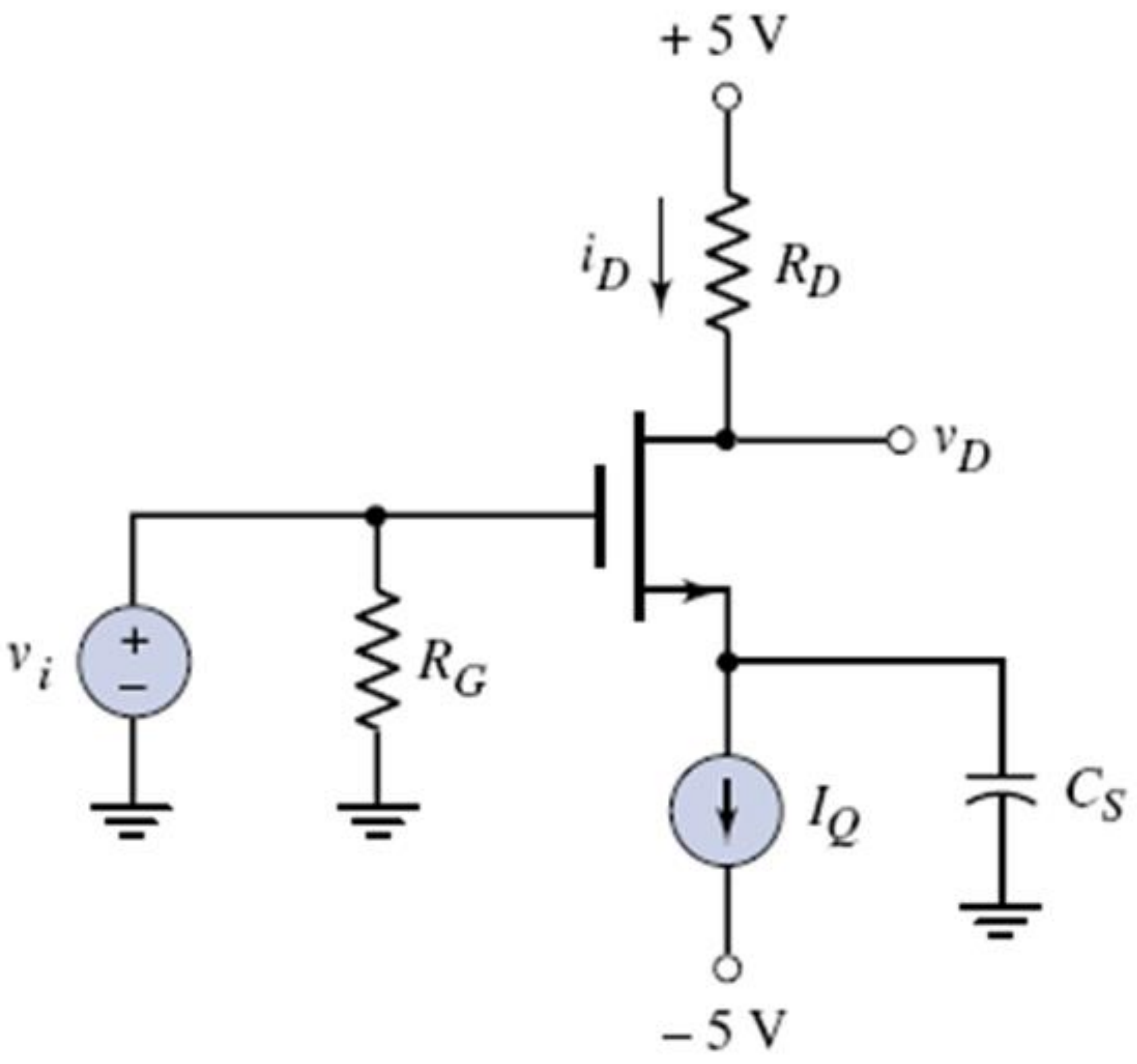


Answer: $g_m = 0.78mA/V$, $A_v = -3$

Class Exercise 3: Common Source

- Determine the resistance R_D and the operating condition of the transistor.

