Journal of the Physical Society of Japan
Vol. 79, No. 10, October, 2010, 104401

Combined Effect of Hall and Ion-Slip Currents on Unsteady MHD Couette Flows in a Rotating System

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(Received May 25, 2010; accepted August 18, 2010; published October 12, 2010)

The unsteady MHD Couette flows of a viscous incompressible electrically conducting fluid between two parallel plates in a rotating system are studied taking hall and ion-slip currents into consideration. The relevant equations are solved analytically using the Laplace transform techniques. A unified closed form analytical expressions for the velocity and the skin friction for the cases; when the magnetic lines of force are fixed relative to the fluid or to the moving plate are derived. The solution obtained shows that the inclusion of Hall and ion-slip currents gives some interesting results. It is found that the influence of the Hall and ion slip parameters have a reducing effect on the magnitude of the secondary velocity especially when the magnetic lines of force are fixed relative to the moving plate. It is also interesting to note that the presence of Hall and ion-slip currents led to an increase in the time it took both the primary and the secondary velocities to achieve their steady state values. On the other hand, the resultant skin friction on the moving plate decreases with an increase in both the Hall and ion-slip parameters when the magnetic field is fixed relative to the fluid, while the opposite behaviour is noticed the magnetic field is fixed relative to the moving plate.

KEYWORDS: MHD, Couette flow, rotating system, Hall and ion-slip currents
DOI: 10.1143/JPSJ.79.104401

1. Introduction

The study of MHD Couette flow has been an active area of research because of its geophysical and astrophysical applications. Couette flow according to Muzychka and Yovanovich[1] can be used as a fundamental method for the measurement of viscosity and as a means of estimating the drag force in many wall driven applications. Katagiri[2] investigated the formation of Couette flow of a viscous, incompressible and electrically conducting fluid between two infinite parallel plane walls in the presence of transverse magnetic field. Since then, many authors Jana *et al.*,[3] Seth and Jana,[4] Singh and Kumar,[5] Mazumdar,[6] Ghosh,[7] Chandran *et al.*,[8] and Singh *et al.*,[9] amongst several others have discussed the MHD Couette flow under different physical effects. Seth *et al.*[10] investigated the unsteady hydromagnetic flow of an electrically conducting viscous incompressible fluid in a rotating system under the influence of a uniform transverse magnetic field when one of the plates is set into motion with a time-dependent velocity in its own plane. Guria *et al.*[11] carried out a study on the unsteady Couette flow in a rotating system. Recently, a study on oscillatory Couette flow in the presence of an inclined magnetic field was carried out by Guria *et al.*,[12] while Das *et al.*[13] studied the unsteady MHD Couette flow in a rotating system. All of these works aasumed a small and moderate values of the magnetic field and therefore ignored the effect of Hall and ion-slip currents in applying the Ohm's law.

However, the current trend for the application of MHD is towards a strong magnetic field, so that the influence of the electromagnetic force is noticeable, Cramer and Pai.[14] Nevertheless, Debnath *et al.*,[15] Mandal and Mandal,[16] Tiwari and Singh,[17] Jana and Data,[18] and Ghosh and Pop[19] have studied the effect of Hall currents under varying conditions. Mazumdar[20] investigated the combined effect of Hall current and rotation on hydromagnetic flow over an oscillating porous plate. Ghosh[21] has studied the effects of Hall current and coriolis force on MHD Couette flow in a rotating system with arbitrary magnetic field. The Hall effects on MHD Couette flow in a rotating system was investigated by Ghosh and Pop.[22] These studies however, negleted the ion-slip current. In many practical applications requiring strong magnetic field there is the need to consider both the Hall and ion-slip currents because of the significant effect they have on the magnitude and direction of the current density and transitively on the magnetic force term. MHD Couette flow with Hall and ion-slip currents has many important engineering applications in power generators, Hall accelerators and flows in channels and ducts, Abo-Eldahab and El-Aziz.[23] Ram[24] studied the effects of Hall and ion-slip currents on free convective heat generating flow in a rotating fluid. The MHD Couette flow with temperature dependent viscosity and the ion-slip was investigated by Attia,[25] similarly, Attia[26] examined the unsteady Couette flow with heat transfer considering ion-slip.

To the best of our knowledge, no work seems to have address the combined effect of Hall and ion-slip currents on the MHD Coutte flow of an electrically conducting incompressible viscous fluid in a rotating system in the presence of a transverse magnetic field. This will be the focus of this present work. The lower plate of the system under consideration is set into motion with a velocity proportional to t'^n, where t' is the time and n is a positive integer, while the upper plate is kept stationary. The criteria for Hall and ion-slip currents effects on the unsteady MHD Couette flow in a rotating system are applied on the Ohm's law. Analytical solutions are then presented for the velocity and the skin friction using the Laplace transform technique for the impulsive motion which corresponds to when $n = 0$. The effects of the various parameters of the problem, e.g., the Hartmann number M, Ekman number E, Hall parameter β_e and ion-slip parameter β_i, are discussed for the two cases

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under consideration (i.e., when the magnetic lines of force are fixed relative to the moving fluid and when fixed to the moving plate).

2. Mathematical Analysis

Consider the motion of a viscous, incompressible and electrically conducting fluid filling the gap between two infinite plates. The two plates are located on the z'-axis at distance h apart and extend from $x' = -\infty$ to ∞ and $y' = -\infty$ to ∞. The fluid flows between the two plates in the x'-direction in the presence of a uniform magnetic field acting perpendicular to the main flow direction. We assume that the magnetic Reynolds number is very small which corresponds to negligible induced magnetic field compared to the externally applied as shown in Pai.[27] Therefore, the uniform magnetic field B is a constant $B \equiv (0, 0, B_0)$ and is considered as the total magnetic field acting on the fluid. The electron-atom collision frequency is assumed to be relatively high, so that the Hall and the ion-slip currents effect cannot be negleted. The combined effect of the Hall and ion-slip currents gives rise to a velocity in the y'-axis. At time $t' \leq 0$, the fluid, the plates and the magnetic lines of force are assumed to be at rest. When $t' > 0$ the lower plates begins to move in its own plane with a velocity Ut'^n where U is a constant and the upper plate remains fixed. The z'-axis is assumed to be normal to the plates. Since the plates are infinite, all physical variables are functions of z' and t' only. It is assumed that no applied and polarisation voltage exists. The equation for the conservation of electric charge $\nabla.J = 0$ results to $J_{z'} =$ constant where $J = (J_{x'}, J_{y'}, J_{z'})$. This constant is assumed to be zero since $J_{z'} = 0$ at the plates which is assumed to be electrically non conducting. Thus, $J_{z'} = 0$ everywhere in the flow.

