

# دوره آموزشی «علم داده» *Data Science Course*




## جلسه چهارم: توزیع‌های پیوسته

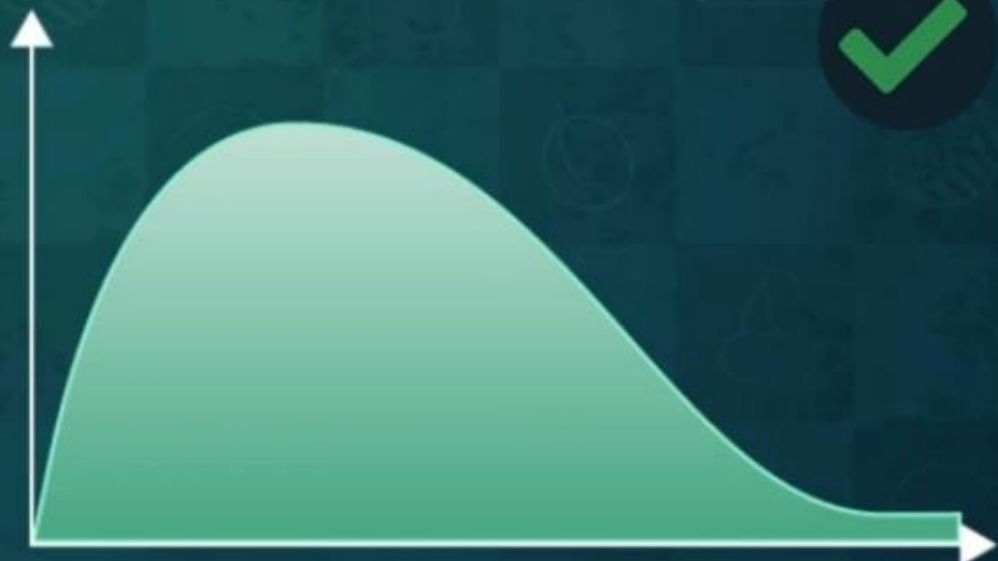
مدرس: محمد فزونی  
عضو هیات علمی دانشگاه گنبد کاووس  
پائیز ۱۳۹۹

# Continuous Distributions

- ◆ Sample space is infinite
- ◆ We cannot record the frequency of each distinct value



Y	P(y)



PDF stands for “*Probability Density Function*”

## Graph of Continuous Distributions



$$f(y) \geq 0$$

The associated probability for every possible value “ $y$ ”

