

دوره آموزشی «علم داده» *Data Science Course*



جلسه چهارم: توزیع های گسسته

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Discrete Distributions

Finitely many distinct outcomes

Y	P(y)



$$P(y) = \frac{1}{y!e}$$

Every unique outcome has a probability assigned to it

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Card Example



$$P(0) + P(1) + P(2) + P(3) = P(y \leq 3)$$

$$b(0) + b(1) + b(2) + b(3) = b(\lambda \leq 3)$$



Die Example

$$P(1) = P(2) = P(3) = P(4) = P(5) = P(6)$$



Main Takeaway

$$X \sim U(a, b)$$

Each outcome is equally likely ♦

♦ Both the mean and the variance are uninterpretable

No predictive power



No predictive power



Bernoulli Distribution

Events with:

- ◆ 1 Trial
- ◆ 2 Possible outcomes



QUIZ

- _____

☐ True
☐ False

VOTE

- ☐ Democratic
☐ Republican

Binomial Distribution

Notation

$$B(n, p)$$

$$X \sim B(10, 0.6)$$

$$\Rightarrow \text{Bern}(p) = B(1, p)$$

Bernoulli vs Binomial Distribution



Quiz

Q1 _____ T/F

Q2 _____ T/F

...

Q10 _____ T/F



◆ Guessing 1 question → Bernoulli event

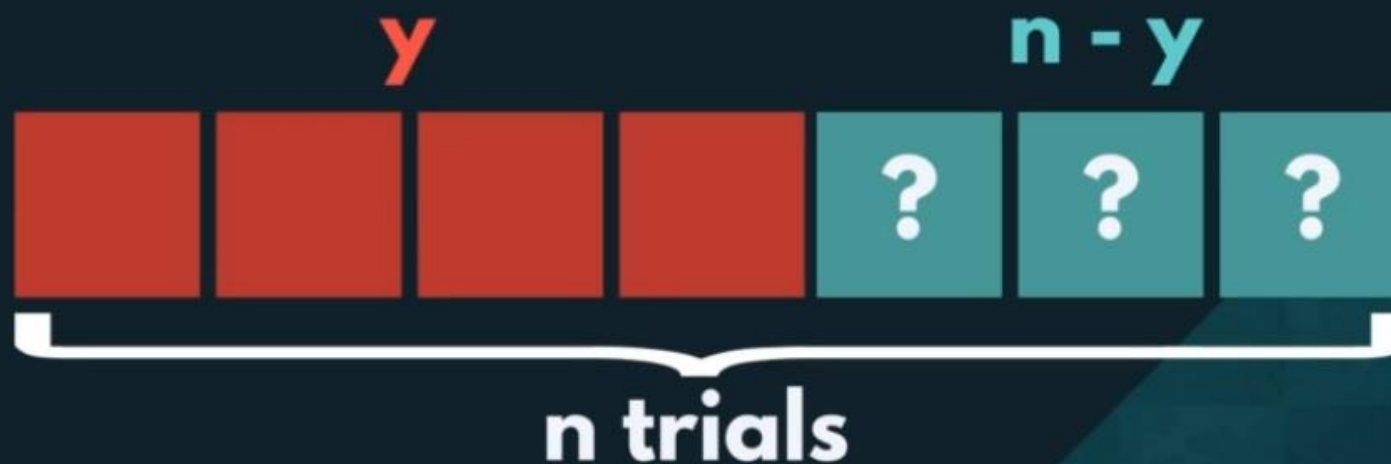
◆ Guessing the entire quiz → Binomial Ev

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The Probability Function

$$P(\text{desired outcome}) = p$$

$$P(\text{alternative outcome}) = 1 - p$$



Exactly y -many times
over the n trials

The Probability Function

$$p(y) = \binom{n}{y} \cdot p^y \cdot (1 - p)^{n - y}$$

GM Example

- A single stock of General Motors

$$p(\uparrow) = 60\% = 0.6$$

$$p(\downarrow) = 40\% = 0.4$$

- 3 Increases in 5 days

$$p(y) = \binom{n}{y} \cdot p^y \cdot (1 - p)^{n - y}$$



ABCD	▼	60.754	
ABCD	▲	4.344	0.3
ABCD	▲	55.543	0.1
ABCD	▼	63.446	0.3
ABCD	▲	32.304	0.30
ABCD	▼	12.324	0.30
ABCD	▼	32.765	0.30
ABCD	▲	21.734	0.304

