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EFFECTS OF HALL AND ION-SLIP CURRENTS ON FREE CONVECTIVE HEAT GENERATING FLOW IN A ROTATING FLUID

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SUMMARY

In this work, the effects of Hall and ion-slip currents on free convective heat generating rarefied gas in a rotating frame of reference are studied for the case of a strong magnetic field imposed perpendicularly to the plate. Expressions for the velocity and temperature fields are obtained, and the effects of the various parameters of the problem, e.g. the Hall parameter, the ion-slip parameter and rarefaction, are discussed through graphs.

KEY WORDS Hall current; ion slip; rarefaction.

INTRODUCTION

The study of magnetohydrodynamic free convective heat generating fluid flows in a slip-flow regime with Hall and ion-slip currents has important engineering applications, e.g. in power generators, MHD accelerators, refrigeration coils, transmission lines, electric transformers and heating elements.

In the last decade, considerable progress has been made in the general theory of rotating fluids (Greenspan, 1968). It is well known that, in a rotating fluid near a flat plate, an Ekman layer exists in which the viscous and Coriolis forces are of the same order of magnitude. The effects of a transverse magnetic field on such a layer was studied by Gupta (1975), Soundalgekar and Pop (1973), and Raptis and Peridikis (1982). Recently Ram (1989, 1990*a* and 1990*b*) has studied the MHD free convective flow in a rotating fluid with Hall currents.

The phenomenon of viscous incompressible gases when their density is slightly reduced owing to low absolute pressure or an increase in temperature, is called rarefaction of the medium. Because of this rarefaction, there is slight departure from continuous flow. The first effect of this was observed as a slip and temperature jump at the plate, and the associated flow regime is called slip flow. Some important effects of gas rarefaction are given by Street (1960).

A survey of the literature reveals that the effects of Hall and ion-slip current on MHD free convection flow of a rarefied gas along a vertical porous plate in a rotating frame of reference have not yet been studied. Hence the object of present work is to study the effects of Hall and ion-slip current on MHD free convection flow in a rotating frame of reference of a rarefied gas along a vertical porous plate. Here, (i) the plate is subjected to a constant suction velocity, (ii) the heat source Q^* is of the type $Q^* = Q(T'_\infty - T')$, (iii) the fluid and the plate are in a state of rigid rotation with a uniform angular velocity $\vec{\Omega}$ about the z'-axis, and (iv) a strong magnetic field of uniform strength is applied transversely to the direction of the flow. Analytical expressions for the velocity and temperature fields are given, and numerical calculations are carried out to illustrate the results graphically.

MATHEMATICAL ANALYSIS

Consider the steady free convective flow of a partially ionized gas past an infinite vertical porous plate. A strong magnetic field B_0 is imposed along the z'-axis. Let the fluid and the plate be in a state of rigid rotation with a uniform angular velocity $\vec{\Omega}$ about the z'-axis.

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Since the plate is infinite in extent and the flow is steady, all the physical variables, except pressure, are functions of z' only. The flow has a low Mach number, and hence the density of the ionized gas can be taken as constant. However, for such a fluid, the Hall and the ion-slip currents significantly affect the flow in the presence of large magnetic fields. The induced magnetic field is neglected, since the magnetic Reynolds number of a partially ionized gas is very small (Shercliff, 1965). Under these conditions, the hydromagnetic flow is governed by the following equations:

$$\frac{\partial w'}{\partial z'} = 0 \tag{1}$$

$$w'\frac{\partial u'}{\partial z'} - 2\Omega v' = \nu\frac{\partial^2 u'}{\partial z'^2} + g\beta^*(T' - T'_\infty) + \frac{1}{\rho} j_{y'} B_0 \tag{2}$$

$$w'\frac{\partial v'}{\partial z'} + 2\Omega u' = \nu\frac{\partial^2 v'}{\partial z'^2} - \frac{1}{\rho} j_{x'} B_0 \tag{3}$$

