Optimal procurement of contract under uncertain process networks

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1 Literature review of related works

To begin our literature review, we examined the chapter that was published in the book Quantitative models for supply chain management, titled "Modeling supply chain contracts: a review" by Tsay et al [1]. The authors present a comprehensive review of supply chain contract models and discuss different types of contracts and the quantitative models used to analyze them, with the aim of providing insights into the design of effective supply chain management. The paper emphasizes the importance of supply chain contracts as a key mechanism for coordination between different entities and state that they are crucial for aligning incentives, sharing risks, and ensuring smooth operations. The authors classify contracts into four categories: price-based contracts, quantity-based contracts, lead time-based contracts, and information sharing contracts. Moreover, Höhn [2] analyzes other contract types that could improve supply chain coodination. However, these authors emphasize the need for more robust models that can accommodate uncertain environments, as well as the importance of considering the human and behavioral aspects of contract negotiation [1]. Additionally, there has not been enough research regarding the optimal selection of contracts.

Advancing to optimally select contracts, Park et al. [3] modeled three different types of quantity-based contracts that a firm might construct with suppliers and customers using disjunctive programming and solved the resulting MILP to determine the optimal contract selection. Moreover, Bansal et al. [4] formulated a multi-period optimization model to minimize total procurement costs in a company. Basing their research on Tsay et al [1], they modeled price-based and quantity-based contracts. Finally, Khalilpour and Karimi [5] created a MILP framework that assists the buyer in choosing the optimal mix of suppliers and contracts in a comprehensive way for the LNG industry, taking into account various factors such as contract durations and timeframes, demand, pricing structures, volume discounts, delivery conditions, shipping expenses, and purchasing commitments [5]. Despite the fact that these research papers achieved great advancements in optimal contract procurement, they have not considered stochastic formulations (only deterministic).

Progressing to incorporate uncertainty in optimal contract acquisition, Rodríguez and Vecchietti [6] tackled the optimization of delivery and purchasing within a supply chain in the context of provision uncertainty, where a discrete probability distribution for the performance of each supplier is proposed to model two possible uncertain situations: In the first case, supplier failures determine the probability that a certain amount will not be delivered. In the second case, suppliers fail to provide the total amount ordered. They model quantity-based contracts with generalized disjunctive programming and solve the optimization problem. Additionally, Feng et al. [7] studied the coordinated contract selection and capacity allocation problem, in a three-tier manufacturing supply chain, with the objective to maximize the manufacturer's profitability [7]. Using a modeling approach based on stochastic programming with recourse, they show how to take into account economic, market, supply, and system uncertainties [7].

While there are multiple researchers who have tried to efficiently model the optimal contract procurement under uncertainty problem, there is still room for improvement. The authors of "Optimal Procurement Contract Selection with Price Optimization under Uncertainty for Process Networks" [8], which is the paper being studied in this project, propose extending the production planning decisions of a chemical process network to include optimal contract selection under uncertainty with suppliers and product selling price optimization. They use three quantity-based contract models: discount after a certain purchased amount, bulk discount, and fixed duration contracts to develop a mixed-integer nonlinear two-stage stochastic programming that accounts for uncertainty in both supply and demand for the planning of the process network [8].

The work presented by Calfa and Grossmann [8] is used in subsequent papers. Dundar [9] develops a MILP to identify minimum-cost approaches for reducing CO2 emissions via co-firing biomass subject to spatially-explicit biomass availability constraints. This MILP is extended to robust MILP model, addressing the uncertainties in power plant boiler installation cost, coal electricity generation cost, as well as the emission rate. Then, they investigate the impact of energy policy-related regulations on biomass demand and procurement cost using a MINLP based on Calfa and Grossmann's work [8]. Zhou-Kangas et al. [10] utilize the optimization model presented in Calfa and Grossmann [8] as the foundation for their work and augment it with three additional objectives for environmental responsibilities and the maintenance of strategic competence of the manufacturer. The additional objectives include: minimizing the environmental impact scores of selected business partners, minimizing the pollution content emitted from the production process, and maximizing the demand in the market for the main products [10]. Differing from Calfa and Grossmann [8], where uncertainty due to future developments was incorporated as stochastic parameters, they consider the uncertainty in the production process.