- $V_T = 0.8V, K = 0.24mA/V^2$
- $I_Q = 250\mu A, V_D = 2.5V$



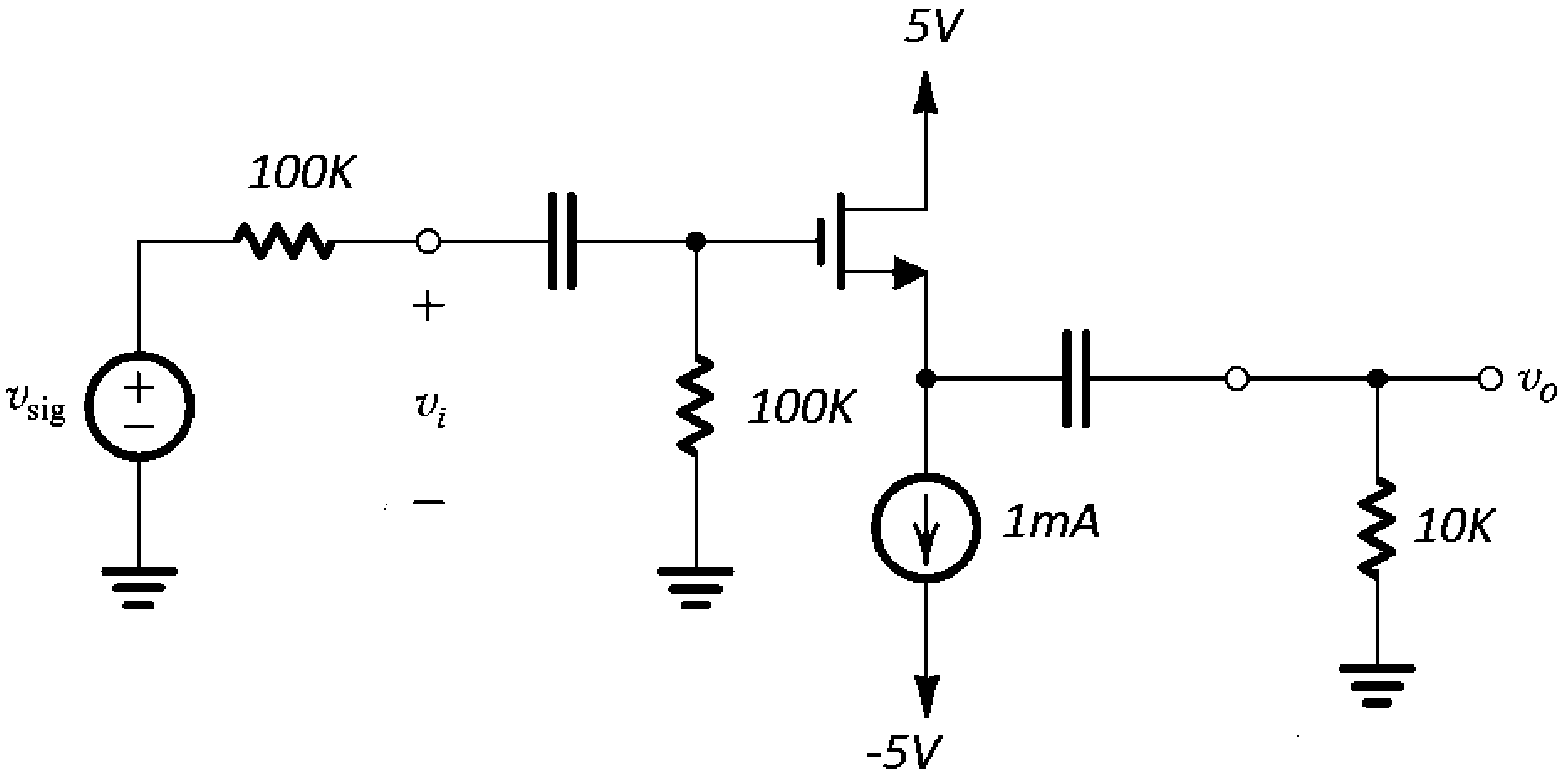
Answer: $R_D = 10K\Omega$, saturation

Class Exercise 4: Common Drain

- Determine g_m.

- Calculate the voltage gain $(A_v = \frac{v_o}{v_i})$.

