دانشگاه صنعتی امیرکبیر
(پلی تکنیک تهران)

Electrical and Electronic Circuits

chapter 13. Operational Amplifier

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عظیم فرقدان

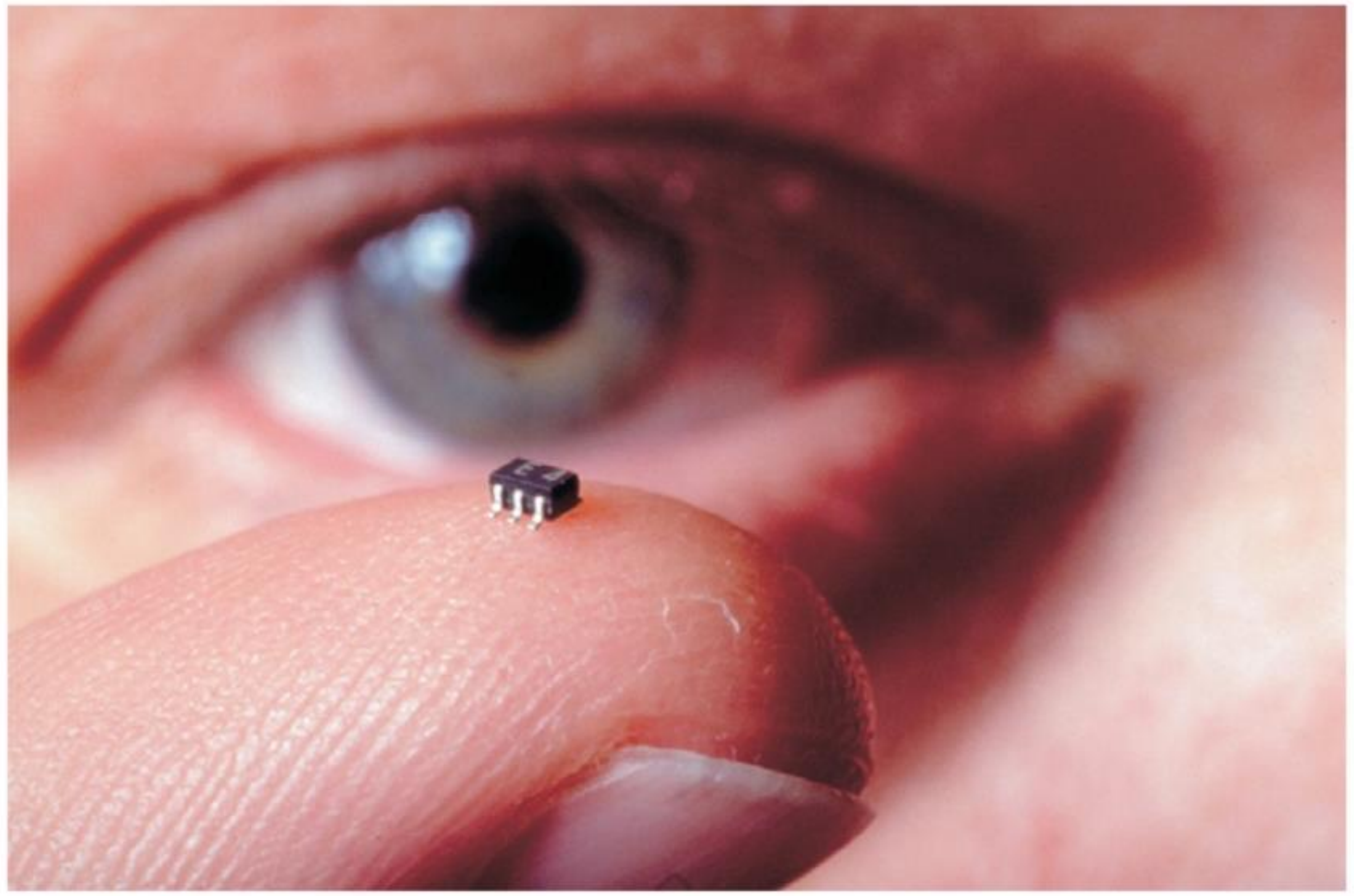
مهر ۱۴۰۳

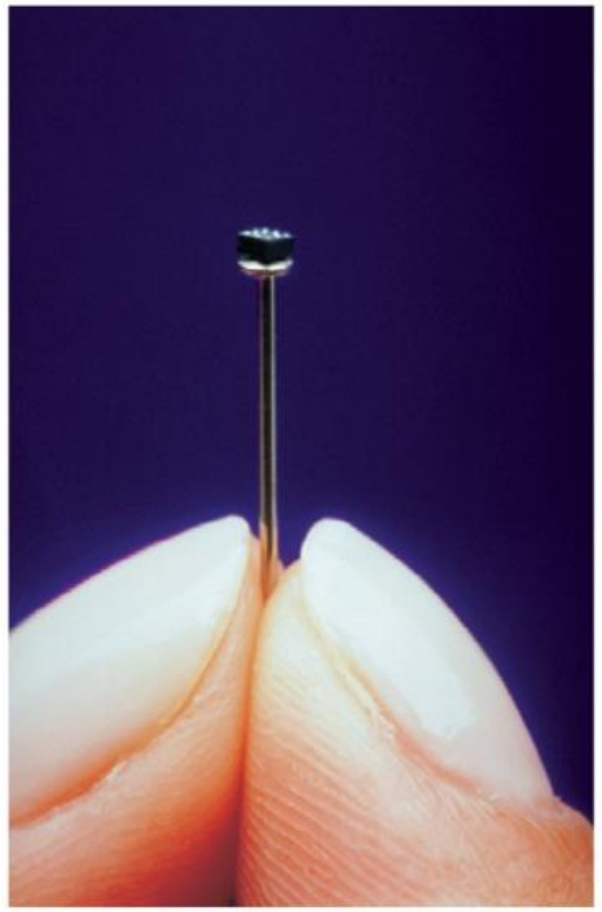
Objectives of the Lecture

- Introduction to Operational Amplifiers
  - Precise Model of Operational Amplifiers
  - Ideal Model
  - Applications of Operational Amplifiers
    - Inverting Amplifier
    - Non-Inverting Amplifier
    - Voltage Follower (Buffer)
    - Multi-Stage Amplifier
    - Ideal Voltage and Current Sources
- Comparator Circuit
- Several Examples

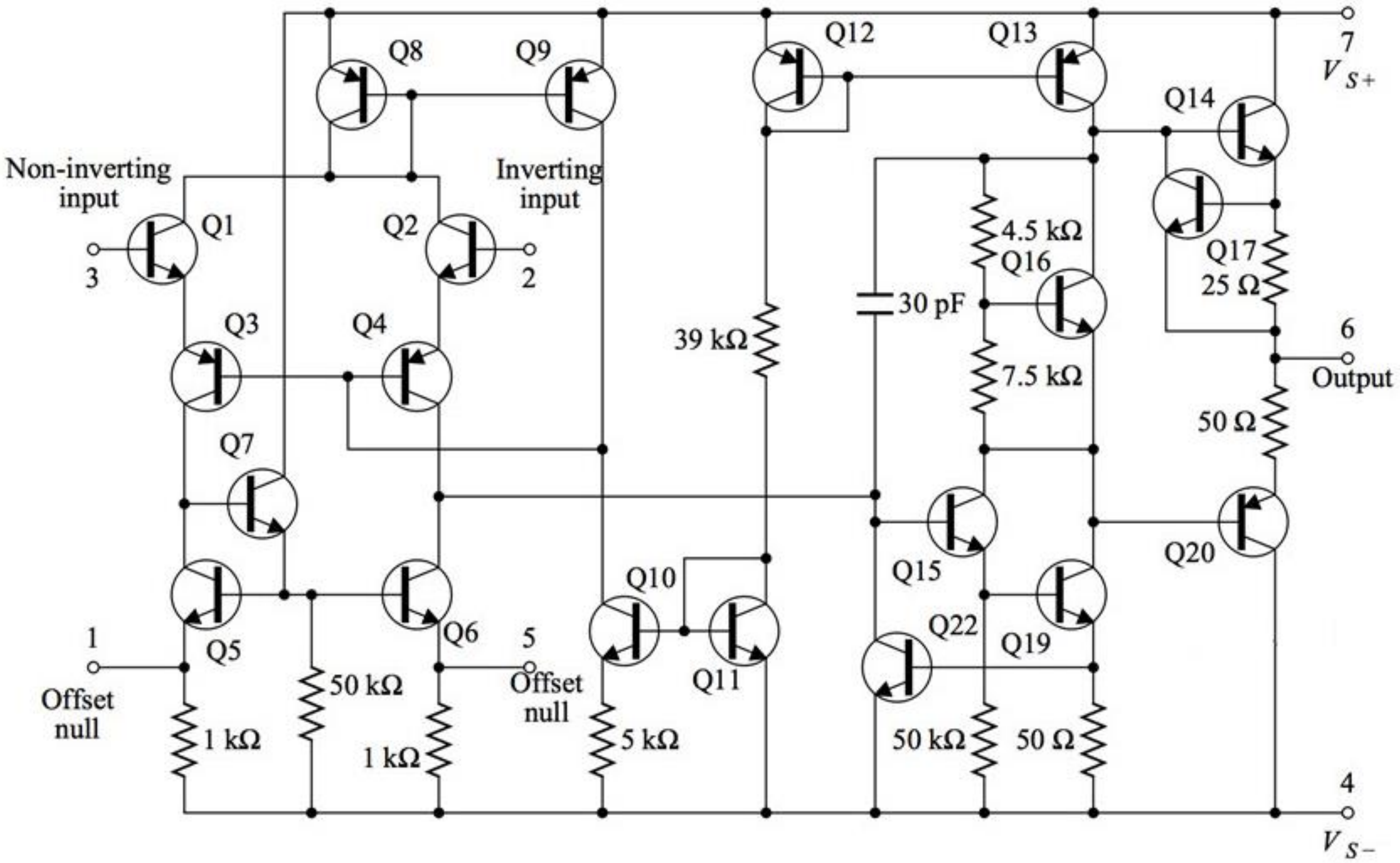
Operational Amplifier (Op-Amp)

- The **Operational Amplifier (Op-Amp)** is an integrated circuit (IC) widely utilized as an amplifier in various applications.
- Its origins date back to the 1940s when it was employed in analog computational circuits for constructing adders, sub tractors, and analog multipliers.





The Internal Circuit of a Typical Operational Amplifier



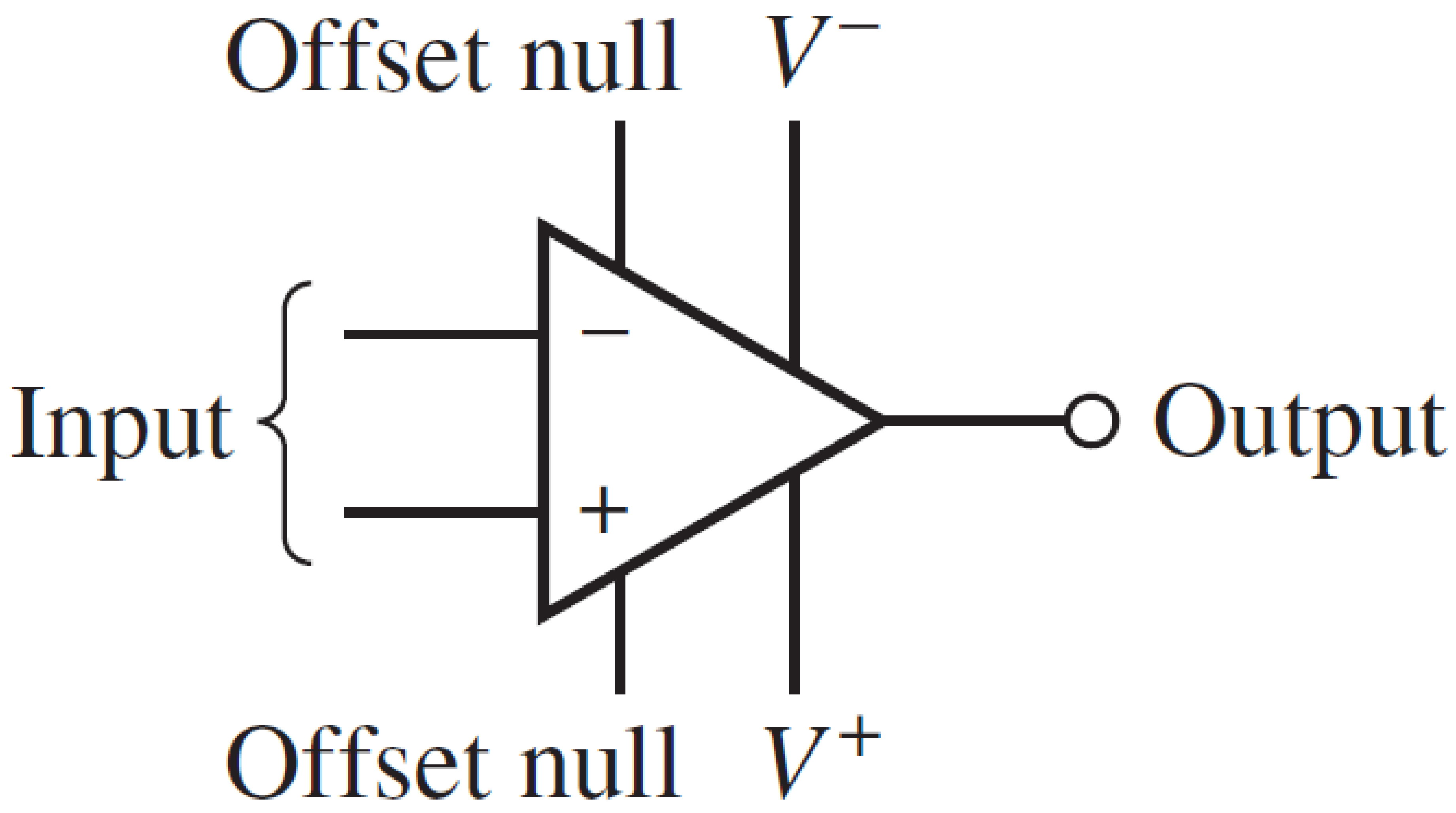
- An operational amplifier (Op-Amp) is generally represented by the following symbol and includes the following pins:

- **Input Pins**: $Input$

- **Output Pin**: $Output$

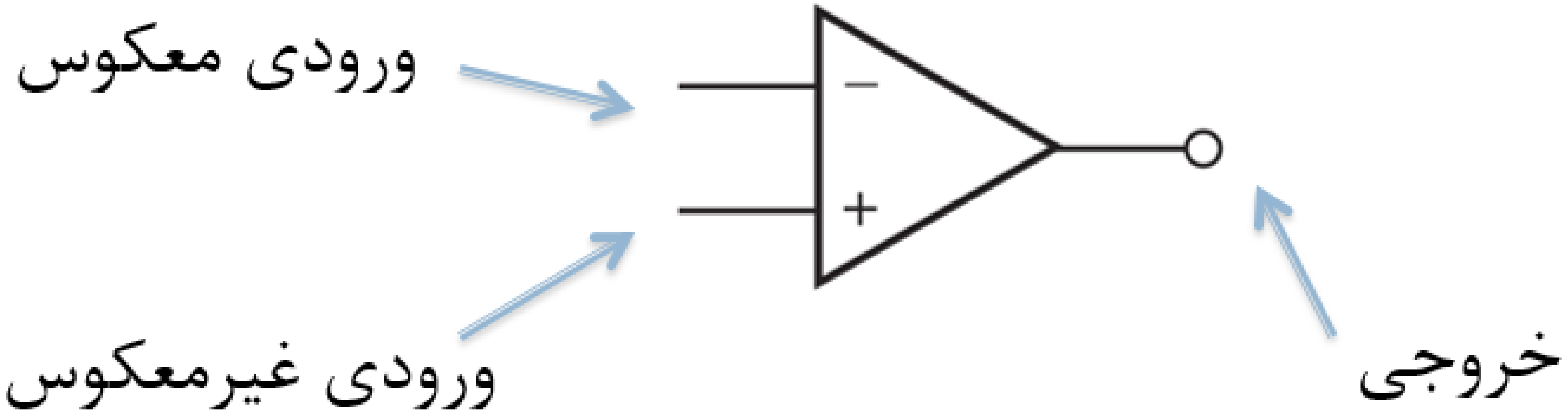
- **Power Supply Pins**: V^+, V^-

- **Offset Adjustment Pins**: $Offset$



Op-Amp Symbol

- ✓ Let us focus on the **input** and **output pins**, assuming the other pins are connected to appropriate voltage levels.
- ✓ An operational amplifier functions as a **differential amplifier**, meaning it amplifies the difference between its input signals.

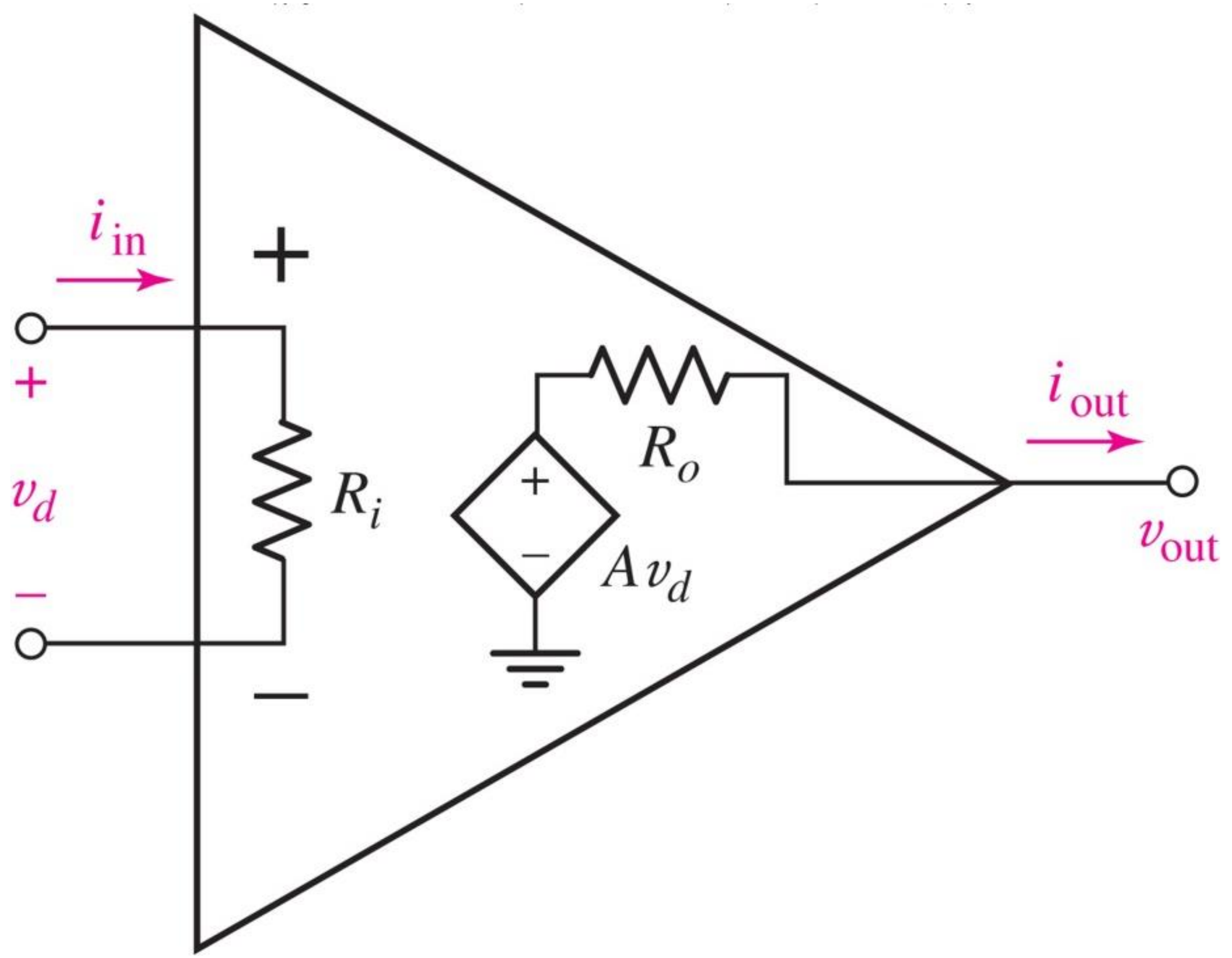


Precise Model of an Op-Amp

- A circuit that operates as a voltage amplifier can be modelled using the following equivalent circuit.

- This model includes:

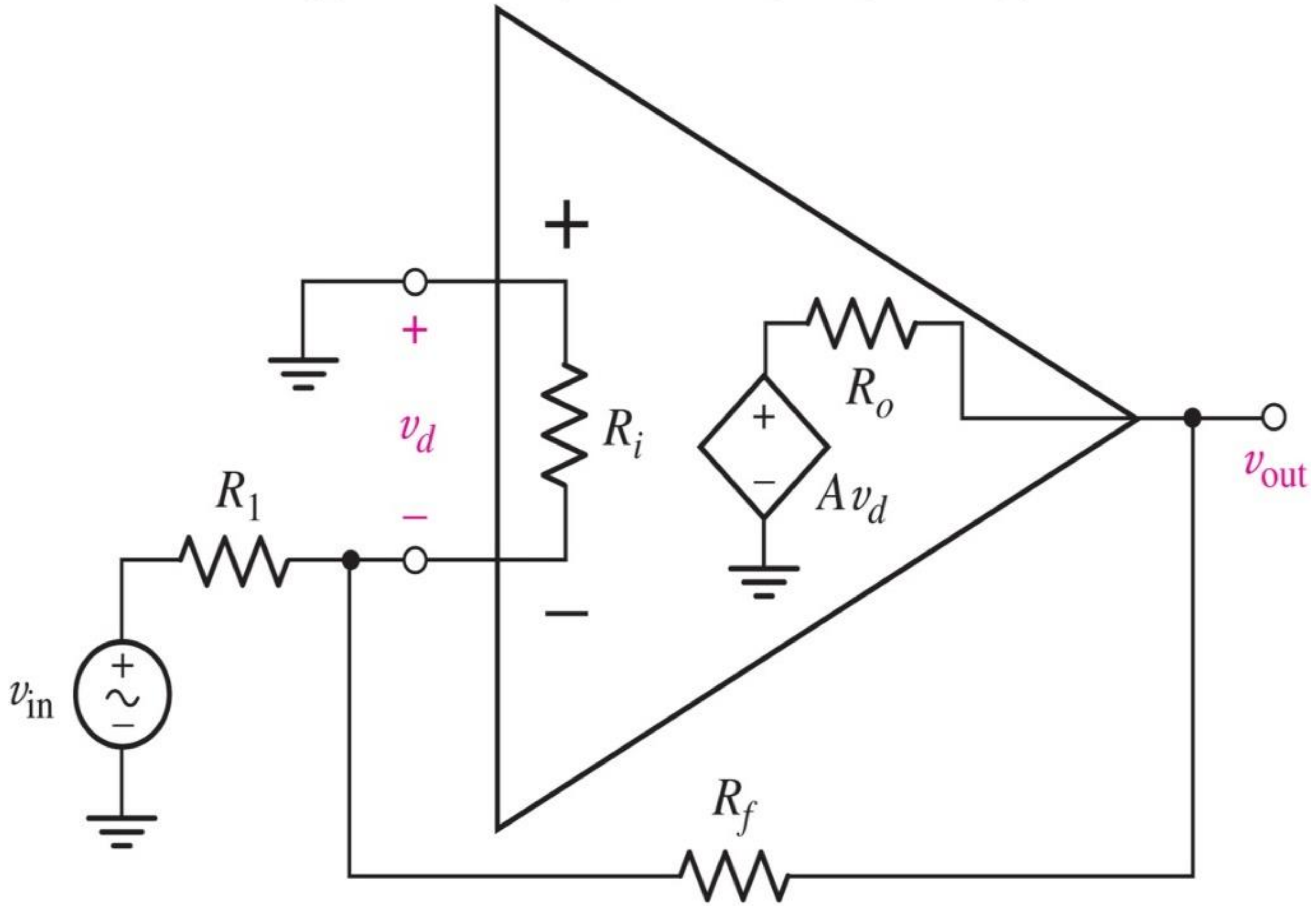
  - **Input Resistance (R_i)**
  - **Output Resistance (R_o)**
  - **Open-Loop Gain (A)**



- ✓ In the circuit below, what is the gain of the amplifier?