2 Problem Statement

The authors address the production planning decisions of a chemical process network that include optimal contract selection under uncertainty with suppliers and product selling price optimization. They use three quantity-based contract models: discount after a certain purchased amount, bulk discount, and fixed duration contracts. The supply chain framework examined in this study is depicted in Figure 1 and can be explained as follows.

The manufacturer possesses one or more production facilities, which may be located in various geographic areas. The primary decisions for the manufacturing firm involve: (1) choosing procurement contracts, and (2) establishing the prices for the products sold. This work adopts a manufacturer-centric perspective, which means that the contract structure (including prices and quantity thresholds) is defined, leaving it up to the manufacturer to decide whether or not to enter into a specific contract for a raw material. Additionally, the manufacturer has the option to use price- or demand-responsive models to determine the pricing of its finished goods [8].

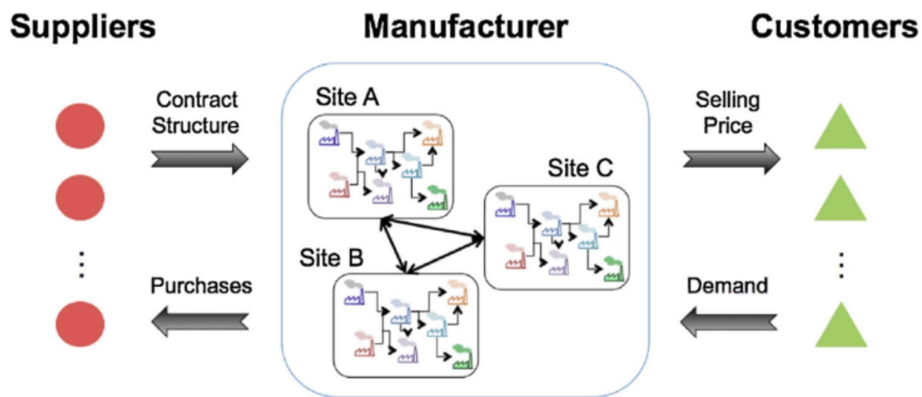

Figure 1: Supply chain structure considered in the paper

There are multiple assumptions made within this optimal contract procurement problem. First, the contract structure is given and fixed. This means that no negotiation between supplier and manufacturer is allowed [8]. In the real world this may not be a truly representative situation because once a contract is set, multiple negotiation rounds can take place before both parties agree to it. However, this assumption is sufficiently accurate for this problem. Second, the operating, inventory, and inter-site transfer costs are given and deterministic [8]. For future work, this assumption can be dropped to include stochastic costs. Finally, a price- or demand-response model is given, which is an efficient way to capture market dynamics. A price-response model expresses demand as function of price, whereas a demand-response model provides the inverse relationship [8].

3 Motivation for and main features of the model

This work is inspired from the paper "Optimal procurement contract selection with price optimization under uncertainty for process networks"[8]. It focuses on dealing with the uncertainty in the market scenarios which are the prices and the demand in the market, which makes the process of a chemical plant uncertain. Persay, the company has a certain budget B and wants to buy a certain raw material R which has a variable cost CP depending on the market scenario and then sells a product P at a variable cost SP. The company can set up a supply chain in broadly the following manners, i) Get discounts on the amount of orders placed. ii)Get bulk discounts on bulk orders. iii) Have fixed length contracts in place for the buying of goods. This is how the goods would be discounted while purchasing, but when it comes to the selling side of goods, the company has to decide for the selling price SP. This will be the first stage variables which we will fix in this two stage stochastic programming solution. Then once we know what amounts will we be purchasing the plant variables can then be solved for, the flows and the production rates.

Now the selling prices can be determined using the demand-response model, which adjusts the prices as a function of demand. This is done using two models - Constant-Elasticity and Logit pricing model. The objective function is the expected value of the profit is the sum of all scenarios. Then we solve the deterministic equivalent model to get the VSS and do the sensitivity analysis. Also we implement the expanding window horizon to solve the problem which solves it a bit faster.