If $V = (u', v', w')$ denotes the velocity components with $w' = 0$ for the case under consideration, then the generalised Ohm's Law with Hall and ion-slip currents under these assumptions according to Sutton and Sherman[28] becomes

$$J = \sigma\left[V \times B_0 - \beta(J \times B_0) + \frac{\beta\beta_{\mathrm{i}}}{B_0}(J \times B_0) \times B_0\right], \quad (1)$$

where β and β_{i} are the Hall factor and the ion-slip parameter respectively. Thus,

$$J_x = \sigma B_0(\gamma u' + \alpha v'), \quad (2)$$
$$J_y = -\sigma B_0(\gamma v' - \alpha u'), \quad (3)$$

where

$$\alpha = \frac{1 + \beta_{\mathrm{e}}\beta_{\mathrm{i}}}{(1 + \beta_{\mathrm{e}}\beta_{\mathrm{i}})^2 + \beta_{\mathrm{e}}^2},$$
$$\gamma = \frac{\beta_{\mathrm{e}}}{(1 + \beta_{\mathrm{e}}\beta_{\mathrm{i}})^2 + \beta_{\mathrm{e}}^2},$$

and $\beta_{\mathrm{e}} = \sigma\beta B_0$ is the Hall parameter. The Navier–Stokes equations of motion in a rotating frame of reference are

$$\frac{\partial u'}{\partial t'} - 2\Omega' v' = \upsilon\frac{\partial^2 u'}{\partial z'^2} + \frac{\sigma B_0^2}{\rho}(\gamma v' - \alpha u'), \quad (4)$$
$$\frac{\partial v'}{\partial t'} + 2\Omega' u' = \upsilon\frac{\partial^2 v'}{\partial z'^2} - \frac{\sigma B_0^2}{\rho}(\gamma u' + \alpha v'), \quad (5)$$

where Ω' is the constant angular velocity along the z'-axis, ρ the density, υ is the kinematic viscosity and σ is the electrical conductivity. The initial and boundary conditions for the present problem are

$$\begin{aligned} &u' = v' = 0,\ 0 \leq z' \leq h \text{ and } t' \leq 0,\\ &u' = Ut'^n,\ v' = 0,\ \text{at } z' = 0, t' > 0;\\ &u' = v' = 0,\ \text{at } z' = h,\ t' > 0. \end{aligned} \quad (6)$$

Equations (4) and (5) are valid when the magnetic lines of force are fixed relative to the fluid. If the magnetic field is also accelerated with the same velocity as the plate, we must account for the relative motion. Thus, eqs. (4) and (5) turn to

$$\frac{\partial u'}{\partial t'} - 2\Omega' v' = \upsilon\frac{\partial^2 u'}{\partial z'^2} - \frac{\sigma B_0^2}{\rho}[\gamma v' - \alpha(u' - KUt'^n)], \quad (7)$$
$$\frac{\partial v'}{\partial t'} + 2\Omega' u' = \upsilon\frac{\partial^2 v'}{\partial z'^2} - \frac{\sigma B_0^2}{\rho}[\gamma(u' - KUt'^n) + \alpha v'], \quad (8)$$

where

$$K = \begin{cases} 0 & \text{when } B_0 \text{ is fixed relative to the fluid} \\ 1 & \text{when } B_0 \text{ is fixed relative to the moving plate} \end{cases}.$$

If we define the complex velocity $q' = u' + \mathrm{i}v'$ subject to the boundary conditions above; eqs. (7) and (8) now becomes

$$\frac{\partial q'}{\partial t'} - 2\mathrm{i}\Omega' q' = \upsilon\frac{\partial^2 q'}{\partial z'^2} - \frac{\sigma B_0^2}{\rho}(\alpha + \mathrm{i}\gamma)q' + \frac{\sigma B_0^2}{\rho}(\alpha + \mathrm{i}\gamma)KUt'^n. \quad (9)$$

The initial and boundary conditions (6) also become

$$\begin{aligned} &q' = 0,\ 0 \leq z' \leq\ h \text{ and } t' \leq 0,\\ &q' = Ut'^n \text{ at } z' = 0,\ t' > 0;\\ &q' = 0,\ \text{at } z' = h,\ t' > 0. \end{aligned} \quad (10)$$

Equations (9) and (10) describe the general flow in that the initial motion of the moving plate is given by a power law in the time variable. The equations are valid for both cases of when the magnetic lines of force are fixed relative to the fluid and when fixed relative to the moving plate. In this paper, we propose to obtain the explicit solution for the impulsive motion which correspond to when $n = 0$ in eq. (9), we introduce the following dimensionless quantities in eq. (9) as

$$z = \frac{z'}{h},\quad q = \frac{q'}{U},\quad t = \frac{\upsilon t'}{h^2},\quad M = B_0 h\sqrt{\frac{\sigma}{\upsilon\rho}},\quad E = \frac{\upsilon}{\Omega' h^2},$$

where z is the dimensionless distance, q is the dimensionless complex velocity, t is the dimensionless time, M and E are the Hartmann and Ekman numbers respectively. After rearrangment we have

$$\frac{\partial^2 q}{\partial z^2} - \frac{\partial q}{\partial t} - \left[M^2(\alpha + \mathrm{i}\gamma) + \frac{2\mathrm{i}}{E}\right]q = -M^2 K(\alpha + \mathrm{i}\gamma) \quad (11)$$

subject to the following dimensionless initial and boundary conditions

$$\begin{aligned} &q = 0,\ 0\ \leq z \leq 1 \text{ and } t\ \leq 0\\ &q = 1 \text{ at } z = 0,\ t > 0;\\ &q = 0,\ \text{at } z = 1,\ t > 0. \end{aligned} \quad (12)$$