# Discrete vs Continuous

The probability for any individual value equal to 0

$$P(X) = 0$$

$$P(x > X) = P(x \geq X)$$

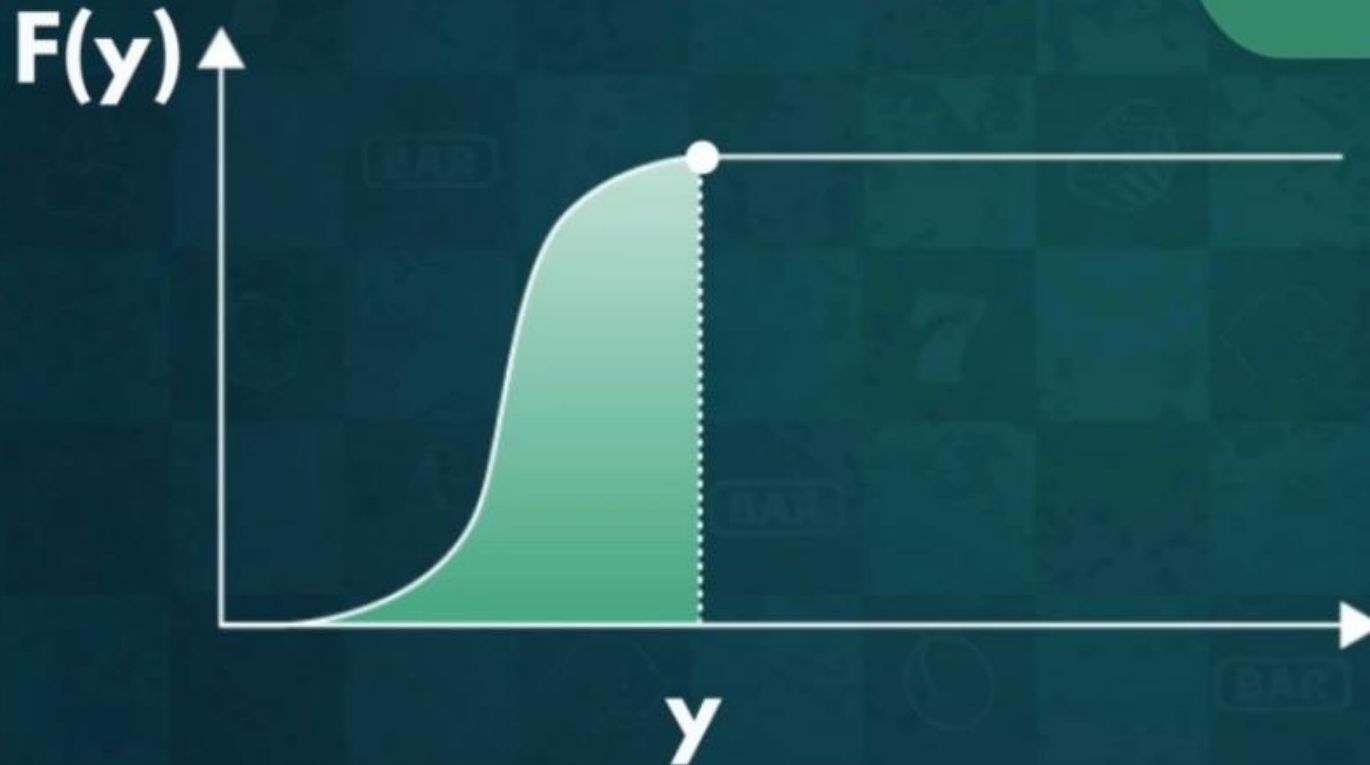
$$P(x < 6 \text{ min}) = P(x \leq 6 \text{ min})$$

