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Primer

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Language Fundamentals

Matrices and Magic Squares

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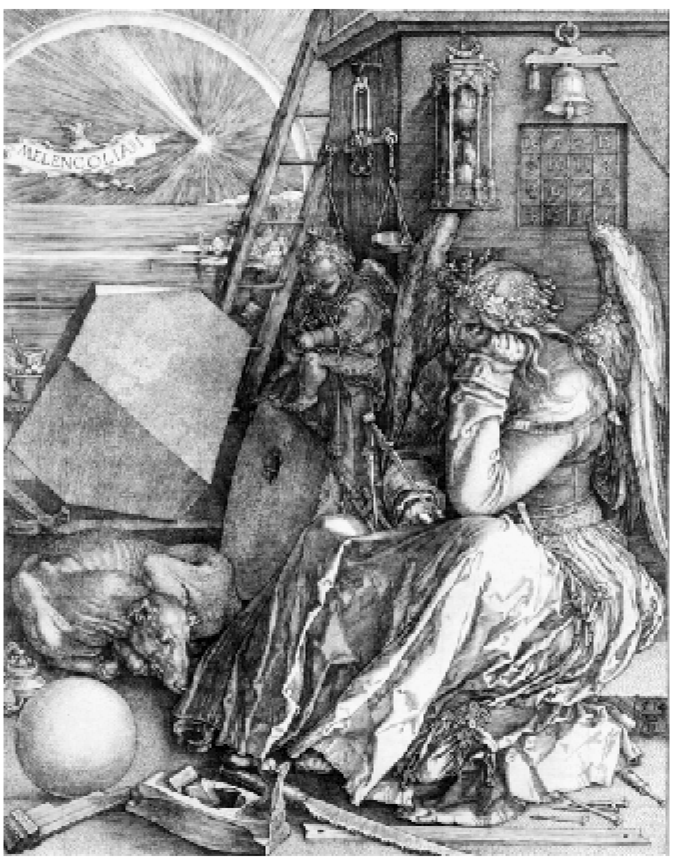
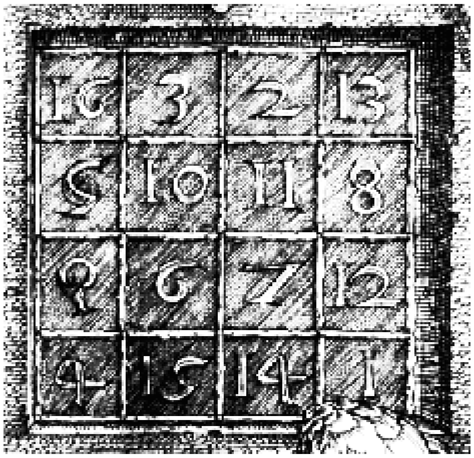
About Matrices

In the MATLAB environment, a matrix is a rectangular array of numbers. Special meaning is sometimes attached to 1-by-1 matrices, which are scalars, and to matrices with only one row or column, which are vectors. MATLAB has other ways of storing both numeric and nonnumeric data, but in the beginning, it is usually best to think of everything as a matrix. The operations in MATLAB are designed to be as natural as possible. Where other programming languages work with numbers one at a time, MATLAB allows you to work with entire matrices quickly and easily. A good example matrix, used throughout this book, appears in the Renaissance engraving Melencolia I by the German artist and amateur mathematician Albrecht Dürer.



This image is filled with mathematical symbolism, and if you look carefully, you will see a matrix in the upper-right corner. This matrix is known as a magic square and was believed by many in Dürer's time to have genuinely magical properties. It does turn out to have some fascinating characteristics worth exploring.



Entering Matrices

The best way for you to get started with MATLAB is to learn how to handle matrices. Start MATLAB and follow along with each example.

You can enter matrices into MATLAB in several different ways:

- Enter an explicit list of elements.
- Load matrices from external data files.
- Generate matrices using built-in functions.
- Create matrices with your own functions and save them in files.

Start by entering Dürer's matrix as a list of its elements. You only have to follow a few basic conventions:

- Separate the elements of a row with blanks or commas.
- Use a semicolon, `;` , to indicate the end of each row.
- Surround the entire list of elements with square brackets, `[ ]`.

To enter Dürer's matrix, simply type in the Command Window

```
A = [16 3 2 13; 5 10 11 8; 9 6 7 12; 4 15 14 1]
```

MATLAB displays the matrix you just entered:

```
A =
    16     3     2    13
     5    10    11     8
     9     6     7    12
     4    15    14     1
```

This matrix matches the numbers in the engraving. Once you have entered the matrix, it is automatically remembered in the MATLAB workspace. You can refer to it simply as `A`. Now that you have `A` in the workspace, take a look at what makes it so interesting. Why is it magic?

sum, transpose, and diag

You are probably already aware that the special properties of a magic square have to do with the various ways of summing its elements. If you take the sum along any row or column, or along either of the two main diagonals, you will always get the same number. Let us verify that using MATLAB. The first statement to try is

```
sum(A)
```

MATLAB replies with

```
ans =
    34    34    34    34
```

When you do not specify an output variable, MATLAB uses the variable `ans`, short for *answer*, to store the results of a calculation. You have computed a row vector containing the sums of the columns of `A`. Each of the columns has the same sum, the *magic* sum, 34.

