

دوره آموزشی «علم داده»

Data Science Course

جلسه سوم:

احتمالات – قضیه بیز

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عضو هیات علمی دانشگاه گنبد کاووس

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#data_science_fozouni



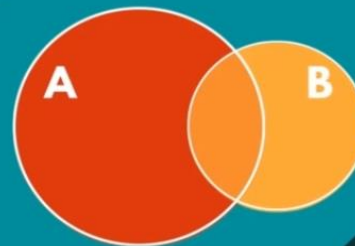
IPM, 2019
Operator
Algebra
Workshop

پیشامد و اشتراک آنها

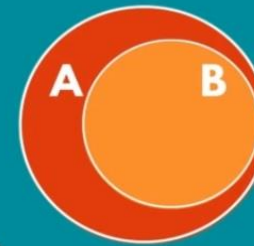
از اشتراک پیشامدها به منظور نشان دادن حالت‌هایی که دو پیشامد بطور همزمان رخ می‌دهند استفاده می‌کنیم

Event ➡ Set of outcomes
(favourable outcomes)

Intersecting



Completely overlapping



یک نمادگذاری

برای ورود به بحث شرطی‌ها،
اولین فرمول، احتمال یافتن
پیشامد A به شرط وقوع B
است.

Notation

Two events: A and B

The probability of getting A ,
if we are given that B has
occurred



$P(A | B)$
"A given B"

Conditional Probabilities

The likelihood of an event occurring,
assuming a different one has already happened

یک مثال ساده اما کاربردی

احتمال وقوع ملکه گشنیز به شرطی که کارت انتخابی

گشنیز باشد چند است؟ $\frac{1}{13}$

Queen of Spades

A → Q ♠

B → ♠

$P(A | B)$



the probability of drawing the Queen of Spades if we know the card is a spade

فرمول احتمال شرطی

یکی از مهمترین بخش‌های قضیه بیز، فرمول احتمال شرطی است

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$





If $P(B) > 0$

Vegetarian Survey

100 men and women are asked if they eat meat



| |  |  | total |
|-------|---|---|-------|
| ♀ | 15 | 32 | 47 |
| ♂ | 29 | 24 | 53 |
| total | 44 | 56 | 100 |

نظرسنجی در خصوص
گیاهخواری

بنظر شما مردان گیاهخوارترند یا زنان؟

Vegetarian Survey



$$P(A|B) \neq P(B|A)$$

Different events

$$P(A|B) = \frac{15}{47}$$



$$P(B|A) = \frac{15}{44}$$

=> It is more likely for a vegetarian to be female,
than for a woman NOT to eat meat

پاسخ به سؤال مطرح شده!!!

قانون احتمال کل

Law of Total Probability

$$\blacklozenge A = B_1 \cup B_2 \cup \dots \cup B_n$$

$$P(A) = P(A|B_1) \times P(B_1) + P(A|B_2) \times P(B_2) \dots$$

قانون احتمال کل در مثال نظرسنجی گیاهخواری

Vegetarian Survey

$$P(\text{🌱}) = P(\text{🌱} | \text{♂}) \times P(\text{♂}) + P(\text{🌱} | \text{♀}) \times P(\text{♀}) =$$
$$= \frac{29}{53} \times \frac{53}{100} + \frac{15}{47} \times \frac{47}{100} = 0.44$$

◆ There is a 44% chance of
someone being vegetarian

| | 🌱 | 🍖 | total |
|-------|----|----|-------|
| ♀ | 15 | 32 | 47 |
| ♂ | 29 | 24 | 53 |
| total | 44 | 56 | 100 |

قانون جمعی

Additive Law

Definition:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

The probability of the union of two sets is equal to the sum of the individual probabilities of each event, minus the probability of their intersection

باز هم مثال نظر سنجی

Vegetarian Survey

$$P(\text{♀} \cup \text{🥗}) = P(\text{♀}) + P(\text{🥗}) - P(\text{♀} \cap \text{🥗}) = \\ = 0.47 + 0.44 - 0.15 = 0.76$$