$y \Rightarrow 3$
 $n \Rightarrow 5$
 $p \Rightarrow 0.6$

GM Example

$$p(y) = \binom{n}{y} \cdot p^y \cdot (1 - p)^{n - y} =$$

$$= C_3^5 \times 0.6^3 \times 0.4^2 =$$

$$= 34.56\%$$

of getting exactly 3 increases

of getting exactly 3 increases

Expected Values

$$E(X) = x_0 \cdot p(x_0) + x_1 \cdot p(x_1) + \dots x_n \cdot p(x_n)$$

$$Y \sim B(n, p)$$

$$E(Y) = p \cdot n$$

Variance and Standard Deviation

$$\begin{aligned}\sigma^2 &= E(y^2) - E(y)^2 = \\ &= n \cdot p \cdot (1 - p)\end{aligned}$$

$$= u \cdot b \cdot (1 - b)$$

Formula

Poisson Distribution is wildly different

$$P(Y) = \frac{\lambda^y e^{-\lambda}}{y!}$$

Its probability function is much different

Expected Value

$$p(y) \rightarrow E(y)$$

$$E(y) = y_0 \frac{\lambda^{y_0} e^{-\lambda}}{y_0!} + y_1 \frac{\lambda^{y_1} e^{-\lambda}}{y_1!} + \dots =$$

$$= \lambda$$

$$= y$$

Variance

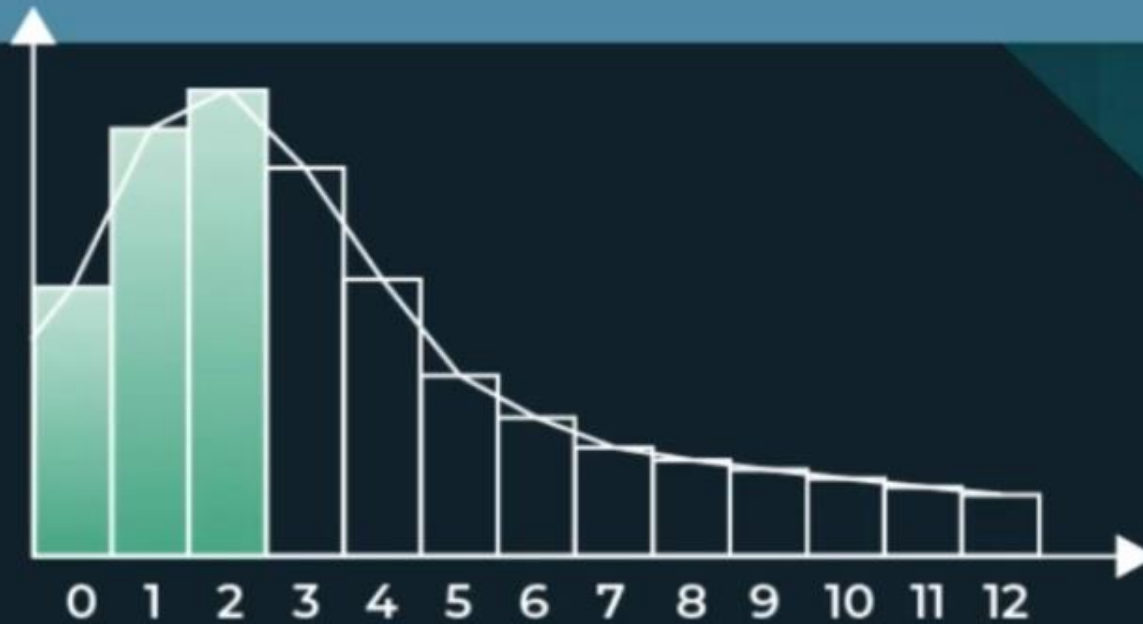
$$\sigma^2 = (y_0 - \mu)^2 + (y_1 - \mu)^2 + \dots =$$

$$= \lambda$$

So we have:

$$\mu = \sigma^2 = \lambda$$

Interval



- ◆ Same steps as discrete distributions
- ◆ The joint probability of all individual elements

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SUBJECT OF THE NEXT VIDEO

Continuous Distributions
In details

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