- $V_T = 0.8V, K = 0.5mA/V^2$



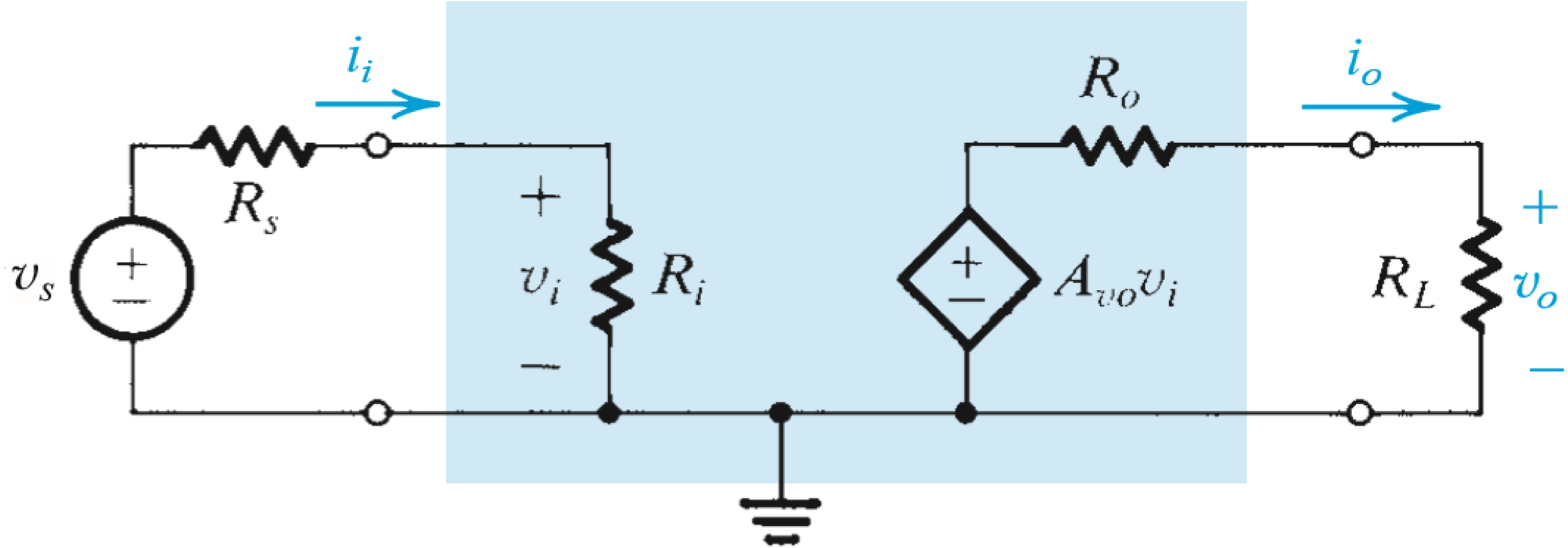
Answer: $g_m = 1mA/V$, $A_v = 0.45$

Input and Output Resistance

In an amplifier, it is typically desired to have:

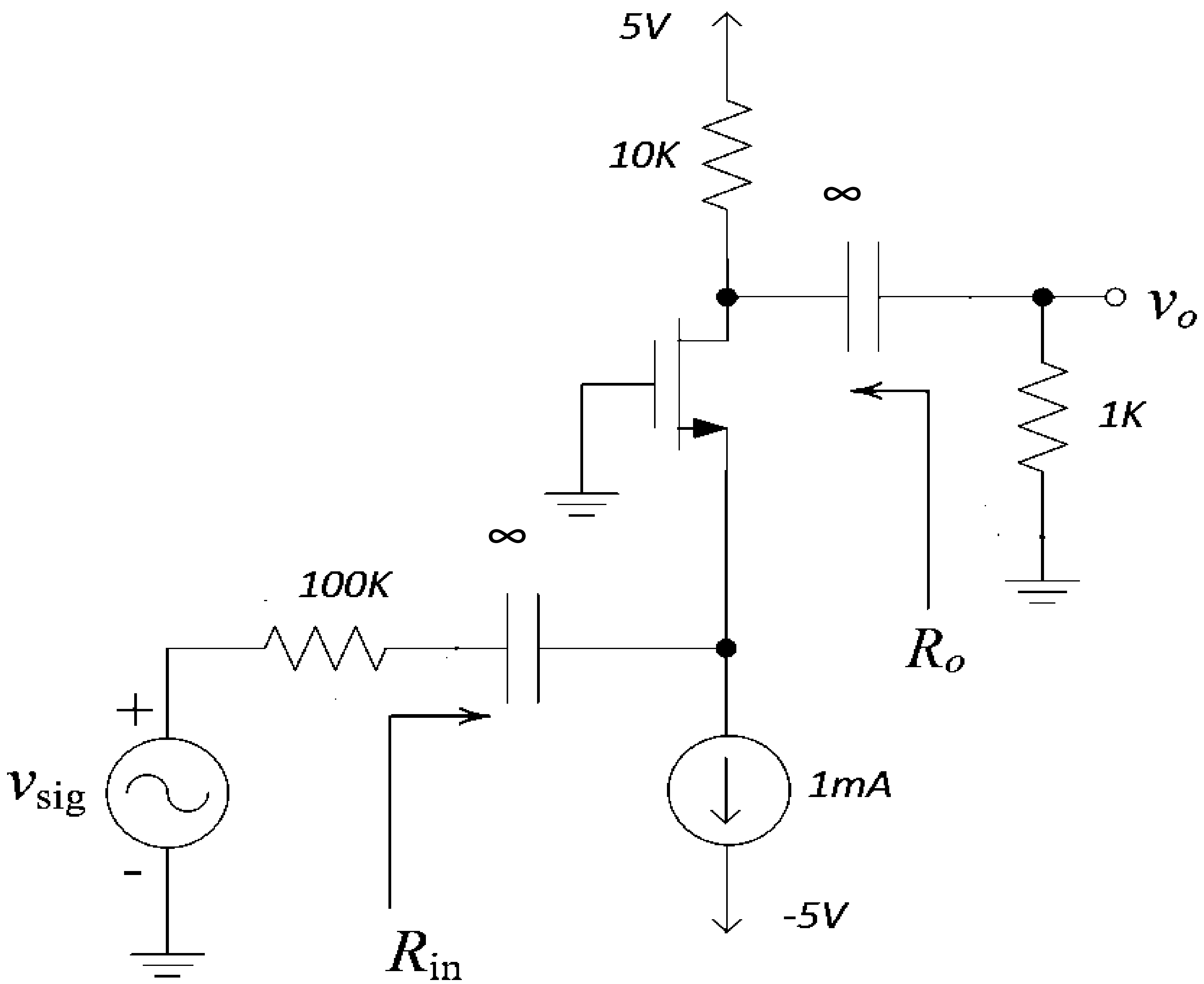
✓ A **high input resistance** R_i to ensure that $v_i \approx v_s$.

✓ A **low output resistance** R_o to ensure that $v_o \approx A_v v_i$.