4 Model Formulation

Let us understand the equations being implemented.

4.1 Equations used in the implementation

The below constraint indicates the amount of chemical j consumed or produced in process i in site s in time t, where $\mu_{i,j,s}$ is the fraction of j being produced/consumed depending on sign of $\mu_{i,j,s}$.

$$W_{i,j,s,t} = |\mu_{i,j,s}| W_{i,j',s,t} \qquad \forall s \in ST, i \in IS_s, j \in J_i, j' \in JM_i, t \in T \tag{1}$$

Material Balance for each chemical j in every site s at time t is given by:

$$V_{j,s,t-1} + \sum_{i \in O_j} W_{i,j,s,t} + P_{j,s,t} = V_{j,s,t} + \sum_{i \in I_j} W_{i,j,s,t} + \sum{}^{s' \in ST}_{s' \neq s} F_{j,s,s',t} + SS_{j,s,t} \qquad \forall s \in ST, j \in J, t \in T \tag{2}$$

Sales amount of each site $SS_{j,s,t}$

$$S_{j,t} = \sum_{s \in ST} SS_{j,s,t} \qquad \forall j \in J, t \in T \tag{3}$$

The constraint below tells about the total purchase amount

$$P_{j,s,t,k} = P^{spot}_{j,s,t,k} + \sum_{c \in C} P^{C}_{j,s,t} \qquad \forall s \in ST, j \in JR, t \in T, k \in K \tag{4}$$

The constraint below indicates the total purchase cost

$$COST_{j,t,k} = \sum_{s \in ST} \left[\alpha^{spot}_{j,s,t,k} P^{spot}_{j,s,t,k} + \sum_{c \in C} COST^{C}_{j,s,t} \right] \qquad \forall j \in JR, t \in T, k \in K \tag{5}$$

The constraint below indicates which type of contract is being chosen.

$$y^{c}_{j,s,t} \leq 1 \quad \forall c \in C \quad where \quad C = d, b, l \tag{6}$$

The constraint below implements the purchase cost of the discount contract

$$COST_{j,s,t,d} = \phi^{d_1}_{j,s,t} P^{d_1}_{j,s,t} + \phi^{d_2}_{j,s,t} P^{d_2}_{j,s,t} \forall j \in JR, s \in ST, t \in T \tag{7}$$

The constraint below implements the purchase amount of all contract schemes in the discount contract.

$$Pd^{d}_{j,s,t} = Pd^{d_1}_{j,s,t} + Pd^{d_2}_{j,s,t} \forall j \in JR, s \in ST, t \in T \tag{8}$$

The constraint below implements the purchase amount of all contract schemes in the discount contract.

$$Pd^{d_1}_{j,s,t} = Pd^{d_{1,1}}_{j,s,t} + Pd^{d_{1,2}}_{j,s,t} \forall j \in JR, s \in ST, t \in T \tag{9}$$

The constraint below implements the upper bound on the amount of discount for the discount contract scheme 1.

$$Pd^{d_1,1}_{j,s,t} \leq y^{d_1}_{j,s,t} \sigma^{d}_{j,s,t} \forall j \in JR, s \in ST, t \in T \tag{10}$$

The constraint below implements the equality for the amount of discount for the discount contract scheme 1 and 2.

$$Pd^{d_1,2}_{j,s,t} = y^{d_2}_{j,s,t} \sigma^{d}_{j,s,t} \forall j \in JR, s \in ST, t \in T \tag{11}$$

The constraint below implements the upper bound on the amount of discount for the discount contract scheme 1.

$$Pd^{d_1,2}_{j,s,t} = y^{d_2}_{j,s,t}\sigma^{d}_{j,s,t}\forall j \in JR, s \in ST, t \in T \tag{12}$$

The constraint below implements the upper bound on the amount of discount for the discount contract scheme 2.