- ◆ There is a 76% chance that a random person from the survey is either **female**, **vegetarian** or **both**

| | 🥗 | 🥩 | total |
|-------|----|----|-------|
| ♀ | 15 | 32 | 47 |
| ♂ | 29 | 24 | 53 |
| total | 44 | 56 | 100 |

قانون ضرب

Multiplication Rule

$$P(A | B) \times P(B) = \frac{P(A \cap B) \times P(B)}{P(B)}$$

$$P(A | B) \times P(B) = \frac{P(A \cap B) \times \cancel{P(B)}}{\cancel{P(B)}}$$

مثالی از قانون ضرب

Cards

1st



B:
NOT drawing a 
on the first try

2nd



A:
Drawing a 
on the second try



نتیجه با کمی محاسبه بدست می آید

$$P(\spadesuit) = \frac{1}{4} = 0.25$$

B: \spadesuit'

$$P(\spadesuit') = 1 - 0.25 = 0.75$$

قانون یا قضیه بیز

Bayes' Rule

Two events: A & B

$$P(A | B) = \frac{P(B | A) \times P(A)}{P(B)}$$

← Conditional probability formula

$$P(A \cap B) = P(B | A) \times P(A)$$

← Multiplication rule

It allows us to find a relationship between the different conditional probabilities of two events

different conditional probabilities of two events

کاربرد قانون بیز چیست؟

Bayes' Rule in Real-Life



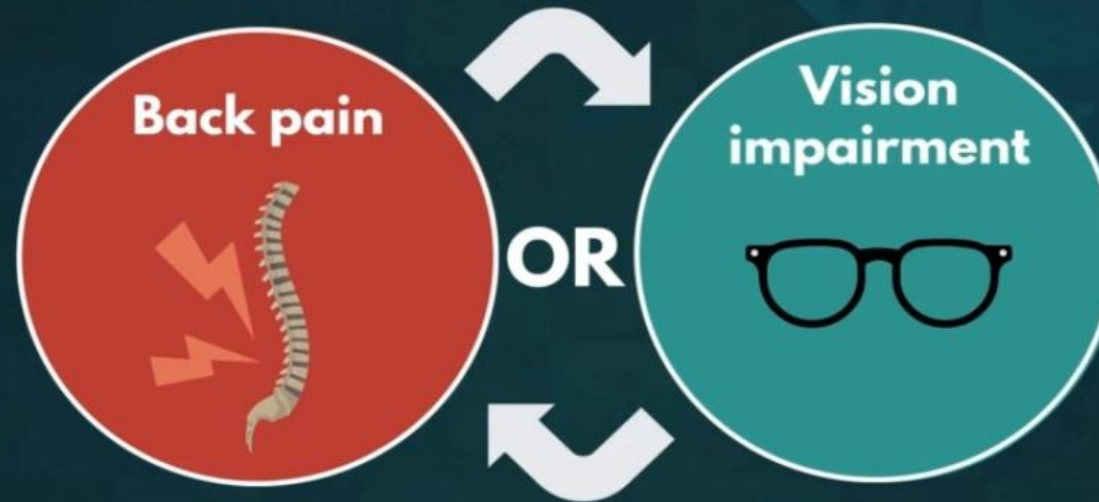
Medical research

- ◆ Trying to find a causal relationship between symptoms
- ◆ Helps us make more reasonable arguments about which one causes the other