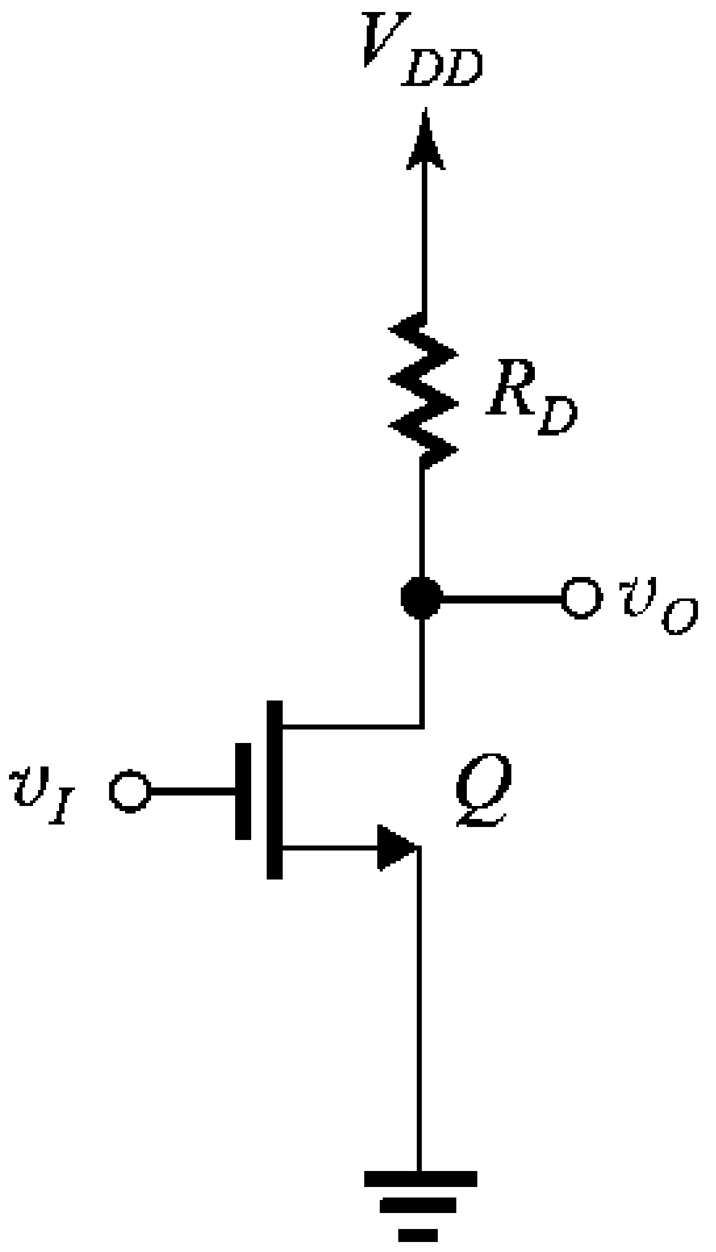


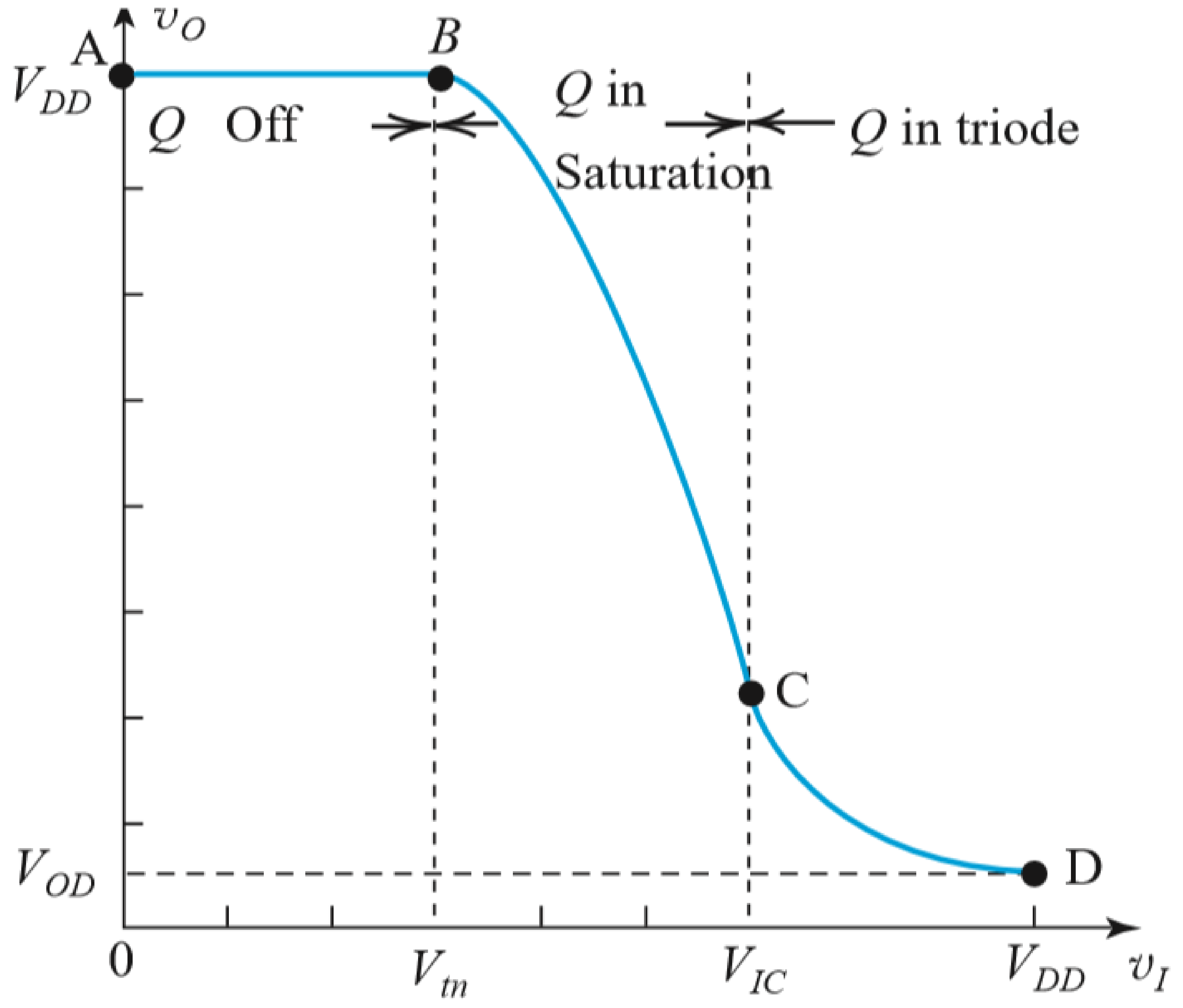
Class Exercise 5