$$Pd^{d_2}_{j,s,t} \leq y^{d_2}_{j,s,t}U^{d}_{j,s,t}\forall j \in JR, s \in ST, t \in T \tag{13}$$

The constraint below implements the lower bound on the amount of discount for the discount contract scheme 2.

$$Pd^{d_2}_{j,s,t} \geq 0\forall j \in JR, s \in ST, t \in T \tag{14}$$

The constraint below implements the logical bound that only one of the two schemes get selected if discount contract is selected.

$$y^{d_1}_{j,s,t} + y^{d_2}_{j,s,t} = y^{d}_{j,s,t}\forall j \in JR, s \in ST, t \in T \tag{15}$$

Similar constraints are being implemented for the bulk discount with different variables, with the only change being the lower bound of the constraint to implement the lower bound on the amount of discount for the discount contract scheme 2, but here it will be for the bulk discount contract type.

$$P^{b_2}_{j,s,t} \geq y^{b_2}_{j,s,t}\sigma^{b}_{j,s,t}\forall j \in JR, s \in ST, t \in T \tag{16}$$

For fixed length contracts, the constraints are a bit different. The below constraints implement purchase cost of fixed length contract

$$COST^{l}_{j,s,t} = \phi^{l_1}_{j,s,t}P^{l_1}_{j,s,t} + \sum_{p \in LC_{2:|LC|}} \sum_{\substack{t' \in T \\ t' \leq t \\ t' \geq t-l_p+1}} \phi^{l_p}_{j,s,t}P^{l_p}_{j,s,t,t'}\forall j \in JR, s \in ST, t \in T \tag{17}$$

The below constraints implements purchase amount of fixed length contract

$$P^{l}_{j,s,t} = P^{l_1}_{j,s,t} + \sum_{p \in LC_{2:|LC|}} \sum_{\substack{t' \in T \\ t' \leq t \\ t' \geq t-l_p+1}} P^{l_p}_{j,s,t,t'}\forall j \in JR, s \in ST, t \in T \tag{18}$$

The below constraints implement a lower bound on purchase amount of fixed length contract

$$y^{l_p}_{j,s,t}\sigma^{l_p}_{j,s,t} \leq P^{l_p}_{j,s,t,t'} \forall j \in JR, s \in ST, p \in LC, (t,t') \in T, t' \leq t, t' \geq t - l_p + 1 \tag{19}$$

The below constraints implement upper bound on the purchase amount of fixed length contract

$$P^{l_p}_{j,s,t,t'} \leq y^{l_p}_{j,s,t} U^{l}_{j,s,t} \forall j \in JR, s \in ST, p \in LC, (t,t') \in T, t' \leq t, t' \geq t - l_p + 1 \tag{20}$$

The below constraints implement a logical constraint to select exactly 1 contract from the scheme of many contracts if fixed duration contract is selected.

$$\sum_{p \in LC} y^{l_p}_{j,s,t} = y^{l}_{j,s,t} \forall j \in JR, s \in ST, t \in T \tag{21}$$

The below constraints implement logical constraint to select exactly 1 contract for the particular duration and not overlap with other contracts in that same period for the fixed duration period.

$$y^{l_p}_{j,s,t} \leq 1 - y^{l_{p'}}_{j,s,t} \forall j \in JR, s \in ST, (p,p') \in LC, (t,t') \in T, t' \leq t, t' \geq t - l_p + 1 \tag{22}$$

The below constraints implements an auxiliary total sales variable.

$$y^{l_p}_{j,s,t} \leq 1 - y^{l_{p'}}_{j,s,t} \forall j \in JR, s \in ST, (p,p') \in LC, (t,t') \in T, t' \leq t, t' \geq t - l_p + 1 \tag{23}$$