How about the row sums? MATLAB has a preference for working with the columns of a matrix, so one way to get the row sums is to transpose the matrix, compute the column sums of the transpose, and then transpose the result.

MATLAB has two transpose operators. The apostrophe operator (for example, `A'`) performs a complex conjugate transposition. It flips a matrix about its main diagonal, and also changes the sign of the imaginary component of any complex elements of the matrix. The dot-apostrophe operator (`A.'`), transposes without affecting the sign of complex elements. For matrices containing all real elements, the two operators return the same result.

So

```
A'
```

produces

```
ans =
    16     5     9     4
     3    10     6    15
     2    11     7    14
    13     8    12     1
```

and

```
sum(A')'
```

produces a column vector containing the row sums

```
ans =
    34
    34
    34
    34
```

For an additional way to sum the rows that avoids the double transpose use the dimension argument for the `sum` function:

```
sum(A,2)
```

produces

```
ans =
    34
    34
    34
    34
```

The sum of the elements on the main diagonal is obtained with the `sum` and the `diag` functions:

```
diag(A)
```

produces

```
ans =
    16
    10
     7
     1
```

and

```
sum(diag(A))
```

produces

```
ans =
    34
```

The other diagonal, the so-called *antidiagonal,* is not so important mathematically, so MATLAB does not have a ready-made function for it. But a function originally intended for use in graphics, `fliplr`, flips a matrix from left to right:

```
sum(diag(fliplr(A)))
ans =
    34
```

You have verified that the matrix in Dürer's engraving is indeed a magic square and, in the process, have sampled a few MATLAB matrix operations. The following sections continue to use this matrix to illustrate additional MATLAB capabilities.

The magic Function

MATLAB actually has a built-in function that creates magic squares of almost any size. Not surprisingly, this function is named `magic`:

```
B = magic(4)
B =
    16     2     3    13
     5    11    10     8
     9     7     6    12
     4    14    15     1
```

This matrix is almost the same as the one in the Dürer engraving and has all the same “magic” properties; the only difference is that the two middle columns are exchanged.

You can swap the two middle columns of `B` to look like Dürer's `A`. For each row of `B`, rearrange the columns in the order specified by 1, 3, 2, 4:

```
A = B(:,[1 3 2 4])
```

```
A =
    16     3     2    13
     5    10    11     8
     9     6     7    12
     4    15    14     1
```

Generating Matrices

MATLAB software provides four functions that generate basic matrices.

`zeros`	All zeros
`ones`	All ones
`rand`	Uniformly distributed random elements
`randn`	Normally distributed random elements

Here are some examples:

```
Z = zeros(2,4)
Z =
     0     0     0     0
     0     0     0     0

F = 5*ones(3,3)
F =
     5     5     5
     5     5     5
     5     5     5

N = fix(10*rand(1,10))
N =
     9     2     6     4     8     7     4     0     8     4

R = randn(4,4)
R =
    0.6353    0.0860   -0.3210   -1.2316
   -0.6014   -2.0046    1.2366    1.0556
    0.5512   -0.4931   -0.6313   -0.1132
   -1.0998    0.4620   -2.3252    0.3792
```

Expressions

In this section…

Variables

Like most other programming languages, the MATLAB language provides mathematical *expressions*, but unlike most programming languages, these expressions involve entire matrices.

MATLAB does not require any type declarations or dimension statements. When MATLAB encounters a new variable name, it automatically creates the variable and allocates the appropriate amount of storage. If the variable already exists, MATLAB changes its contents and, if necessary, allocates new storage. For example,

```
num_students = 25
```

creates a 1-by-1 matrix named `num_students` and stores the value 25 in its single element. To view the matrix assigned to any variable, simply enter the variable name.

Variable names consist of a letter, followed by any number of letters, digits, or underscores. MATLAB is case sensitive; it distinguishes between uppercase and lowercase letters. `A` and `a` are *not* the same variable.

Although variable names can be of any length, MATLAB uses only the first `N` characters of the name, (where `N` is the number returned by the function `namelengthmax`), and ignores the rest. Hence, it is important to make each variable name unique in the first `N` characters to enable MATLAB to distinguish variables.

```
N = namelengthmax
N =
    63
```

Numbers

MATLAB uses conventional decimal notation, with an optional decimal point and leading plus or minus sign, for numbers. Scientific notation uses the letter `e` to specify a power-of-ten scale factor. Imaginary numbers use either `i` or `j` as a suffix. Some examples of legal numbers are

```
3              -99            0.0001
9.6397238      1.60210e-20    6.02252e23
1i             -3.14159j      3e5i
```

MATLAB stores all numbers internally using the *long* format specified by the IEEE® floating-point standard. Floating-point numbers have a finite *precision* of roughly 16 significant decimal digits and a finite *range* of roughly 10^{-308} to 10^{+308}.