$$w'\frac{\partial T'}{\partial z'} = \frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial z'^2} + \frac{\mu}{\rho C_p} + \left[\left(\frac{\partial u'}{\partial z'}\right)^2 + \left(\frac{\partial v'}{\partial z'}\right)^2\right] + Q(T'_\infty - T') \tag{4}$$

where u', v' and w' are the velocity components in the x', y' and z' directions, respectively, g is the acceleration due to gravity, β^* is the coefficient of volume expansion, ν is the kinematic viscosity, ρ is the density of the ionized gas, k is the thermal conductivity, C_p is the specific heat of the gas at constant pressure, and finally T' and T'_∞ are the temperatures in the boundary layer and the free stream, respectively, and $j_{x'}$ and $j_{y'}$ are the current density components.

The boundary conditions are $u' = L_1\,\partial u'/\partial z'$, $v' = L_1\,\partial v'/\partial z'$, and $T' - T'_w = L_2\,\partial T'/\partial z'$ at $z' = 0$.

$$u' = 0,\ v' = 0,\ T' \to T'_\infty \text{ as } z' \to \infty \tag{5}$$

where L_1 is the velocity slip coefficient, L_2 is the temperature jump coefficient, and other physical quantities have their usual meanings.

The equation of conservation of electric charge $\nabla\cdot\vec{J} = 0$ gives $j_{z'}$ = constant, where $\vec{J} = (j_{x'}, j_{y'}, j_{z'})$. This constant is assumed to be zero, since $j_{z'} = 0$ everywhere in the flow.

The expressions for the current-density components $j_{x'}$ and $j_{y'}$, as obtained from the generalized Ohm's law (Sutton and Sherman, 1965), are given by

$$\begin{aligned} j_{x'} &= [\alpha(E_{x'} + B_0 v') - \beta(E_{y'} - B_0 u')]\sigma \\ j_{y'} &= [\alpha(E_{y'} - B_0 u') + \beta(E_{x'} + B_0 v')]\sigma \end{aligned} \tag{6}$$

where

$$\alpha = (1 + \beta_e\beta_i)/[(1 + \beta_e\beta_i)^2 + \beta_e^2]$$

$$\beta = \beta_e/[(1 + \beta_e\beta_i)^2 + \beta_e^2]$$

$$\beta_e = \omega_e\tau_e \quad \text{(Hall parameter)}$$

$$\beta_i = \omega_i\tau_i \quad \text{(ion-slip parameter)}$$

and $\vec{E} = (E_{x'}, E_{y'}, E_{z'})$ (the electric field components). Here ω_e and ω_i are electron and ion-cyclotron frequencies, τ_e and τ_i are collision frequencies, and σ is the electrical conductivity of the gas.

On integrating equation (1), we have

$$w' = -w_0\ (w_0 > 0) \tag{7}$$

where w_0 is the constant suction velocity at the plate, and the negative sign indicates that the suction velocity is directed towards the plate.