$$P(x = 6) = 0$$



# Cumulative Distribution Function (CDF)

$$F(y) = P(Y \leq y)$$

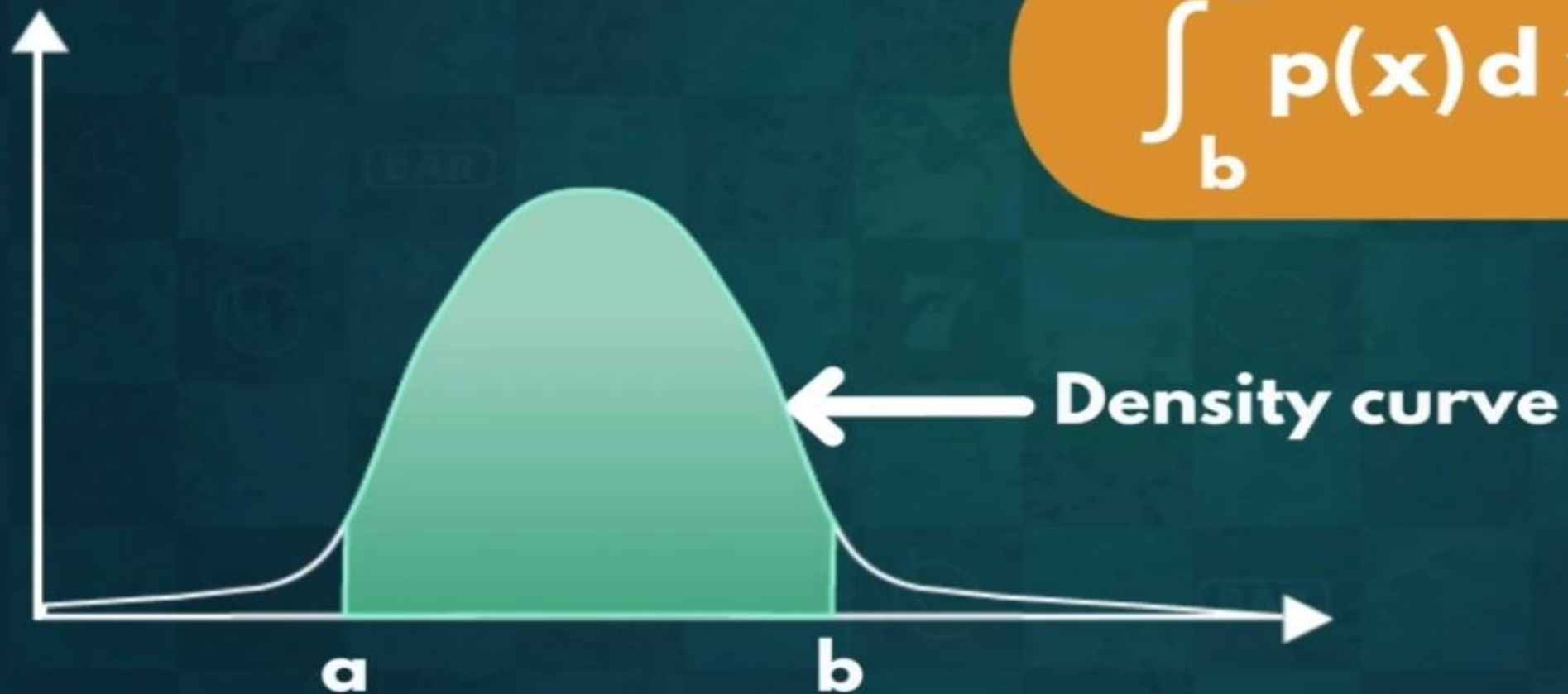




# Probability of Intervals

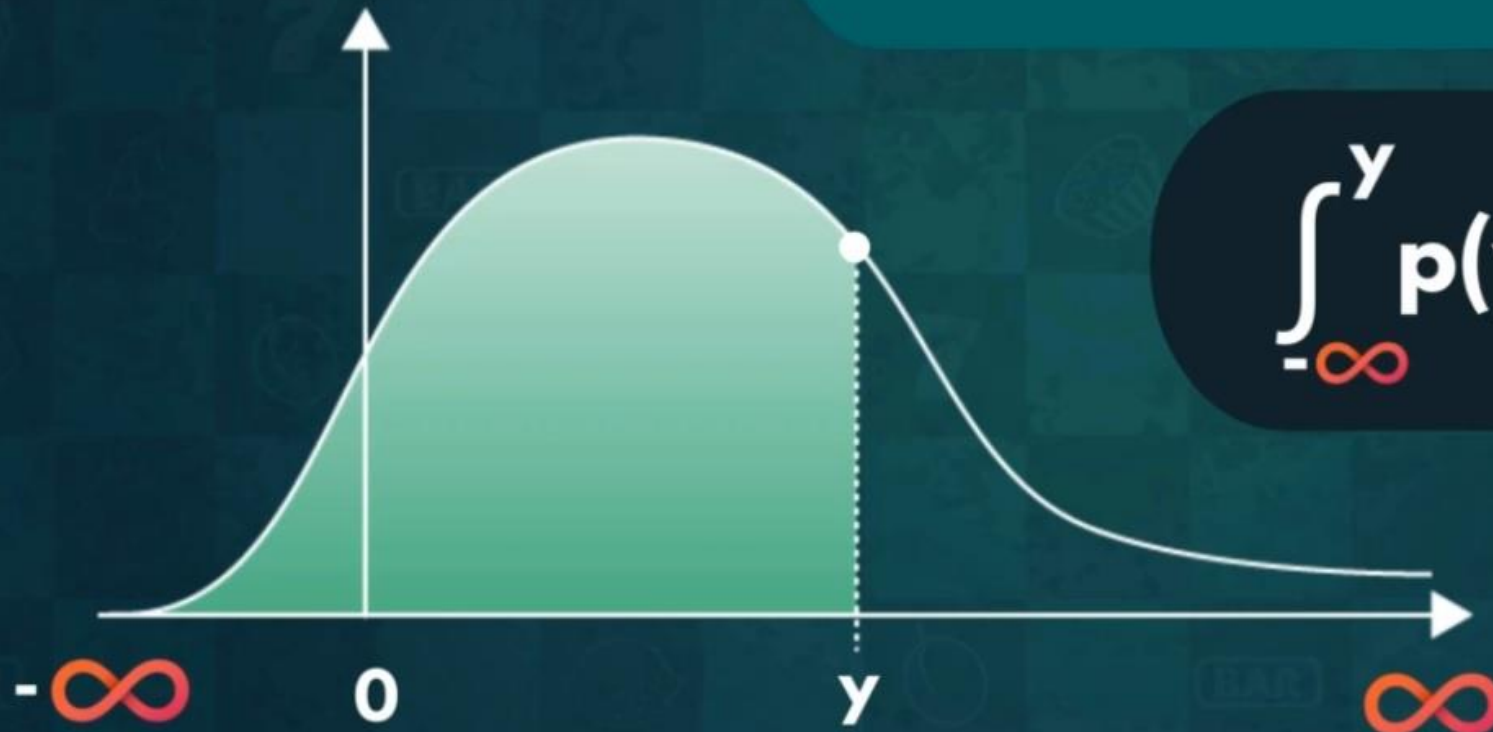
$$p(b > x > a)$$

$$\int_b^a p(x) dx$$



## CDF vs PDF

Probability of the interval  $(-\infty; y)$



$$\int_{-\infty}^y p(y) dy$$

## CDF vs PDF

PDF  $\xrightarrow{\text{Integral}}$  CDF

$$\int_{-\infty}^y p(y) dy = F(y)$$

PDF  $\xleftarrow{\text{Derivative}}$  CDF

$$p(y) = F(y) \frac{d}{dy}$$