Numbers represented in the double format have a maximum precision of 52 bits. Any double requiring more bits than 52 loses some precision. For example, the following code shows two unequal values to be equal because they are both truncated:

```
x = 36028797018963968;
y = 36028797018963972;
x == y
ans =
      1
```

Integers have available precisions of 8-bit, 16-bit, 32-bit, and 64-bit. Storing the same numbers as 64-bit integers preserves precision:

```
x = uint64(36028797018963968);
y = uint64(36028797018963972);
x == y
ans =
      0
```

MATLAB software stores the real and imaginary parts of a complex number. It handles the magnitude of the parts in different ways depending on the context. For instance, the `sort` function sorts based on magnitude and resolves ties by phase angle.

```
sort([3+4i, 4+3i])
ans =
   4.0000 + 3.0000i   3.0000 + 4.0000i
```

This is because of the phase angle:

```
angle(3+4i)
ans =
    0.9273
angle(4+3i)
ans =
    0.6435
```

The “equal to” relational operator == requires both the real and imaginary parts to be equal. The other binary relational operators > <, >=, and <= ignore the imaginary part of the number and consider the real part only.

Matrix Operators

Expressions use familiar arithmetic operators and precedence rules.

`+`	Addition
`-`	Subtraction
`*`	Multiplication
`/`	Division
`\`	Left division
`^`	Power
`'`	Complex conjugate transpose
`( )`	Specify evaluation order

Array Operators

When they are taken away from the world of linear algebra, matrices become two-dimensional numeric arrays. Arithmetic operations on arrays are done element by element. This means that addition and subtraction are the same for arrays and matrices, but that multiplicative operations are different. MATLAB uses a dot, or decimal point, as part of the notation for multiplicative array operations.

The list of operators includes

`+`	Addition
`-`	Subtraction
`.*`	Element-by-element multiplication
`./`	Element-by-element division
`.\`	Element-by-element left division
`.^`	Element-by-element power
`.'`	Unconjugated array transpose

If the Dürer magic square is multiplied by itself with array multiplication

```
A.*A
```

the result is an array containing the squares of the integers from 1 to 16, in an unusual order:

```
ans =
   256     9     4   169
    25   100   121    64
    81    36    49   144
    16   225   196     1
```

Building Tables

Array operations are useful for building tables. Suppose `n` is the column vector

```
n = (0:9)';
```

Then

```
pows = [n  n.^2  2.^n]
```

builds a table of squares and powers of 2:

```
pows =
     0     0     1
     1     1     2
     2     4     4
     3     9     8
     4    16    16
     5    25    32
     6    36    64
     7    49   128
     8    64   256
     9    81   512
```

The elementary math functions operate on arrays element by element. So

```
format short g
x = (1:0.1:2)';
logs = [x log10(x)]
```

builds a table of logarithms.

```
logs =
     1.0           0
     1.1     0.04139
     1.2     0.07918
     1.3     0.11394
     1.4     0.14613
     1.5     0.17609
     1.6     0.20412
     1.7     0.23045
     1.8     0.25527
     1.9     0.27875
     2.0     0.30103
```

Functions

MATLAB provides a large number of standard elementary mathematical functions, including `abs`, `sqrt`, `exp`, and `sin`. Taking the square root or logarithm of a negative number is not an error; the appropriate complex result is produced automatically. MATLAB also provides many more advanced mathematical functions, including Bessel and gamma functions. Most of these functions accept complex arguments. For a list of the elementary mathematical functions, type

```
help elfun
```

For a list of more advanced mathematical and matrix functions, type

```
help specfun
help elmat
```

Some of the functions, like `sqrt` and `sin`, are *built in*. Built-in functions are part of the MATLAB core so they are very efficient, but the computational details are not readily accessible. Other functions are implemented in the MATLAB programing language, so their computational details are accessible.

There are some differences between built-in functions and other functions. For example, for built-in functions, you cannot see the code. For other functions, you can see the code and even modify it if you want.

Several special functions provide values of useful constants.

`pi`	3.14159265...
`i`	Imaginary unit, $\sqrt{-1}$
`j`	Same as `i`
`eps`	Floating-point relative precision, $\varepsilon = 2^{-52}$
`realmin`	Smallest floating-point number, 2^{-1022}
`realmax`	Largest floating-point number, $(2-\varepsilon)2^{1023}$
`Inf`	Infinity

`NaN`	Not-a-number

Infinity is generated by dividing a nonzero value by zero, or by evaluating well defined mathematical expressions that *overflow*, that is, exceed `realmax`. Not-a-number is generated by trying to evaluate expressions like `0/0` or `Inf-Inf` that do not have well defined mathematical values.

