Diskrete Mathematik - Übungen SW02

David Jäggli

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Inhaltsverzeichnis

1 Funktionen

I.)
$\mathbb{D} = \mathbb{R}$
$\mathbb{W} = \mathbb{N}$

Korrektur I.)
$\mathbb{D} =$ alle binäre Strings
$\mathbb{W} = 2\mathbb{N}$

II.)
Gegeben:
$f(x) = x^2 + 1$
$g(x) = x + 2$
whereas $\mathbb{R} \mapsto \mathbb{R}$

Gesucht:

1. $f \circ g$
2. $g \circ f$
3. $f + g$
4. $f \cdot g$

$f \circ g = f(x+2) = (x+2)^2 + 1 = \underline{x^2 + 4x + 5}$
$g \circ f = g(x^2+1) = (x^2+1) + 2 = \underline{x^2 + 3}$
$f + g = (x^2+1) + (x+2) = \underline{x^2 + x + 3}$
$f \cdot g = (x^2+1) \cdot (x+2) = \underline{x^3 + 2x^2 + x + 2}$

2 Folgen, Reihen, Summen- und Produktzeichen

III.)

a) $\prod\limits_{i=0}^{10} i = 0$

b) $\prod\limits_{i=5}^{8} i = 1680$

c) $\prod\limits_{i=1}^{100} (-1)^i = -1$

d) $\prod\limits_{i=1}^{10} 2 = 2^{10}$

Korrektur:

c) anscheinend gilt $\prod\limits_{i=1}^{n} (-1)^i = (-1)^{\frac{n\cdot(n+1)}{2}}$ wodurch

$$\prod_{i=1}^{100} (-1)^i = (-1)^{\frac{100\cdot 101}{2}} = (-1)^{5050} = 1$$

IV.)

a) $\sum_{j=0}^{8} 3 \cdot 2^j = 3 \cdot (1+2+4+8+16+32+64+128+256) = 3 \cdot (2^9 - 1) = 1533$

b) $\sum_{k=1}^{8} 2^k = 2^9 - 2 = 510$

c) $\sum_{l=2}^{8} (-3)^l = (-3)^2 + (-3)^3 + (-3)^4 + (-3)^5 + (-3)^6 + (-3)^7 + (-3)^8 = 4923$

d) $\sum_{i=0}^{8} 2 \cdot (-3)^i = 2 \cdot (c + 1 - 3) = 9842$

Korrektur:

Forgot about $\sum_{k=0}^{n} x^k = \frac{x^{n+1}-1}{x-1}$

V.)

$$\sum_{k=99}^{200} k^3 = \sum_{k=1}^{200} k^3 - \sum_{k=1}^{98} k^3 = \frac{200^2 \cdot 201^2}{4} - \frac{98^2 \cdot 99^2}{4} = 380477799$$

VI.)

a) $\sum_{j=0}^{4} j! = 1 + 1 + 2 + 6 + 24 = 34$

b) $\prod_{j=0}^{4} j! = 1 \cdot 1 \cdot 2 \cdot 6 \cdot 24 = 288$

VII.)

$$\sum_{j=1}^{n} (a_j - a_{j-1}) = a_n - a_0$$

$\sum_{j=1}^{n} (a_j - a_{j-1}) = (a_1 - a_0) + (a_2 - a_1) \ldots (a_{n-1} - a_{n-2}) + (a_n - a_{n-1})$
$= a_1 - a_0 + a_2 - a_1 + \cdots + a_{n-1} - a_{n-2} + a_n - a_{n-1}$
$= -a_0 + a_1 - a_1 + a_2 - a_2 + \cdots + a_{n-2} - a_{n-2} + a_{n-1} - a_{n-1} + a_n$
$= a_n - a_0$