- ➢ $KCL_1: \frac{-v_d-v_{in}}{R_1}+\frac{-v_d-v_{out}}{R_f}+\frac{-v_d}{R_i}=0$

- ➢ $KCL_2: \frac{v_{out}+v_d}{R_f}+\frac{v_{out}-Av_d}{R_o}=0$



✓ By solving the system of equations and eliminating v_d, we obtain:

➢ $A_v = \frac{v_{out}}{v_{in}} = \left[\frac{R_o+R_f}{R_o-AR_f}\left(1+\frac{R_1}{R_f}+\frac{R_1}{R_i}\right)-\frac{R_1}{R_f}\right]^{-1}$

➢Example Solution for the Inverting Amplifier Using LM741 Specifications

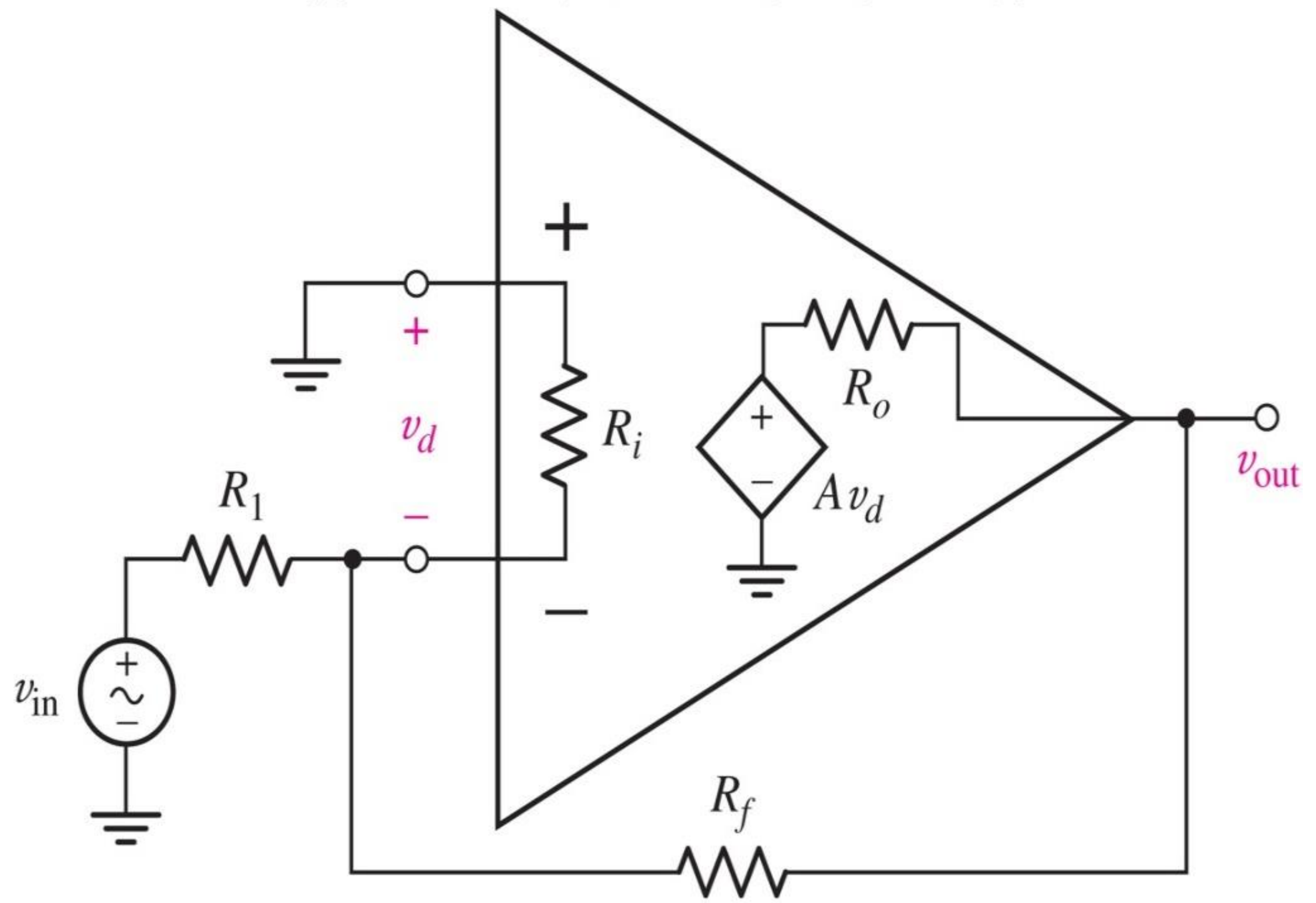
➢ $A = 200000$

➢ $R_i = 2M\Omega$

➢ $R_o = 75\Omega$

➢ If $R_f = 47$ and $R_1 = 4.7$

➢ Then: $A_v = -9.999$



Advantages of Using an Op-Amp as a Voltage Amplifier

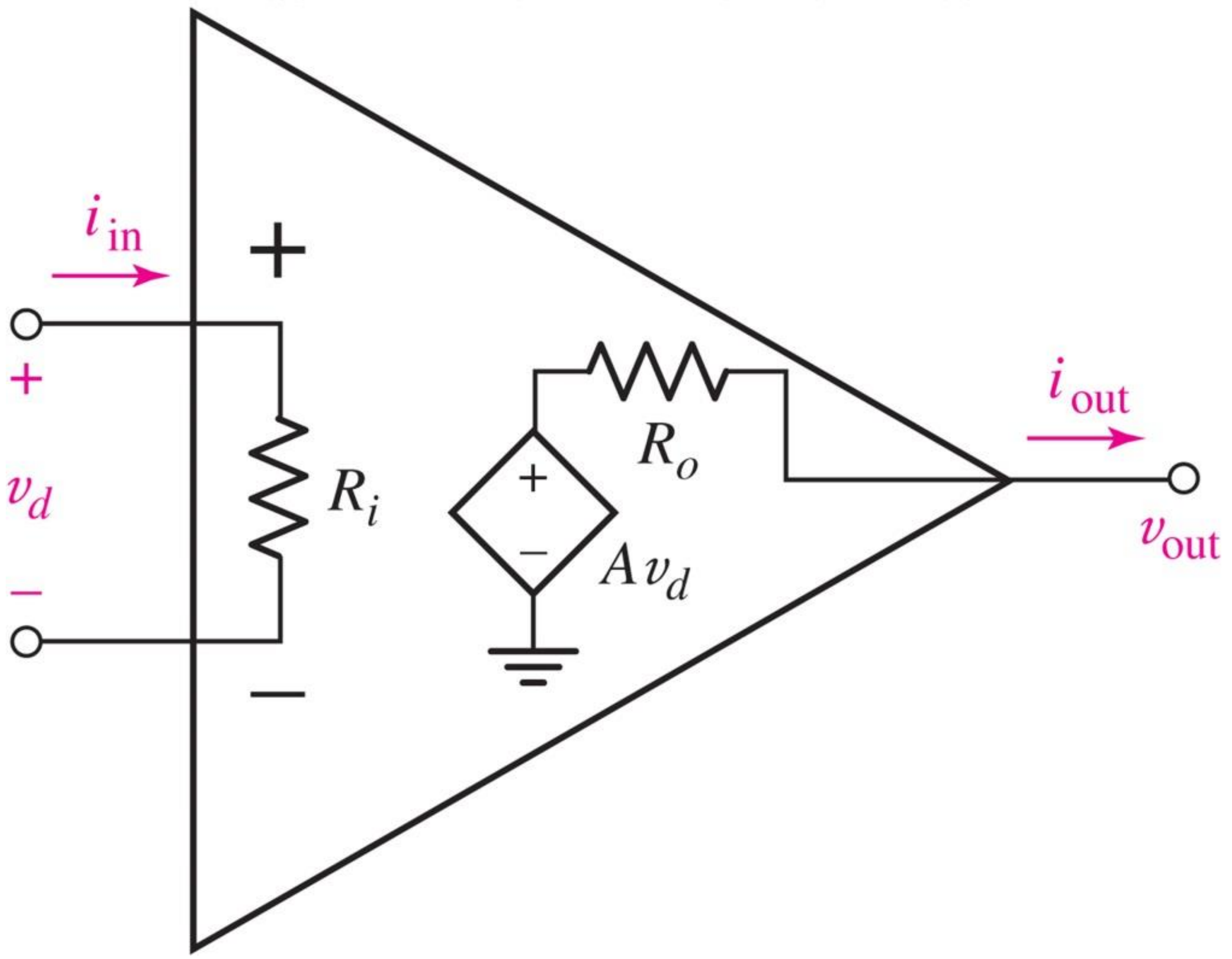
- ✓ **High Input Resistance**:(in the range of mega-ohms to tera-ohms)
  - ➢ To ensure that the maximum voltage of the source is applied across its input.
- ✓ **Low Output Resistance**:(in the range of a few ohms to a few tens of ohms)
  - ➢ to ensure that the entire output voltage of the amplifier is delivered to the load.
- ✓ **Adjustable Gain**:(achieved by applying feedback resistance):
  - ➢ As observed in the circuit shown, provided that A is sufficiently large, the gain of the amplifier can be approximated with high accuracy as $Av \approx -\frac{R_f}{R_1}$.

Ideal Model

- In the ideal case, when $A = \infty$, $R_i = \infty$, and $R_o = 0$, the behaviour of the operational amplifier can be described as follows:

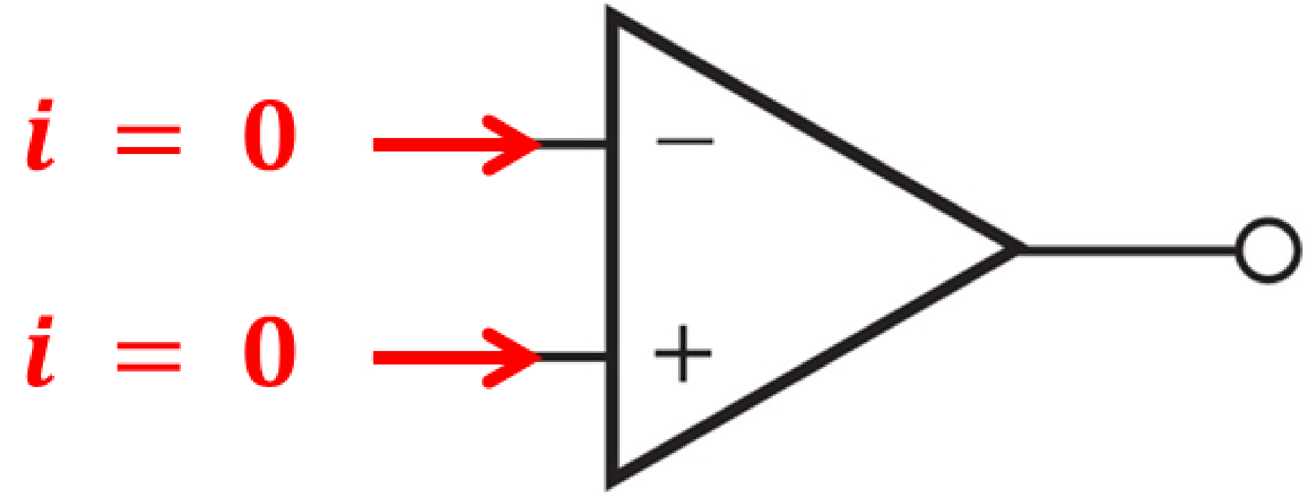
- Since v_{out} has a finite value (it cannot exceed the supply voltage), $v_d = \frac{v_{out}}{A} \approx 0$.

- As a result, $i_{in} \approx 0$.

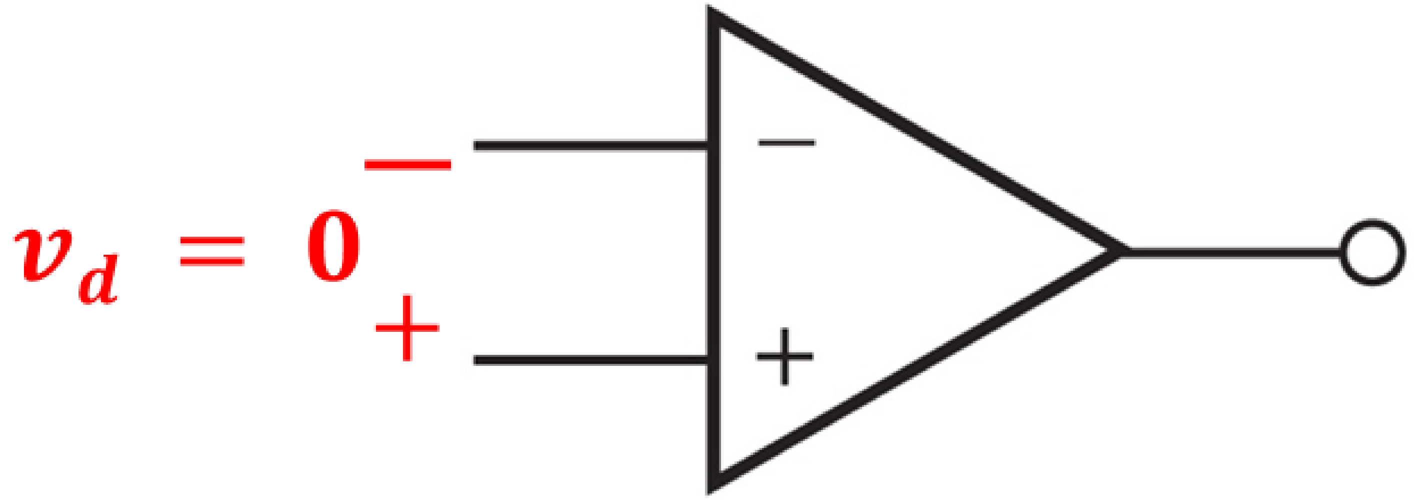


Rules of the Ideal Model:

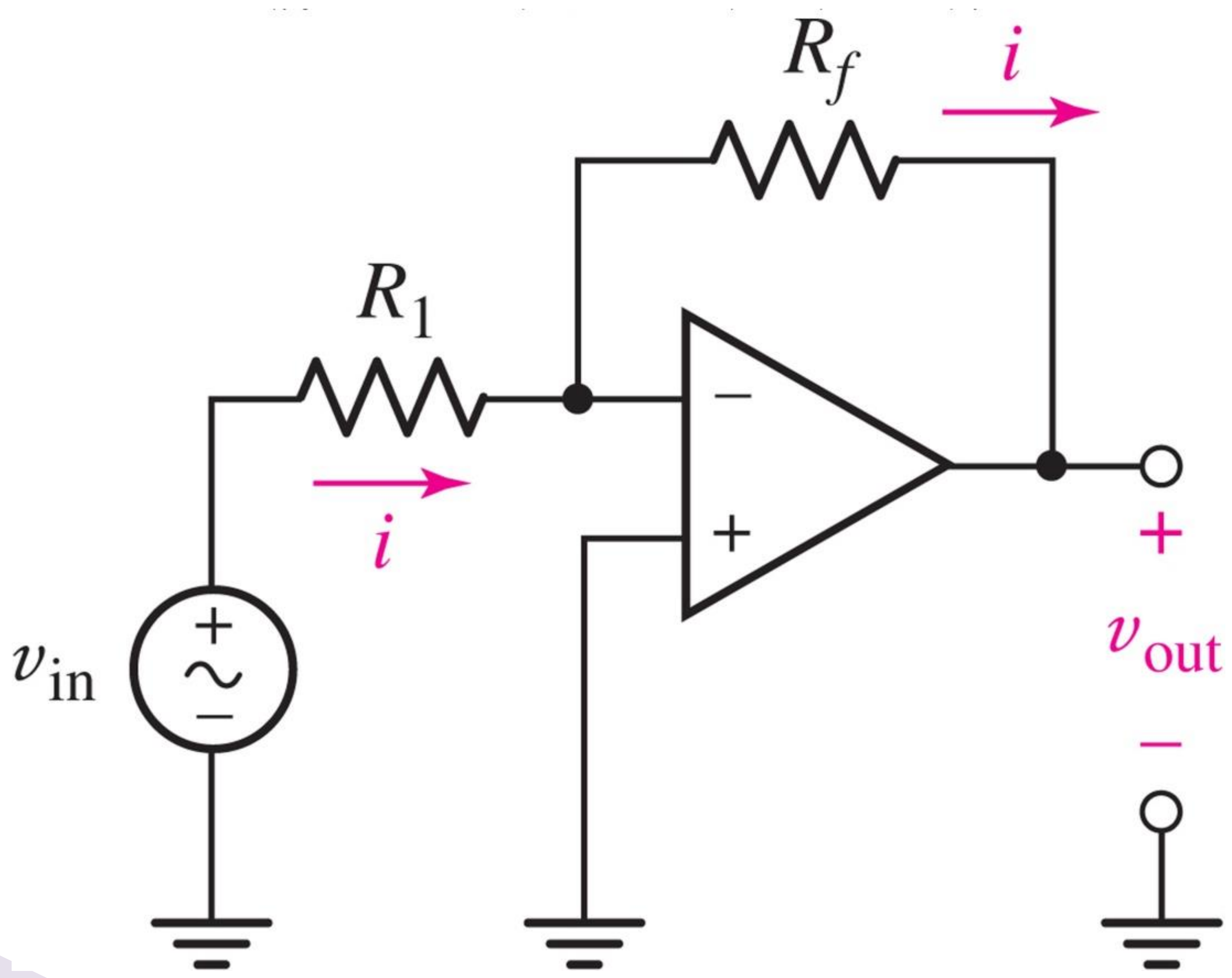
1.No current flows through the input terminals.



2.The voltage difference between the two input terminals is zero.



✓ By applying KVL and using the rules of the ideal operational amplifier, we have:

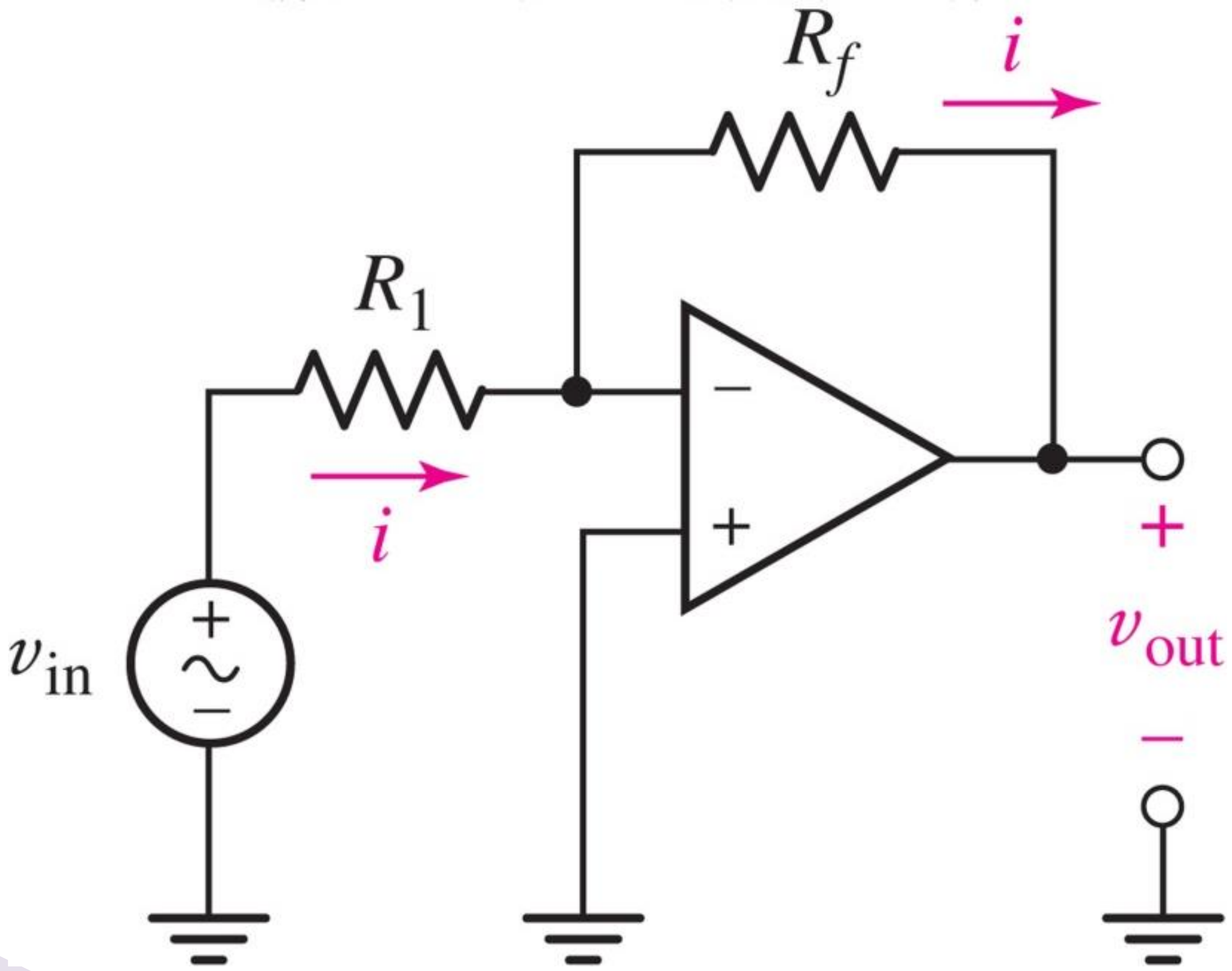


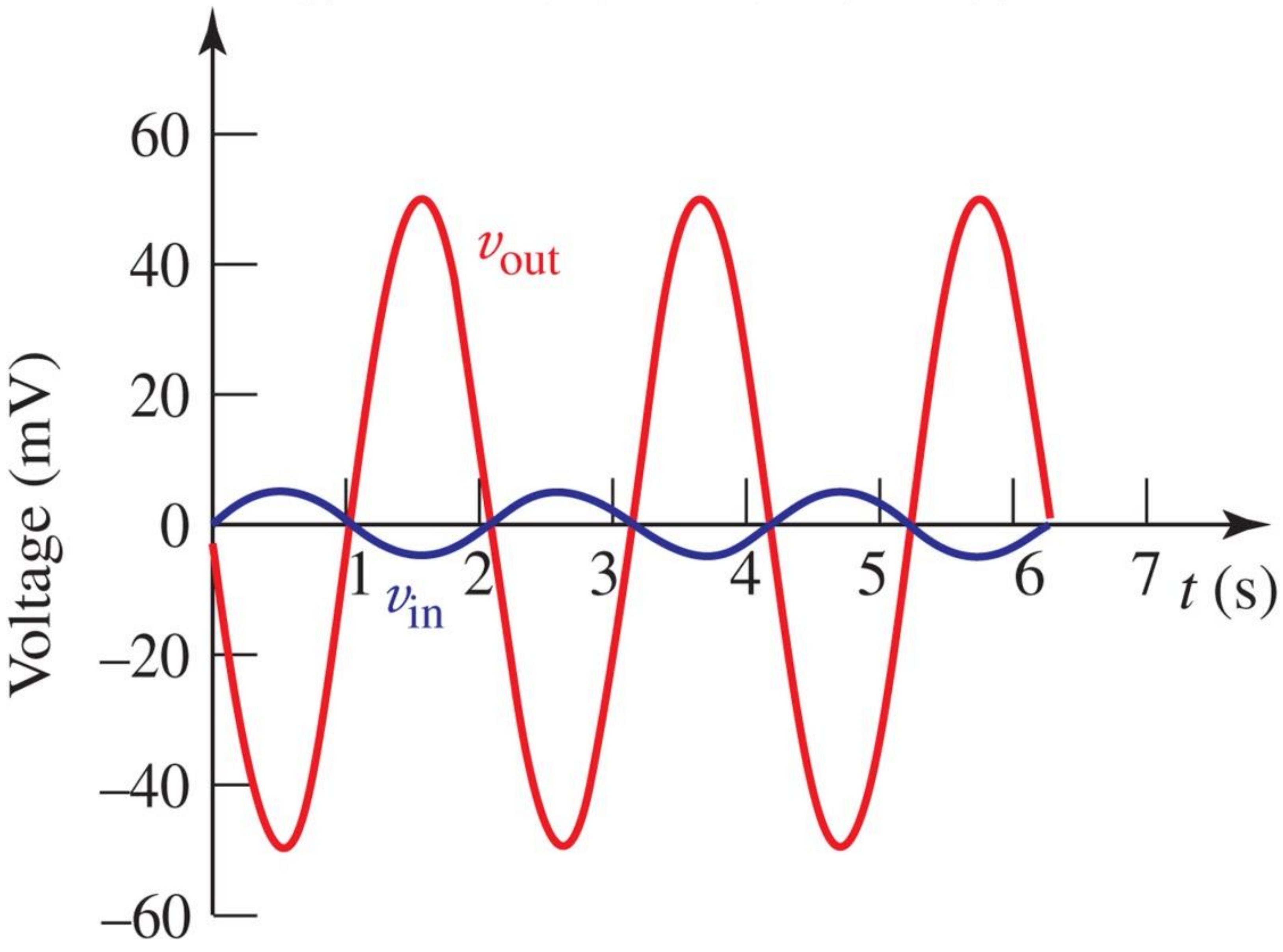
$$v_{out} = -\frac{R_f}{R_1} v_{in}$$

Example

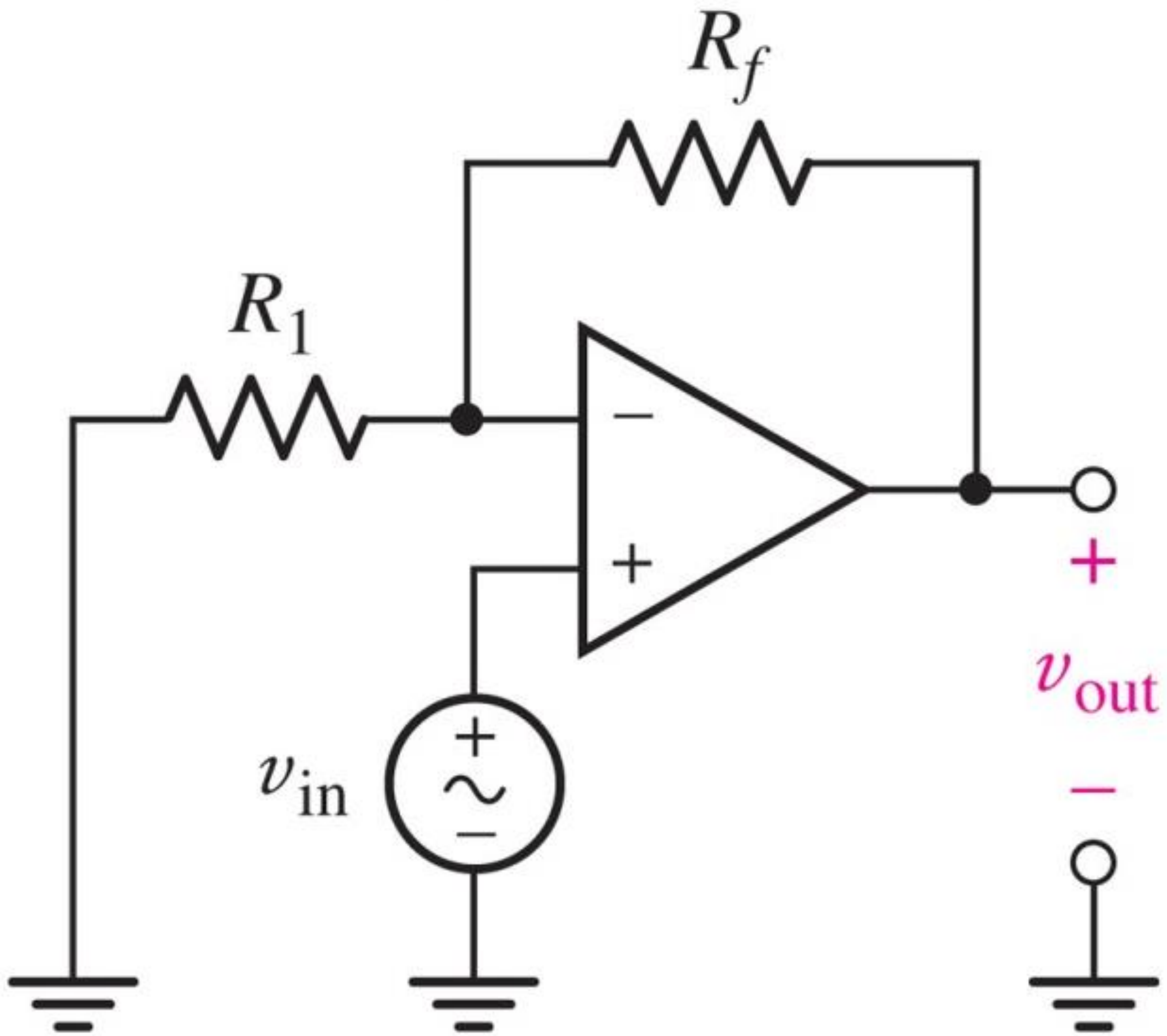
- $v_{in}(t) = 5\sin 3t\ mV, R_f = 47K\Omega, R_1 = 4.7K\Omega$

$$\boldsymbol{v_{out}(t) = -50\ sin\ 3t\ mV}$$



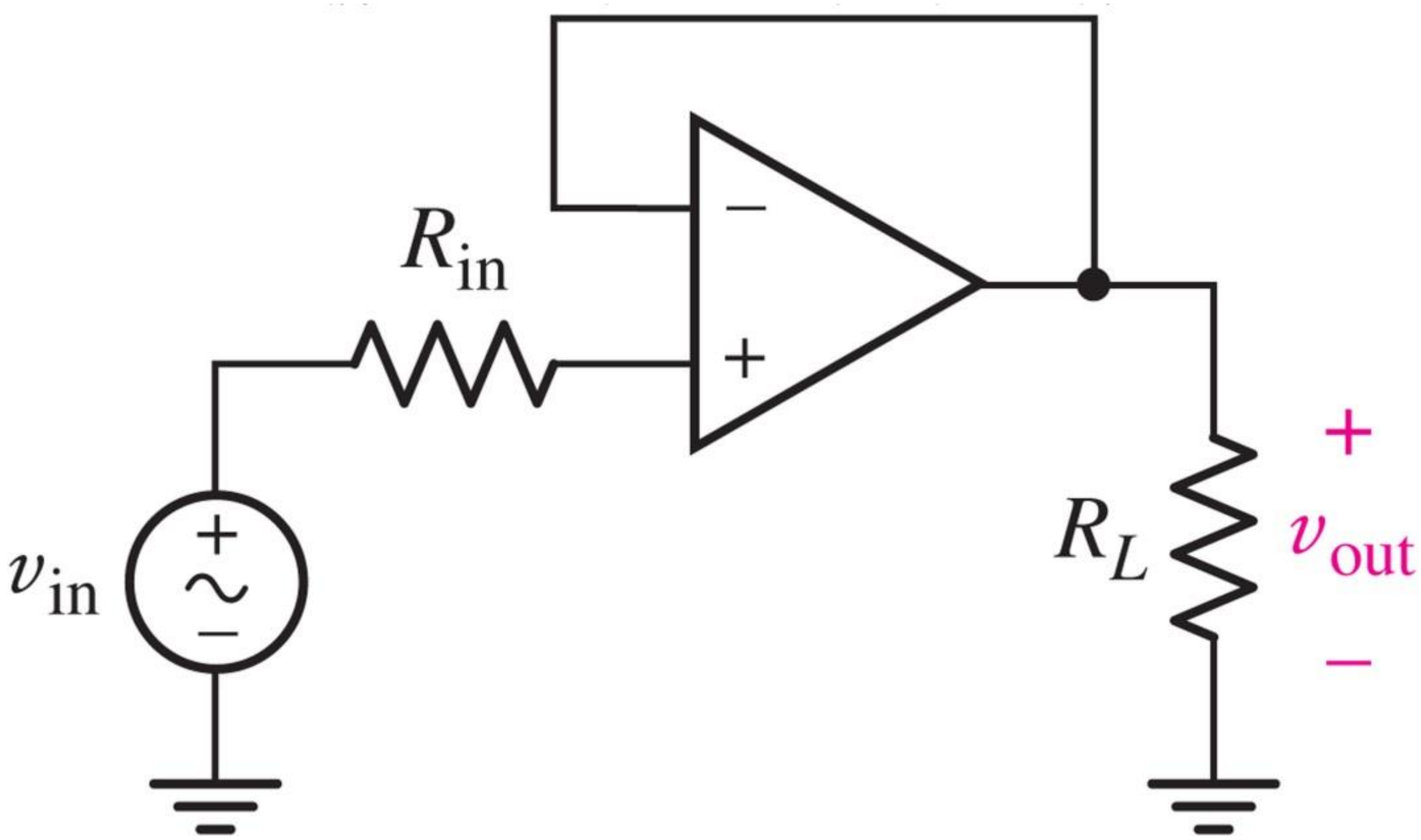


Non-Inverting Amplifier



$$v_{out} = \left(1 + \frac{R_f}{R_1}\right) v_{in}$$

Voltage Follower (Buffer)