The function names are not reserved. It is possible to overwrite any of them with a new variable, such as

```
eps = 1.e-6
```

and then use that value in subsequent calculations. The original function can be restored with

```
clear eps
```

Examples of Expressions

You have already seen several examples of MATLAB expressions. Here are a few more examples, and the resulting values:

```
rho = (1+sqrt(5))/2
rho =
    1.6180

a = abs(3+4i)
a =
     5

z = sqrt(besselk(4/3,rho-i))
z =
   0.3730+ 0.3214i

huge = exp(log(realmax))
huge =
  1.7977e+308

toobig = pi*huge
toobig =
   Inf
```

Entering Commands

In this section...

The format Function

The `format` function controls the numeric format of the values displayed. The function affects only how numbers are displayed, not how MATLAB software computes or saves them. Here are the different formats, together with the resulting output produced from a vector `x` with components of different magnitudes.

Note To ensure proper spacing, use a fixed-width font, such as Courier.

```
x = [4/3 1.2345e-6]

format short

   1.3333    0.0000

format short e

   1.3333e+000  1.2345e-006

format short g

   1.3333  1.2345e-006

format long

   1.33333333333333   0.00000123450000

format long e

   1.333333333333333e+000    1.234500000000000e-006

format long g

   1.33333333333333               1.2345e-006

format bank

   1.33          0.00

format rat

   4/3          1/810045

format hex
```

```
   3ff5555555555555   3eb4b6231abfd271
```

If the largest element of a matrix is larger than 10^3 or smaller than 10^{-3}, MATLAB applies a common scale factor for the short and long formats.

In addition to the `format` functions shown above

```
format compact
```

suppresses many of the blank lines that appear in the output. This lets you view more information on a screen or window. If you want more control over the output format, use the `sprintf` and `fprintf` functions.

Suppressing Output

If you simply type a statement and press **Return** or **Enter**, MATLAB automatically displays the results on screen. However, if you end the line with a semicolon, MATLAB performs the computation, but does not display any output. This is particularly useful when you generate large matrices. For example,

```
A = magic(100);
```

Entering Long Statements

If a statement does not fit on one line, use an ellipsis (three periods), `...`, followed by **Return** or **Enter** to indicate that the statement continues on the next line. For example,

```
s = 1 -1/2 + 1/3 -1/4 + 1/5 - 1/6 + 1/7 ...
      - 1/8 + 1/9 - 1/10 + 1/11 - 1/12;
```

Blank spaces around the =, +, and - signs are optional, but they improve readability.

Command Line Editing

Various arrow and control keys on your keyboard allow you to recall, edit, and reuse statements you have typed earlier. For example, suppose you mistakenly enter

```
rho = (1 + sqt(5))/2
```

You have misspelled `sqrt`. MATLAB responds with

```
Undefined function 'sqt' for input arguments of type 'double'.
```

Instead of retyping the entire line, simply press the ↑ key. The statement you typed is redisplayed. Use the ← key to move the cursor over and insert the missing `r`. Repeated use of the ↑ key recalls earlier lines. Typing a few characters, and then pressing the ↑ key finds a previous line that begins with those characters. You can also copy previously executed statements from the Command History.

Indexing

In this section...

Subscripts

The element in row `i` and column `j` of `A` is denoted by `A(i,j)`. For example, `A(4,2)` is the number in the fourth row and second column. For the magic square, `A(4,2)` is `15`. So to compute the sum of the elements in the fourth column of `A`, type

```
A(1,4) + A(2,4) + A(3,4) + A(4,4)
```

This subscript produces

```
ans =
     34
```

but is not the most elegant way of summing a single column.

It is also possible to refer to the elements of a matrix with a single subscript, `A(k)`. A single subscript is the usual way of referencing row and column vectors. However, it can also apply to a fully two-dimensional matrix, in which case the array is regarded as one long column vector formed from the columns of the original matrix. So, for the magic square, `A(8)` is another way of referring to the value `15` stored in `A(4,2)`.

If you try to use the value of an element outside of the matrix, it is an error:

```
t = A(4,5)
```

```
Index exceeds matrix dimensions.
```

Conversely, if you store a value in an element outside of the matrix, the size increases to accommodate the newcomer:

```
X = A;
X(4,5) = 17
```

```
X =
    16     3     2    13     0
     5    10    11     8     0
     9     6     7    12     0
     4    15    14     1    17
```

The Colon Operator

The colon, `:`, is one of the most important MATLAB operators. It occurs in several different forms. The expression

```
1:10
```

is a row vector containing the integers from 1 to 10:

```
1     2     3     4     5     6     7     8     9    10
```