$$E(y)$$

**$P(y) = 0 \rightarrow$  We can't apply the summation formula**

$$E(y) = \int_{-\infty}^{\infty} y p(y) dy$$

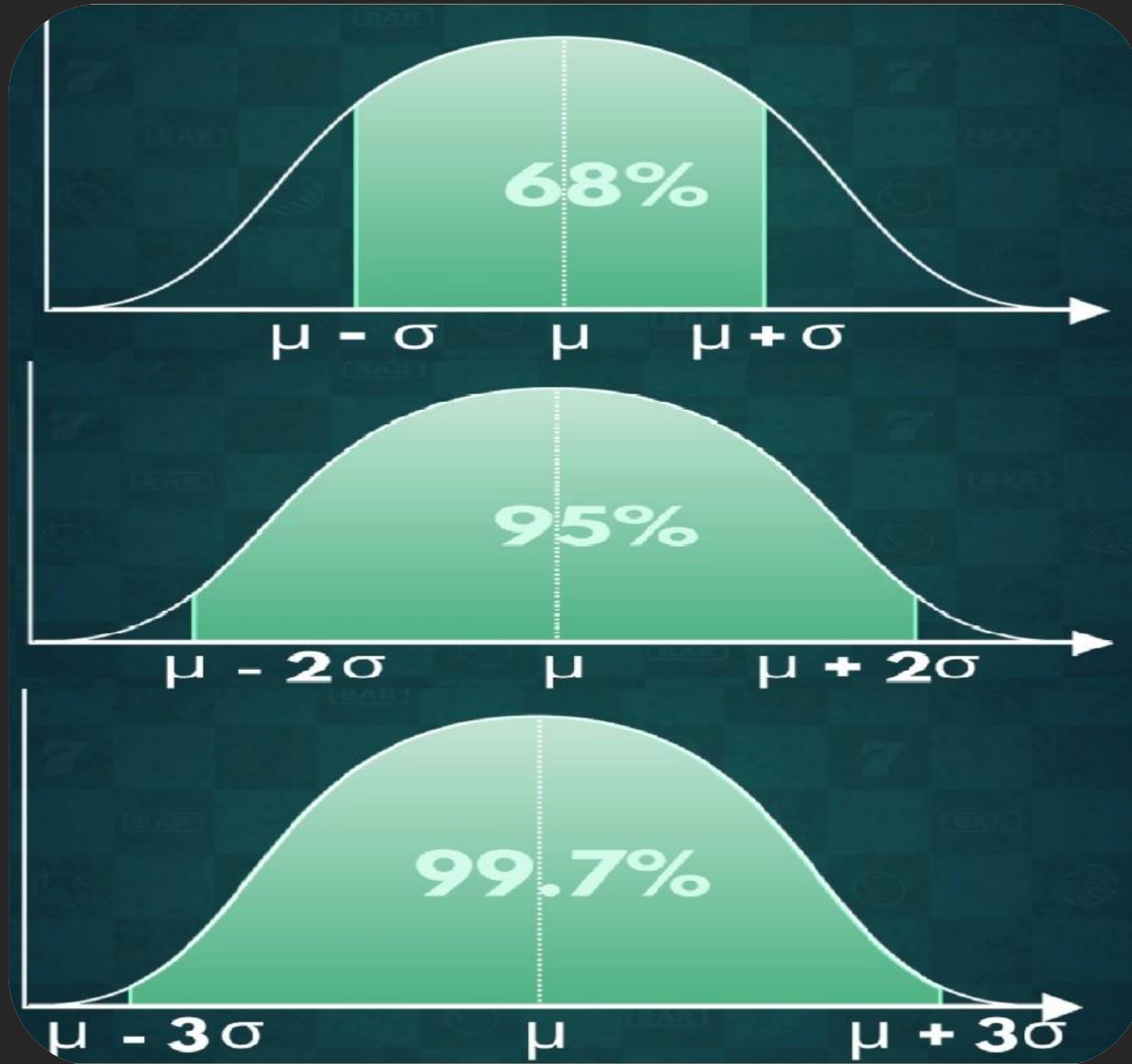
## *Expected Value and Variance*

**$P(y) = 0 \rightarrow$  We can't apply the summation formula**

$$E(y) = \int_{-\infty}^{\infty} y p(y) dy$$

**Same variance formula**

$$\text{Var}(y) = E(y^2) - E(y)^2$$



# Standardizing

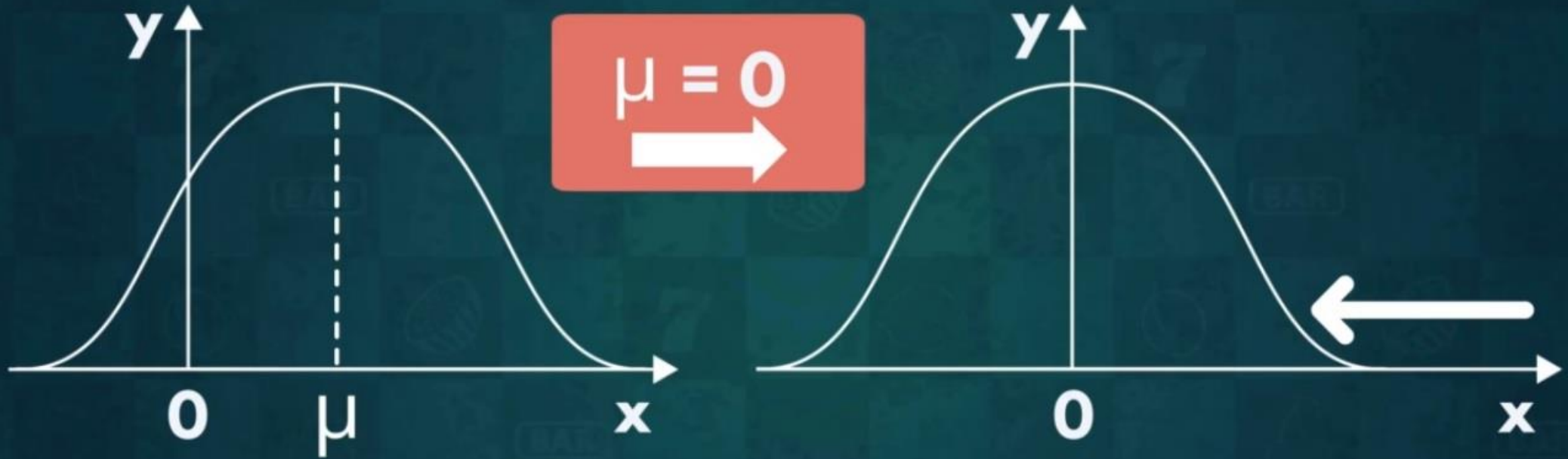
A special kind of transformation

$\mu$

$$E(X) = 0$$