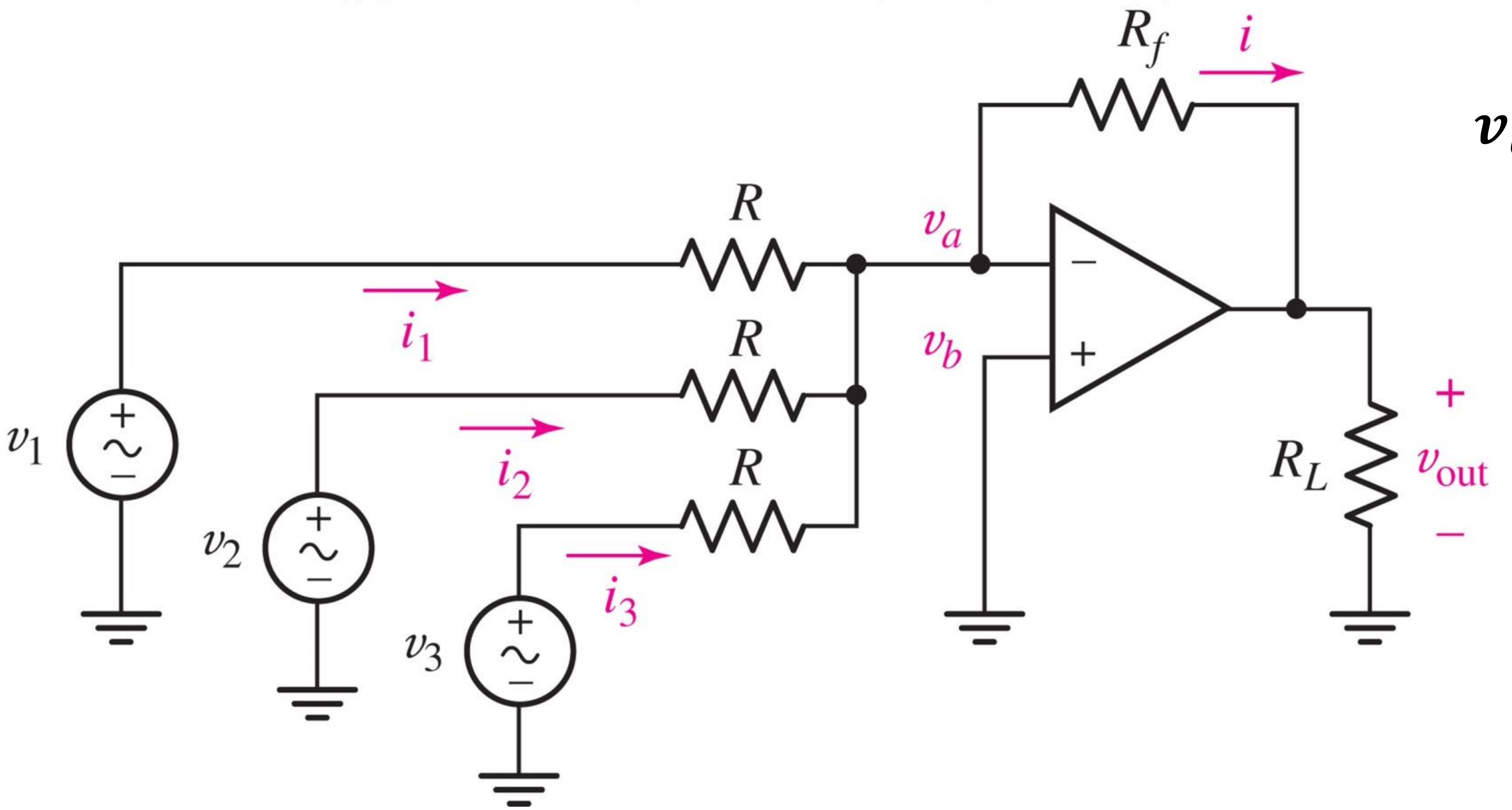


$$v_{out}(t) = vin(t)$$

1.The output voltage of the buffer is independent of the load resistance (R_L). The buffer can supply the required current to maintain a constant output voltage, even if R_L changes.

2.This circuit eliminates the effect of the source's input resistance by presenting a very high input resistance to the source and a very low output resistance to the load.

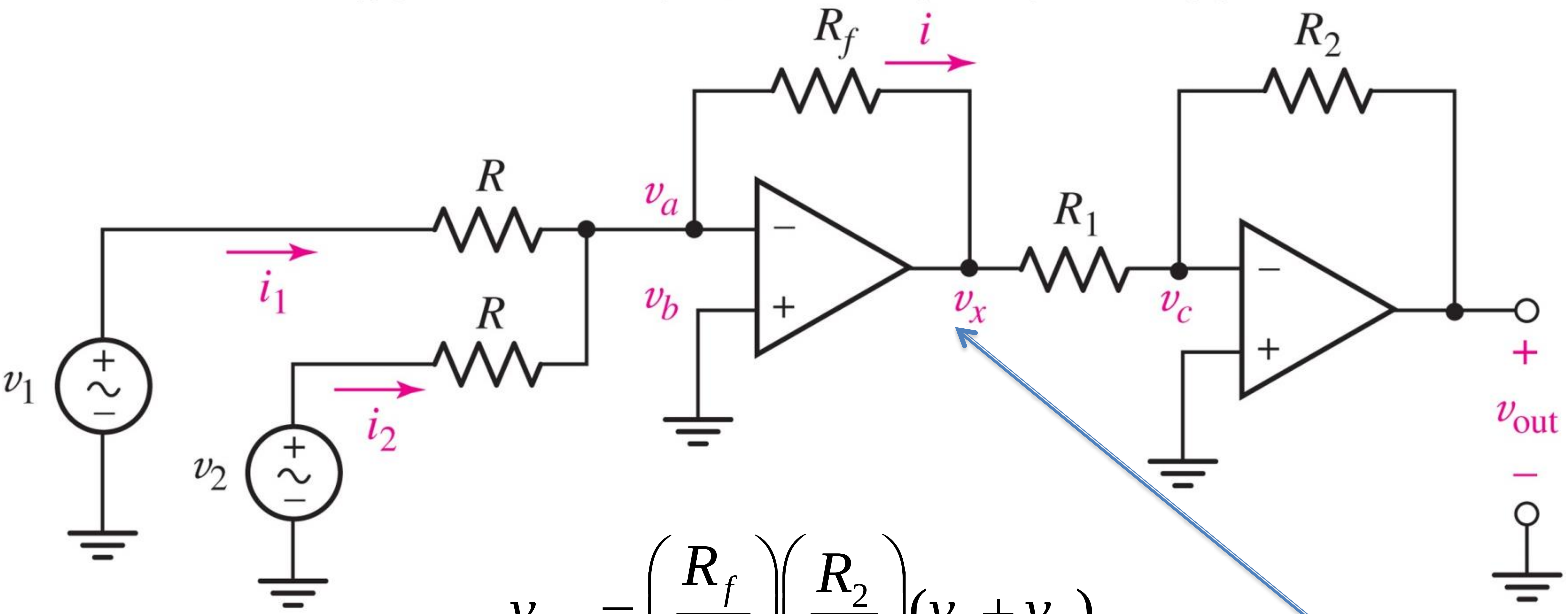
Analog Summing Circuit



$$\boldsymbol{v_{out} = -\frac{R_f}{R}(v_1 + v_2 + v_3)}$$

✓ This circuit performs the addition of input signals and amplifies the result by a factor of $-\frac{R_f}{R}$.

Cascading Multiple Operational Amplifiers

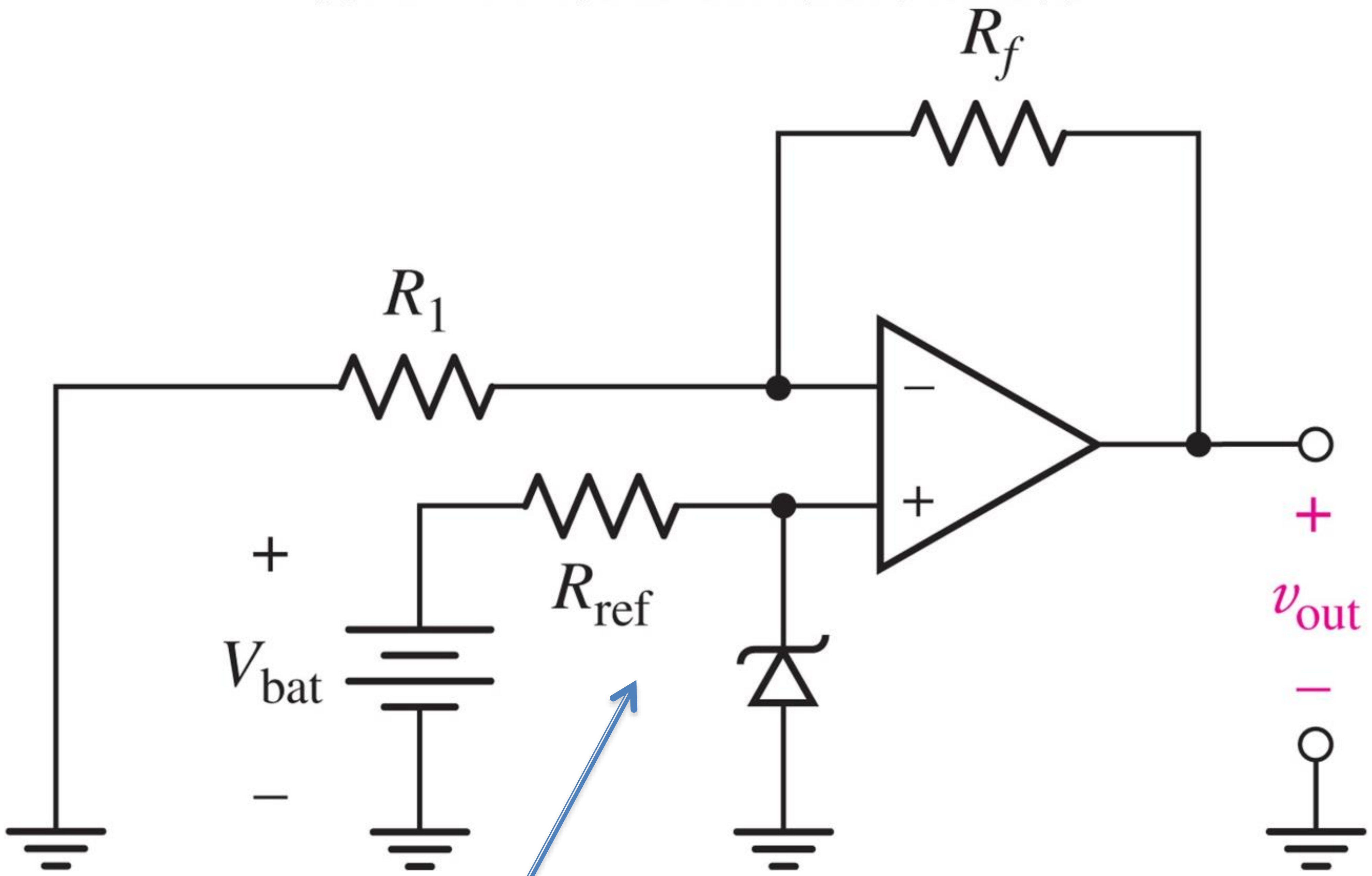


$$v_{out} = \left(\frac{R_f}{R}\right)\left(\frac{R_2}{R_1}\right)(v_1 + v_2)$$

- ✓ The output voltage of each stage is independent of the subsequent stage. **(Why?)**
- ✓ As a result, **Op-Amp** can be cascaded without any alteration in their individual gains.

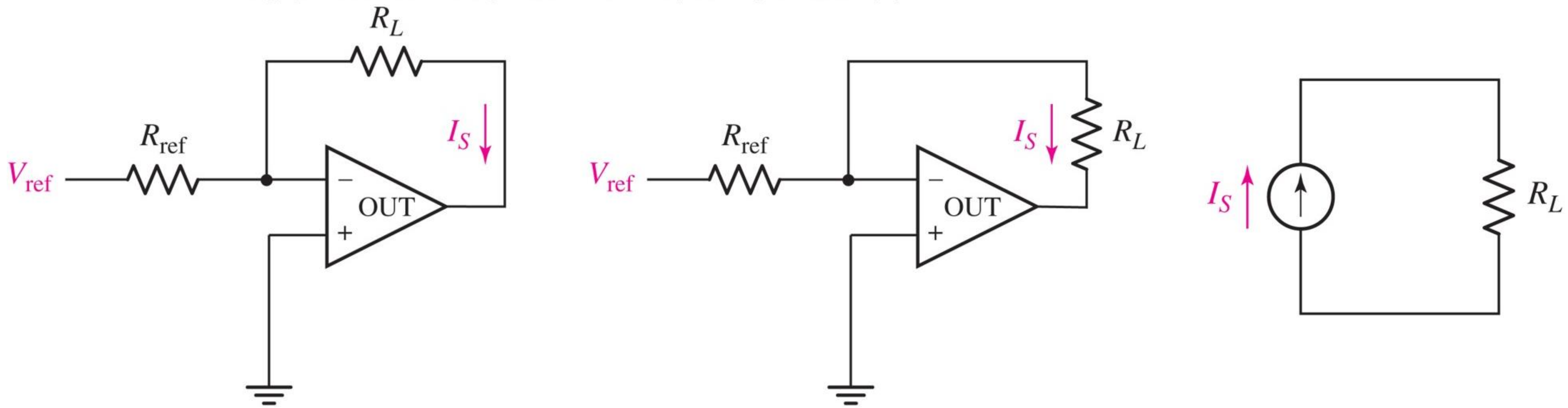
Op-Amp as an Ideal Voltage Source

$$v_{out} = \left(1 + \frac{R_f}{R_1}\right) V_Z$$



This circuit resembles the voltage regulator using a Zener diode, which we have encountered before.

Op-Amp as an Ideal Current Source



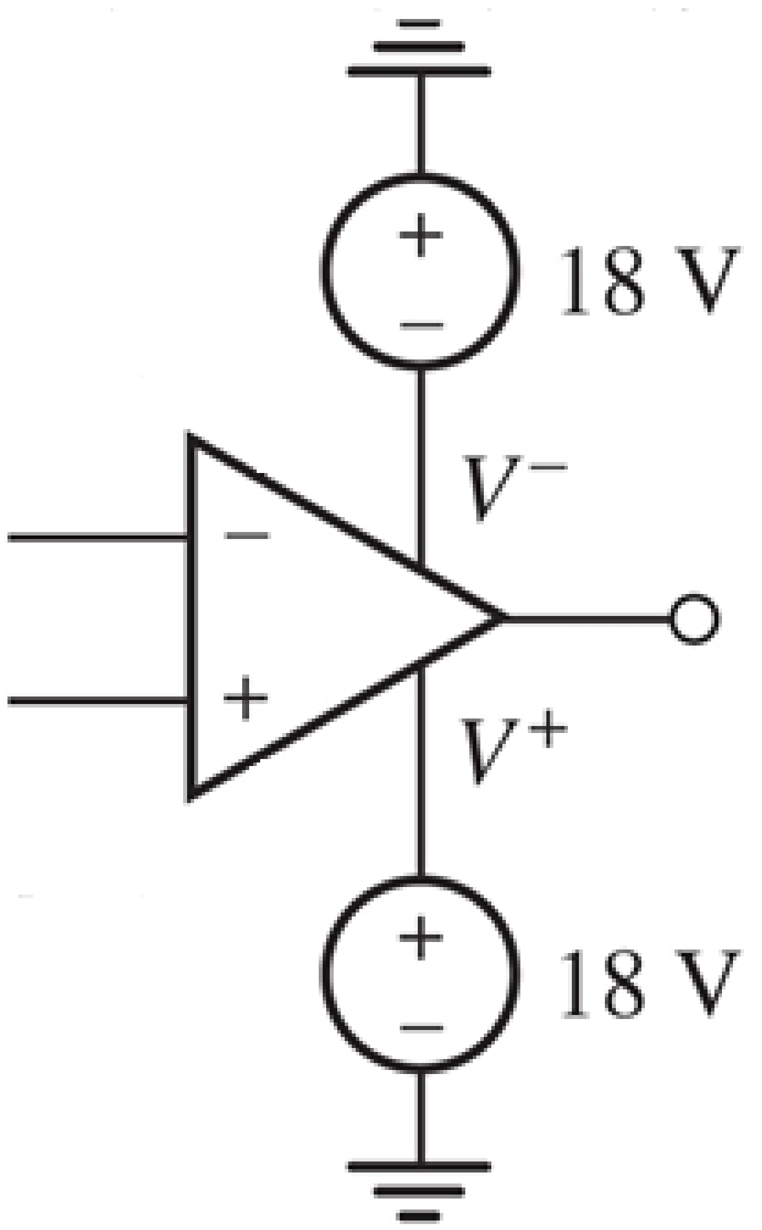
- Using a reference voltage V_{ref} and a resistor V_{ref}, an ideal current source can be constructed with a current:

$$I_S = \frac{V_{ref}}{R_{ref}}$$

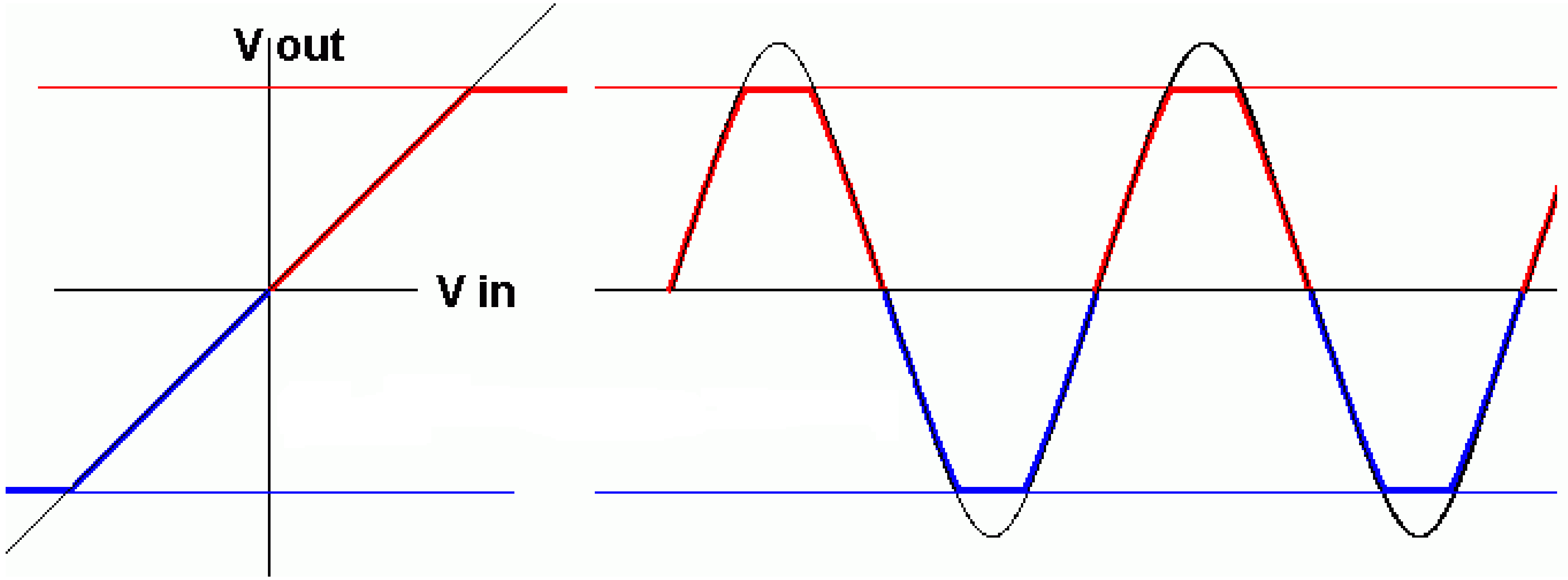
- This current is independent of the load resistance R_L,

Power Supplies for Op-Amp

- An operational amplifier requires power to function and perform amplification. Therefore, it must be connected to a power supply.
- Typically, equal and opposite voltage values are applied to the V^+ and V^- terminals. These voltage levels usually range between ±5V and ±24V.
- The ground of the power supplies must be connected to the input and output ground reference of the circuit to ensure proper operation and signal integrity.



- ✓ If the input signal is such that the amplified output exceeds V^+ or falls below V^-, the output will saturate.
- ✓ In this state, the output voltage remains clamped at V^+ or V^-, depending on the direction of the input signal.



The output signal enters saturation and becomes clipped

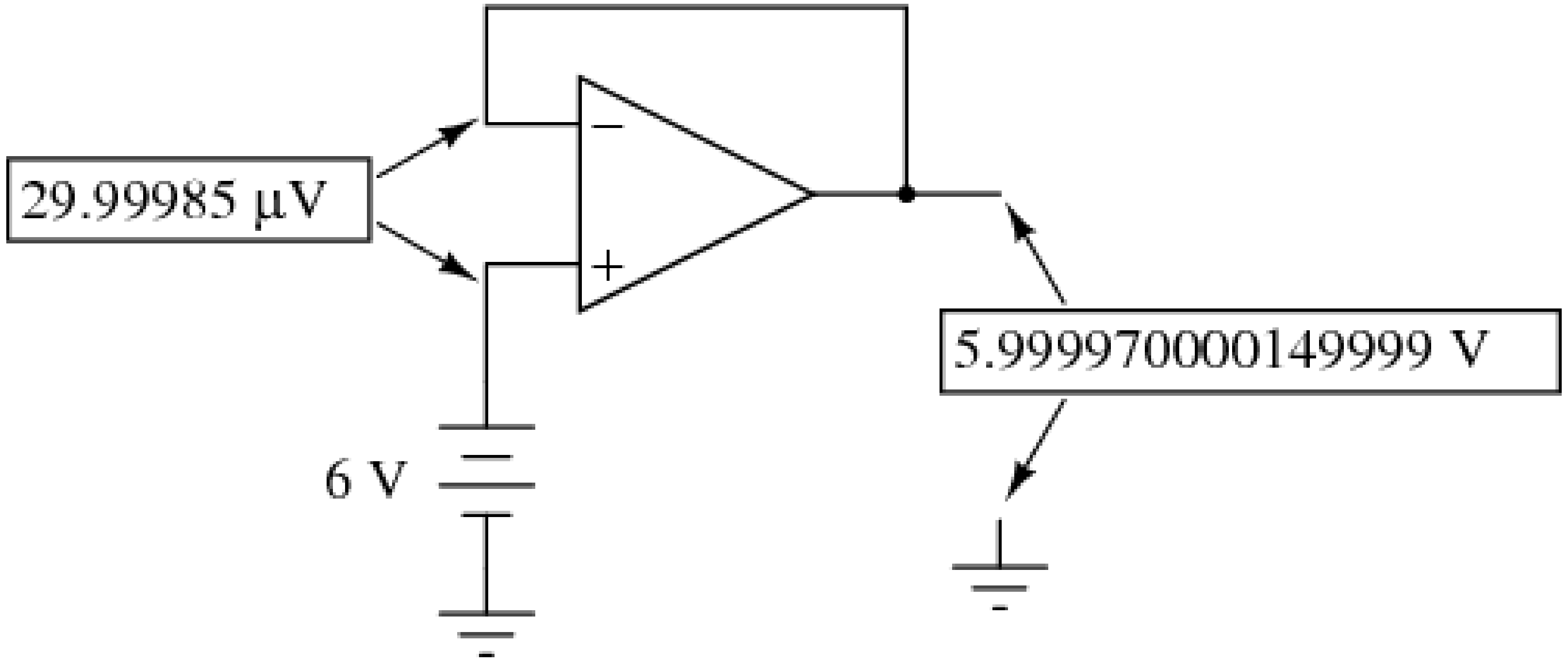
Role of Negative Feedback

- The role of negative feedback is to transform a large and undefined gain into a fixed, controlled gain.