The solution of eq. (11) can be obtained by using the Laplace transform technique. Define the following transform variables

$$Q(z,r)=\int_0^\infty q(z,t)\mathrm{e}^{-rt}\,\mathrm{d}t, \quad (r>0),$$

where r is the Laplace parameter, and taking the Laplace transform of eq. (11) with the corresponding boundary conditions (12) we obtain the following ordinary differential equation

$$\frac{\mathrm{d}^2Q}{\mathrm{d}z^2}-(m+r)Q=-\frac{M^2K(\alpha+\mathrm{i}\gamma)}{r}, \tag{13}$$

where $m=M^2(\alpha+\mathrm{i}\gamma)+2\mathrm{i}/E$. The boundary condition now becomes

$$Q=\begin{cases} r^{-1} & \text{at } z=0 \\ 0 & \text{at } z=1 \end{cases}. \tag{14}$$

The solution of eqs. (13) and (14) is

$$Q(z,r)=\frac{1}{r}\sum_{k=0}^{\infty}(\mathrm{e}^{-a\delta}-\mathrm{e}^{-b\delta}) + \frac{M^2K(\alpha+\mathrm{i}\gamma)}{r\delta^2}\left\{1-\sum_{k=0}^{\infty}(-1)^k[\mathrm{e}^{-c\delta}+\mathrm{e}^{-d\delta}]\right\}, \tag{15}$$

where $\delta=\sqrt{m+r}$, $a=2k+z$, $b=2k+2-z$, $c=k+z$, and $d=k+1-z$. We can then obtain the velocity distribution $q(z,t)$ on inversion of the above equation

$$\begin{aligned} q(z,t)=&\sum_{k=0}^{\infty}[f_1(t,a,m)-f_1(t,b,m)] \\ &+\lambda_1\mathrm{e}^{-mt}\sum_{k=0}^{\infty}(-1)^k[f_2(t,c,m) \\ &+f_2(t,d,m)]-\lambda_1\sum_{k=0}^{\infty}(-1)^k[f_1(t,c,m) \\ &+f_1(t,d,m)]+\lambda_1(1-\mathrm{e}^{-mt}), \end{aligned} \tag{16}$$

where λ_1 and the functionals f_1 and f_2 are set in the Appendix. The primary and the secondary velocities are obtained by separating the real and imaginary parts of the complex equations obtained in eq. (16) using MATLAB.

2.1 Skin friction

The skin friction τ is defined as $\partial q/\partial z$, which is obtained by differentiating eq. (16) with respect to z:

$$\begin{aligned} \tau=\frac{\partial q}{\partial z}=&\sum_{k=0}^{\infty}[f_3(t,a,m)-f_3(t,b,m)] \\ &+\lambda_1\mathrm{e}^{-mt}\sum_{k=0}^{\infty}(-1)^k[f_4(t,c,m) \\ &+f_4(t,d,m)]-\lambda_1\sum_{k=0}^{\infty}(-1)^k[f_3(t,c,m) \\ &+f_3(t,d,m)]. \end{aligned} \tag{17}$$

The axial τ_x and the transverse τ_y skin frictions can be obtained by separating the real and imaginary parts in eq. (17) using MATLAB:

$$\tau=\tau_x+\mathrm{i}\tau_y. \tag{18}$$

Equation (17) represents the general expression for the skin friction between different fluid layers within the channel. On the lower plate ($z=0$) we have the skin friction to be

$$\begin{aligned} \tau_0&=\left.\frac{\partial q}{\partial z}\right|_{z=0} \\ &=\sum_{k=0}^{\infty}[f_3(t,a_1,m)-f_3(t,b_1,m)] \\ &\quad+\lambda_1\mathrm{e}^{-mt}\sum_{k=0}^{\infty}(-1)^k[f_4(t,c_1,m)+f_4(t,d_1,m)] \\ &\quad-\lambda_1\sum_{k=0}^{\infty}(-1)^k[f_3(t,c_1,m)+f_3(t,d_1,m)], \end{aligned} \tag{19}$$

where $a_1=2k$, $b_1=2k+2$, $c_1=k$, and $d_1=k+1$. The functionals f_3 and f_4 are defined in the Appendix while on the upper plate ($z=1$), we have

$$\begin{aligned} \tau_1&=\left.\frac{\partial q}{\partial z}\right|_{z=1} \\ &=\sum_{k=0}^{\infty}[f_3(t,a_2,m)-f_3(t,b_2,m)] \\ &\quad+\lambda_1\mathrm{e}^{-mt}\sum_{k=0}^{\infty}(-1)^k[f_4(t,c_2,m)+f_4(t,d_2,m)] \\ &\quad-\lambda_1\sum_{k=0}^{\infty}(-1)^k[f_3(t,c_2,m)+f_3(t,d_2,m)], \end{aligned} \tag{20}$$

where $a_2=2k+1$, $b_2=2k+1$, $c_2=k+1$, $d_2=k$ are defined in the Appendix.
The resultant skin friction is obtained by taking the absolute values of the axial τ_x and the transverse τ_y. The results are then computed and are presented in §3.