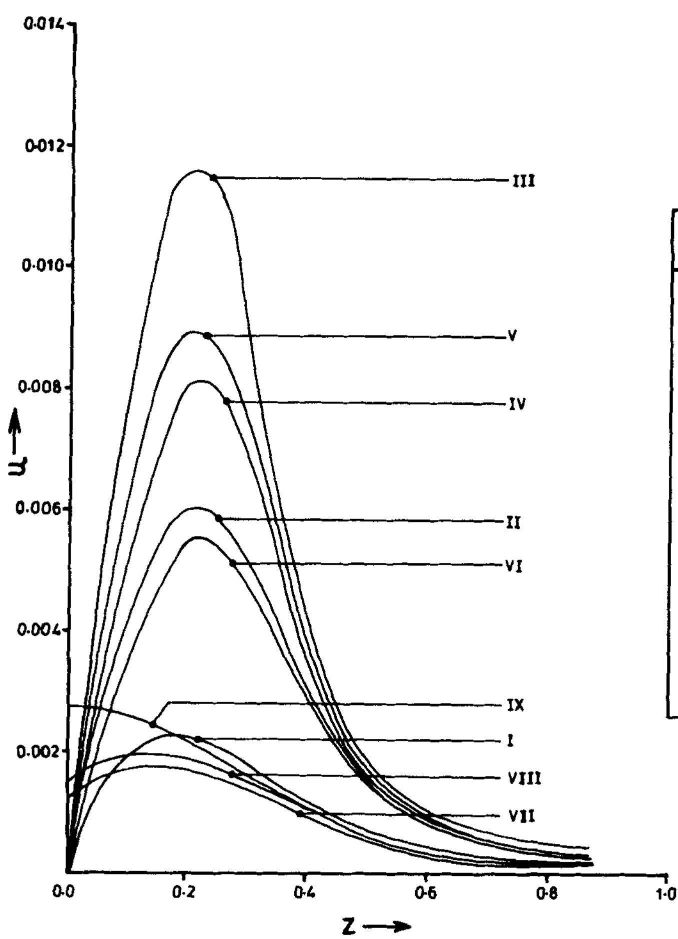
We introduce the following dimensionless quantities

$$
\begin{aligned}
& u = \frac{u'}{w_0}, \quad v = \frac{v'}{w_0}, \ z = \frac{w_0 z'}{\nu}, \ \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty} \\
& Gr = \frac{\nu g \beta (T'_w - T'_\infty)}{w_0^3}, \quad Er = \frac{\Omega \nu}{w_0^2}, \\
& M^2 = \frac{\sigma B_0^2 \nu}{\rho w_0^2}, \quad Pr = \frac{\rho \nu C_p}{k} \\
& \delta = \frac{Q \nu^2}{K w_0^2}, \ Ec = \frac{w_0^2}{C_p (T'_w - T'_\infty)} \\
& E_x = E_{x'}/B_0 w_0, \quad E_y = E_{y'}/B_0 w_0, \quad j_x = j_{x'}/\sigma B_0 w_0, \\
& j_y = j_{y'}/\sigma B_0 w_0
\end{aligned}
\tag{8}
$$

in equations (2), (3) and (4), we get nondimensional equations of the problem as

$$
Q'' + Q' - (2Eri + H_0^2)Q = H_0^2 Ei - Gr\,\theta \tag{9}
$$

$$
\theta'' + Pr\,\theta' - \delta\theta = -Pr\,E_c(Q' \cdot \bar{Q}') \tag{10}
$$



	β_e	β_i	δ	
I	0.5	0·2	0·3	$h_1 = 0$, $h_2 = 0$
II	1.0	0·2	0·3	
III	1·5	0·2	0·3	
IV	1·0	0·3	0·3	
V	1·0	0·4	0·3	
VI	1·0	0·2	0·4	
VII	0·5	0·2	0·3	$h_1 = 0·3$, $h_2 = 0·7$
VIII	1·0	0·2	0·3	
IX	1·5	0·2	0·3	

Figure 1. Primary velocity u profile

where $Q = u + iv$, $\vec{E} = E_x + iE_y$, $H_0^2 = M^2\alpha_0$, $\alpha_0 = (\alpha + i\beta)$, primes denote differentiation with respect to z, and a bar denotes the complex conjugate of the corresponding quantities. The corresponding boundary conditions of the problem are

$$\begin{aligned} &Q = h_1 \frac{\partial Q}{\partial z}, \quad \theta = 1 + h_2 \frac{\partial \theta}{\partial z} \quad \text{at } z = 0 \\ &Q = 0, \ \theta = 0 \quad \text{as} \quad z \to \infty \end{aligned} \tag{11}$$