$$\text{Var}(X) = 1$$

# Standardizing



$$y = f(x)$$

$$\lambda = t(x)$$

$$y = f(x - \mu)$$

$$\lambda = t(x - h)$$



# Standardizing



$$y = f(x - \mu)$$

$$y = f\left(\frac{x - \mu}{\sigma}\right)$$

$$\lambda = t(x - h)$$

$$\lambda = t\left(\frac{Q}{x - h}\right)$$

# Student's T Distribution

Small sample size approximation  
of a Normal Distribution

Certain characteristics + Sufficient data = Normal distribution

Certain characteristics + Sufficient data = Student's T distribution

# Applications *of chi-squared*

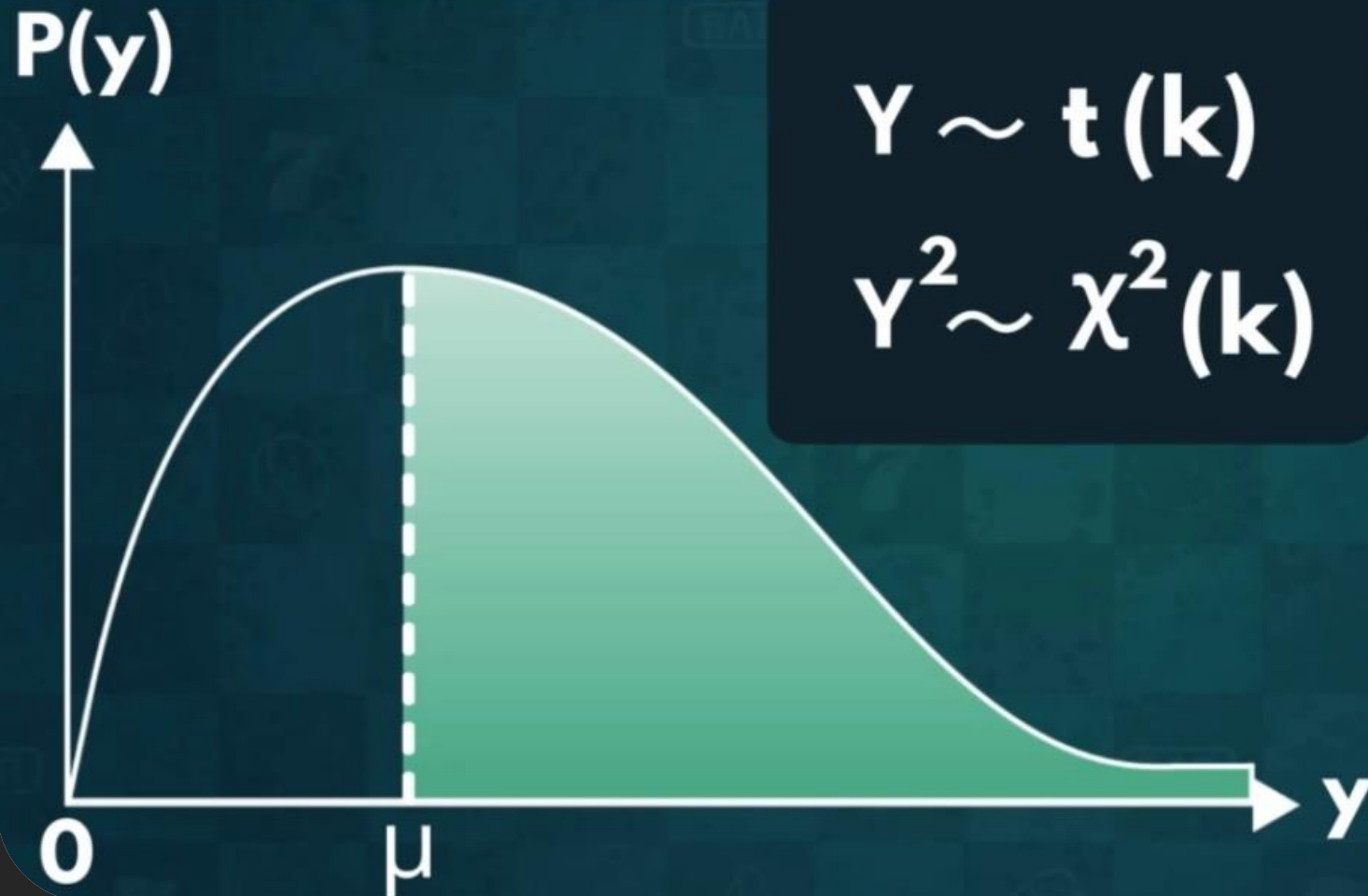
**Few events in real life**

**Statistical analysis**

- ◆ **Hypothesis testing**
- ◆ **Computing confidence intervals**

**Goodness of fit**

# Chi-Squared Distribution



$$Y \sim t(k)$$

$$Y^2 \sim \chi^2(k)$$

$$X \sim \chi^2(k)$$

$$\sqrt{X} \sim t(k)$$

**Asymmetric**

# Transformation

$$\left. \begin{array}{l} Y \sim \text{Exp}(\lambda) \\ X = \ln(Y) \end{array} \right\} \Rightarrow X \sim N(\mu, \sigma^2)$$