2.2 Steady state

As time increases, the response of the velocity with respect to time becomes insignificant. It implies that the flow velocity has attained its steady state value at that time. The expression for the velocity distribution $\bar{q}$ at this state is obtained by setting $\partial q/\partial t$ in eq. (11) to zero. We then obtain the following ordinary differential equation

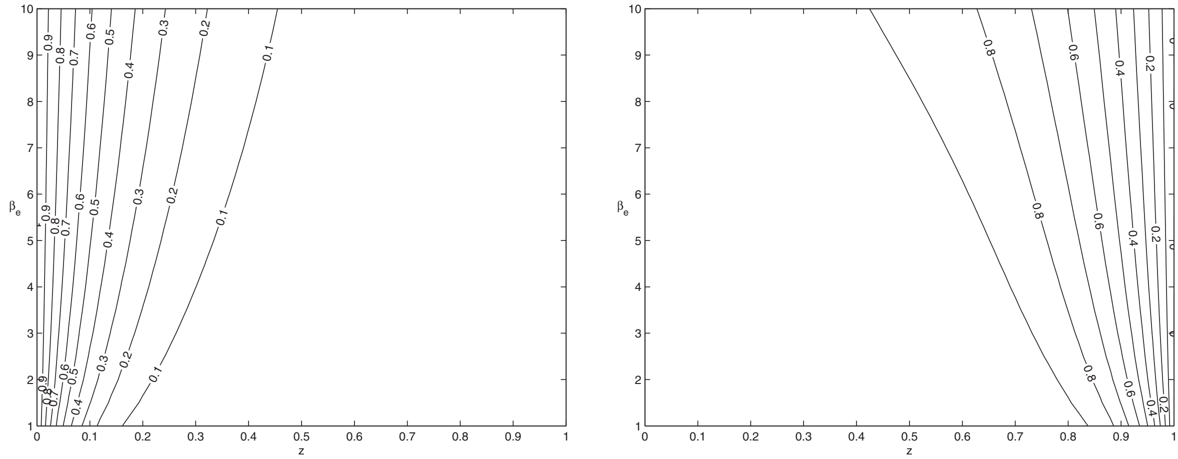
$$\frac{\mathrm{d}^2\bar{q}}{\mathrm{d}z^2}-m\bar{q}=-M^2K(\alpha+\mathrm{i}\gamma). \tag{21}$$

Equation (21) is then solved with eq. (12) to obtain the steady state velocity

$$\bar{q}(z)=\frac{\mathrm{e}^{-\xi z}-\mathrm{e}^{-\xi(2-z)}}{1-\mathrm{e}^{-2\xi}} + \frac{M^2K(\alpha+\mathrm{i}\gamma)}{\xi^2}\left[1-\frac{\mathrm{e}^{-\xi z}+\mathrm{e}^{-\xi(1-z)}}{1+\mathrm{e}^{-\xi}}\right], \tag{22}$$

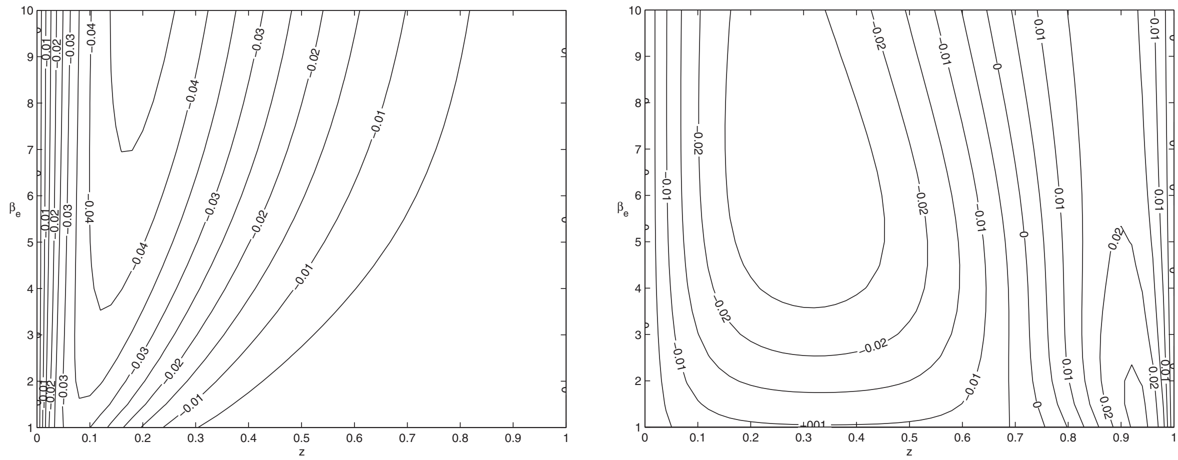
where $\xi=\sqrt{m}$ The corresponding skin friction τ_s for the steady state is obtained by differentiating eq. (22) with respect to z:

$$\begin{aligned} \tau_s&=\frac{\partial\bar{q}}{\partial z} \\ &=-\xi\frac{(\mathrm{e}^{-\xi z}+\mathrm{e}^{\xi z})}{1-\mathrm{e}^{-2\xi}}+\frac{M^2K(\alpha+\mathrm{i}\gamma)}{\xi}\left(\frac{\mathrm{e}^{-\xi z}-\mathrm{e}^{\xi z}}{1+\mathrm{e}^{-\xi}}\right). \end{aligned} \tag{23}$$



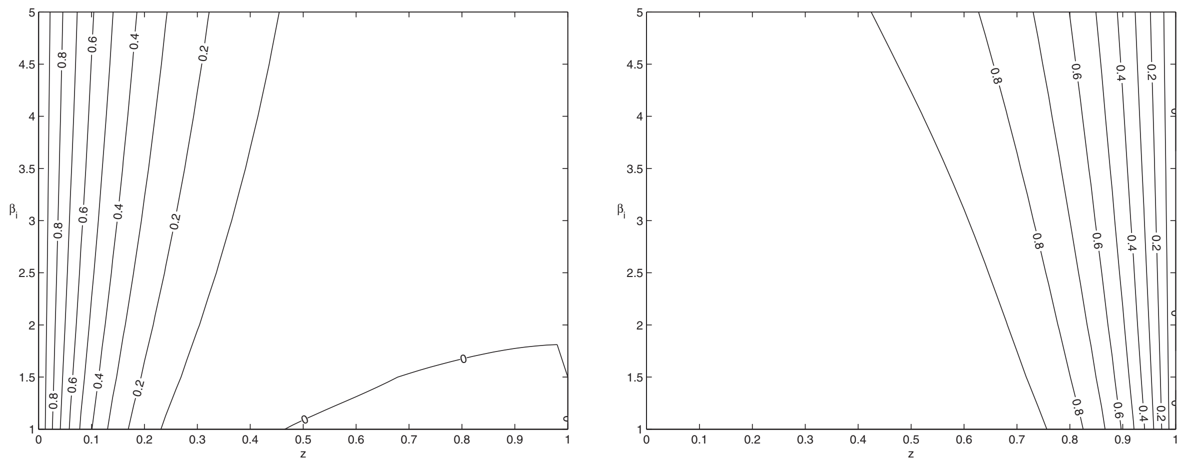
(a) (b)