where

$$h_1 = \frac{L_1 w_0}{v} \quad \text{and} \quad h_2 = \frac{L_2 w_0}{v}$$

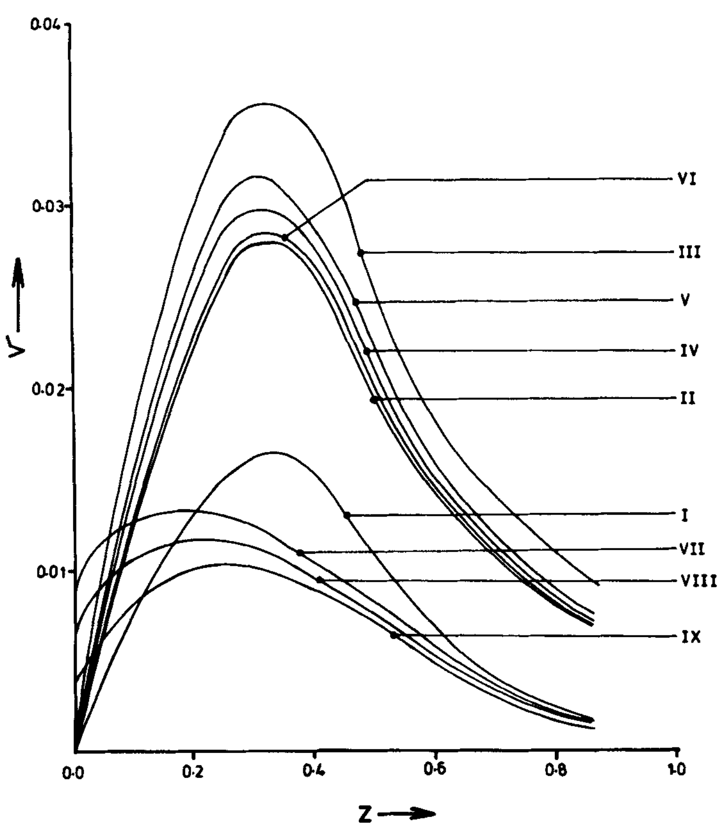
We now consider further the case of a short circuit problem in which the applied electric field $\vec{E} = 0$ (Meyer, 1958). With this assumption, equation (9) reduces to

$$Q'' + Q' - M_1 Q = -Gr\theta \tag{12}$$

where $M_1 = (2Eri + H_0^2)$.

The system of equations (10) and (12) is nonlinear, and in order to obtain a solution we expand Q and θ in powers of Eckert number Ec, assuming that it is very small. This is justified in low-speed incompressible flow. Hence

$$\begin{aligned} Q &= Q_0 + EcQ_1 + 0(Ec^2) \\ \theta &= \theta_0 + Ec\theta_1 + 0(Ec^2) \end{aligned} \tag{13}$$



	β_e	β_i	σ	
I	0.5	0.2	0.3	$h_1 = 0$, $h_2 = 0$
II	1.0	0.2	0.3	
III	1.5	0.2	0.3	
IV	1.0	0.3	0.3	
V	1.0	0.4	0.3	
VI	1.0	0.2	0.4	
VII	0.5	0.2	0.3	$h_1 = 0.3$, $h_2 = 0.7$
VIII	1.0	0.2	0.3	
IX	1.5	0.2	0.3	

Figure 2. Secondary velocity v profile

On substituting equation (13) into equations (10) and (12), and equating the coefficients of different powers of Ec, we obtain the solution under modified boundary conditions as

$$Q_0 = A_3 e^{-R_2 z} - A_2 e^{-R_1 z}$$

$$Q_1 = A_{18} e^{-R_2 z} - A_{13} e^{-R_1 z} - A_{14} e^{-(R_1+\bar{R}_2)z} - A_{15} e^{-(R_1+R_2)z} + A_{16} e^{-2R_1 z} + A_{17} e^{-(R_2+\bar{R}_2)z}$$

$$\theta_0 = A_1 e^{-R_1 z}$$

$$\theta_1 = A_{12} e^{-R_1 z} + A_8 e^{-(R_1+\bar{R}_2)z} + A_9 e^{-(R_1+R_2)z} - A_{10} e^{-2R_1 z} - A_{11} e^{-(R_2+\bar{R}_2)z}$$

where the constants are defined in the Appendix.