- Why Don't We Apply Feedback to the Positive Terminal?

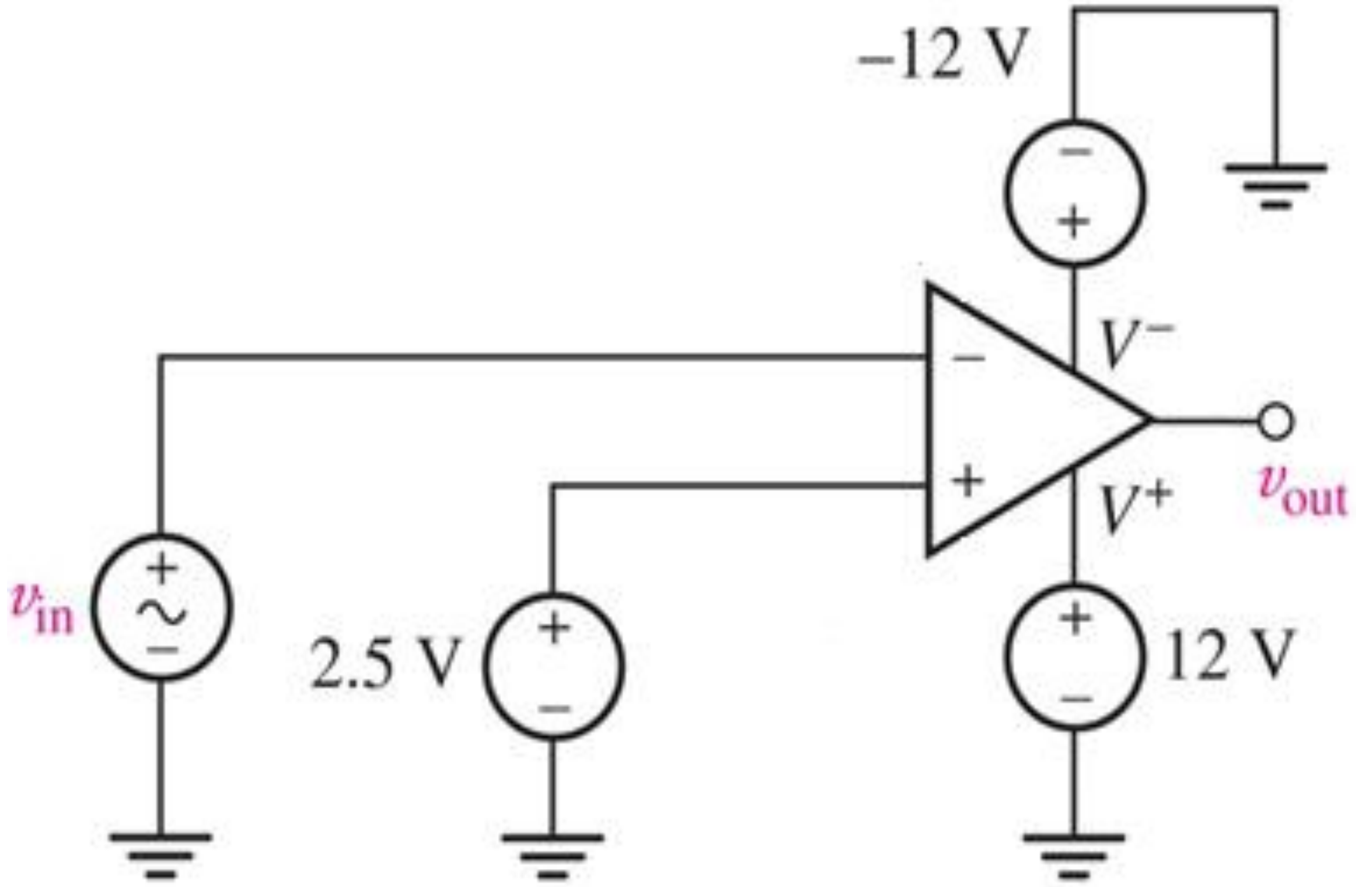
$$\boldsymbol{v_+} \uparrow \;\rightarrow\; \boldsymbol{v_d} \uparrow \;\xrightarrow{\boldsymbol{v_{out}} = \boldsymbol{A v_d}}\; \boldsymbol{v_{out}} \uparrow \;\rightarrow\; \boldsymbol{v_-} \uparrow \;\rightarrow\; \boldsymbol{v_d} \downarrow \;\; \ldots$$

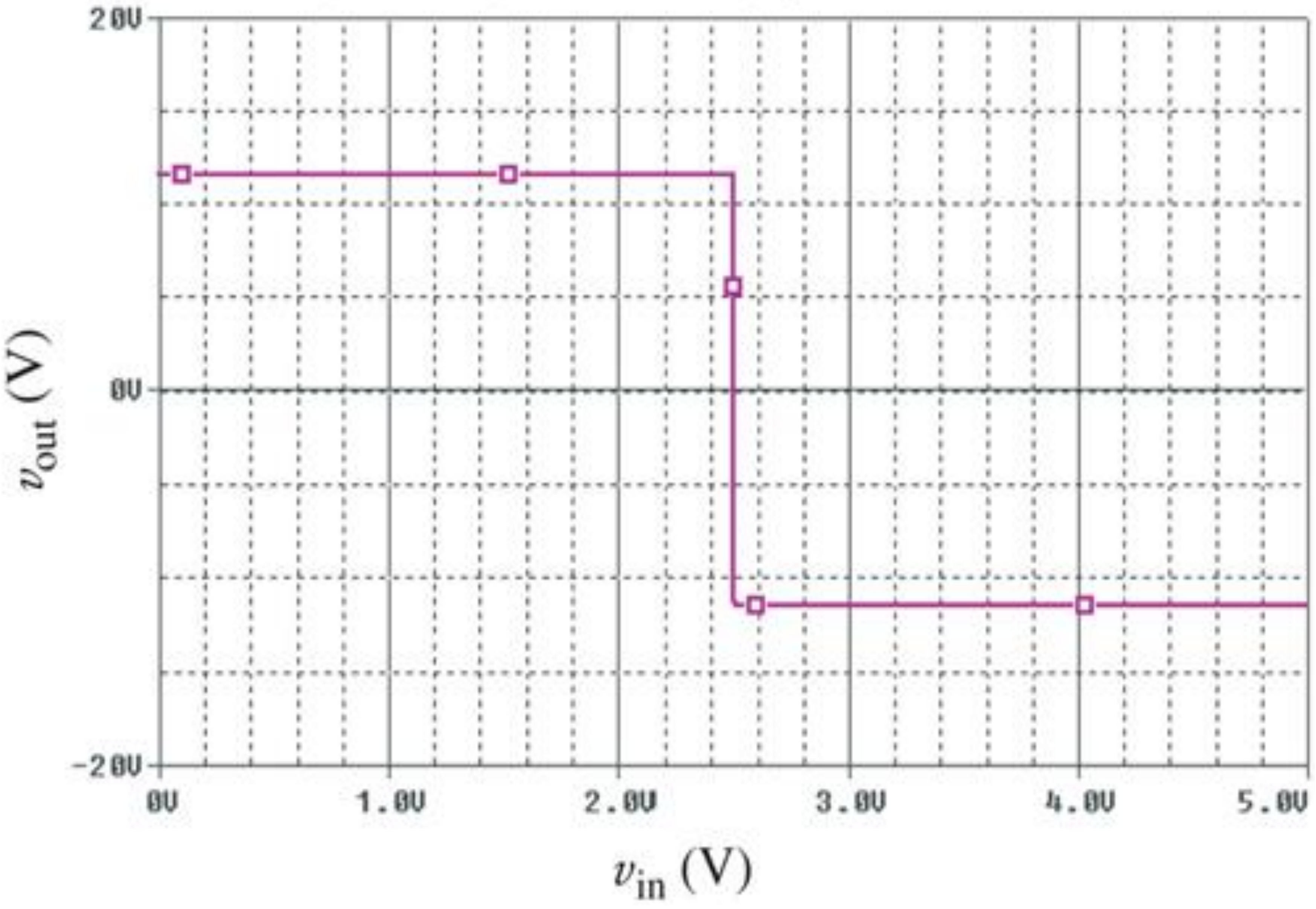
- Thus, the negative feedback stabilizes the operation by keeping v_d close to zero



- An operational amplifier can be used as a voltage comparator when operated in open-loop configuration (without feedback).

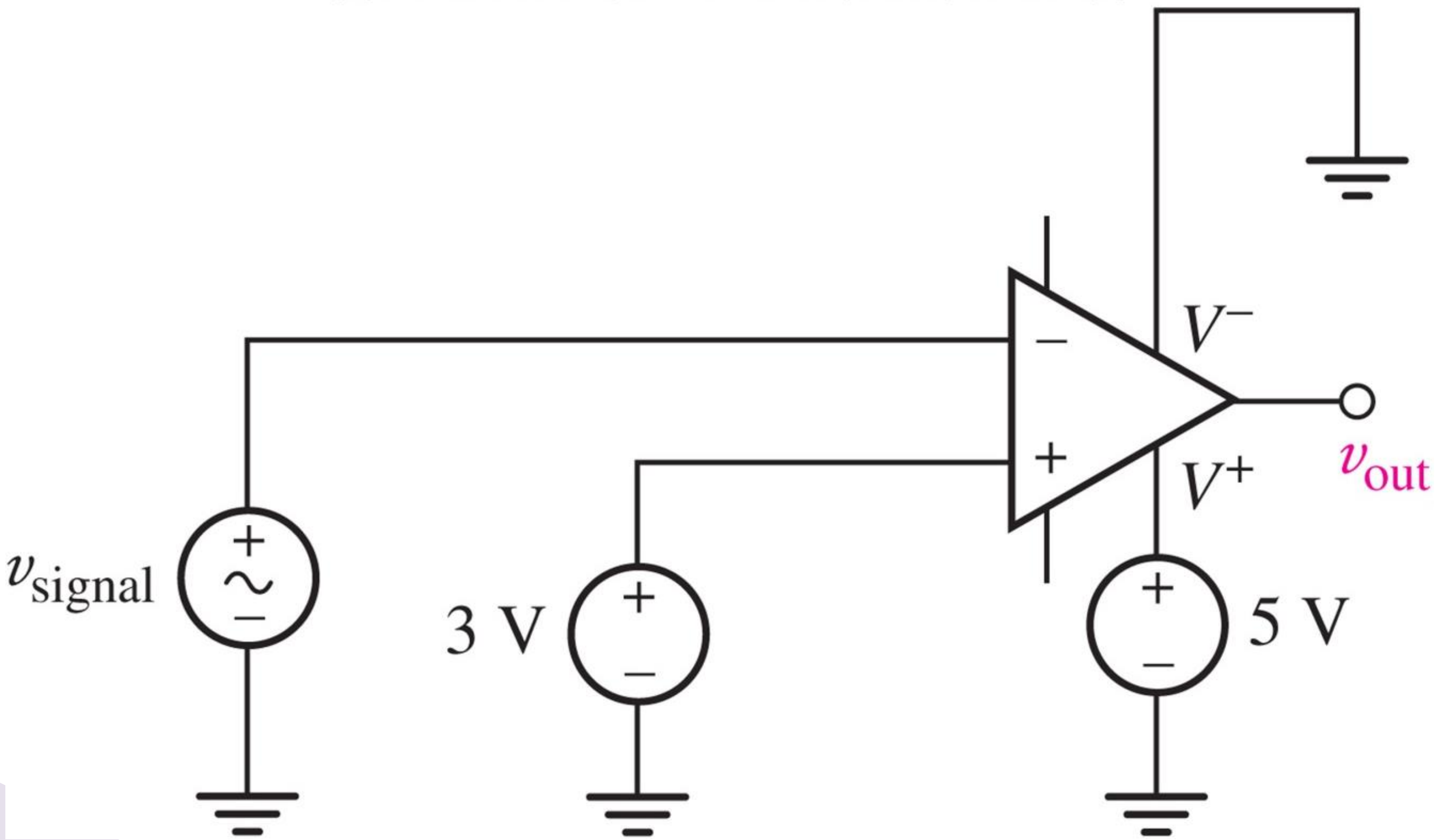
$$v_{out} = \begin{cases} 12 & v_{in} < 2.5 \\ -12 & v_{in} > 2.5 \end{cases}$$

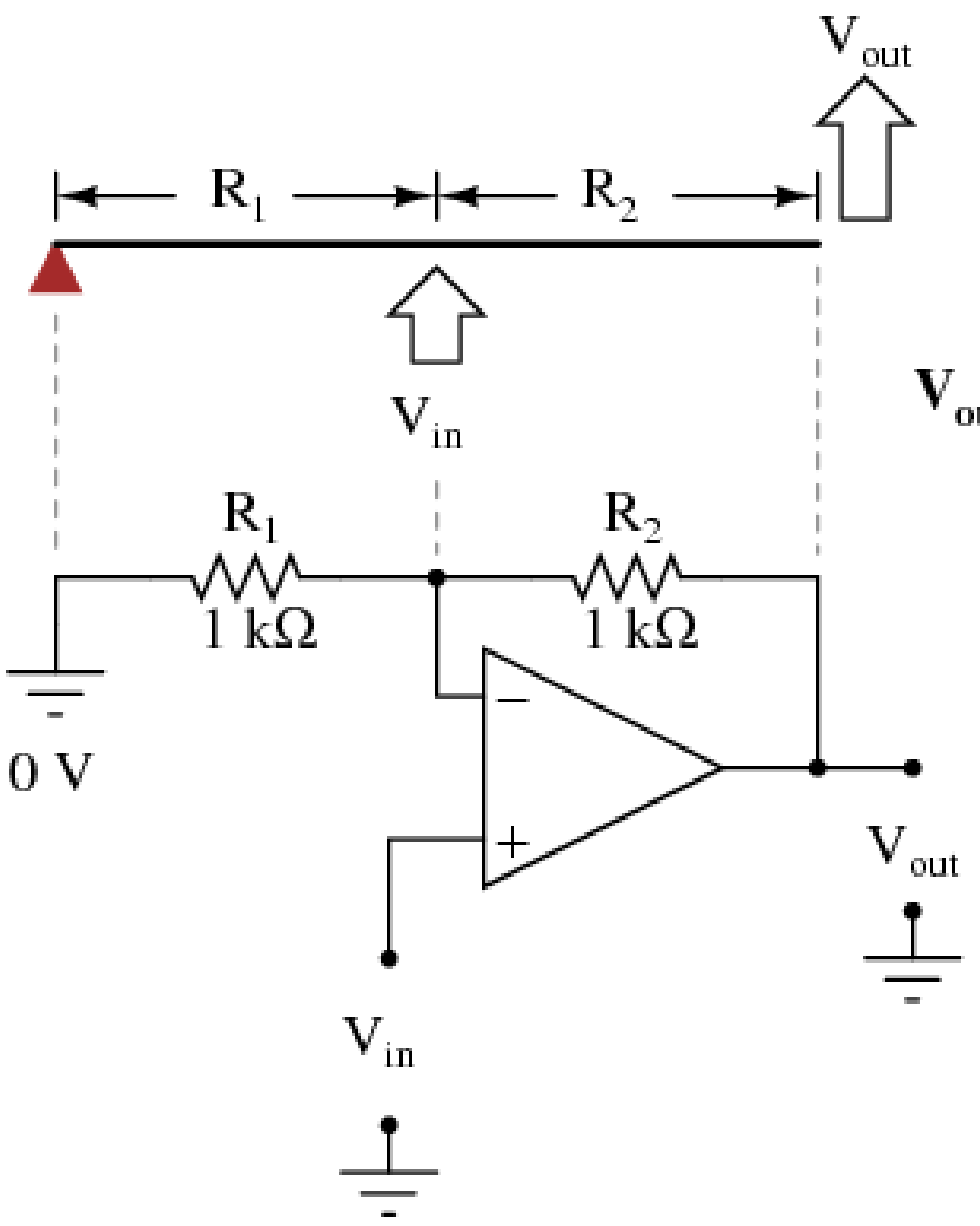




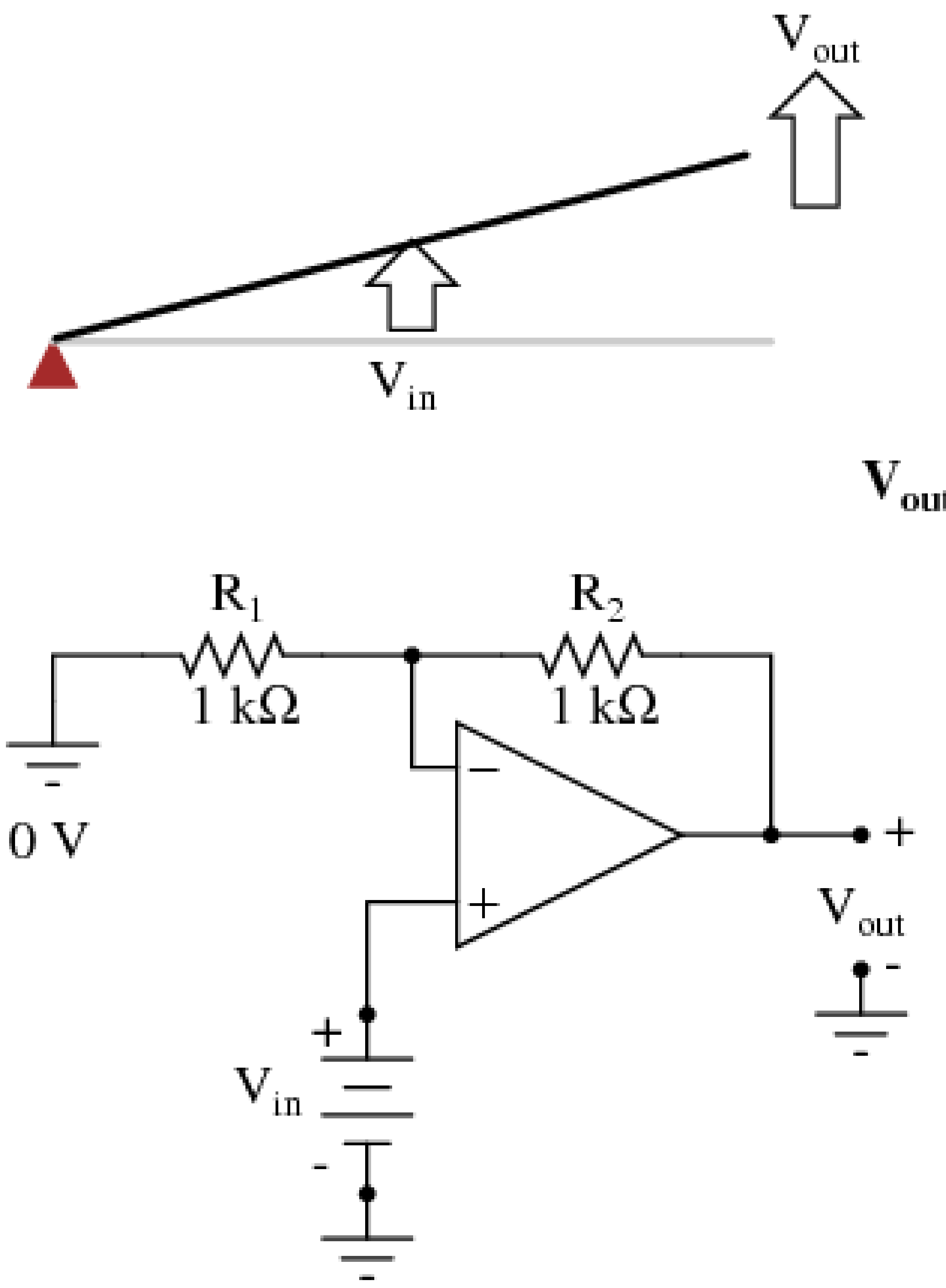
Example

Assume we have a temperature sensor that generates a voltage ranging from 0 to 5 volts for temperatures between 0 and 100 degrees Celsius. Design a circuit that provides a logical output of 1 when the temperature is below 60 degrees.