Fig. 1. Primary velocity profile u showing the effect of Hall current in the presence of ion-slip current with $K = 0$ and 1 represented by (a) and (b) respectively with fixed values of $M = 35.0$, $E = 1.0$, $t = 0.1$, $\beta_i = 5.0$, and $\beta_e \in [1.0, 10.0]$.



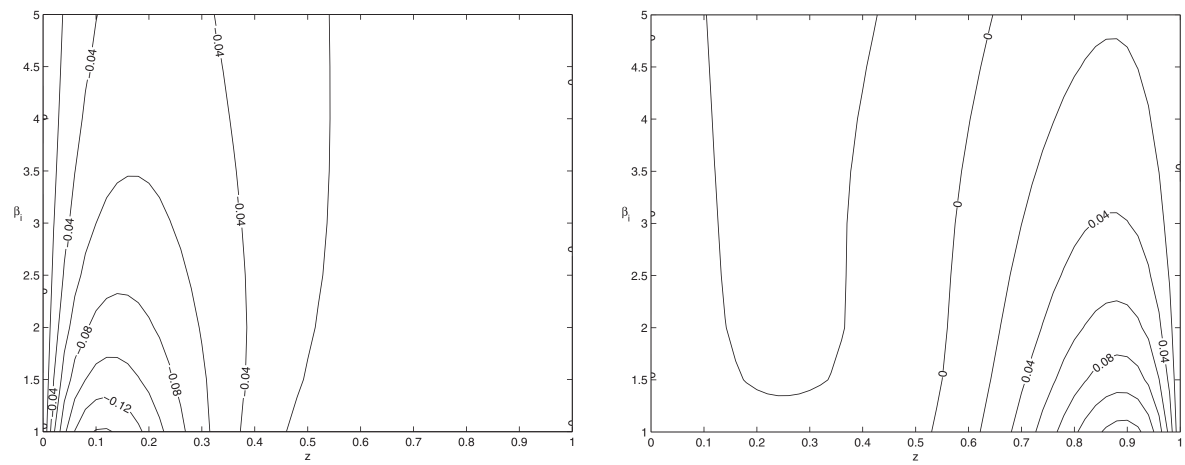
(a) (b)

Fig. 2. Secondary velocity profile v showing the effect of Hall current in the presence of ion-slip current with $K = 0$ and 1 represented by (a) and (b) respectively with fixed values of $M = 35.0$, $E = 1.0$, $t = 0.1$, $\beta_i = 5.0$, and $\beta_e \in [1.0, 10.0]$.



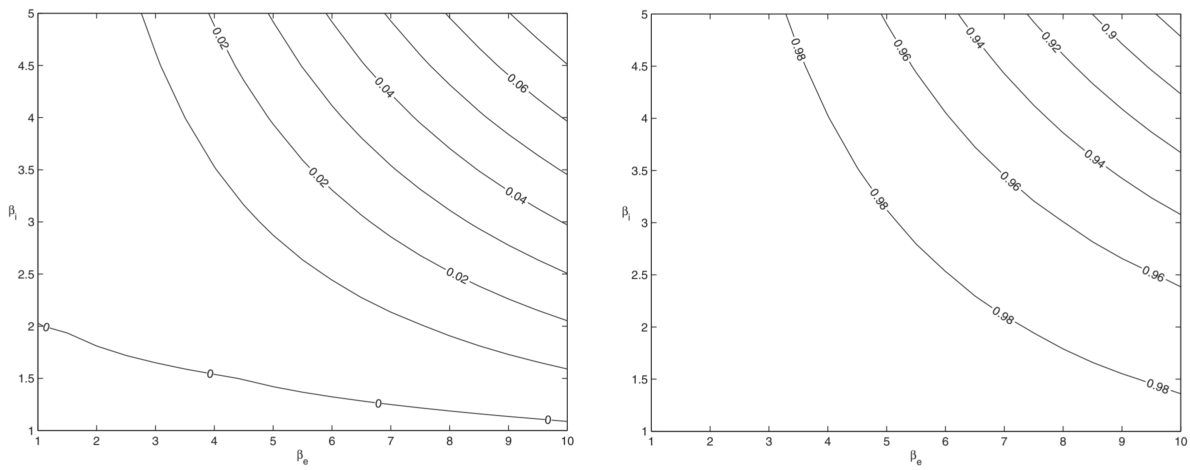
(a) (b)

Fig. 3. Primary velocity u showing the effect of ion-slip current with $K = 0$ and 1 represented by (a) and (b) respectively with fixed values of $M = 35.0$, $E = 1.0$, $t = 0.1$, $\beta_e = 10.0$, and $\beta_i \in [1.0, 5.0]$.