DISCUSSION

To study the behaviour of the velocity and temperature profiles, curves are drawn for various values of the parameters that describe the flow. The results obtained for steady flow are displayed in Figures 1–3 for $M = 5{\cdot}0$, $Gr = 5{\cdot}0$, $Pr = 0{\cdot}71$,. $Ec = 0{\cdot}01$, and $Er = 0{\cdot}3$, and for different values of the Hall parameter β_e, the ion-slip parameter β_i and the heat source parameter δ. Here the Prandtl number is taken to be equal to 0·71, which corresponds physically to that of air. The Eckert number Ec, which may be interpreted as the addition of heat due to viscous dissipation takes the values 0·01 which is more appropriate for incompressible fluids. The Grashof number Gr takes a positive value, and this case corresponds to the cooling of the surface. The magnetic parameter M is chosen arbitrarily as 5·0, which signifies a strong magnetic field. For rarefied gas, the velocity slip coefficient and temperature jump coefficient are taken as 0·3 and 0·7, respectively.

From Figures 1 and 2 for $Gr > 0$ (in the presence of cooling of the plate by free convection currents). It is seen that (i) there is rise in both the velocity fields with the increase of the Hall parameter β_e and the ion-slip

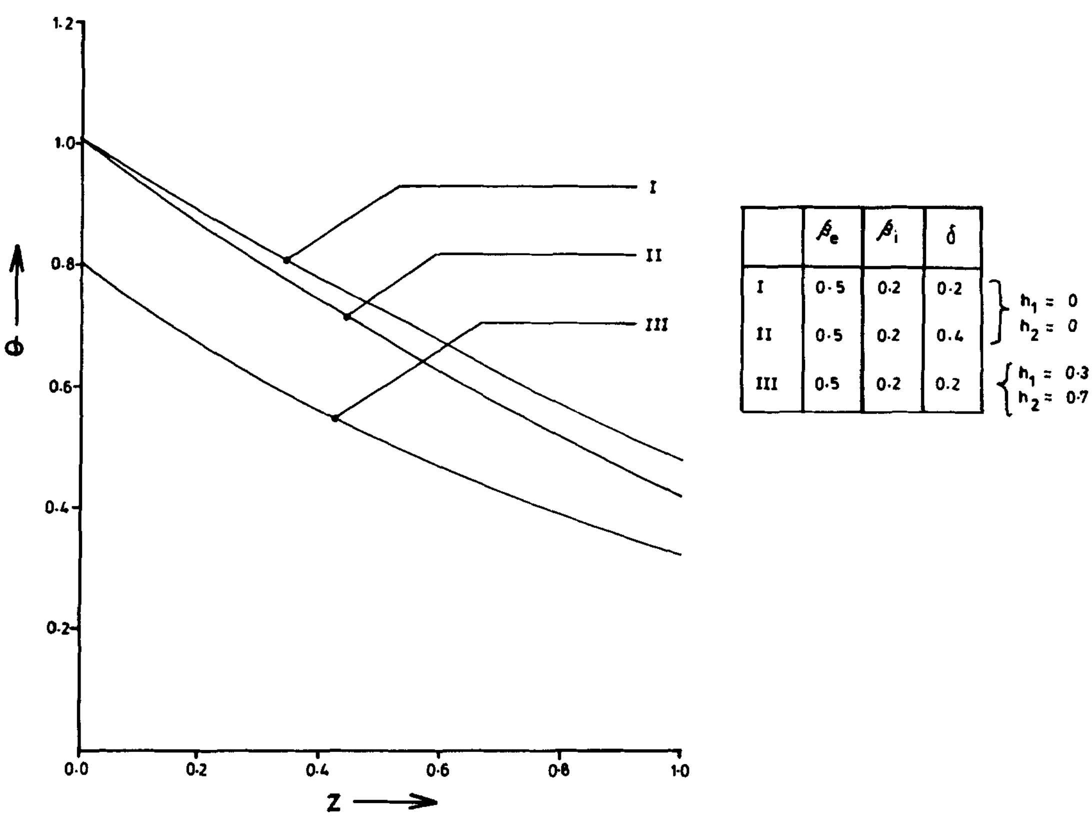


Figure 3. Temperature profiles

parameter β_i, (ii) with the increase in heat source parameter δ, the primary velocity u decreases whereas the secondary velocity v increases, and (iii) the primary velocity and secondary velocity for nonrarefied gas are greater than those of rarefied gas. From Figure 3, we observe that (i) the