تحقیقات در حوزه پزشکی

Bayes' Rule in Real-Life



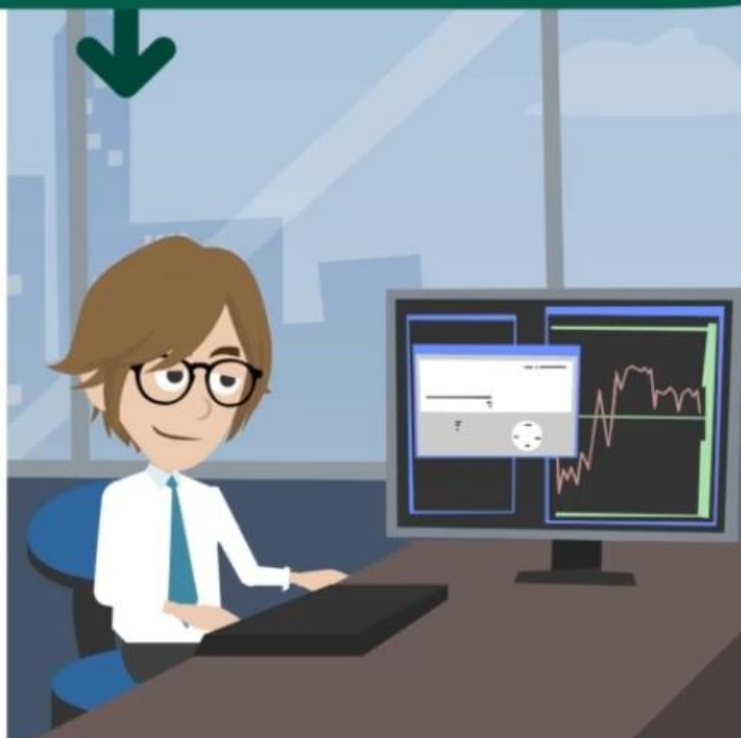
$$P(VI|BP) = 67\%$$

$>$

$$P(BP|VI) = 41\%$$

Bayes' Rule in Real-Life

Deteriorating effect on individual's eyesight



No other underlying factor that would suggest incoming back pains



مقایسه کار و انتخاب بین تجربه یا نمرات بالا

Bayes' Rule in Real-Life

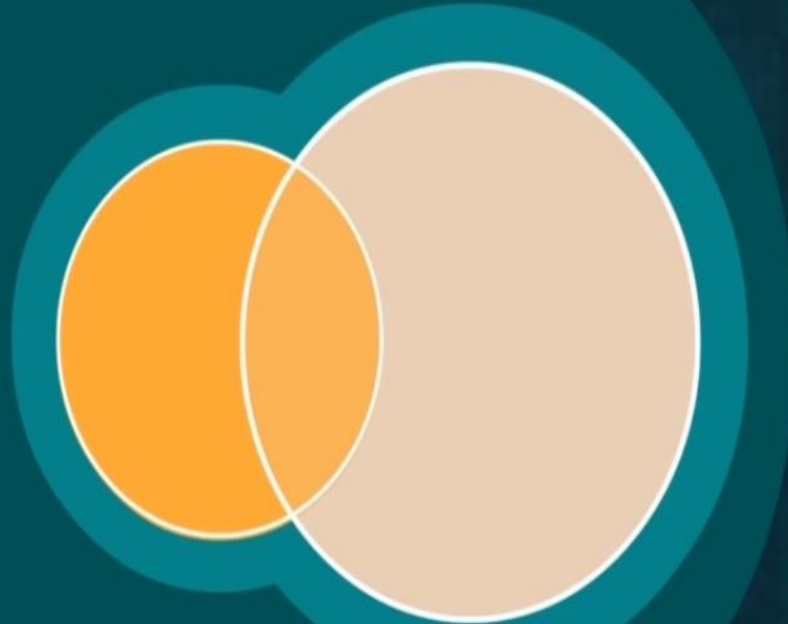
◆ 200 successful candidates

$$P(\text{EXP}) = 45\%$$

$$P(A+) = 60\%$$