The objective function for the Constant-Elasticity pricing model is given by,

$$\begin{aligned} max \quad \mathbb{E}[PROFIT] = \sum_{k} \Pi_k \Bigg[\sum_{j \in JP} \sum_{t \in T} (\beta_2' S_{j,t}^{1-\frac{1}{E_j}} + S_{j,t}\epsilon k) + \sum_{s \in ST} \sum_{j \in JR} \sum_{t \in T} \alpha^{spot}_{j,s,t,k} COST_{j,s,t,k} \\ - \sum_{s \in ST} \sum_{i \in IS_s} \sum_{j \in JM_i} \sum_{t \in T} \delta_{i,s,t} W_{i,j,s,t,k} - \sum_{j \in J} \sum_{s \in ST} \sum_{t \in T} \xi_{j,s,t} V_{j,s,t,k} - \sum_{s \in ST} \sum_{\substack{s' \in ST \\ s' \neq s}} \sum_{j \in J} \sum_{t \in T} \eta_{j,s,s',t} F_{j,s,s',t,k} \Bigg] \end{aligned} \tag{24}$$

The objective function for the logit pricing model is given by,

$$\begin{aligned} max \quad \mathbb{E}[PROFIT] = \sum_{k} \Pi_k \Bigg[\sum_{j \in JP} \sum_{t \in T} (\frac{1}{\beta_5} ln(\frac{\beta_3 - S_{j,t}}{S_{j,t}} - \beta_4) + S_{j,t}\epsilon k) + \sum_{s \in ST} \sum_{j \in JR} \sum_{t \in T} \alpha^{spot}_{j,s,t,k} COST_{j,s,t,k} \\ - \sum_{s \in ST} \sum_{i \in IS_s} \sum_{j \in JM_i} \sum_{t \in T} \delta_{i,s,t} W_{i,j,s,t,k} - \sum_{j \in J} \sum_{s \in ST} \sum_{t \in T} \xi_{j,s,t} V_{j,s,t,k} - \sum_{s \in ST} \sum_{\substack{s' \in ST \\ s' \neq s}} \sum_{j \in J} \sum_{t \in T} \eta_{j,s,s',t} F_{j,s,s',t,k} \Bigg] \end{aligned} \tag{25}$$

For this motivating example the problem size is

BLOCKS OF EQUATIONS 27 SINGLE EQUATIONS 762

BLOCKS OF VARIABLES 24 SINGLE VARIABLES 797

NON ZERO ELEMENTS 2,481 NON LINEAR N-Z 12

CODE LENGTH 60 DISCRETE VARIABLES 60

5 Case Study

We implemented the case study on the motivating example given in the library. Our approach involved solving the problem using the expanding time horizon approach.

Expanding time horizon refers to the process of extending the time frame in which an individual or organization makes decisions or plans for the future. It means increasing the amount of time that is taken into consideration when making a decision or forecasting future outcomes. This approach for solving the problem will allow avoiding one large problem but solve multiple smaller problems.

How we implement the expanding time horizon is, we initially have a region of a horizon of 2 time periods and then we solve the two time period horizon with rigorous constraints, which we will be calling an active period and the rest periods are solved with relaxations. Then once we have the solution for the first time period, we fix that solution for the first problem and then solve an expanding window of time horizons, with the remaining being solved with relaxations. There is one increment in each iteration of the length of the horizon window. This procedure is done till the last time period is solved with the rigorous constraints. The below figure shows how our approach works.