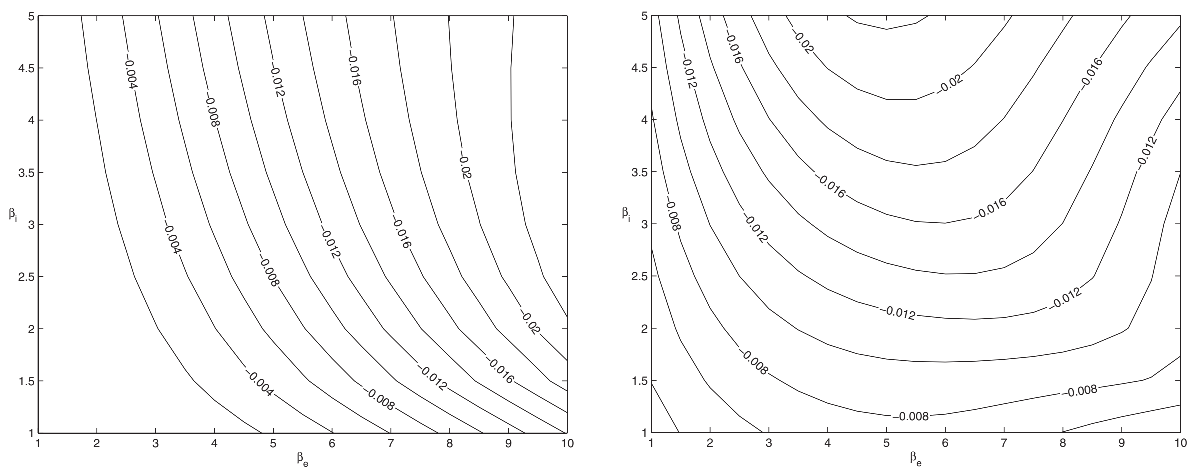
(a) (b)

Fig. 4. Secondary velocity v showing the effect of ion-slip current with $K = 0$ and 1 represented by (a) and (b) respectively with fixed values of $M = 35.0$, $E = 1.0$, $t = 0.1$, $\beta_e = 10.0$, and $\beta_i \in [1.0, 5.0]$.



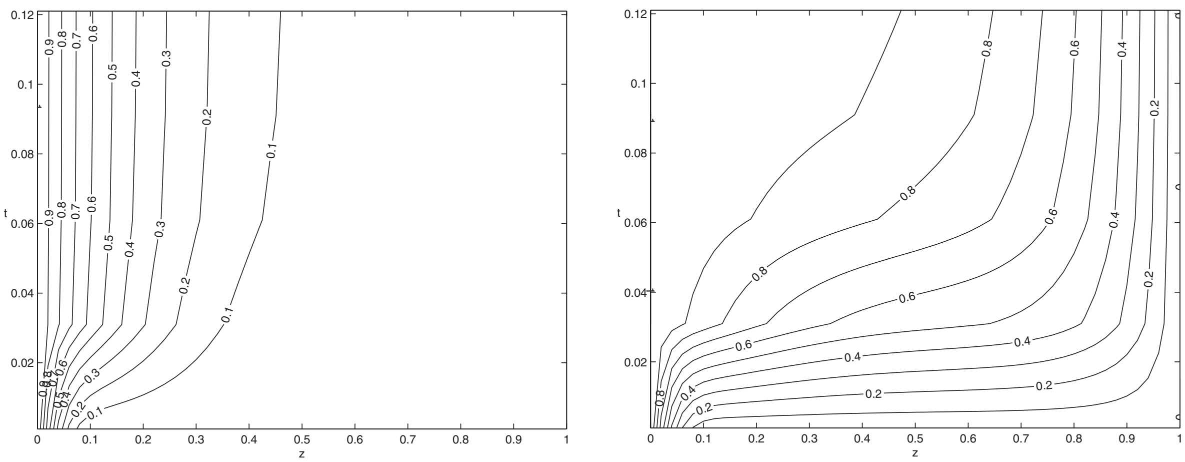
(a) (b)

Fig. 5. Primary velocity u showing the effect of Hall current over the ion-slip current with $K = 0$ and 1 represented by (a) and (b) respectively with fixed values of $z = 0.5$, $t = 0.1$, $M = 35.0$, and $E = 1.0$.



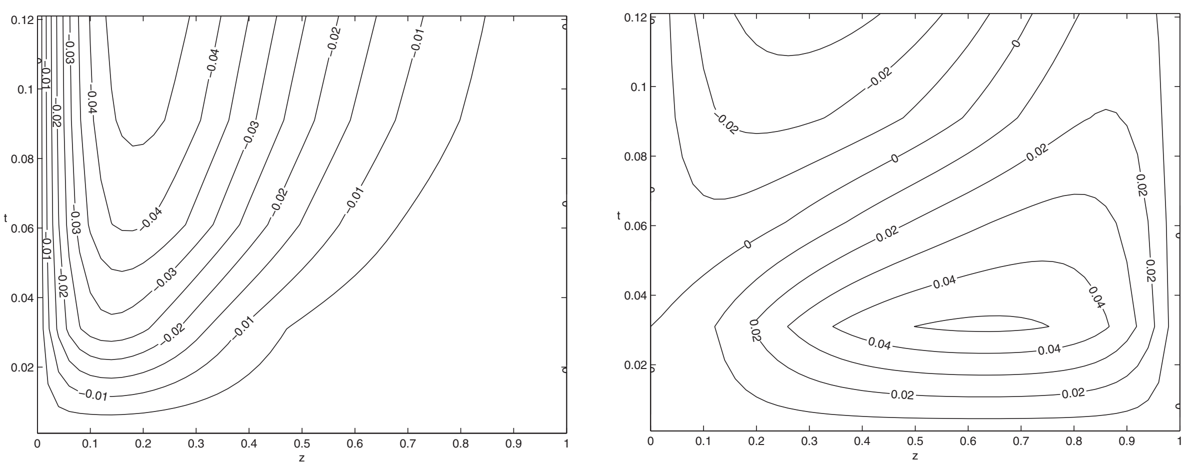
(a) (b)

Fig. 6. Secondary velocity v showing the effect of Hall current over the ion-slip current with $K = 0$ and 1 represented by (a) and (b) respectively with fixed values $z = 0.5$, $t = 0.1$, $M = 35.0$, and $E = 1.0$.