To obtain nonunit spacing, specify an increment. For example,

```
100:-7:50
```

is

```
100    93    86    79    72    65    58    51
```

and

```
0:pi/4:pi
```

is

```
0    0.7854    1.5708    2.3562    3.1416
```

Subscript expressions involving colons refer to portions of a matrix:

```
A(1:k,j)
```

is the first `k` elements of the `j`th column of `A`. Thus,

```
sum(A(1:4,4))
```

computes the sum of the fourth column. However, there is a better way to perform this computation. The colon by itself refers to *all* the elements in a row or column of a matrix and the keyword `end` refers to the *last* row or column. Thus,

```
sum(A(:,end))
```

computes the sum of the elements in the last column of `A`:

```
ans =
     34
```

Why is the magic sum for a 4-by-4 square equal to 34? If the integers from 1 to 16 are sorted into four groups with equal sums, that sum must be

```
sum(1:16)/4
```

which, of course, is

```
ans =
     34
```

Concatenation

Concatenation is the process of joining small matrices to make bigger ones. In fact, you made your first matrix by concatenating its individual elements. The pair of square brackets, `[]`, is the concatenation operator. For an example, start with the 4-by-4 magic square, `A`, and form

```
B = [A  A+32; A+48  A+16]
```

The result is an 8-by-8 matrix, obtained by joining the four submatrices:

```
B =

    16     3     2    13    48    35    34    45
     5    10    11     8    37    42    43    40
     9     6     7    12    41    38    39    44
     4    15    14     1    36    47    46    33
    64    51    50    61    32    19    18    29
    53    58    59    56    21    26    27    24
    57    54    55    60    25    22    23    28
    52    63    62    49    20    31    30    17
```

This matrix is halfway to being another magic square. Its elements are a rearrangement of the integers `1:64`. Its column sums are the correct value for an 8-by-8 magic square:

```
sum(B)

ans =
   260   260   260   260   260   260   260   260
```

But its row sums, `sum(B')'`, are not all the same. Further manipulation is necessary to make this a valid 8-by-8 magic square.

Deleting Rows and Columns

You can delete rows and columns from a matrix using just a pair of square brackets. Start with

```
X = A;
```

Then, to delete the second column of `X`, use

```
X(:,2) = []
```

This changes `X` to

```
X =
    16     2    13
     5    11     8
     9     7    12
     4    14     1
```

If you delete a single element from a matrix, the result is not a matrix anymore. So, expressions like

```
X(1,2) = []
```

result in an error. However, using a single subscript deletes a single element, or sequence of elements, and reshapes the remaining elements into a row vector. So

```
X(2:2:10) = []
```

results in

```
X =
    16     9     2     7    13    12     1
```

Scalar Expansion

Matrices and scalars can be combined in several different ways. For example, a scalar is subtracted from a matrix by subtracting it from each element. The average value of the elements in our magic square is 8.5, so

```
B = A - 8.5
```

forms a matrix whose column sums are zero:

```
B =
      7.5     -5.5     -6.5      4.5
     -3.5      1.5      2.5     -0.5
      0.5     -2.5     -1.5      3.5
     -4.5      6.5      5.5     -7.5

sum(B)

ans =
     0     0     0     0
```

With scalar expansion, MATLAB assigns a specified scalar to all indices in a range. For example,

```
B(1:2,2:3) = 0
```

zeros out a portion of B:

```
B =
      7.5        0        0      4.5
     -3.5        0        0     -0.5
      0.5     -2.5     -1.5      3.5
     -4.5      6.5      5.5     -7.5
```

Logical Subscripting

The logical vectors created from logical and relational operations can be used to reference subarrays. Suppose X is an ordinary matrix and L is a matrix of the same size that is the result of some logical operation. Then X(L) specifies the elements of X where the elements of L are nonzero.

This kind of subscripting can be done in one step by specifying the logical operation as the subscripting expression. Suppose you have the following set of data:

```
x = [2.1 1.7 1.6 1.5 NaN 1.9 1.8 1.5 5.1 1.8 1.4 2.2 1.6 1.8];
```

The NaN is a marker for a missing observation, such as a failure to respond to an item on a questionnaire. To remove the missing data with logical indexing, use isfinite(x), which is true for all finite numerical values and false for NaN and Inf:

```
x = x(isfinite(x))
x =
  2.1 1.7 1.6 1.5 1.9 1.8 1.5 5.1 1.8 1.4 2.2 1.6 1.8
```

Now there is one observation, `5.1`, which seems to be very different from the others. It is an *outlier*. The following statement removes outliers, in this case those elements more than three standard deviations from the mean:

```
x = x(abs(x-mean(x)) <= 3*std(x))
x =
  2.1 1.7 1.6 1.5 1.9 1.8 1.5 1.8 1.4 2.2 1.6 1.8
```

For another example, highlight the location of the prime numbers in Dürer's magic square by using logical indexing and scalar expansion to set the nonprimes to 0. (See “The magic Function” on page 2-5.)

```
A(~isprime(A)) = 0

A =
     0     3     2    13
     5     0    11     0
     0     0     7     0
     0     0     0     0
```

The find Function

The `find` function determines the indices of array elements that meet a given logical condition. In its simplest form, `find` returns a column vector of indices. Transpose that vector to obtain a row vector of indices. For example, start again with Dürer's magic square. (See “The magic Function” on page 2-5.)

```
k = find(isprime(A))'
```

picks out the locations, using one-dimensional indexing, of the primes in the magic square:

```
k =
     2     5     9    10    11    13
```

Display those primes, as a row vector in the order determined by `k`, with

```
A(k)

ans =
     5     3     2    11     7    13
```

When you use `k` as a left-side index in an assignment statement, the matrix structure is preserved:

```
A(k) = NaN

A =
    16   NaN   NaN   NaN
   NaN    10   NaN     8
     9     6   NaN    12
     4    15    14     1
```

Types of Arrays

In this section...