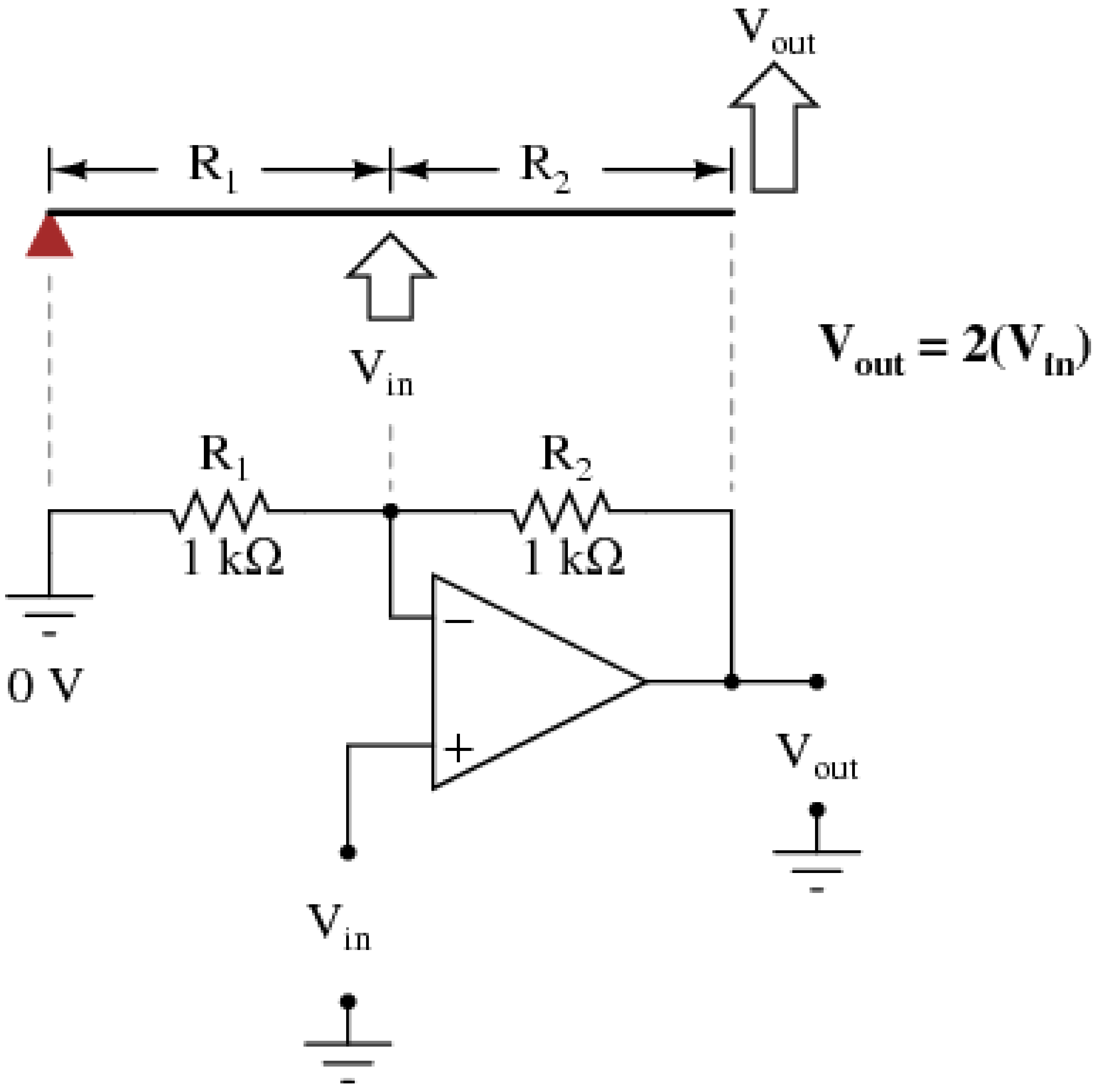


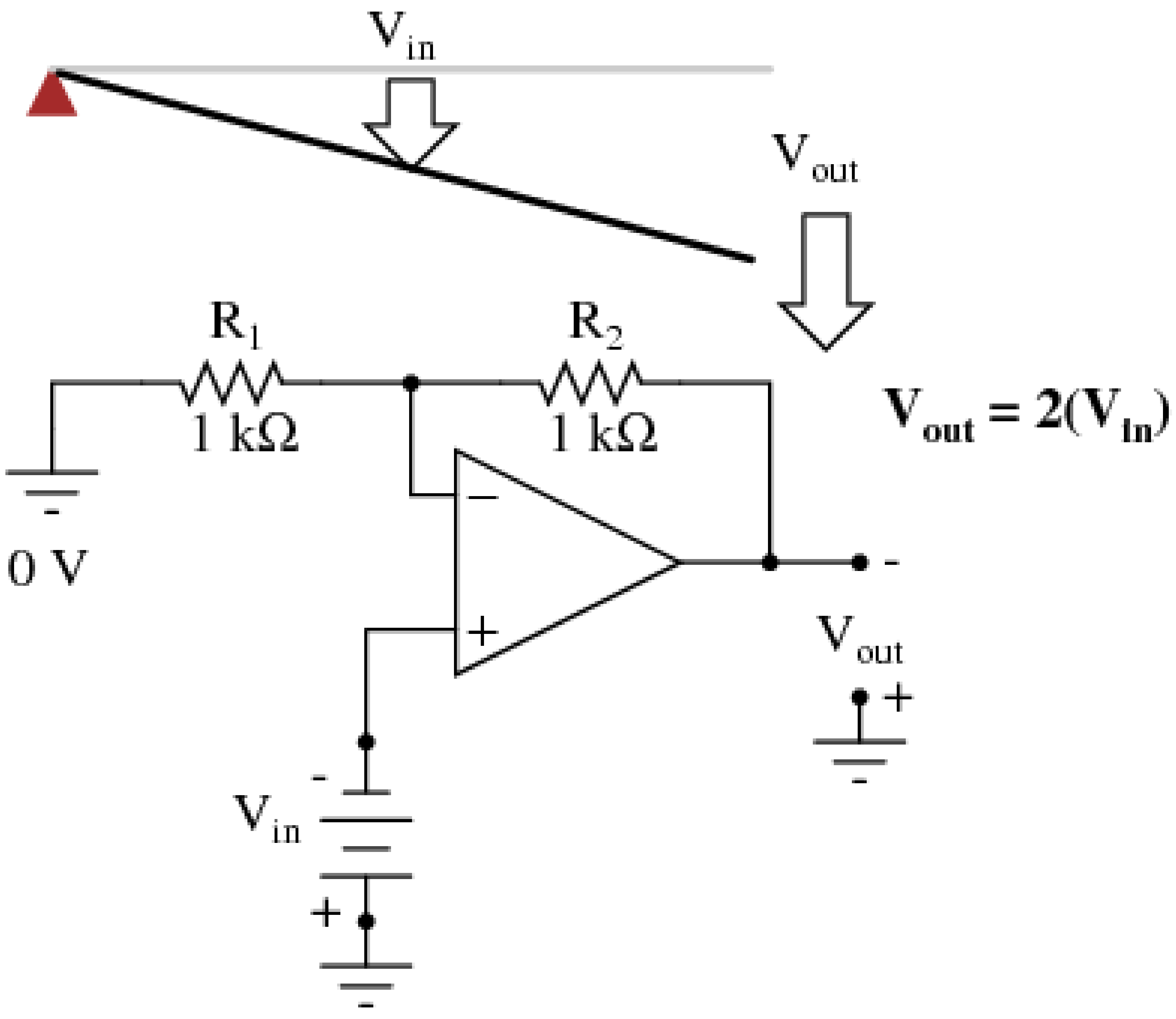
$$V_{out} = 2(V_{in})$$



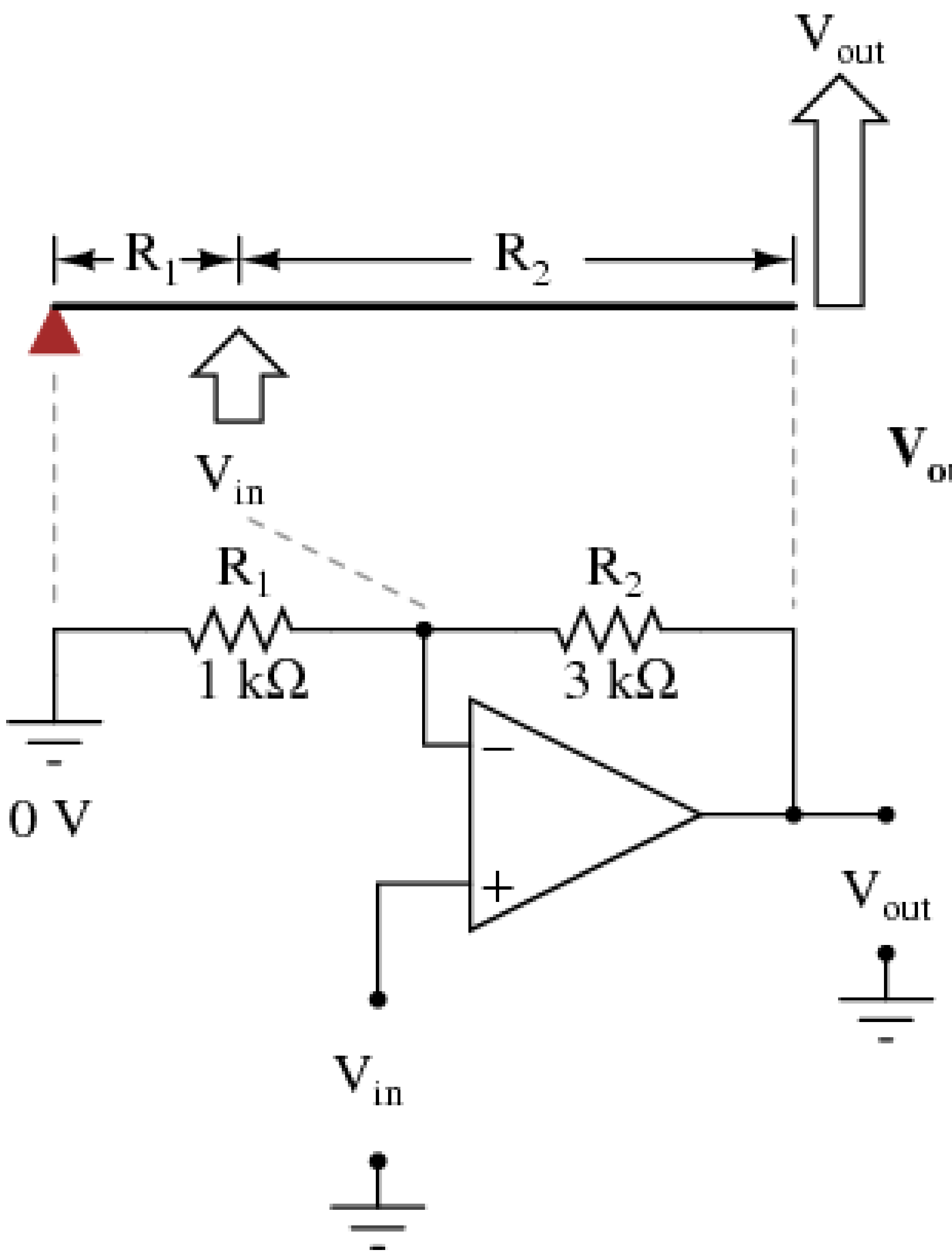
$$V_{out} = 2(V_{in})$$

Similarity of Non-Inverting Amplifier to a Lever

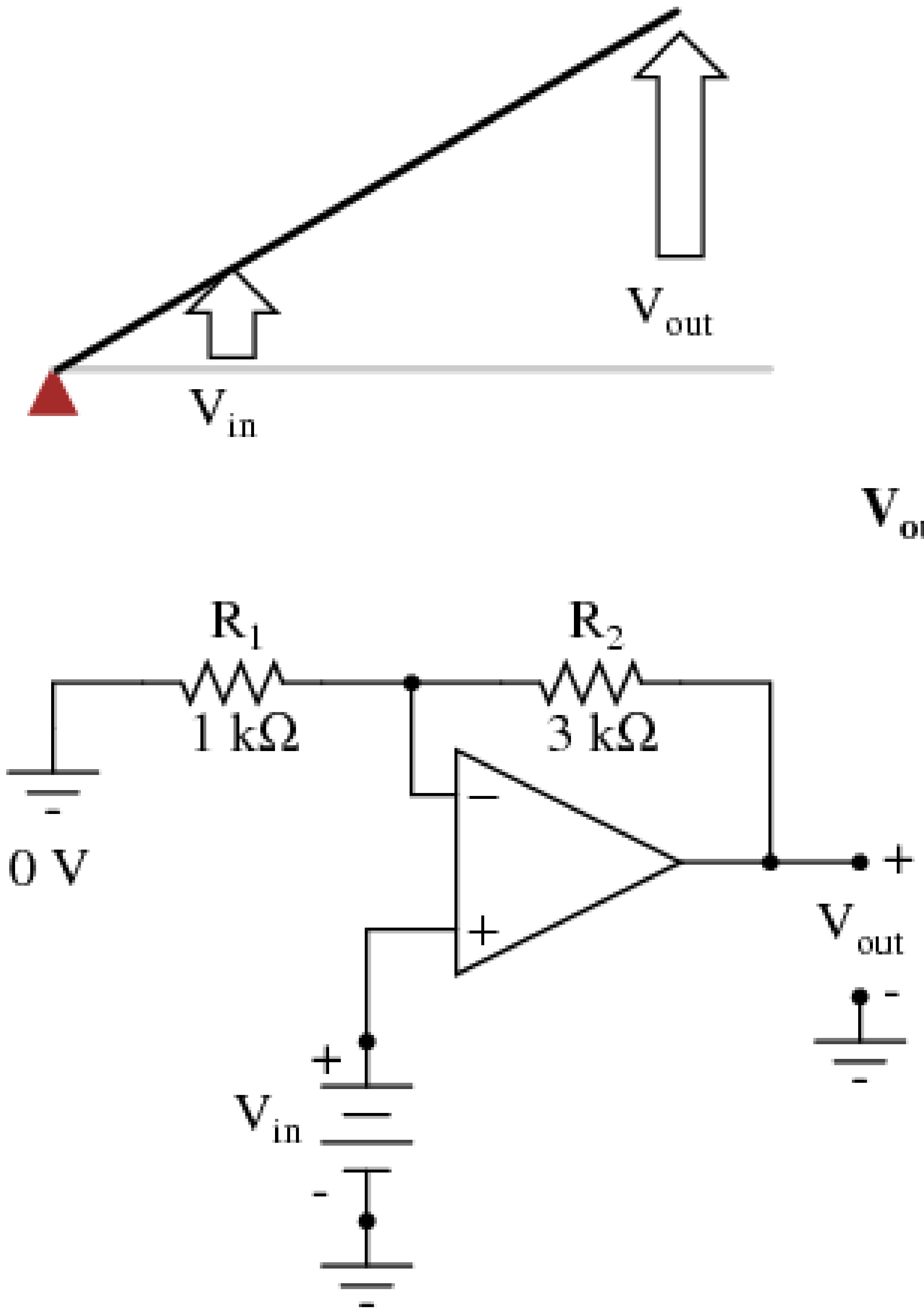




Similarity of Non-Inverting Amplifier to a Lever



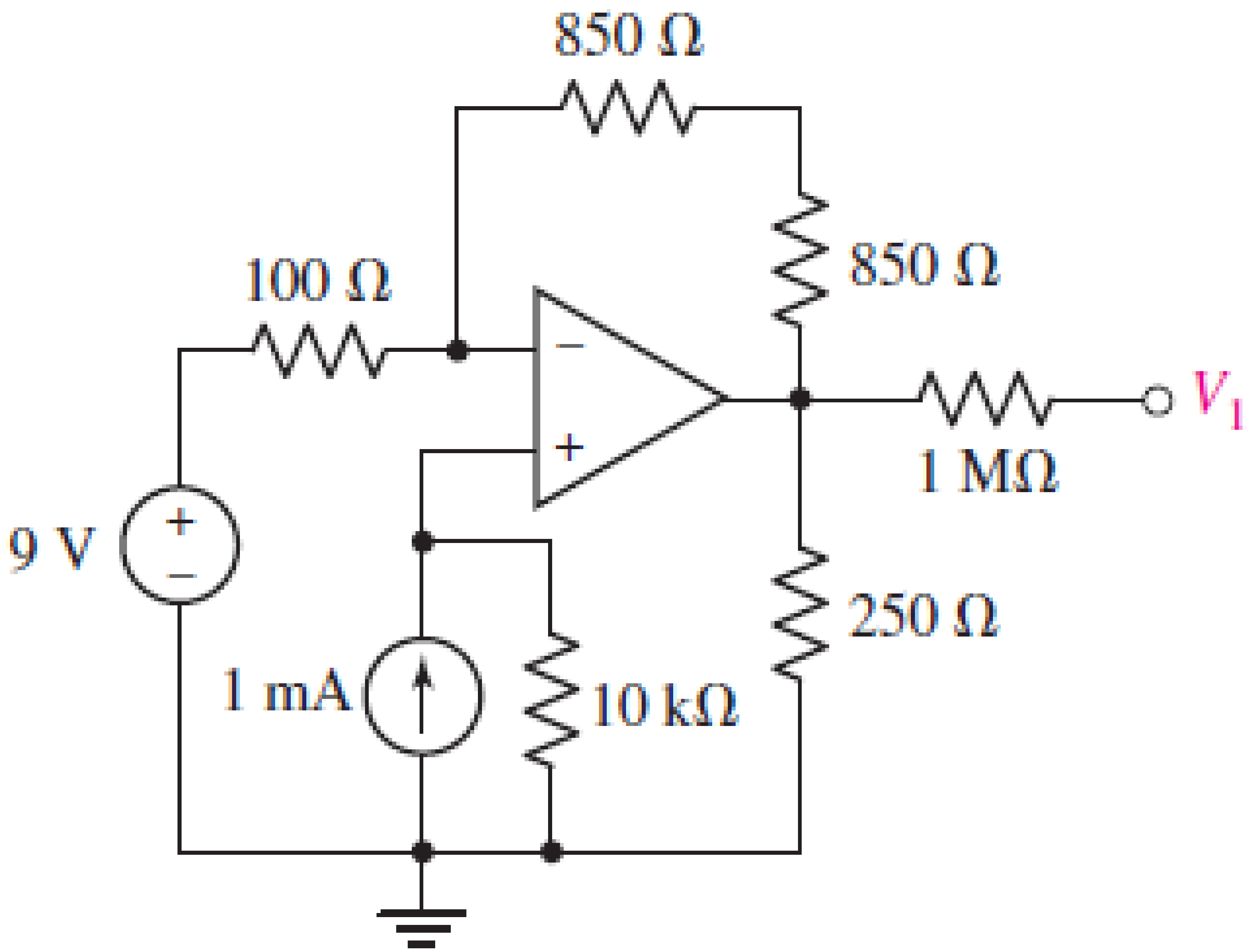
$V_{out} = 4(V_{in})$

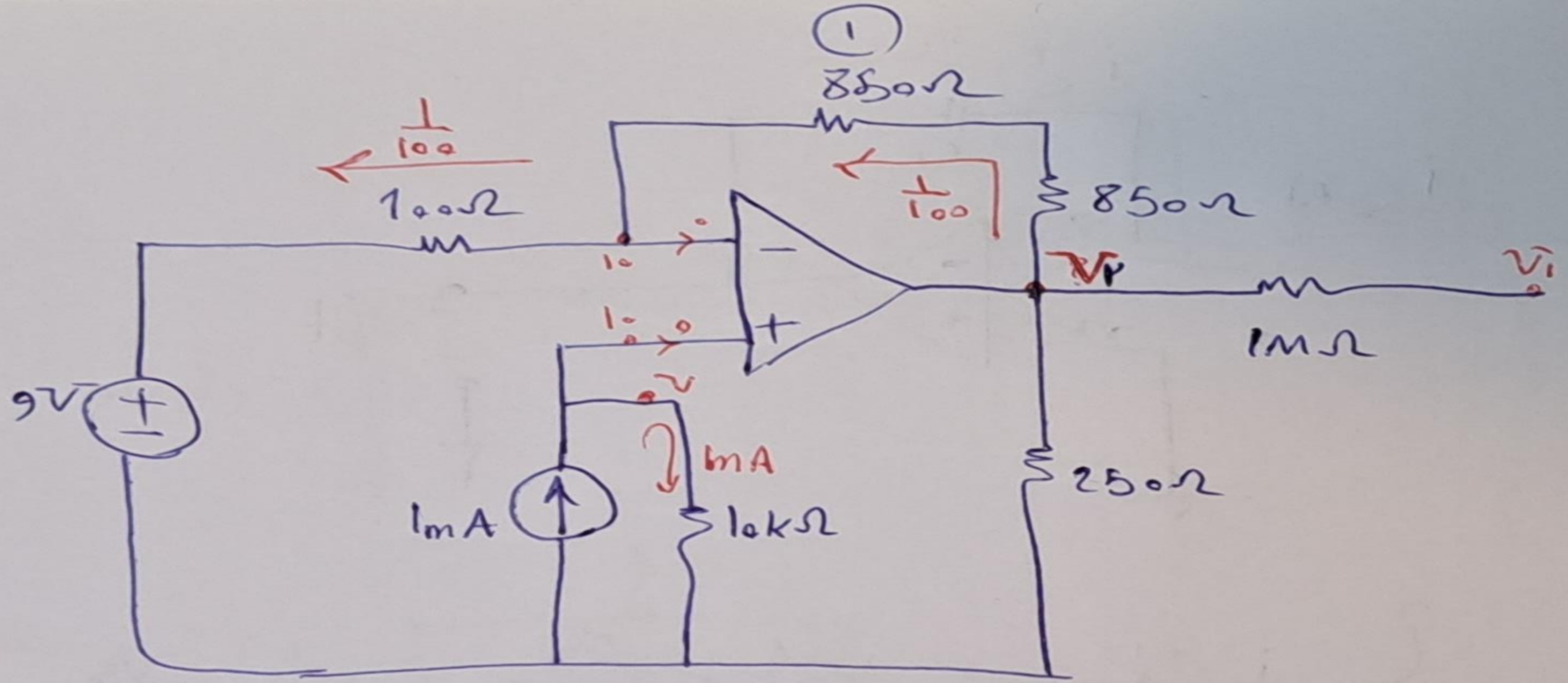


$V_{out} = 4(V_{in})$

Class Exercise 1

V_1=?





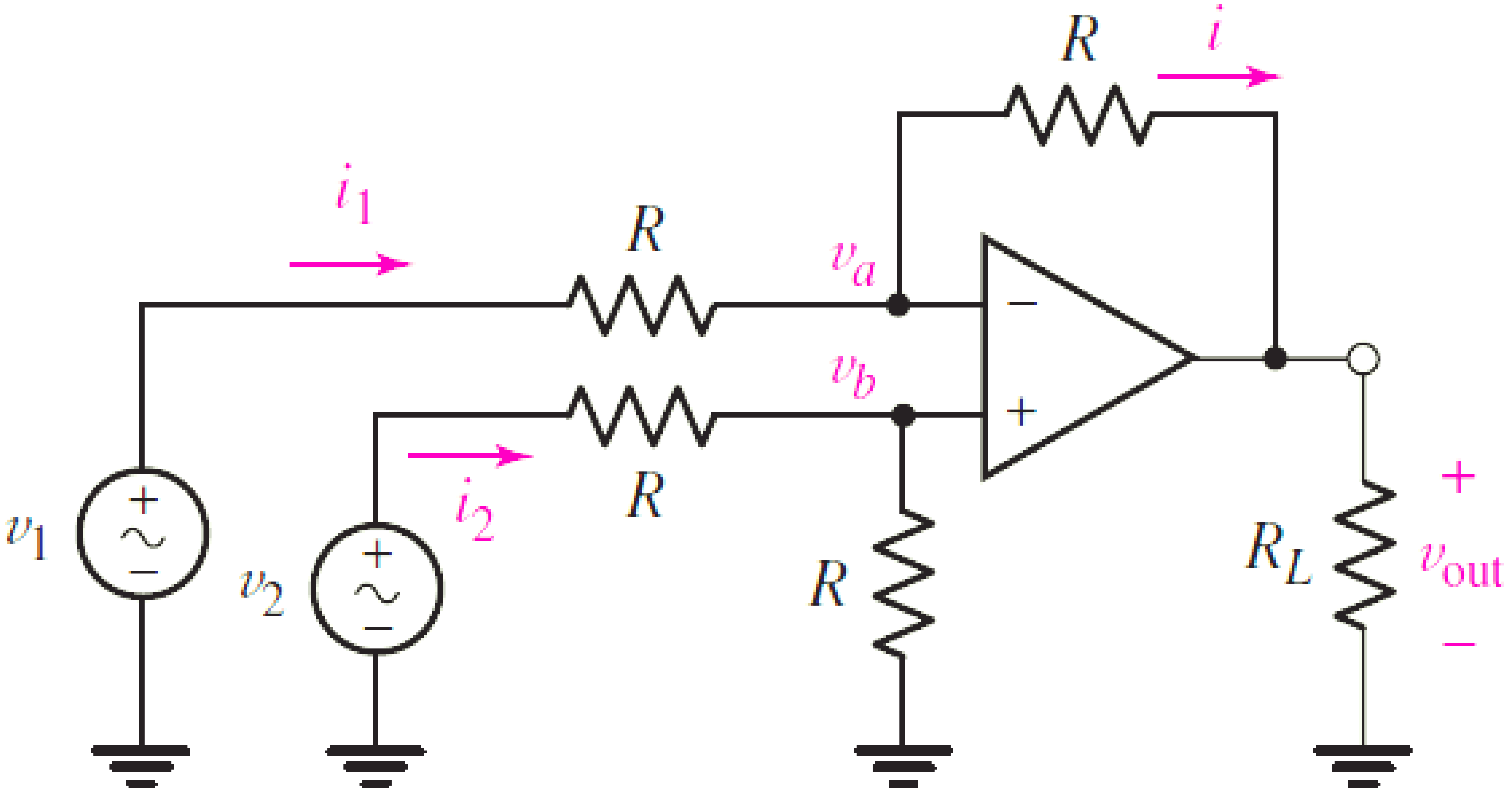
$$V = 10k \times 1m = 10v$$

$$\frac{V_2 - 10}{1700} = \frac{1}{100} \Rightarrow V_2 - 10 = 17 \Rightarrow V_2 = 27v$$

چونکه V_1 روی هواست و هیچ جریانی از آن عبور نمی‌کند $V_1 = V_2$ و برابر ۲۷v هستند.

Class Exercise 2

$V_{out} = ?$



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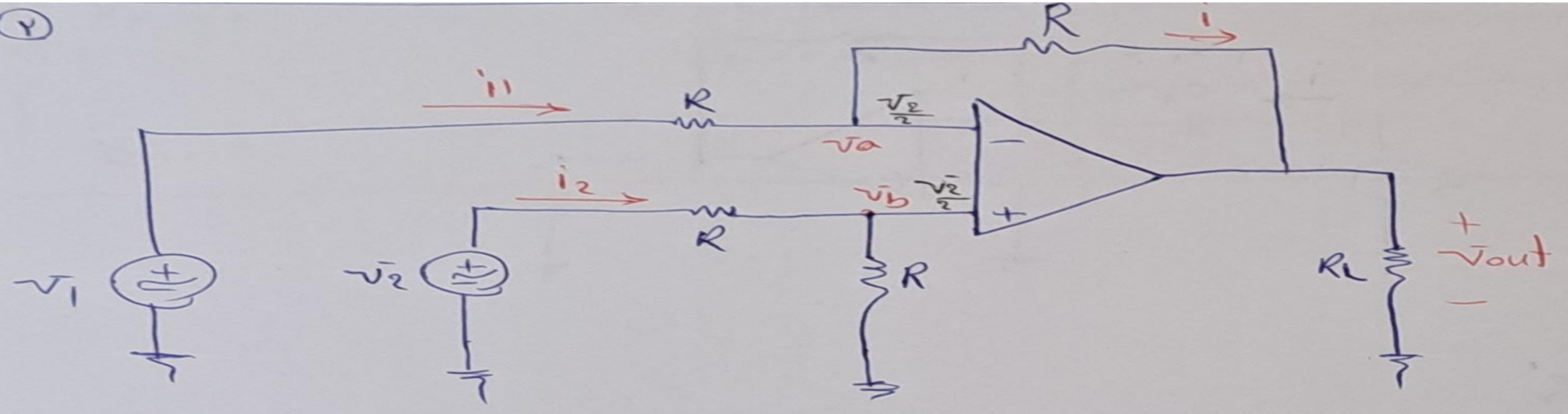

$V_b = \frac{V_2}{2}$ تقسیم ولتاژ بین دو تا R

$$i_1 = \frac{V_1 - \frac{V_2}{2}}{R}$$

$$i_1 = i = \frac{\frac{V_2}{2} - V_{out}}{R}$$

$$\frac{V_1 - \frac{V_2}{2}}{\cancel{R}} = \frac{\frac{V_2}{2} - V_{out}}{\cancel{R}}$$

$$\boxed{V_{out} = V_1 - V_2}$$

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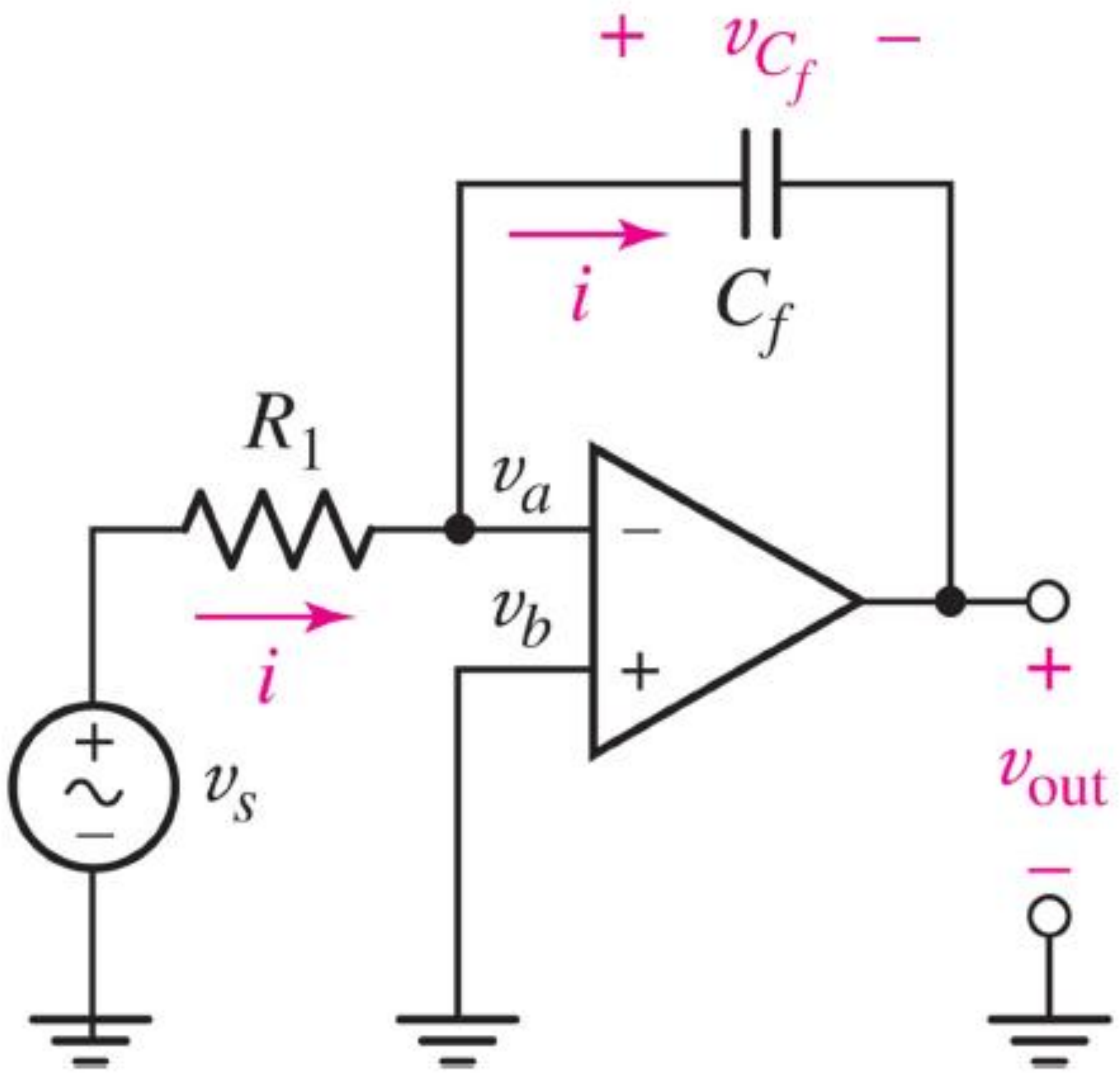
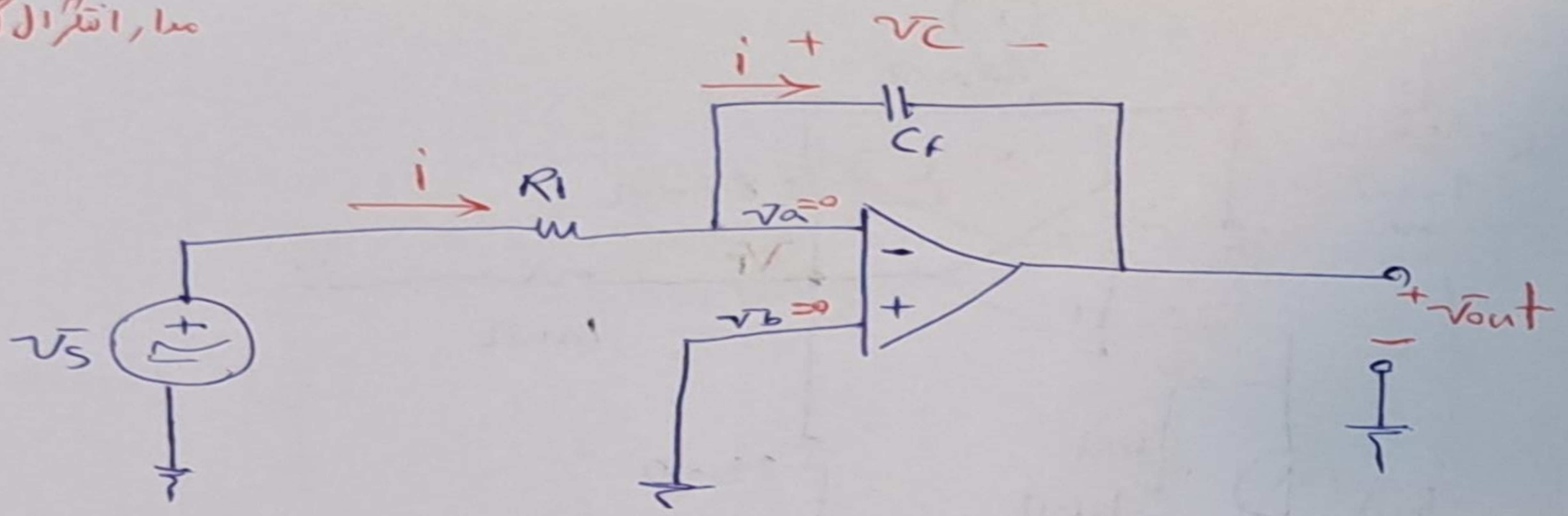
V_{out} =?