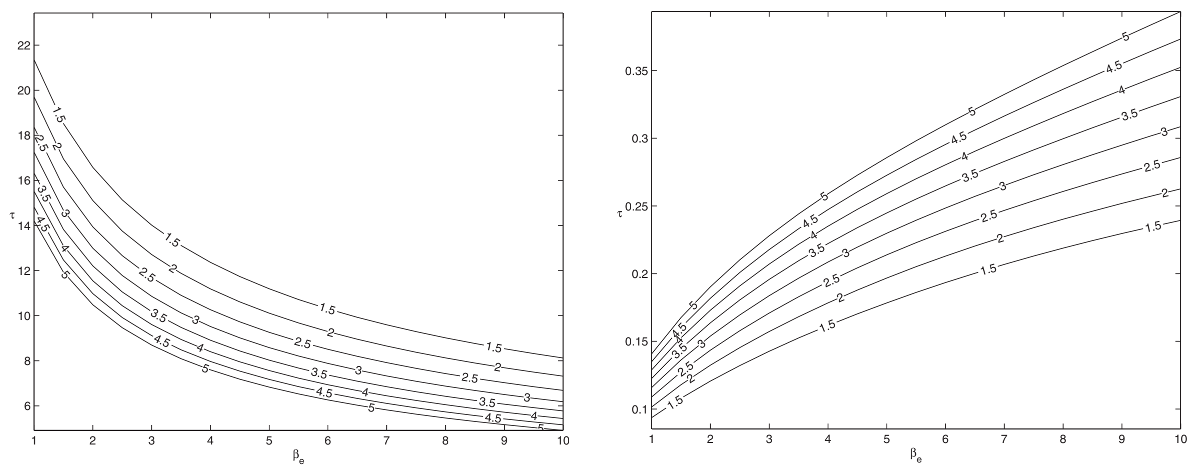
(a) (b)

Fig. 7. Primary velocity u showing the effect of time with $K = 0$ and 1 represented by (a) and (b) respectively with fixed values of $M = 35.0$, $E = 1$, $t = 0.4$, $\beta_e = 10.0$, $\beta_i = 5.0$, and $t \in [0.01, 0.15]$.



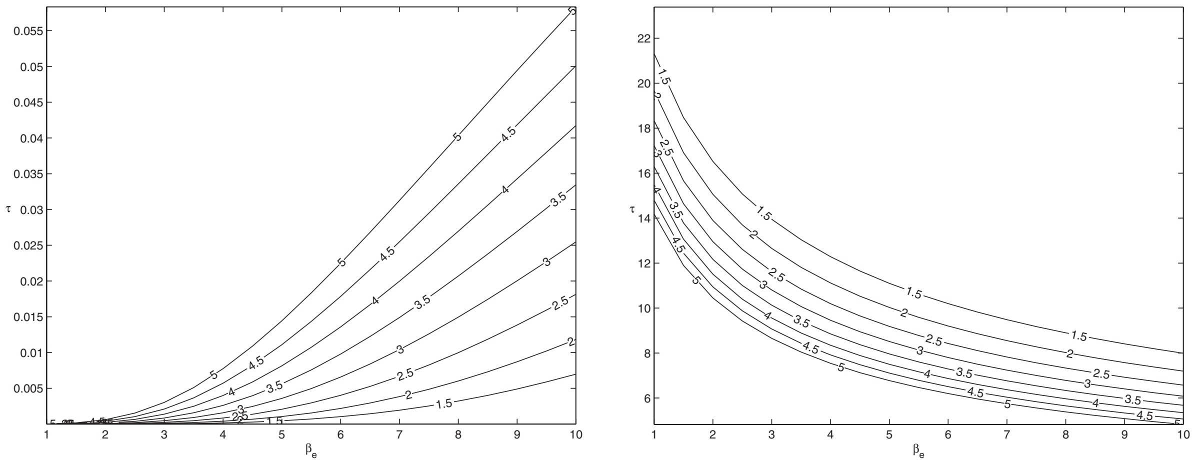
(a) (b)

Fig. 8. Secondary velocity v showing the effect of time with $K = 0$ and 1 represented by (a) and (b) respectively with fixed values of $M = 35.0$, $E = 1$, $t = 0.4$, $\beta_e = 10.0$, $\beta_i = 5.0$, and $t \in [0.01, 0.15]$.



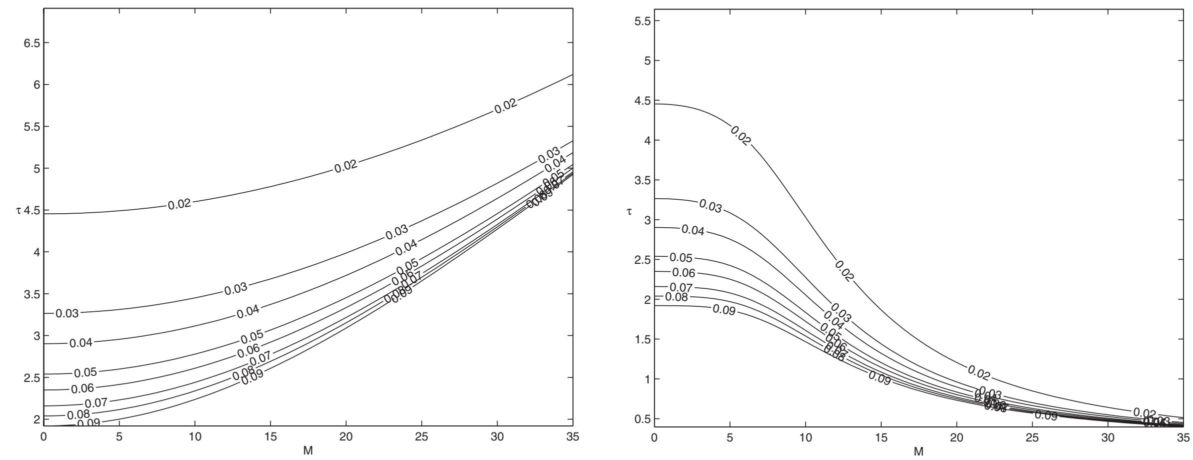
(a) (b)

Fig. 9. The resultant skin friction at $z = 0$ showing the effect of Hall and ion-slip parameter with $K = 0$ and 1 represented by (a) and (b) respectively with fixed values of $E = 1$, $M = 35.0$, and $t = 0.1$.