Multidimensional Arrays

Multidimensional arrays in the MATLAB environment are arrays with more than two subscripts. One way of creating a multidimensional array is by calling `zeros`, `ones`, `rand`, or `randn` with more than two arguments. For example,

```
R = randn(3,4,5);
```

creates a 3-by-4-by-5 array with a total of `3*4*5 = 60` normally distributed random elements.

A three-dimensional array might represent three-dimensional physical data, say the temperature in a room, sampled on a rectangular grid. Or it might represent a sequence of matrices, $A^{(k)}$, or samples of a time-dependent matrix, $A(t)$. In these latter cases, the (i, j)th element of the kth matrix, or the t_kth matrix, is denoted by `A(i,j,k)`.

MATLAB and Dürer's versions of the magic square of order 4 differ by an interchange of two columns. Many different magic squares can be generated by interchanging columns. The statement

```
p = perms(1:4);
```

generates the 4! = 24 permutations of `1:4`. The `k`th permutation is the row vector `p(k,:)`. Then

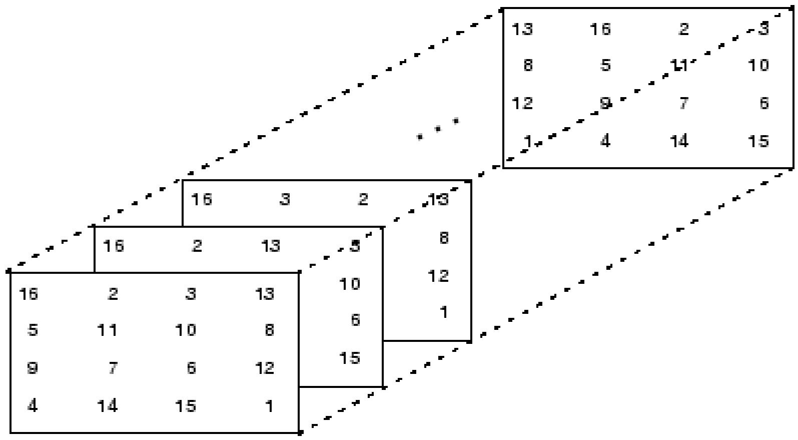
```
A = magic(4);
M = zeros(4,4,24);

for k = 1:24
   M(:,:,k) = A(:,p(k,:));
end
```

stores the sequence of 24 magic squares in a three-dimensional array, `M`. The size of `M` is

```
size(M)

ans =
     4     4    24
```



Note The order of the matrices shown in this illustration might differ from your results. The `perms` function always returns all permutations of the input vector, but the order of the permutations might be different for different MATLAB versions.

The statement

```
sum(M,d)
```

computes sums by varying the `d`th subscript. So

```
sum(M,1)
```

is a 1-by-4-by-24 array containing 24 copies of the row vector

```
34    34    34    34
```

and

```
sum(M,2)
```

is a 4-by-1-by-24 array containing 24 copies of the column vector

```
34
34
34
34
```

Finally,

```
S = sum(M,3)
```

adds the 24 matrices in the sequence. The result has size 4-by-4-by-1, so it looks like a 4-by-4 array:

```
S =
   204   204   204   204
   204   204   204   204
   204   204   204   204
   204   204   204   204
```

Cell Arrays

Cell arrays in MATLAB are multidimensional arrays whose elements are copies of other arrays. A cell array of empty matrices can be created with the `cell` function. But, more often, cell arrays are created by enclosing a miscellaneous collection of things in curly braces, `{}`. The curly braces are also used with subscripts to access the contents of various cells. For example,

```
C = {A sum(A) prod(prod(A))}
```

produces a 1-by-3 cell array. The three cells contain the magic square, the row vector of column sums, and the product of all its elements. When `C` is displayed, you see

```
C = 
    [4x4 double]    [1x4 double]    [20922789888000]
```

This is because the first two cells are too large to print in this limited space, but the third cell contains only a single number, 16!, so there is room to print it.