$$v_{out} = -\frac{1}{R_1 C_f}\int_0^t v_s \, dt' - v_{C_f}(0)$$

۳ مدار انتگرال گیر



$$i_0 = \frac{V_S}{R_1}$$

$$\cancel{V_{cf}' = \frac{V_s}{R C_f}} \qquad V_{cf}' = \frac{V_S}{R\,C_f}$$

جریان خازن $i = C_f V_{cf}'$

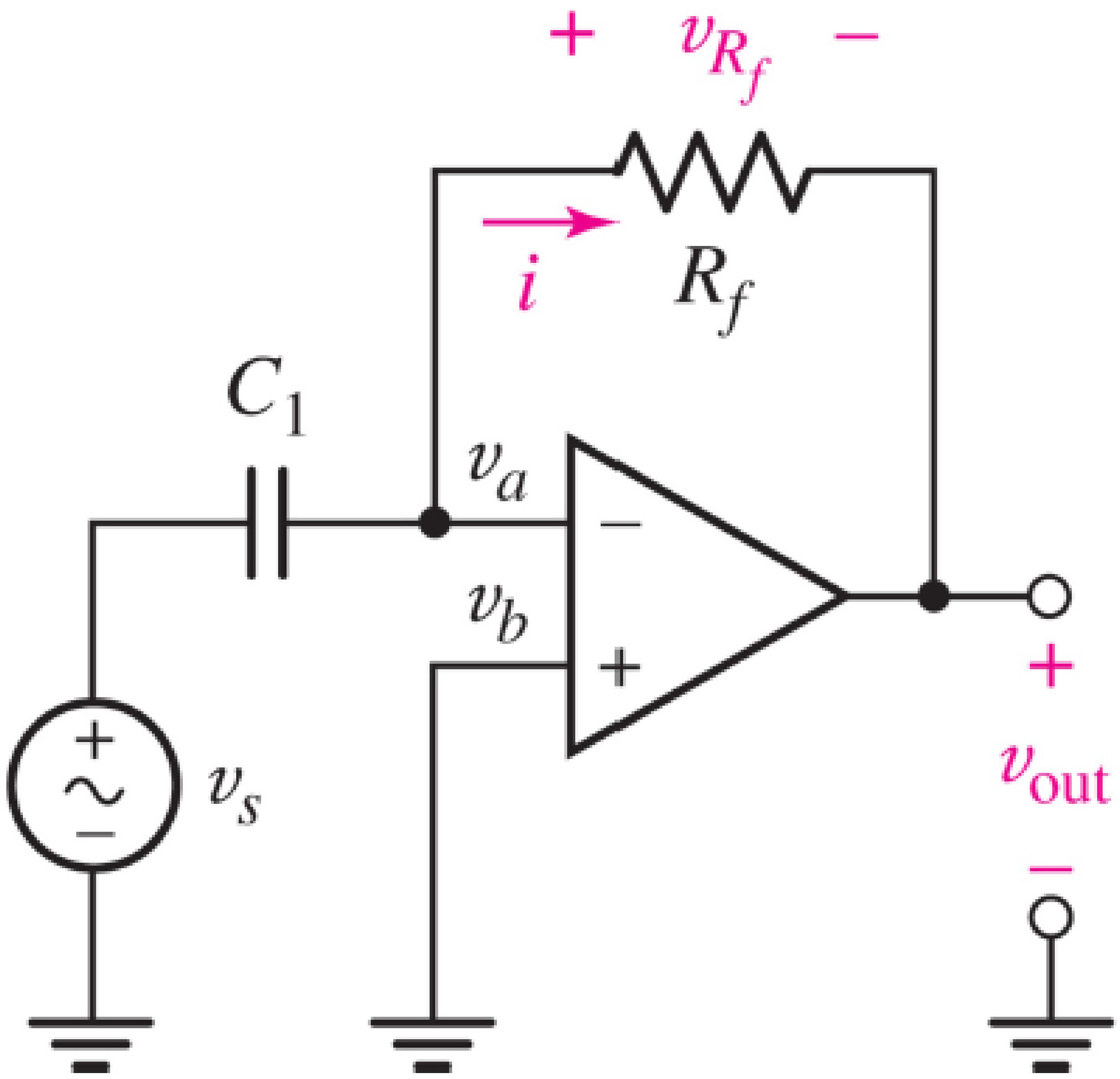
$$V_{cf} = \frac{1}{C_f}\int_0^t i + V_{cf}(0^+)$$

$$-V_{out} - V_{cf} = 0 \Rightarrow V_{out} = -V_{cf}$$

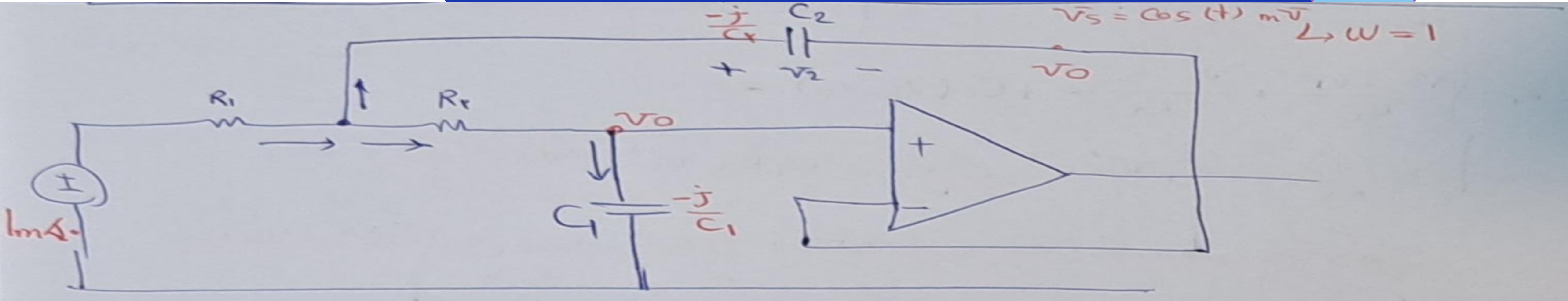
اگر انتگرال بگیریم $\Rightarrow$ $V_{out} = \dfrac{-1}{C_f}\displaystyle\int \frac{V_s}{R_1}\,dt - V_{cf}(0^+) = \dfrac{-1}{R\,C_f}\displaystyle\int_0^t v_s\,dt - V_{cf}(0^+)$

Class Exercise 4

V_{out} =?



$$v_{out} = -C_1 R_f \frac{dv_s}{dt}$$



$$
KCL\ V_1: \begin{cases} \dfrac{I_m - V_1}{R_1} + \dfrac{V_1 - V_o}{R_2} + \dfrac{V_1 - V_o}{-j/C_2} \\[2ex] \dfrac{V_1 - V_o}{R_2} = \dfrac{V_o}{-j/C_1} \end{cases}
$$

با استفاده از این معادله می‌توان V_1 را حذف کرد $\Leftarrow$

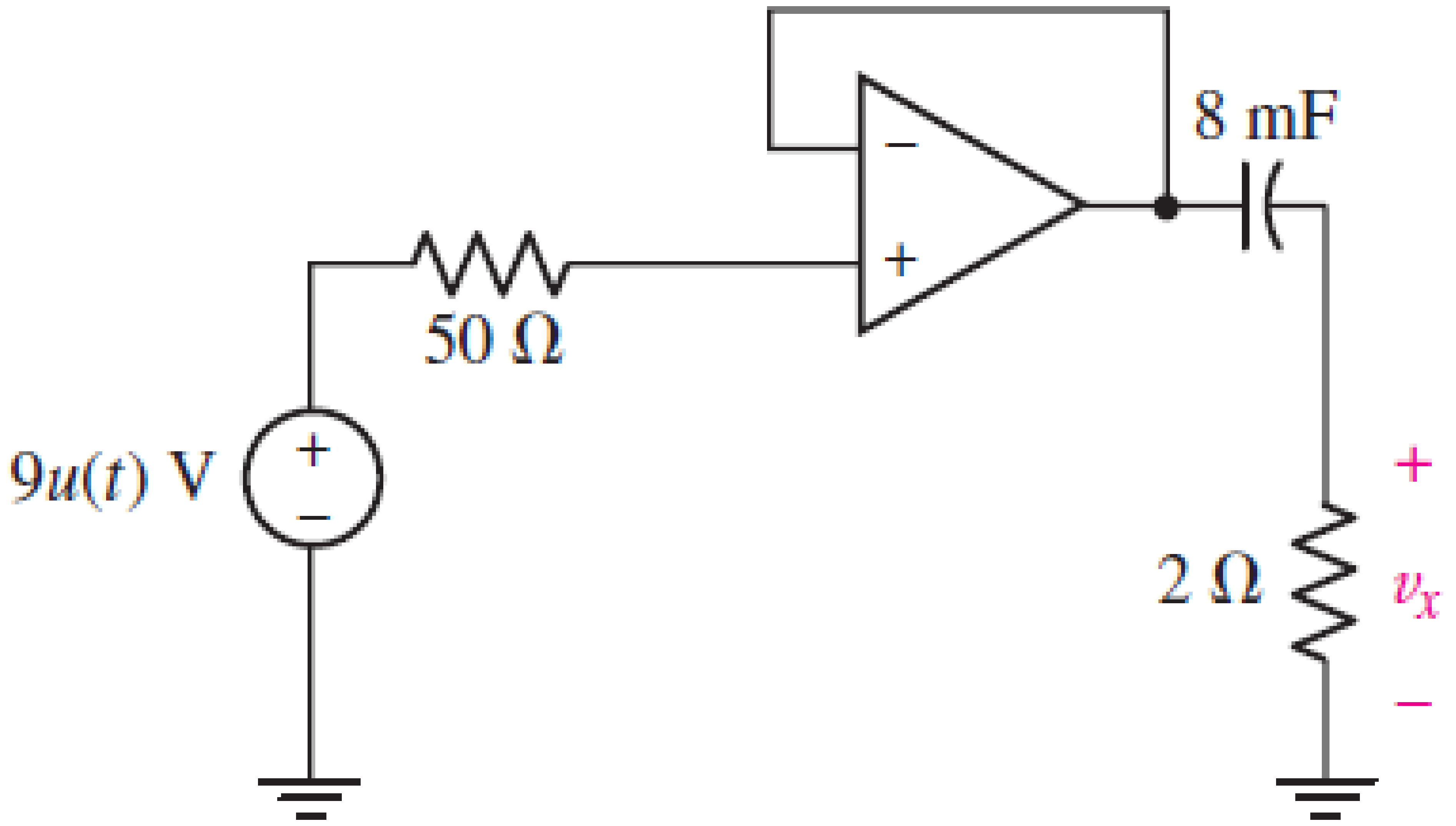
با جایگذاری در رابطهٔ دوم می‌توان V_o را بر حسب مقادیر ثابت بدست آورد.

V_o یک عدد مختلط $\longleftarrow$ که باید به حوزه زمان برده شود.

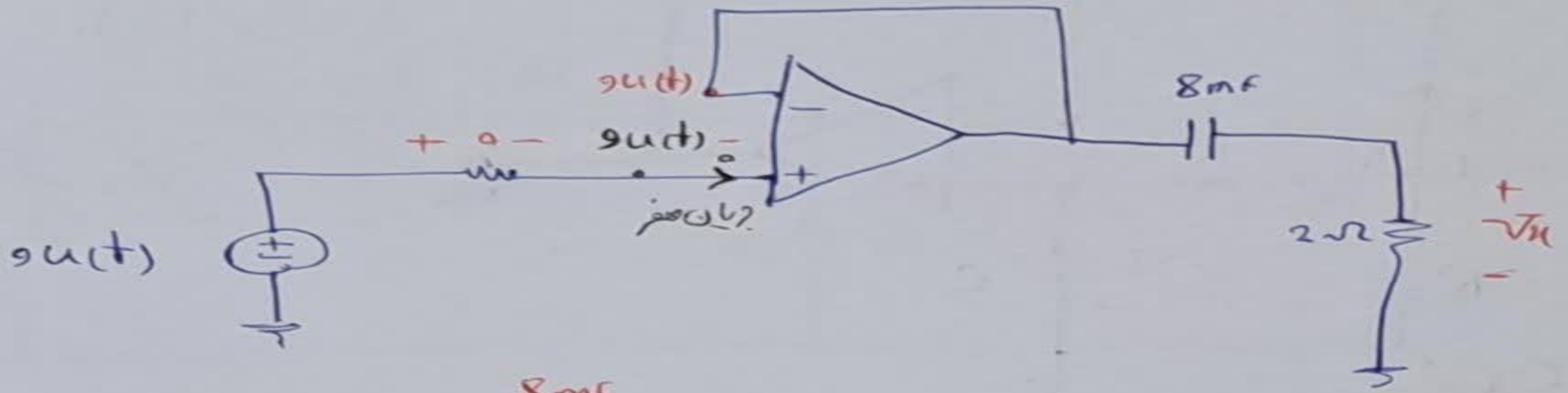
$$
\boxed{A e^{j\theta} \longrightarrow A\cos(t+\theta)}
$$

Class Exercise 5

v_x=?



سؤال ۵



هم‌ارز با مدار

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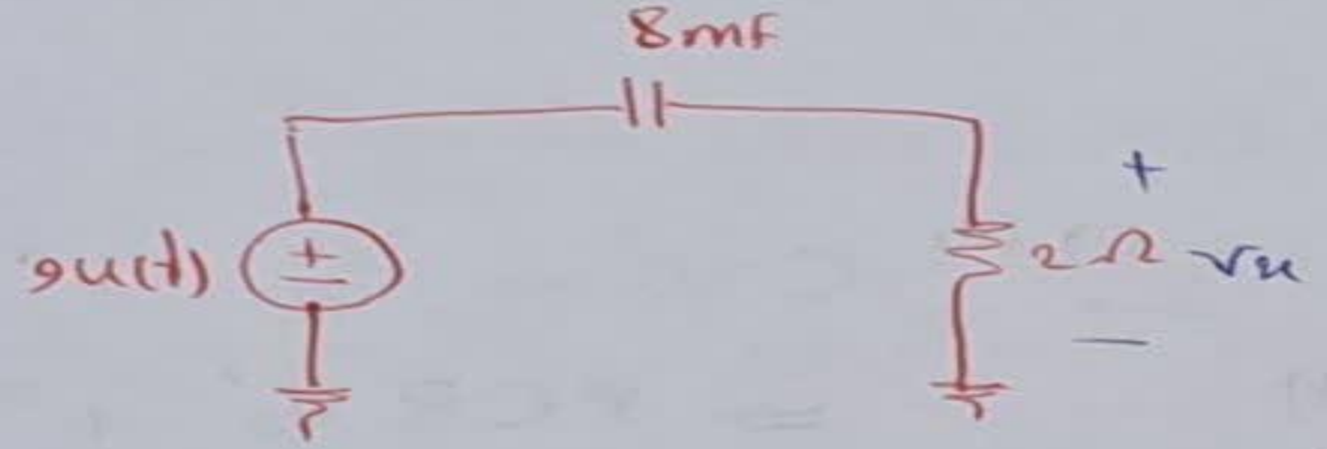
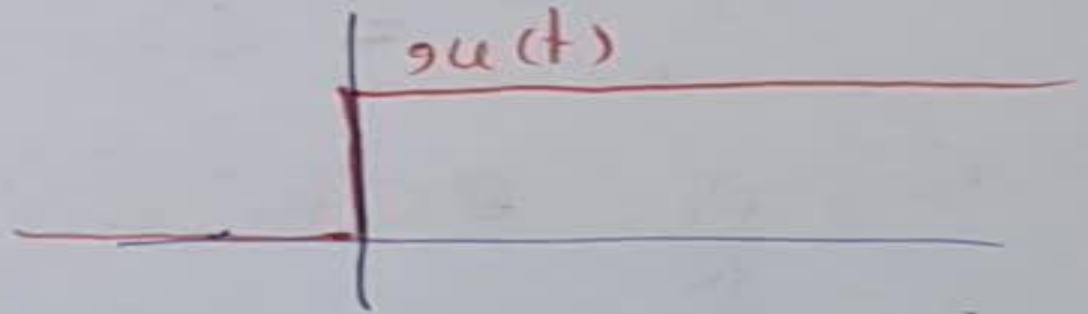




$$v_x = \cancel{v_\infty}^{\,0} + (\cancel{v_0}^{\,9} - \cancel{v_\infty}^{\,0})\, e^{-\frac{t}{16m}} = 9e^{-\frac{t}{16m}}$$

در لحظه صفر منفی خازن در مدار نیست در نتیجه $v_c(0^-) = 0$

$v_c(0^-) = v_c(0^+)$ و هر دو صفر هستند

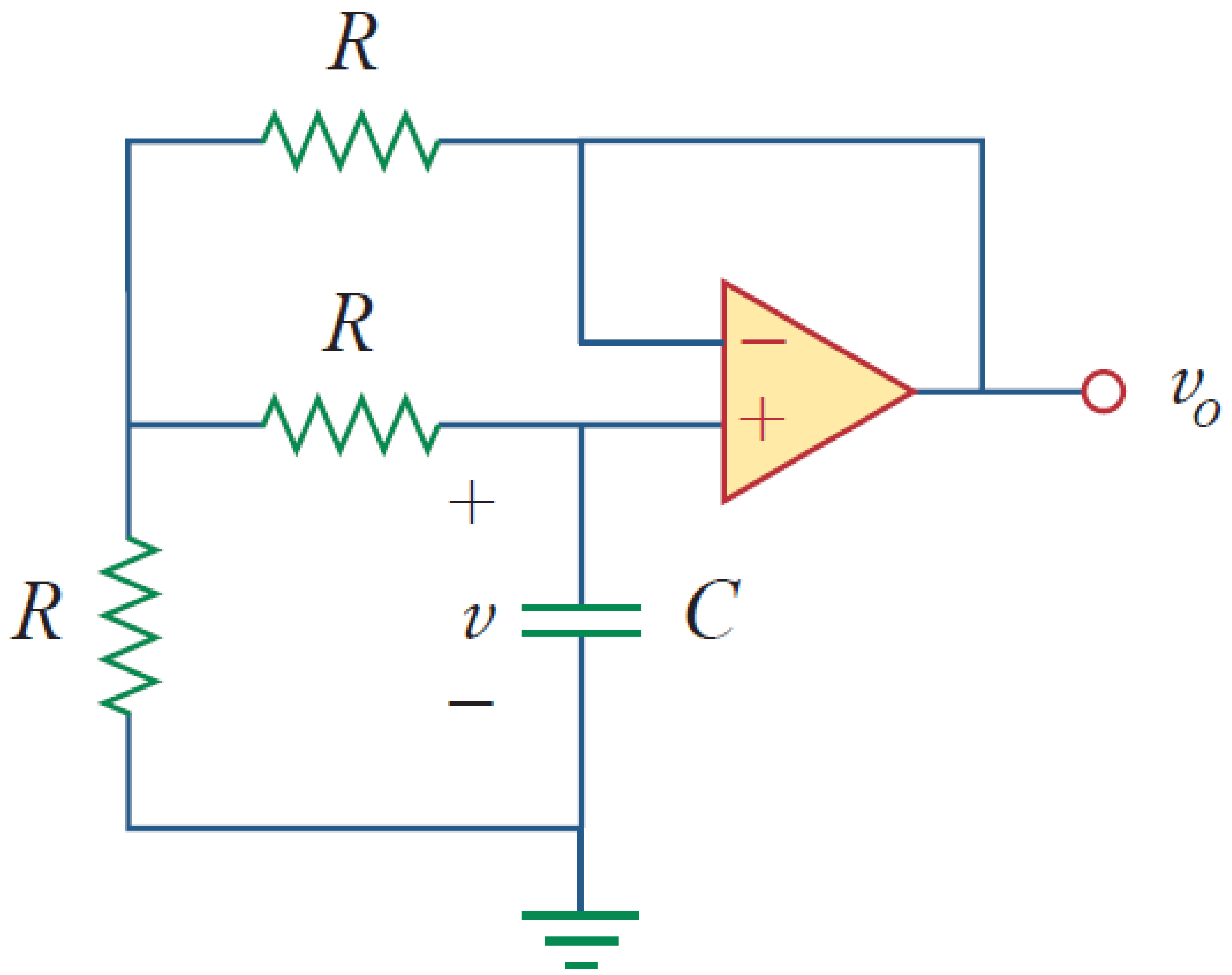
در نتیجه در لحظه صفر مثبت ۹ v روی دو سر مقاومت است

$v_\infty = 0$ → ولتاژ دو سر خازن مدار باز شده

در نتیجه $v_\infty = 0$

$v_x(0^+) = 9$

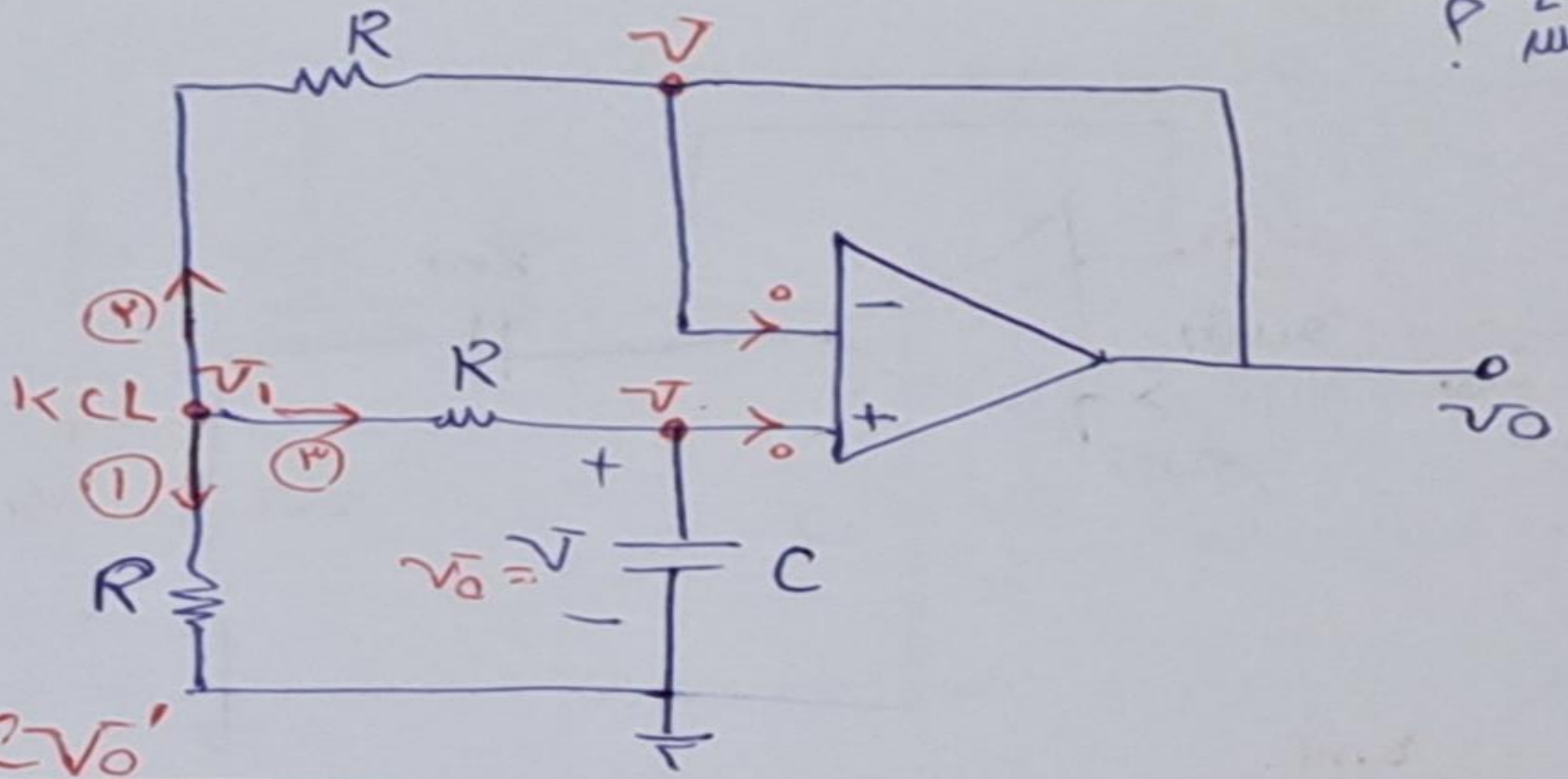
If the initial voltage across the capacitor at time $t = 0$ is 10 volts, determine the output voltage v_o.