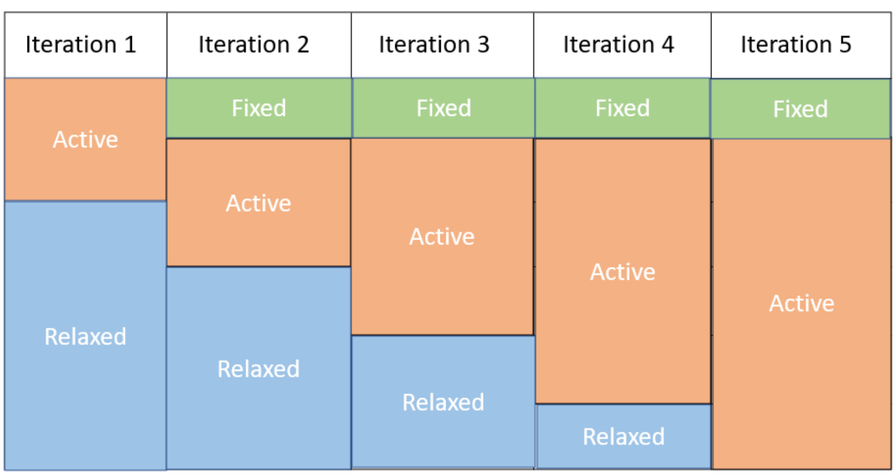

Figure 2: Expanding window horizon approach

This approach proved to be beneficial as it resulted in around a 55 min time reduction (from the max cycles time) of the time used to solve the original MINLP problem (solution was not found). It also reported in nearly the same objective function (in the tolerance limit of 0.5%) and it happened because of a crossover in the solution.

Further suggestions would be include rolling time horizon/ shrinking time horizon or other decomposition techniques, which would make solving this problem faster.

The solution time is 6089.625 seconds for solving the entire MINLP as a whole, while the solution time for the 2761.8 seconds for solving the rolling time horizon with 2 active windows. The model is run on 11^{th} Gen Intel(R) Core(TM) i7-1165G7 @ 2.80GHz 2.80 GHz and 16GB of RAM.

6 Sensitivity Analysis

6.1 Value of Stochastic Solution

One of the objective ways to get the sensitivity analysis done in stochastic programming is to do the Value of Stochastic Solution.

Let us understand the meaning of Value of Stochastic solution (VSS). VSS is the metric to quantify the potential benefits of using stochastic optimization techniques to solve problems that involve uncertainty. This helps because of the error, the deterministic solution might not be able to give us an optimal solution, but the stochastic model would take into account the uncertainty and yield us an optimal solution.

6.2 VSS Study

We have applied VSS on a few scenarios and the entire problem with uniform probability distribution, by removing the uncertainty in the first stage decision variables and make it deterministic. The results for scenario 10 indicate that the, VSS = 860.313 - 187.091 = 673.222 which is around 78.2%. The results for scenario 2 indicate that the, VSS = 230.563 - 196.376 = 34.187 which is around 14.82%. The results for the entire problem with uniform probability distribution is given by, VSS = 290.676 - 180.577= 110.099 which is about 37.87%.

Due to the time constraint (most scenarios took 2 hours to run), we were able to calculate the VSS for only the even scenarios. The below plot gives a better understanding of how the VSS varies with scenarios and the whole problem.

7 Mistakes in code

The major mistake that we found in the code apart from the data which we assume was written correctly was in the objective function of the models.

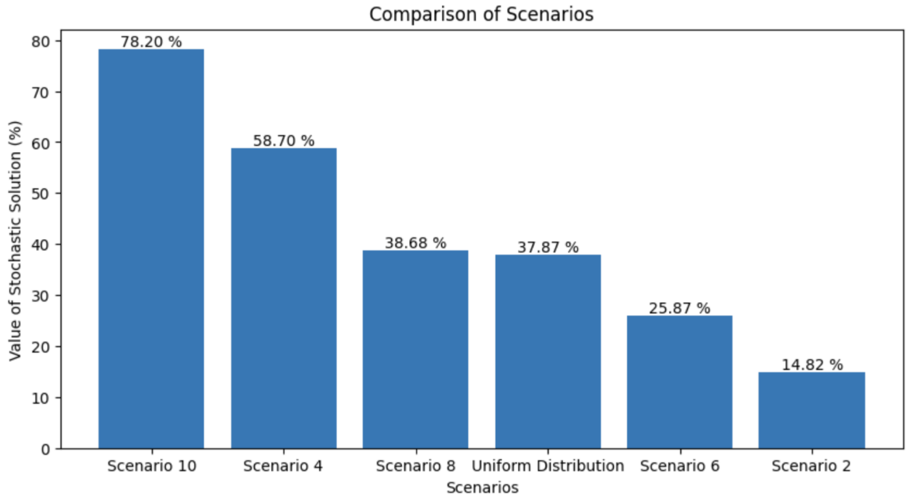

Figure 3: VSS for the whole problem and even scenarios

The authors solve 2 Demand-response models which have two different objective function which mainly vary in the DRM model. The authors here write the same variable PROFIT for both the models which gets overridden everytime with the logit pricing model. Thus, one should run the code with the two different variable names. This would get us two solutions and let us decide which solution is better.