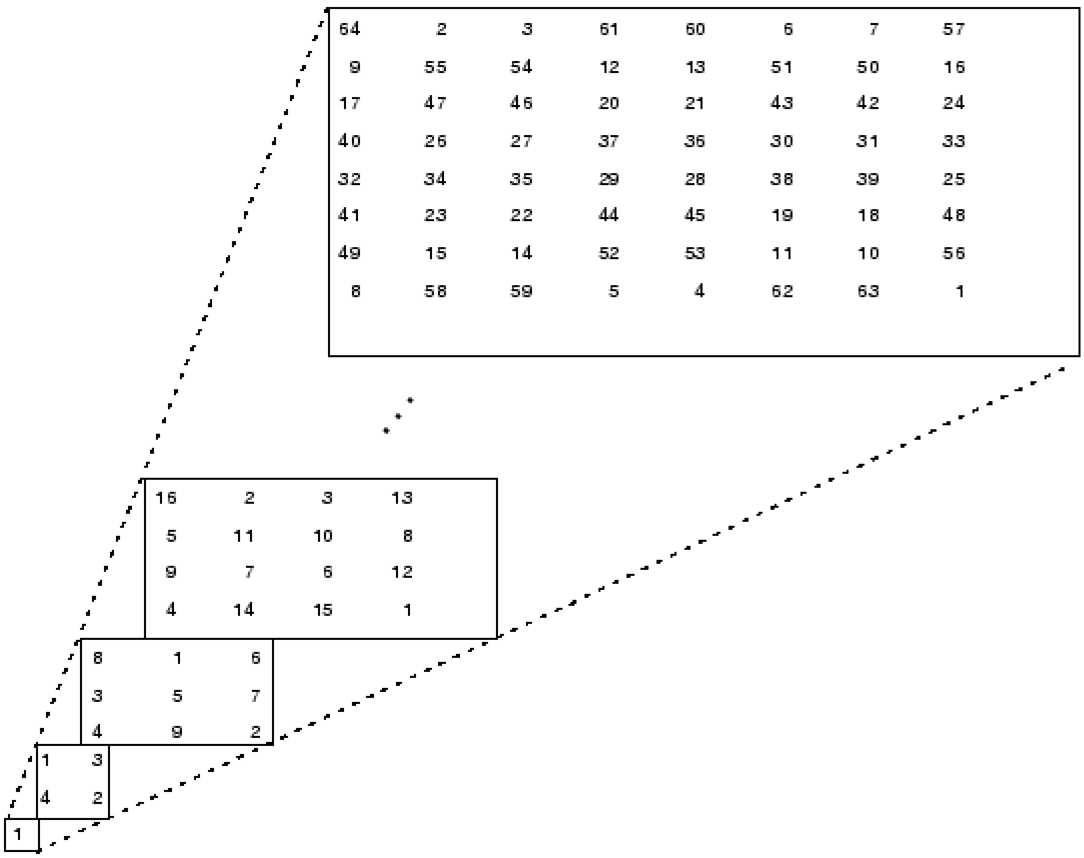
Here are two important points to remember. First, to retrieve the contents of one of the cells, use subscripts in curly braces. For example, `C{1}` retrieves the magic square and `C{3}` is 16!. Second, cell arrays contain *copies* of other arrays, not *pointers* to those arrays. If you subsequently change `A`, nothing happens to `C`.

You can use three-dimensional arrays to store a sequence of matrices of the *same* size. Cell arrays can be used to store a sequence of matrices of *different* sizes. For example,

```
M = cell(8,1);
for n = 1:8
   M{n} = magic(n);
end
M
```

produces a sequence of magic squares of different order:

```
M = 
    [           1]
    [ 2x2  double]
    [ 3x3  double]
    [ 4x4  double]
    [ 5x5  double]
    [ 6x6  double]
    [ 7x7  double]
    [ 8x8  double]
```



You can retrieve the 4-by-4 magic square matrix with

```
M{4}
```

Characters and Text

Enter text into MATLAB using single quotes. For example,

```
s = 'Hello'
```

The result is not the same kind of numeric matrix or array you have been dealing with up to now. It is a 1-by-5 character array.

Internally, the characters are stored as numbers, but not in floating-point format. The statement

```
a = double(s)
```

converts the character array to a numeric matrix containing floating-point representations of the ASCII codes for each character. The result is

```
a =
    72    101    108    108    111
```

The statement

```
s = char(a)
```

reverses the conversion.

Converting numbers to characters makes it possible to investigate the various fonts available on your computer. The printable characters in the basic ASCII character set are represented by the integers `32:127`. (The integers less than 32 represent nonprintable control characters.) These integers are arranged in an appropriate 6-by-16 array with

```
F = reshape(32:127,16,6)';
```

The printable characters in the extended ASCII character set are represented by `F+128`. When these integers are interpreted as characters, the result depends on the font currently being used. Type the statements

```
char(F)
char(F+128)
```

and then vary the font being used for the Command Window. To change the font, on the **Home** tab, in the **Environment** section, click **Preferences** > **Fonts**. If you include tabs in lines of code, use a fixed-width font, such as `Monospaced`, to align the tab positions on different lines.