temperature field decreases with an increase of heat source parameter δ, and (ii) the temperature field for nonrarefied gas is greater than that of rarefied gas. Thus we conclude that the rarefication causes a decrease in the velocity fields and the temperature field.

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APPENDIX

Definitions of constants used in mathematical analysis:

$$R_1 = \tfrac{1}{2}[Pr + \sqrt{(Pr^2 + 4\delta)}],\ R_2 = \tfrac{1}{2}[1 + \sqrt{(1 + 4M_1)}]$$

$$A_1 = 1/(1 + R_1 h_2),\ A_2 = GrA_1/[R_1^2 - R_1 - M_1]$$

$$A_3 = A_2(1 + R_1 h_1)/(1 + R_2 h_1),\ A_4 = A_3\bar{A}_3 R_2 \bar{R}_2$$

$$A_5 = A_2\bar{A}_2 R_1^2,\ A_6 = R_1 R_2 \bar{A}_2 A_3$$

$$A_7 = R_1\bar{R}_2 A_2\bar{A}_3,\quad A_8 = A_7/[(R_1 + \bar{R}_2)^2 - (R_1 + \bar{R}_2) - \delta]$$

$$A_9 = A_6/[(R_1 + R_2)^2 - (R_1 + R_2) - \delta],\quad A_{10} = A_5/(4R_1^2 - 2R_1 - \delta)$$

$$A_{11} = A_4/[(R_2 + \bar{R}_2)^2 - (R_2 + \bar{R}_2) - \delta]$$

$$\begin{aligned} A_{12} = {} & [A_{11}(1 + h_2(R_2 + \bar{R}_2)) + A_{10}(1 + 2R_1 h_2) - A_9(1 + h_2(R_1 + R_2)) \\ & - A_8(1 + h_2(R_1 + \bar{R}_2))]/(1 + R_2 h_2) \end{aligned}$$

$$A_{13} = GrA_{12}/(R_1^2 - R_1 - M_1),\ A_{14} = GrA_8/[(R_1 + \bar{R}_2)^2 - (R_1 + \bar{R}_2) - M_1]$$

$$A_{15} = GrA_9/[(R_1 + R_2)^2 - (R_1 + R_2) - M_1]$$

$$A_{16} = GrA_{10}/[4R_1^2 - 2R_1 - M_1]$$

$$A_{17} = GrA_{11}/[(R_2 + \bar{R}_2)^2 - (R_2 + \bar{R}_2) - M_1]$$

$$\begin{aligned} A_{18} = {} & [A_{13}(1 + R_1 h_1) + A_{14}(1 + h_1(R_1 + \bar{R}_2)) + A_{15}(1 + h_1(R_1 + R_2)) \\ & - A_{16}(1 + 2R_1 + h_1) - A_{17}(1 + h_1(R_2 + \bar{R}_2))]/(1 + h_1 R_2) \end{aligned}$$