سؤال ⑥

اگر ولتاژ خازن در لحظه t=0 برابر ۱۰ ولت باشد ؟

$V(0^+) = 10 \qquad V_o = ?$



$V_o = V$

$$\frac{V_1}{R} + \overbrace{\frac{V_1 - V_o}{R} + \frac{V_1 - V_o}{R}}^{2CV_o'} \qquad (1)$$

جریان ③

$$\frac{V_1 - V_o}{R} = CV_o' \qquad (2)$$

$$\frac{V_1}{R} = \cancel{2}CV_o' + \frac{V_o}{R} \qquad (3)$$

(۳) در (۱) $\Rightarrow$

$$CV_o' + \frac{V_o}{R} + 2CV_o' = 0$$

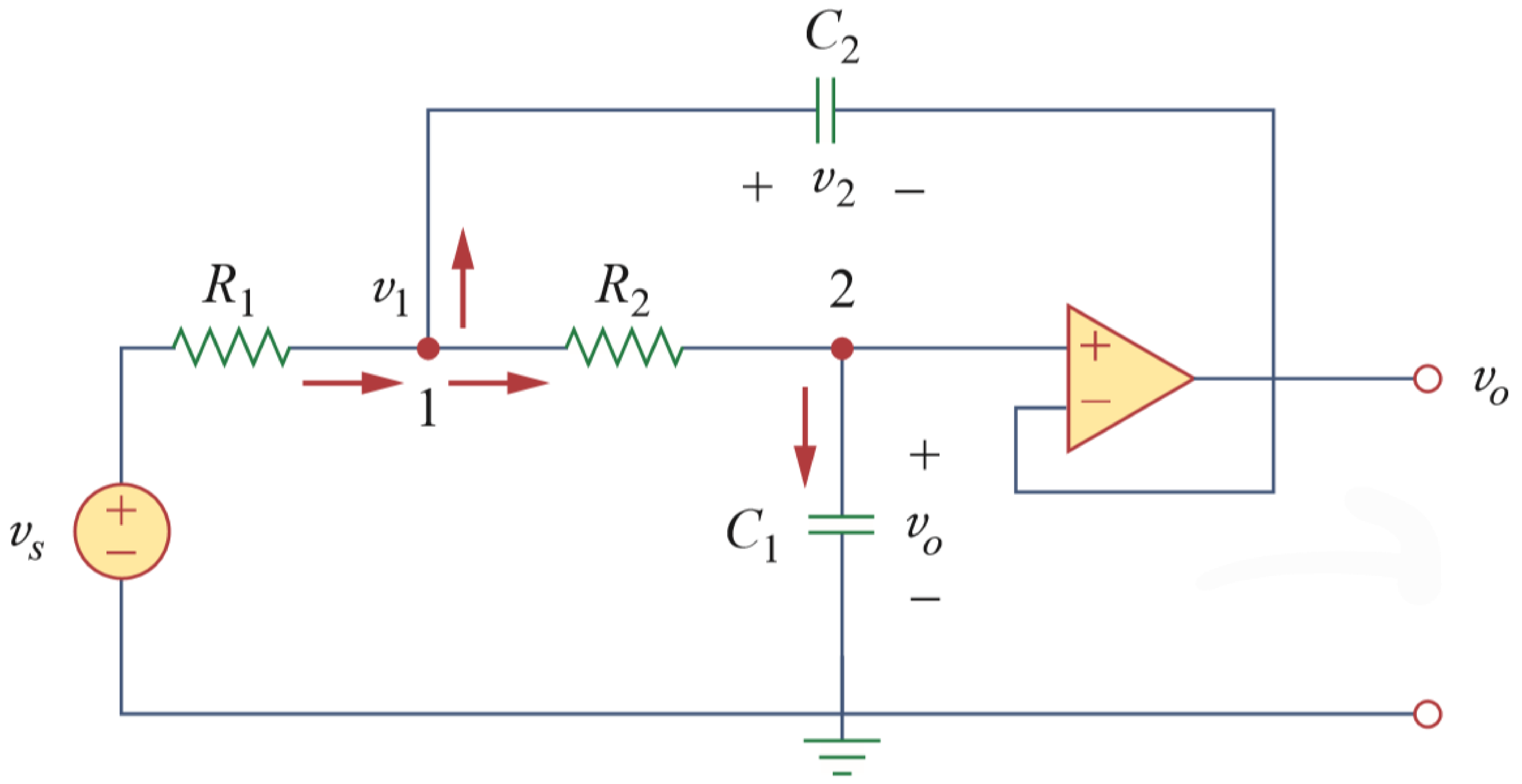
$$\Rightarrow 3CRV_o' + V_o = 0$$

$$\downarrow$$

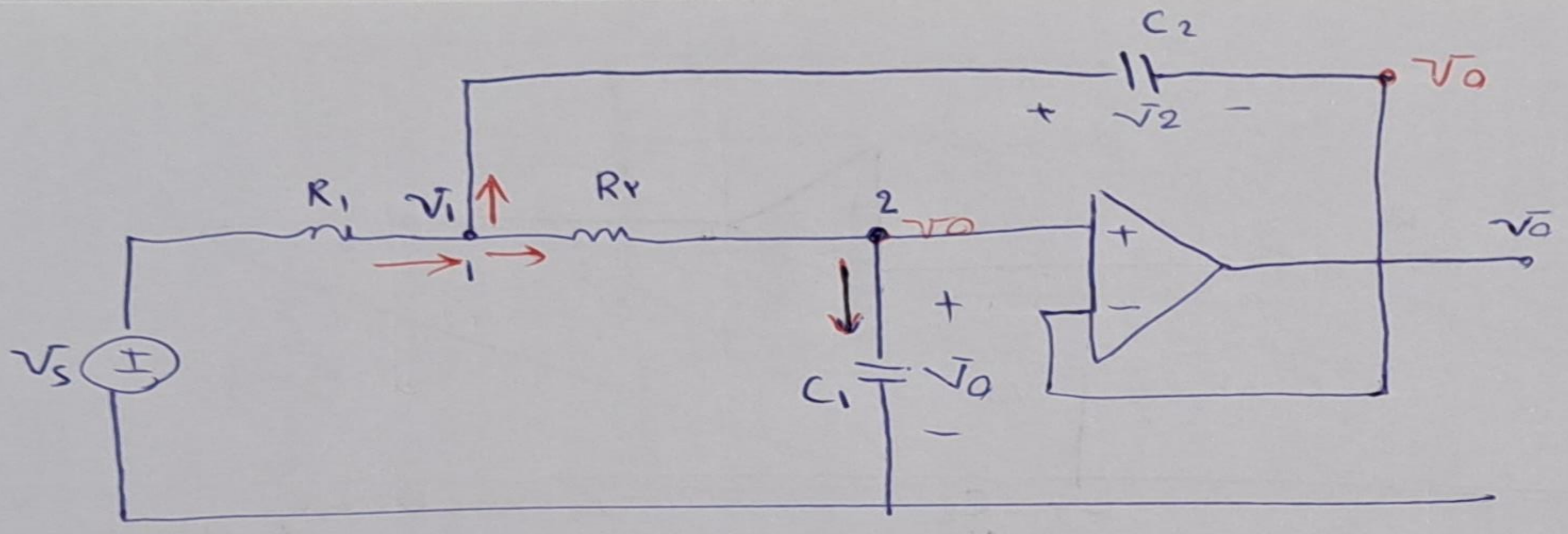
$$3CRS + 1 = 0 \Rightarrow S = \frac{-1}{3RC}$$

$$\Rightarrow V_o = 10\, e^{\frac{-t}{3RC}}$$

- $v_s = 10u(t)mV, \;\; R_1 = R_2 = 10K\Omega, C_1 = 20\mu F, \; C_2 = 100\mu F$
- $v_o = ?$



۷



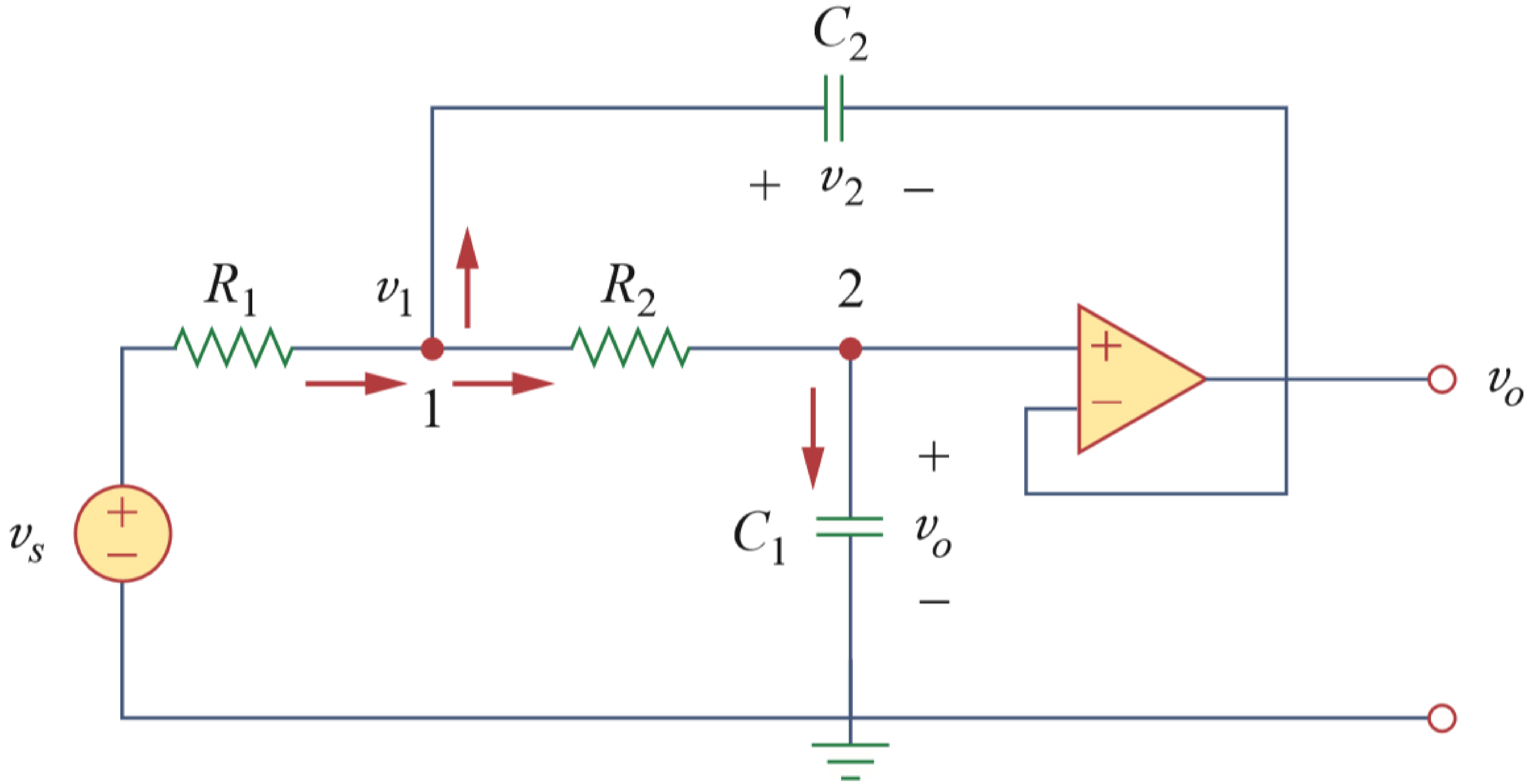
$$\text{KCL } v_1: \begin{cases} \dfrac{v_s - v_1}{R_1} + \dfrac{v_1 - v_0}{R_2} + C_2 (v_1 - v_0)' = 0 \\[2ex] \dfrac{v_1 - v_0}{R_2} = C_1 v_0' \end{cases} \Rightarrow \boxed{v_1 = C_1 R_2 v_0' + v_0}$$

یک دستگاه معادله داریم می‌توان v_1 را حذف کرد و v_0 را بر حسب v_s نوشت.

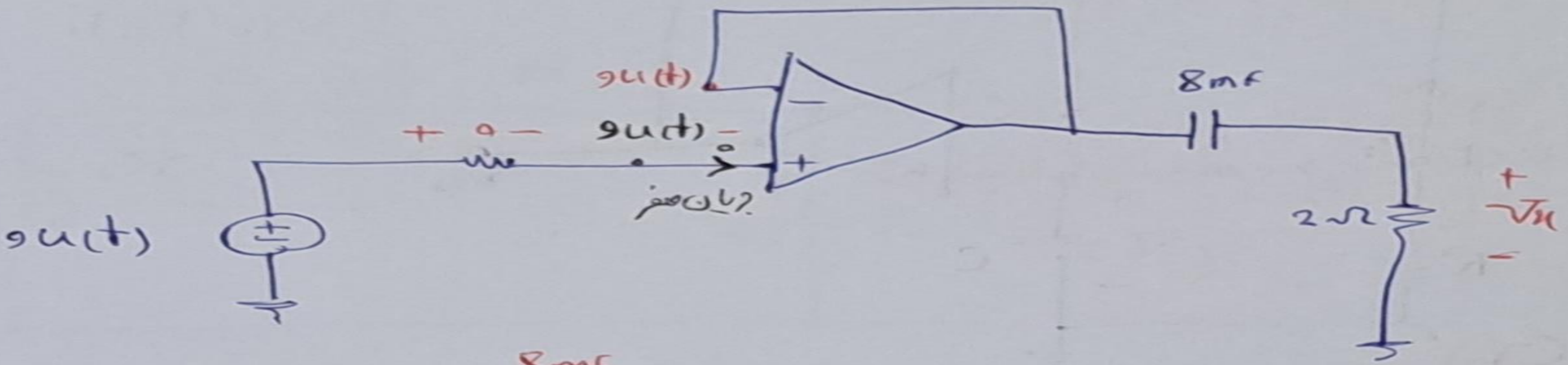
ادامه آن مشابه میان‌ترم (مدار مرتبه دوم) پس باید یاد بگیریم مدار مرتبه دوم را چگونه حل کنیم.

Class Exercise 8

- $v_s = \cos(t)mV,\ R_1 = R_2 = 10K\Omega,\ C_1 = 20\mu F,\ C_2 = 100\mu F$
- $v_o = ?$ (Use phasors).

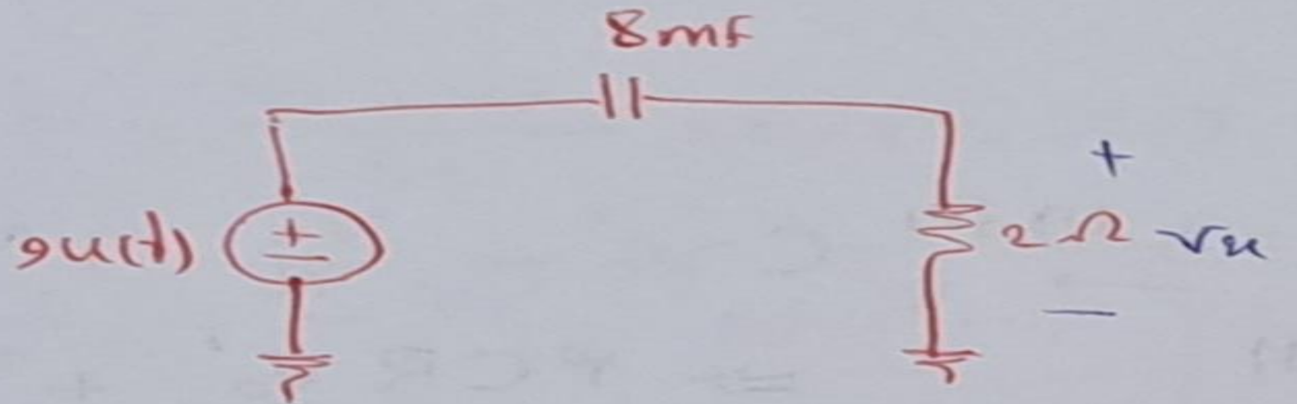


سؤال ۵



معادل با مدار

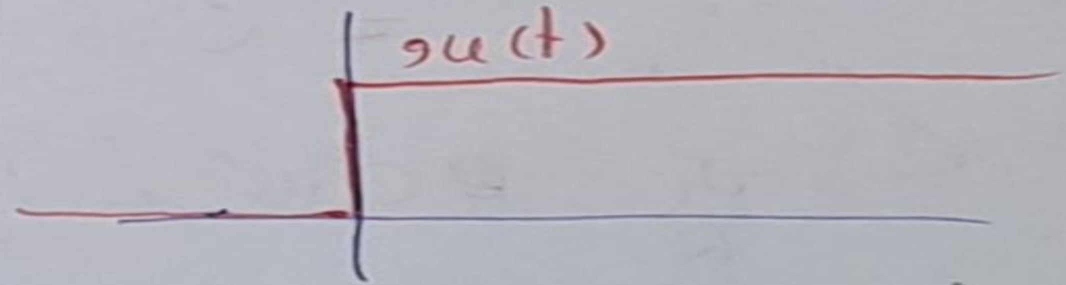
$\equiv$



$$v_R = \cancel{v_\infty}^{\,0} + (\cancel{v_0}^{\,9} - \cancel{v_\infty}^{\,0})\, e^{\frac{-t}{16m}} = 9e^{\frac{-t}{16m}}$$

$v_\infty = 0 \Rightarrow$ ولتاژ دو سر خازن مدار باز شده در نتیجه $v_\infty = 0$

$v_R(0^+) = 9$



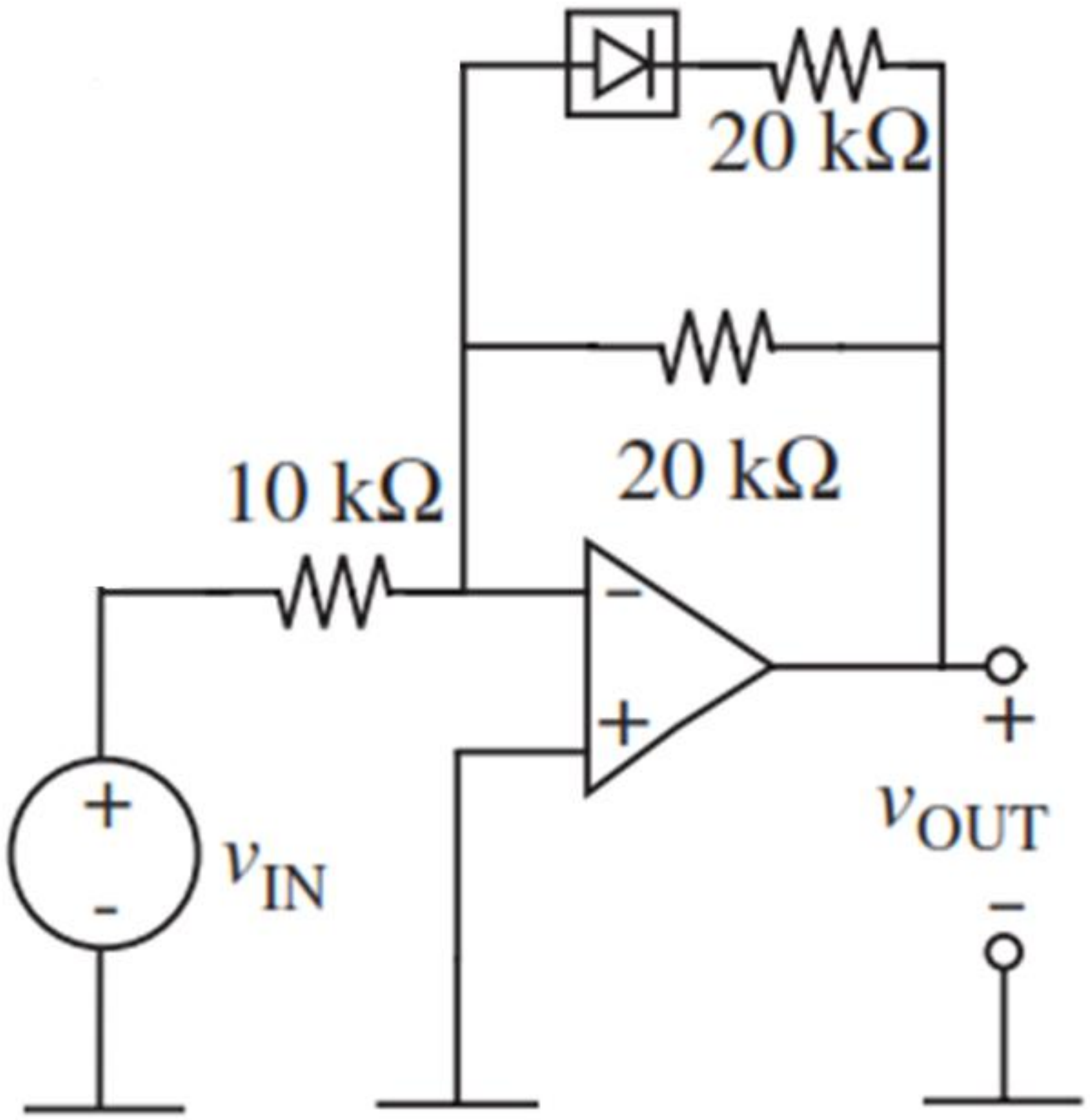
در لحظه صفر منفی خازن در مدار نیست در نتیجه $v_c(0^-) = 0$

$v_c(0^-) = v_c(0^+)$ و هر دو صفر هستند

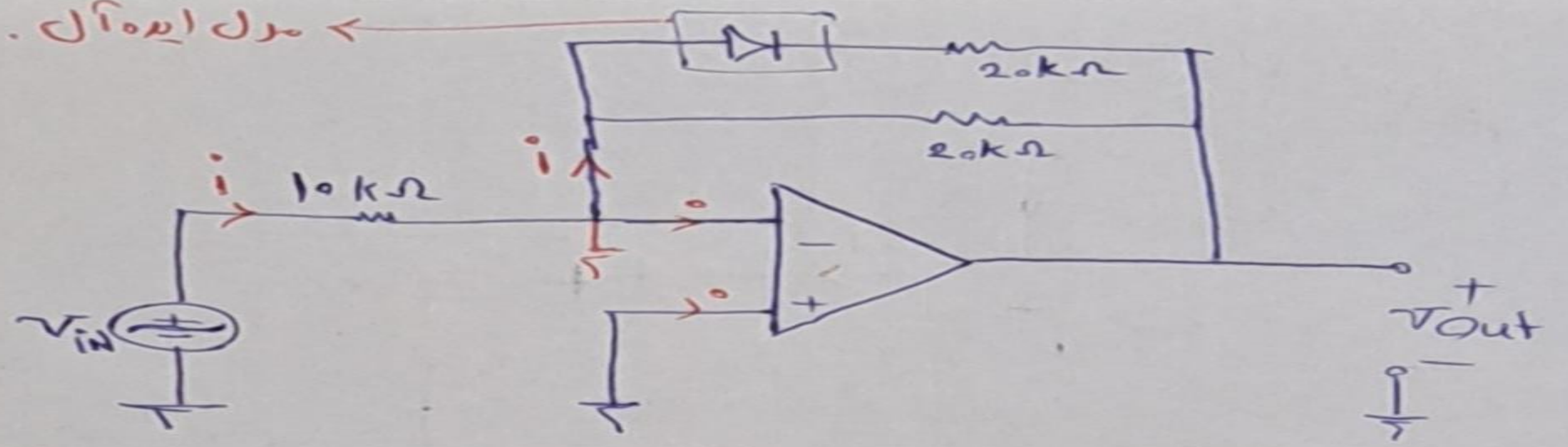
در نتیجه در لحظه صفر مثبت ۹ ولت روی دو سر مقاومت است

Class Exercise 9

If $v_{in} = \sin(200\pi t)$, plot the output voltage v_{out}.