- One of the most common transformations

- One of the most common transformations



# Logistic Distribution

Location

Scale Parameter

**Logistic ( $\mu, S$ )**

**$Y \sim \text{Logistic}(6, 3)$**

# Tennis Example

Expectation

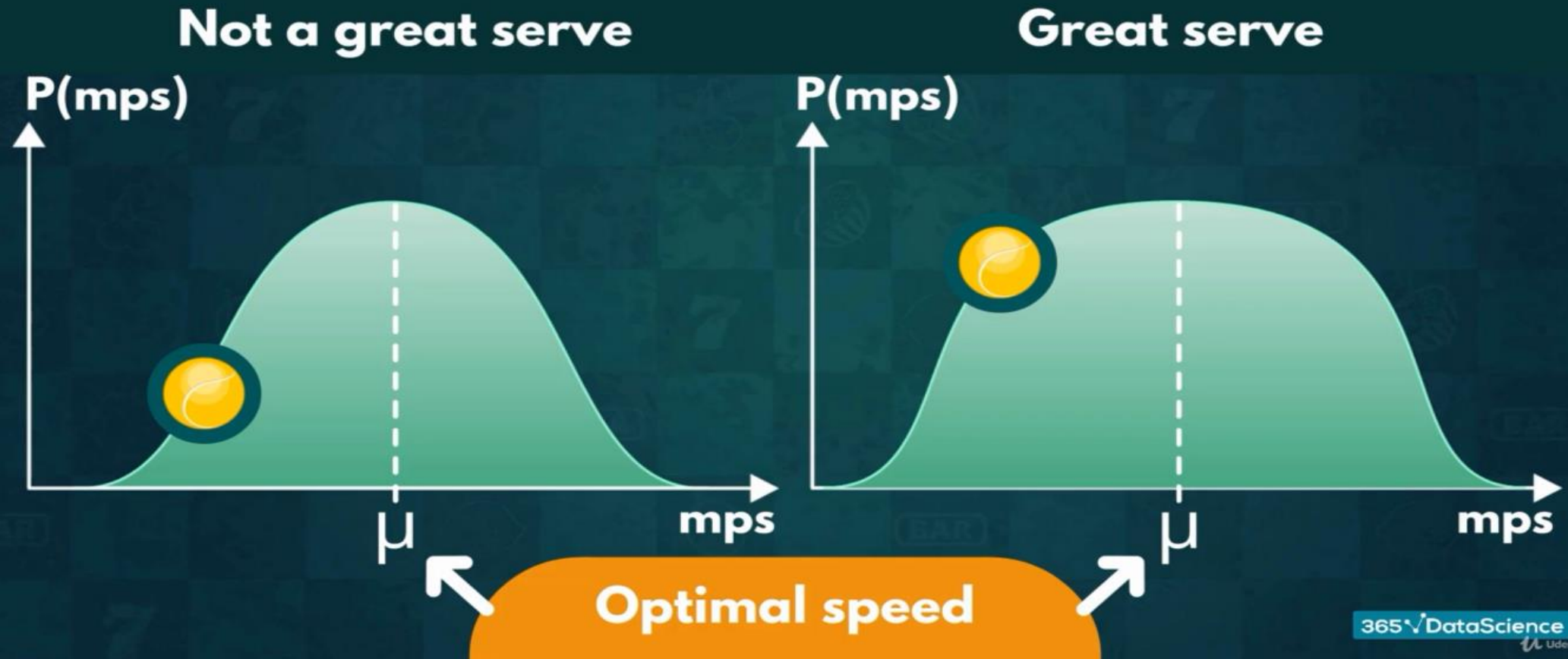


Reality



We cannot  
assume a  
linear  
relationship

# Tennis Example



# SUBJECT OF THE NEXT VIDEO

## *Conditionals in Python*

# درباره من

Mohammad Fozouni (Ph.D.)  
Dep. of Math & Statistics  
Gonbad Kavous University

- [fozouni@hotmail.com](mailto:fozouni@hotmail.com)
- <https://m-fozouni.ir>
- <http://profs.gonbad.ac.ir/fozouni/en>
- <https://www.aparat.com/elmedade>

#data\_science\_fozouni



IPM, 2019  
Operator  
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