HISTORY

The sequence is named after the Italian mathematician “Leonardo of Pisa”, who was known as “Fibonacci”. He introduced this sequence to the Western world in his book "Liber Abaci," published in 1202. However, the sequence had been previously described in Indian mathematics as early as 200 BC in the works of the mathematician “Pingala” and later in the writings of “Virahanka” and “Gopala”.

Fibonacci initially presented the sequence through a problem about rabbit population growth.

"A certain man put a pair of rabbits in a place surrounded on all sides by a wall. How many pairs of rabbits can be produced from that pair in a year if it is supposed that every month each pair begets a new pair which from the second month on becomes productive?"

Pictorial Explanation:

Month	**Rabbit Pairs**	**Explanation**
1	1	Initial pair
2	1	Initial pair matures
3	2	Initial pair produces a new pair
4	3	Initial pair produces another pair, first offspring mature
5	5	Two pairs produce new pairs
6	8	Three pairs produce new pairs
7	13	...
8	21	...
9	34	...
10	55	...
11	89	...

12	144	...

Recursion

“Recursion” is a technique in programming and mathematics where a function calls itself directly or indirectly. In the context of the Fibonacci sequence, it allows us to compute Fibonacci numbers straightforwardly by defining the problem in terms of smaller instances of the same problem.

Recursive Algorithm for Fibonacci Sequence

Here’s a simple recursive function to calculate Fibonacci numbers:

1. **Base Case**:
   If (n = 0), return (0).
2. **Base Case:**
   If (n = 1), return (1).
3. **Recursive Case:**
   For (n > 1), return (F(n) = F(n-1) + F(n-2)).

Visualization of Recursion

The decision tree below illustrates how recursive calls expand as the Fibonacci function processes (F(5)):

```
          F(5)
         /    \
       F(4)     F(3)
      /   \     /   \
    F(3)   F(2) F(2)   F(1)
    / \    / \
  F(2) F(1) F(1) F(0)
   / \
 F(1) F(0)
```

Mathematical Proofs of Fibonacci Sequence

Binet's Formula

One mathematical proof related to the Fibonacci sequence is “Binet's Formula” which expresses the (n)-th Fibonacci number using the golden ratio (phi):

$$F(n) = \frac{\phi^n - (1-\phi)^n}{\sqrt{5}}$$

where $\phi = \frac{1+\sqrt{5}}{2} \approx 1.618.$

Proof by Induction

The Fibonacci sequence can also be proven using the principle of induction.

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Statement: For n ≥ 0, the Fibonacci sequence defined by F(0) =0,F(1)=1, and F(n)=F(n-1)+F(n-2) is valid.

Base Case:

Form =0: F(0) = 0.

Form = 1: F(1) = 1.

Inductive Step:

Assume that the statement is true for n = k and n = k - 1 (inductive hypothesis). We need to prove it form = k + 1:

F(k+1)=F(k)+F(k-1)

By the inductive hypothesis, this holds true because both F(k) and F(k - 1) are defined as per the Fibonacci sequence.
By induction, the Fibonacci sequence holds for all n.