8 Recommendations

8.1 Multi objective Model formulation

One recommendation that we would like to present is the use of multi objective model formulation.

The authors solve two models which are more or less similar with the only difference being the objective function which is different for different DRM models. The two models can be further combined using a multi-objective formulation. This could be done in multiple ways, like weighted sum method, ϵ-Constraint, Weighted metric method. Of this, we shall give an example of the weighted metric method as it is most suitable for our problem (since our problem is non-convex),

$$\begin{aligned} min \quad & l_p(x) = \left(\sum_{m=1}^{M} w_m |f_m(x) - z_m^*|^P \right)^{\frac{1}{P}} \\ \text{s.t} \quad & g_j(x) \geq 0, \quad j = 1, 2, \ldots, J \\ & h_k(x) = 0, \quad k = 1, 2, \ldots, K \\ & x_i^{(L)} \leq x_i \leq x_i^{(U)} \quad i = 1, 2, \ldots, n \end{aligned} \tag{26}$$

where w is the weights those get multiplied to the distance metric of the best solution to the current solution. M is the number of objective functions, j is the number of inequality constraints, i is the number of equality constraints and n is the number of variables. The above example is referred from [11].

8.2 Decomposition methods

Decomposition methods could be applied to solve the problem in a much nicer manner than solving it all at once. While this may yield in a relaxed solution, we hypothesize that it will solve the problem faster.

8.3 Solver

The solution uses CONOPT. We can use CONOPT4 and GUROBI10, the latest solvers, to solve NLP and MILP respectively. This could be an approach to check with different solvers the solution time of the problem.

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9 APPENDIX

i process or chemical plant

j chemical

s site

t time period

Sets

I set of processes or chemical plants

I_j set of processes that consume chemical j

IS_s set of processes that belong to site s

JR set of raw materials JP set of products

J_i set of chemicals involved in process i

JM_i set of main products of process i

O_j set of processes that produce chemical j

ST set of sites

T set of time periods

Parameters

$a^L_{j,s,t}$,$a^U_{j,s,t}$ lower and upper bounds on availability of raw material j to site s at time period t

$\alpha^{spot}_{j,s,t}$ spot market price of raw material j to site s at time period t

$\delta_{i,s,t}$ operating cost of process i in site s at time period t

$\mu_{i,j,s}$ mass factor of product j in process i in site s

$\xi_{j,s,t}$ inventory cost of chemical j in site s at time period t

$\eta_{j,s,s',t}$ inter-site transfer cost of chemical j from site s to site $s^{'}$ at time period t

$Q_{i,s,t}$ production capacity of process i in site s at time period t

$V^U_{j,s,t}$ upper bound on inventory of chemical j in site s at time period t

$F^U_{j,s,s',t}$ upper bound on inter-site transfer of chemical j from site s to site $s^{'}$ at time period t

Variables

$P_{j,s,t}$ purchase amount of raw material j by site s at time period t

$SS_{j,s,t}$ sales amount of product j from site s at time t

$S_{j,t}$ aggregated sales amount of product j for all sites at time t

$F_{j,s,s',t}$ inter-site transfer amount of product j from site s to site $s^{'}$ at time t

$V_{j,s,t}$ inventory level of chemical j in site s at time t

$W_{i,j,s,t}$ amount of chemical j consumed or produced in process i in site s at time t

PROFIT objective function variable representing the profit of the production planning model

$SALES_{j,t}$ sales of product j in time period t

$COST_{j,t}$ purchase costs of raw material j in time period t

$P^{spot}_{j,s,t}$ spot market component of purchase amounts of raw material j by site s in time period t

$P^{c}_{j,s,t}$ contract component of purchase amounts of raw material j by site s in time period t and contract type c

$COST^{c}_{j,s,t}$ procurement cost for raw material j in site s at time period t and contract type c