- Find g_m and A_v
- Find R_{in}, R_o
- $V_T = 0.8V, K = 0.5mA/V^2$

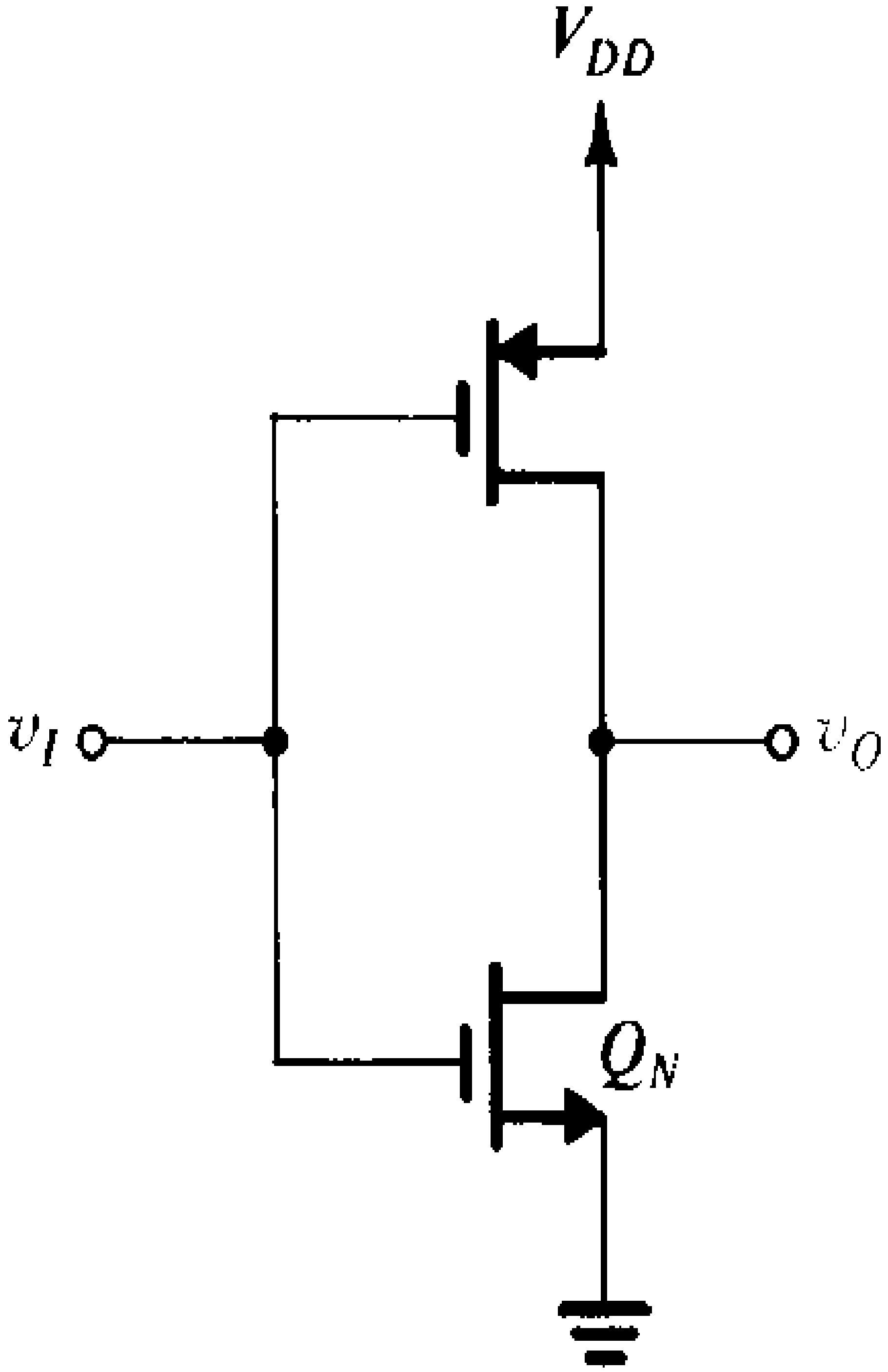