$$P(A+ | \text{EXP}) = 50\%$$

$$P(\text{EXP} | A+) = ?$$



احتمالات هم به تجربه وزن خاصی می ده

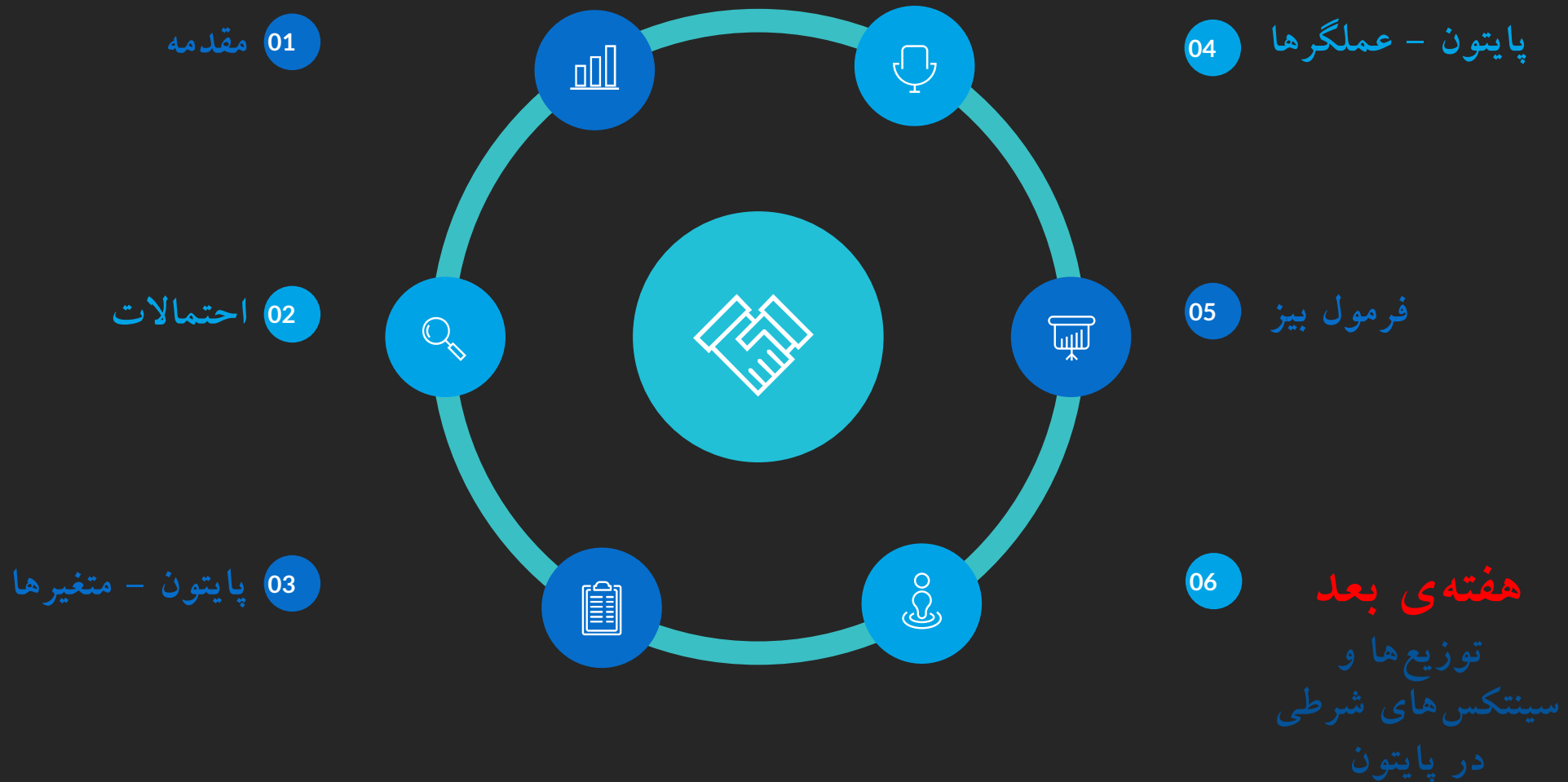
Bayes' Rule in Real-Life

$$P(\text{EXP} \mid \text{A+}) = 0.375 < P(\text{A+} \mid \text{EXP}) = 0.5$$

Candidates who had internships are more likely to also have a high GPA

more likely to also have a high GPA

خب، چی گفتیم و قراره چی بگیم



Thanks For Watching & Stay Tuned



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