Concatenation with square brackets joins text variables together. The statement

```
h = [s, ' world']
```

joins the characters horizontally and produces

```
h =
   Hello world
```

The statement

```
v = [s; 'world']
```

joins the characters vertically and produces

```
v =
   Hello
   world
```

Notice that a blank has to be inserted before the `'w'` in `h` and that both words in `v` have to have the same length. The resulting arrays are both character arrays; `h` is 1-by-11 and `v` is 2-by-5.

To manipulate a body of text containing lines of different lengths, you have two choices—a padded character array or a cell array of character vectors. When creating a character array, you must make each row of the array the same length. (Pad the ends of the shorter rows with spaces.) The `char` function does this padding for you. For example,

```
S = char('A','rolling','stone','gathers','momentum.')
```

produces a 5-by-9 character array:

```
S =
A
rolling
stone
gathers
momentum.
```

Alternatively, you can store the text in a cell array. For example,

```
C = {'A';'rolling';'stone';'gathers';'momentum.'}
```

creates a 5-by-1 cell array that requires no padding because each row of the array can have a different length:

```
C =
    'A'
    'rolling'
    'stone'
    'gathers'
    'momentum.'
```

You can convert a padded character array to a cell array of character vectors with

```
C = cellstr(S)
```

and reverse the process with

```
S = char(C)
```

Structures

Structures are multidimensional MATLAB arrays with elements accessed by textual *field designators*. For example,

```
S.name = 'Ed Plum';
S.score = 83;
S.grade = 'B+'
```

creates a scalar structure with three fields:

```
S =
     name: 'Ed Plum'
    score: 83
    grade: 'B+'
```

Like everything else in the MATLAB environment, structures are arrays, so you can insert additional elements. In this case, each element of the array is a structure with several fields. The fields can be added one at a time,

```
S(2).name = 'Toni Miller';
S(2).score = 91;
S(2).grade = 'A-';
```

or an entire element can be added with a single statement:

```
S(3) = struct('name','Jerry Garcia',...
               'score',70,'grade','C')
```

Now the structure is large enough that only a summary is printed:

```
S =
1x3 struct array with fields:
    name
    score
    grade
```

There are several ways to reassemble the various fields into other MATLAB arrays. They are mostly based on the notation of a *comma-separated list*. If you type

```
S.score
```

it is the same as typing

```
S(1).score, S(2).score, S(3).score
```

which is a comma-separated list.

If you enclose the expression that generates such a list within square brackets, MATLAB stores each item from the list in an array. In this example, MATLAB creates a numeric row vector containing the `score` field of each element of structure array `S`:

```
scores = [S.score]
scores =
    83    91    70

avg_score = sum(scores)/length(scores)
avg_score =
   81.3333
```

To create a character array from one of the text fields (`name`, for example), call the `char` function on the comma-separated list produced by `S.name`:

```
names = char(S.name)
names =
   Ed Plum
   Toni Miller
   Jerry Garcia
```

Similarly, you can create a cell array from the `name` fields by enclosing the list-generating expression within curly braces:

```
names = {S.name}
names =
    'Ed Plum'    'Toni Miller'    'Jerry Garcia'
```

To assign the fields of each element of a structure array to separate variables outside of the structure, specify each output to the left of the equals sign, enclosing them all within square brackets:

```
[N1 N2 N3] = S.name
N1 =
   Ed Plum
N2 =
   Toni Miller
N3 =
   Jerry Garcia
```

Dynamic Field Names

The most common way to access the data in a structure is by specifying the name of the field that you want to reference. Another means of accessing structure data is to use dynamic field names. These names express the field as a variable expression that MATLAB evaluates at run time. The dot-parentheses syntax shown here makes `expression` a dynamic field name:

```
structName.(expression)
```

Index into this field using the standard MATLAB indexing syntax. For example, to evaluate `expression` into a field name and obtain the values of that field at columns `1` through `25` of row `7`, use

```
structName.(expression)(7,1:25)
```

Dynamic Field Names Example

The `avgscore` function shown below computes an average test score, retrieving information from the `testscores` structure using dynamic field names:

```
function avg = avgscore(testscores, student, first, last)
for k = first:last
   scores(k) = testscores.(student).week(k);
end
avg = sum(scores)/(last - first + 1);
```

You can run this function using different values for the dynamic field `student`. First, initialize the structure that contains scores for a 25-week period:

```
testscores.Ann_Lane.week(1:25) = ...
  [95 89 76 82 79 92 94 92 89 81 75 93 ...
   85 84 83 86 85 90 82 82 84 79 96 88 98];

testscores.William_King.week(1:25) = ...
  [87 80 91 84 99 87 93 87 97 87 82 89 ...
   86 82 90 98 75 79 92 84 90 93 84 78 81];
```

Now run `avgscore`, supplying the students name fields for the `testscores` structure at run time using dynamic field names:

```
avgscore(testscores, 'Ann_Lane', 7, 22)
ans =
   85.2500

avgscore(testscores, 'William_King', 7, 22)
ans =
   87.7500
```