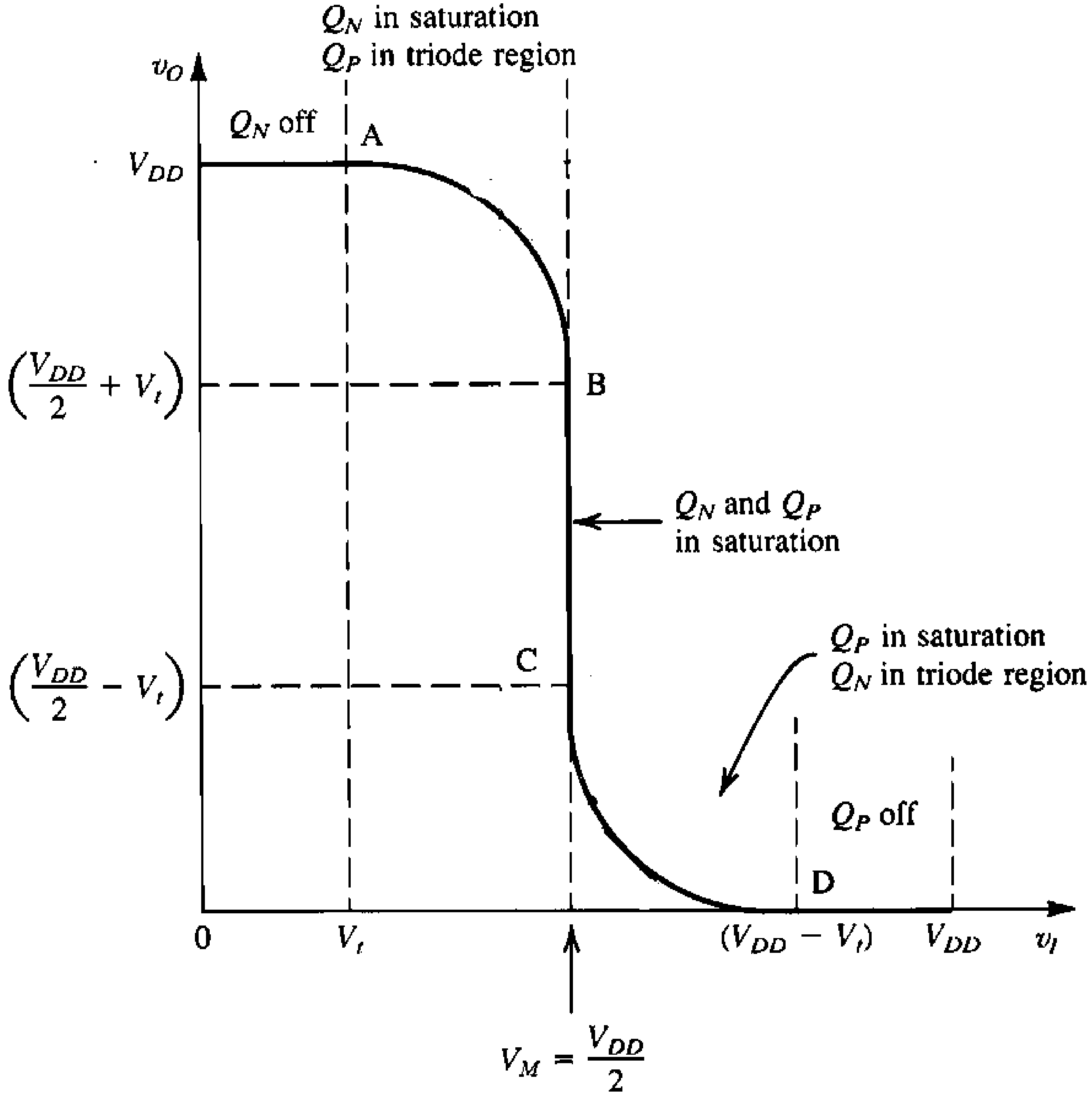
NMOS Inverter with Resistive Load





CMOS Inverter Gate:





Thanks