سؤال ۹ . مدل ایده‌آل ←



$V_{in} = \sin(200\pi t)$

دیود ایده‌آل

$v=0$, $i>0$

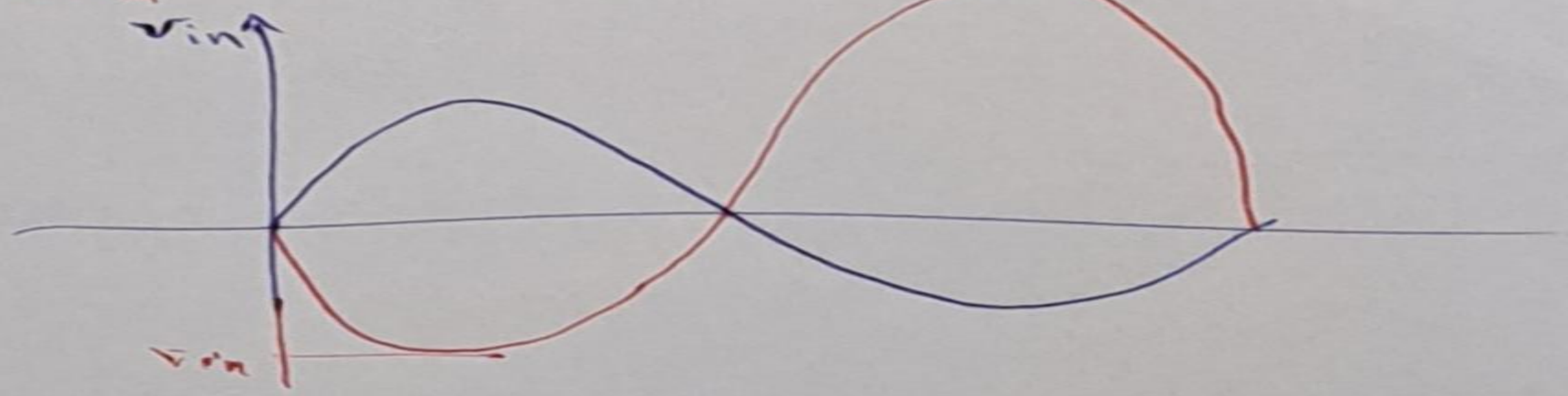
$v<0$, $i=0$

با حالت فرض ← دیود { روشن / خاموش

با استدلال می‌کنیم که اگر دیود بخواهد روشن باشد باید جریان آن مثبت باشد جریان آن هم

$i = \frac{V_{in}}{10k}$

پس هر جا که V_{in} مثبت باشد i مثبت خواهد بود در نتیجه



با دیود وصل ← $20 \| 20 = 10$

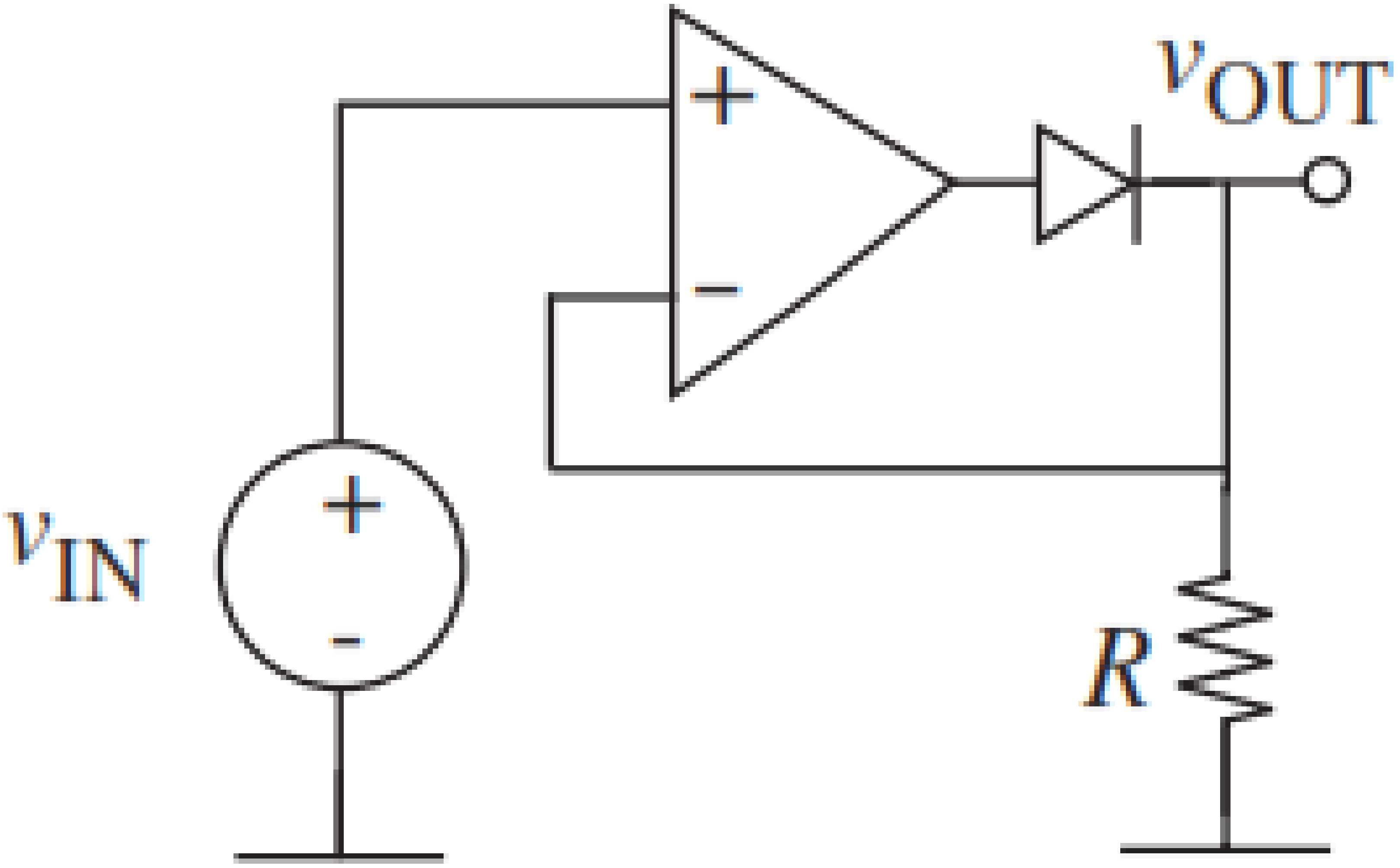
$V_{out} = \frac{-10k}{10k} V_{in}$

با دیود قطع 20

$V_{out} = -2 V_{in}$

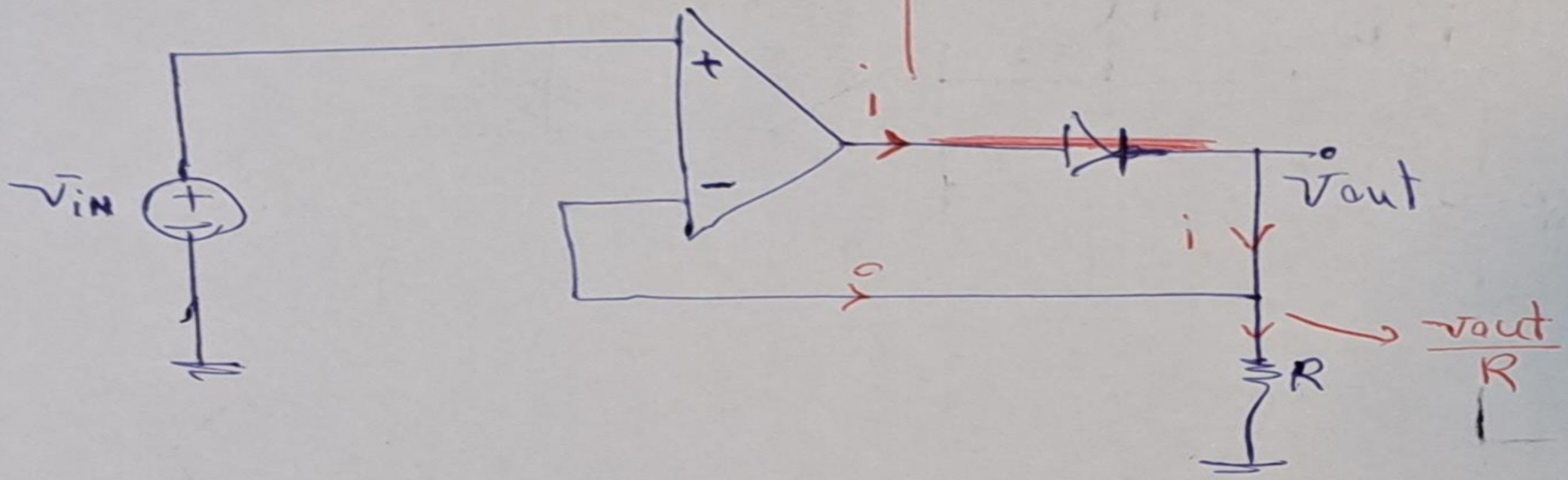
Class Exercise 10

- Plot the diagram of v_{out} as a function of v_{in}.



سؤال ۱۰

روشن یا خاموش بودن به این جریان وابسته شد که تا الان در مورد آن حرف نزده بودیم.



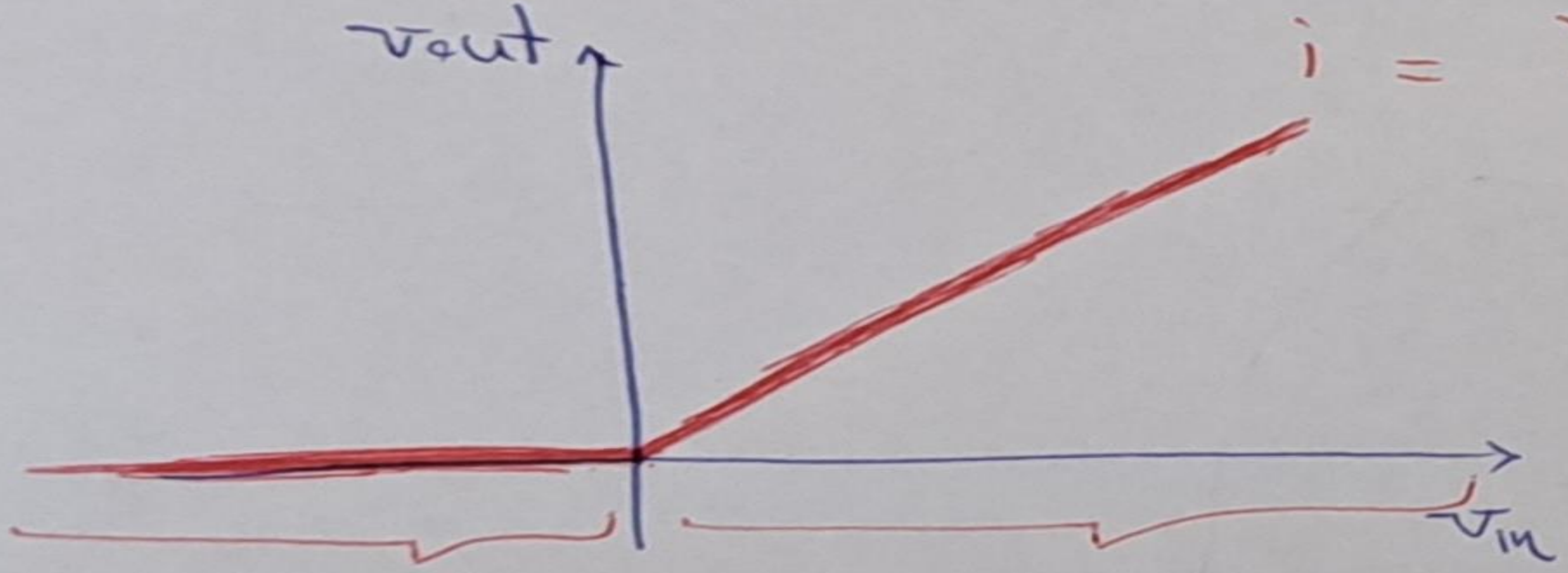
فرض کنیم D روشن باشد. در نتیجه $V_{out} = V_{in}$

$i = \frac{V_{out}}{R} = \frac{V_{in}}{R}$

V_{in} که همیشه مثبت نیست.

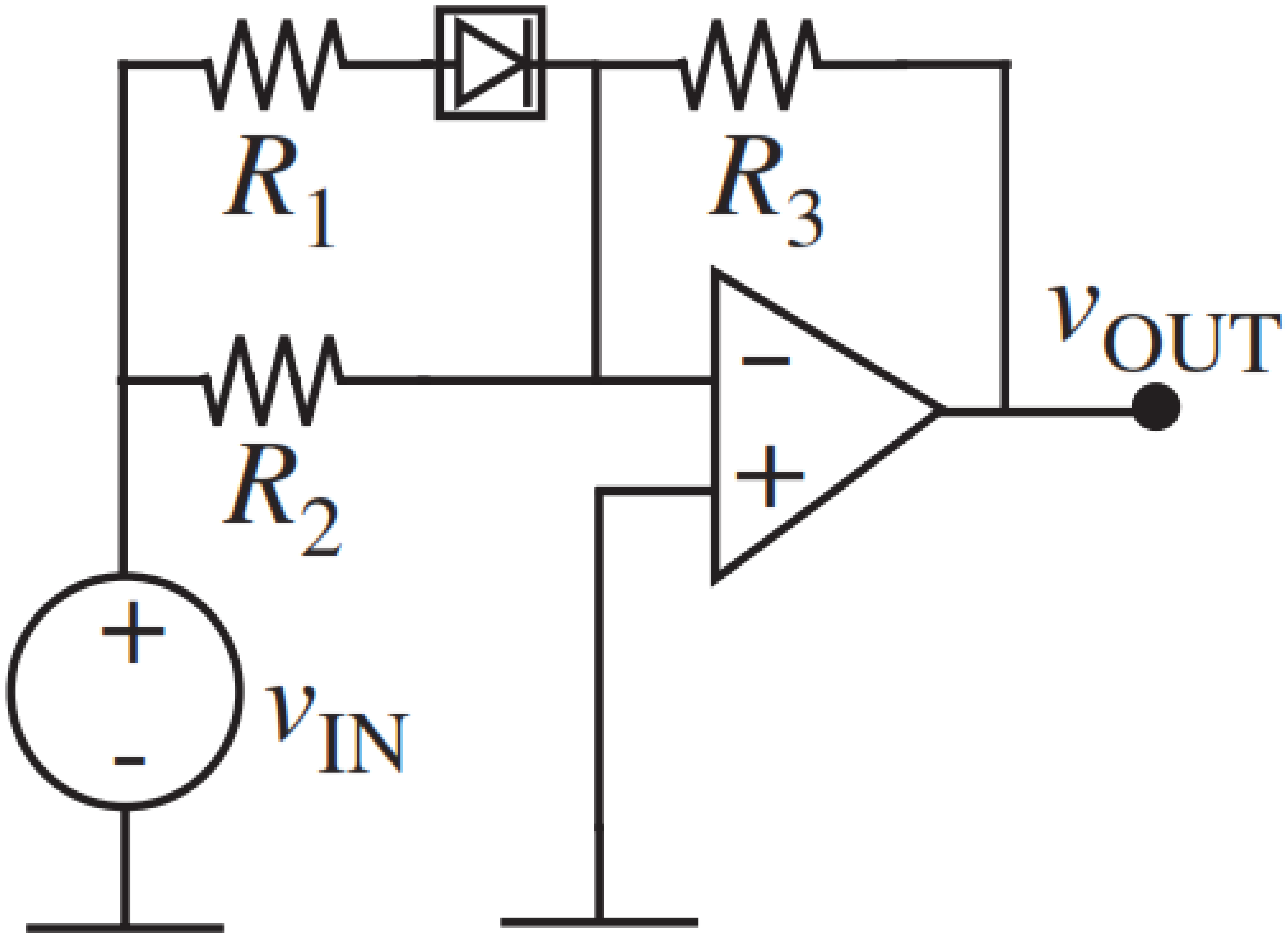
$i > 0$ شرط روشن بودن

در نتیجه برای $V_{in} > 0$، می‌توان $V_{out} = V_{in}$

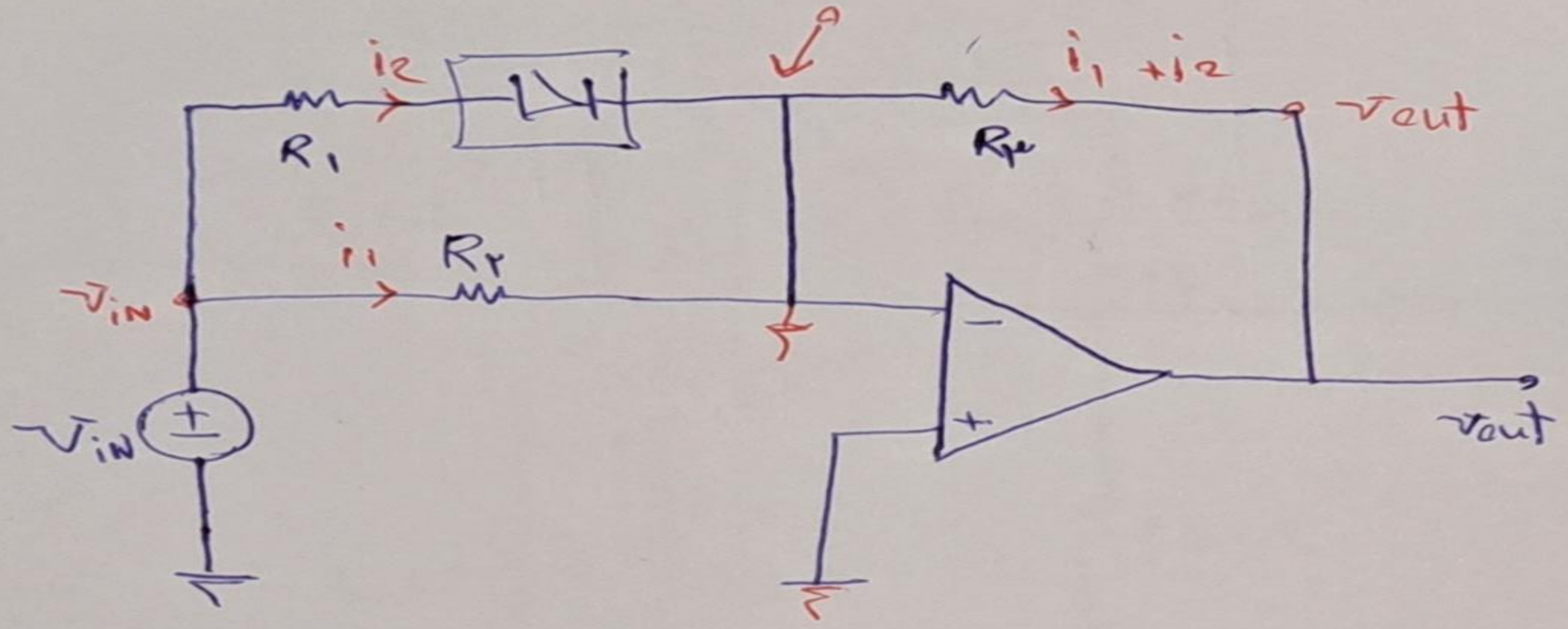


Class Exercise 11

- Plot the diagram of v_{out} as a function of v_{in}.



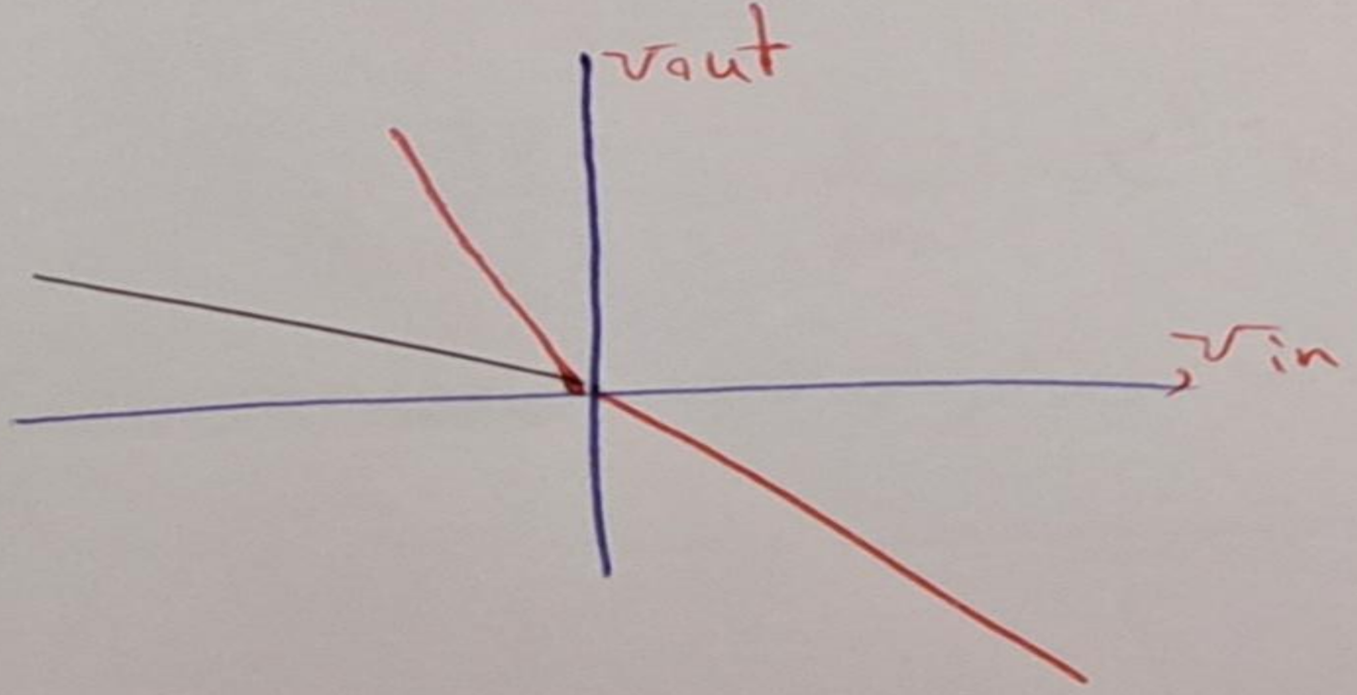
11



فرض کنیم D روشن باشد.

KCL: $\frac{0 - v_{in}}{R_1 \| R_2} + \frac{0 - v_{out}}{R_3} = 0 \Rightarrow v_{out} = \frac{-R_3}{R_1 \| R_2} v_{in}$

← تقویت کننده معکوس کننده



شرط روشن بودن $i_2 > 0$ ← $v_{in} > 0$

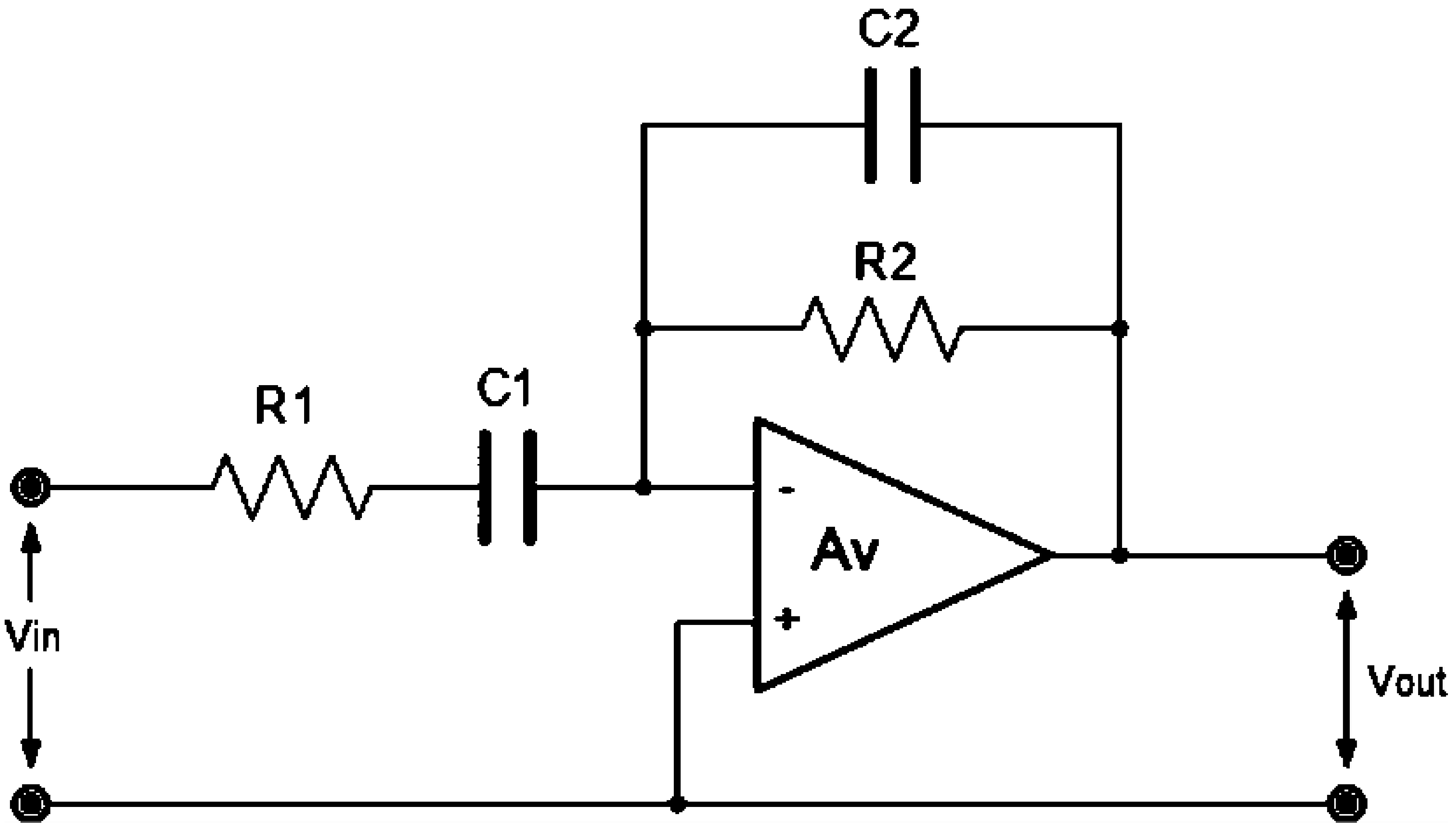
شرط خاموش $v_{in} < 0$ ←

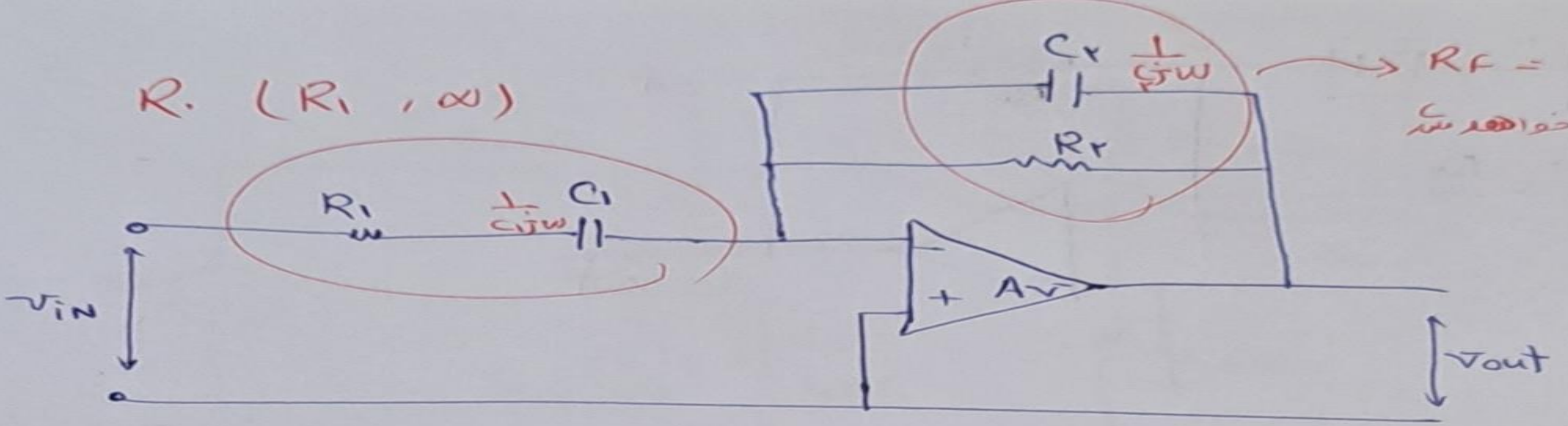
$v_{out} = \frac{-R_3}{R_2} v_{in}$

Class Exercise

This is an active filter (using an op-amp in a frequency filter). Calculate:

- ✓ The type of filter
- ✓ The filter gain
- ✓ The cut-off frequency





$R.\ (R_1\ ,\ \infty)$

$R_F = (0\ ,\ R_2)$

$\omega \to \infty$ اتصال کوتاه و $R_F = 0$ خواهد شد

$V_{out} = \frac{-R_F}{R_s} V_{in}$

یک تقویت کننده معکوس کننده است.

مثل یک مقاومت متغیر با فرکانس.

که R_F و R در فرکانس های مختلف تغییر می کنند.

$$\boxed{V_{out} = \frac{-R_F}{R_1} V_{in}} \longrightarrow$$

فرکانس در بی نهایت

$R_F = 0$

$R = R_1$

$\Rightarrow A = \frac{0}{R_1} = 0$

در فرکانس پایین

$\Rightarrow A = \frac{R_2}{\infty} = 0$

$\frac{1}{\sqrt{2}}$

ادامه آن مثل قبل خواهد شد A را داریم در واقع تابع انتقال را داریم

بگذاریم و فرکانس قطع را حساب می کنیم.

Thanks