Math13 - Introduction to Statistics, Fall 2015

Exercises 2; Probability Basics and Counting Principles

1) Suppose that X takes on one of the values 1, 2, 3, 4, or 5.
If $P(X < 3) = 0.4$ and $P(X \geq 4) = 0.5$, find:

a. $P(X = 3) = 0.5 - 0.4 = 0.1$
b. $P(X < 4) = 1 - 0.5 = 0.5$

2) If A and B are events such that: $0 < P(A) < 1$, and $0 < P(B) < 1$; select the correct answer:

1.	$P(A) / 2 = 0.6$	always true	sometimes true	never true
2.	$P(A \text{ and } A^c) = 0$	always true	sometimes true	never true
3.	$P(A \text{ or } A^c) = 0$	always true	sometimes true	never true
4.	$P(A \text{ and } B) \leq P(A)$	always true	sometimes true	never true
5.	$P(A \mid A^c) = 1$	always true	sometimes true	never true
6.	$P(B \mid A) = P(B)$	always true	sometimes true	never true
7.	$P(A \text{ or } B) = P(A) + P(B)$	always true	sometimes true	never true
8.	$P(A) = P(B)$	always true	sometimes true	never true
9.	$P(A \mid B) - P(A) = 0$	always true	sometimes true	never true
10.	$P(A \mid B)\, P(B) = P(B \mid A)\, P(A)$	always true	sometimes true	never true

4) Suppose that A and B are events such that $P(A) = 0.4$, $P(B) = 0.5$, and $P(A \cap B) = 0.2$.

a. Are A and B mutually exclusive?
b. Are A and B independent?

5) When two balanced dice are rolled, 36 equally likely outcomes are possible. Let Z denote the sum of the dice.

a. What are the possible values of Z?
   2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12
b. Find $P(Z = 7)$
   6 / 36
c. Find the probability distribution of Z. Leave your probabilities in fraction form.

d. Construct a graph of the probability distribution.

Exercises 2; Probability Basics and Counting Principles

6) Suppose that A and B are independent events with P(A) = 0.8, and $P(B^c) = 0.4$

a. Find $P(A \cap B)$
b. $P(A \cup B)$
c. P(B)
d. $P(A^c \cap B)$

7) A total of 500 married working couples were polled about whether their annual salaries exceeded $75,000. The following information was obtained:

		Husband	
		Less than $75,000	More than $75,000
Wife	Less than $75,000	212	198
	More than $75,000	36	54

a. What is the probability that the husband earns less than $75,000?

b. What is the conditional probability that the wife earns more than $75,000 given that the husband earns more than this amount?

c. What is the conditional probability that the wife earns more than $75,000 given that the husband earns less than this amount?

8) The committee on Student Life did a survey of 417 students regarding satisfaction with student government and class standing. The results follow:

	Freshman	Sophomore	Junior	Senior	***Total***
Not satisfied	17	19	23	12	***71***
Neutral	61	35	32	38	***166***
Satisfied	23	49	43	65	***180***
Total	***101***	***103***	***98***	***115***	***417***

Find the probability that a student selected at random is:

a. Satisfied =
b. Junior =
c. Satisfied, given that the student is a senior =
d. Neutral and freshman =
e. Senior, given satisfied =
f. Neutral or satisfied =
g. At least a sophomore =

9) A club has 90 members: 50 are lawyers and 50 are liars. Everyone is either a lawyer or a liar. Consider the experiment of randomly selecting a member. Let **A** be the event of selecting a lawyer. Let **B** be the event of selecting a liar.

a. What is P(A), the probability that a randomly selected member is a lawyer?

b. What is P(B), the probability that a randomly selected member is a liar?

c. What is $P(B^c)$, the probability that a randomly selected member is not a liar?

d. What is $P(A \cap B)$, the probability that a randomly selected member is both a lawyer and a liar?

e. What is P(A | B), the probability of randomly selecting a lawyer given that the member is a liar?

f. What is $P(A \cap B^c)$, the probability that a randomly selected member is both a lawyer but not a liar?

g. What is $P(B^c \mid A)$, the probability that a lawyer is not a liar?

10) Suppose that 25% of a forest consists of trees of species A, 40% of species B, and 35% of species C.

a. What is the probability that the tree selected at random will be of species A?

b. What is the probability that the tree selected at random will not be of species A?

c. If it is known that the tree is not of species A, what is the probability that it will be of species B?

11) An automobile license plate consists of three numbers followed by three letters. How many license plates are possible if:

a. repetition of numbers and letters is allowed?

b. no repetition of numbers and letters is allowed? (e.g. 112ABC, 278ABA, 112ABA are not allowed)

c. no repetition is allowed, the first digit must be either 1, 2 or 3, and the first letter cannot be a vowel?

12) Suppose that license plates are given out in random fashion. Mr. and Mrs. Brown (a two car family) went to get automobile license plates. If repetition is allowed, what is the probability that their plates:

a. would end with the same letter?
b. begin with the same digit?
c. begin with the same digit and end with the same letter?