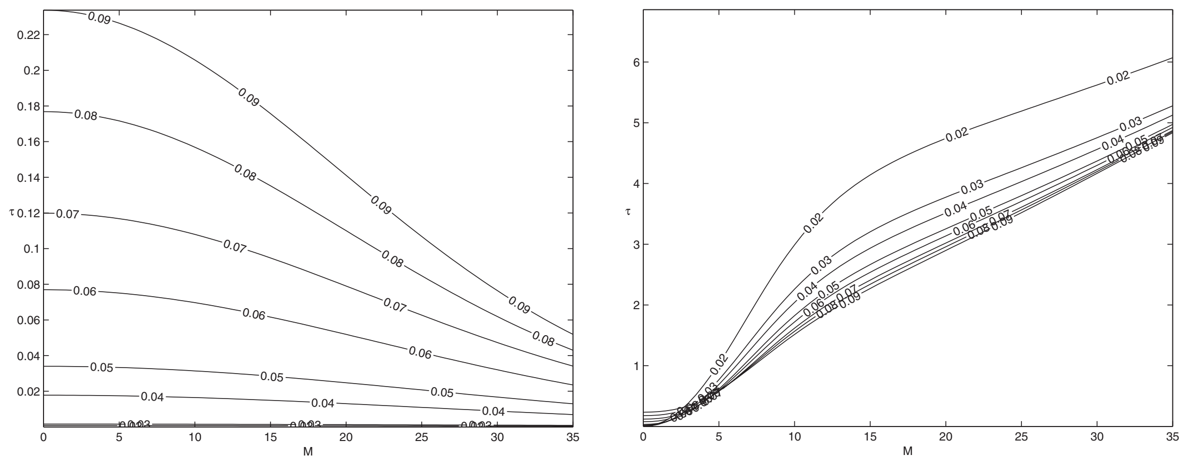
(a) (b)

Fig. 10. The resultant skin friction at $z = 1$ showing the effect of Hall and ion-slip parameter with $K = 0$ and 1 represented by (a) and (b) respectively with fixed values of $E = 1$, $M = 35.0$, and $t = 0.1$.



(a) (b)

Fig. 11. The resultant skin friction at $z = 0$ showing the effect of Hartmann number and time with $K = 0$ and 1 represented by (a) and (b) respectively with fixed values of $E = 1$, $\beta_e = 10.0$, $\beta_i = 5.0$, and $t \in [0.01, 0.15]$.



(a) (b)

Fig. 12. The resultant skin friction at $z = 1$ showing the effect of Hartmann number and time with $K = 0$ and 1 represented by (a) and (b) respectively with fixed values of $E = 1$, $\beta_e = 10.0$, $\beta_i = 5.0$, and $t \in [0.01, 0.15]$.

3. Results and Discussion

In order to discuss the results, some numerical calculations are carried out for the dimensionless velocities and the skin friction. The general expressions for the velocity and skin friction are given by eqs. (16) and (17). The solution given in the equations are in complex form; so with MATLAB, one can separate both the velocity and skin friction into real and imaginary parts and then calculate the numerical values of the primary and secondary velocities and the associated skin frictions. Contour and line graphs of the velocity profiles and skin friction are presented in Figs. 1–12 to reveal the influences of the governing parameters. The value of the Ekman number E is fixed throughout as one in order to concentrate on the effect of the Hall and ion slip currents. Figure 1 shows the effect of the Hall current in the presence of ion-slip current on the primary velocity for fixed values of $M = 35.0$, $E = 1.0$, $t = 0.1$, and $\beta_{\mathrm{i}} = 5.0$. From Fig. 1(a), the primary velocity u is observed to increase as the Hall current increases when the magnetic field is fixed relative to the fluid ($K = 0$), the reverse is the case when the magnetic field is fixed relative to the moving plate ($K = 1$).

In Fig. 2(a), the secondary velocity reduces as the Hall current increases when $K = 0$, while for $K = 1$, three distinctive regions can be noticed. Near the moving lower plate, it is observed that v decreases with an increase in the Hall parameter, while at the middle of the channel it increase. A reversal of this trend is noticed as we move towards the stationary plate. Figures 3 and 4 show the effect of the ion-slip current on the velocity profiles for fixed values of $M = 35.0$, $E = 1.0$, $t = 0.1$, and $\beta_{\mathrm{e}} = 10.0$. The effect of the ion-slip parameter on the primary veloicty as observed in Fig. 3 is similar to effect the Hall current have on the primary velocity as seen in Fig. 1. However, the secondary velocity v when the magnetic field is fixed relative to the fluid ($K = 0$), is seen in Fig. 4(a) to increase with the ion-slip parameter close to the plate at $z = 0$ (i.e., the moving plate). As the ion-slip parameter is increased further and close to the stationary plate, the effect of the ion slip on the flow behaviour is reduced to the barest minimum. An opposite behaviour is noticed when the magvetic field is fixed relative to the moving plate ($K = 1$) as observed in Fig. 4(b). The primary velocity u is seen in Fig. 5 to increase with both the Hall and the ion-slip currents when $K = 0$, while when $K = 1$, it decreases with both Hall and ion-slip currents. The behaviour of the secondary velocity with respect to Hall and ion-slip current is depicted in Fig. 6. The ion-slip current is a decreasing function of the secondary velocity at all times, the Hall current behaves in a similar manner when $K = 0$. However, the behaviour of the Hall current towards the secondary velocity when $K = 1$ is somehow erratic, it reduces with Hall current close to the moving plate until it reaches zero at the middle of the plate and started increasing at the other half of the channel. Figure 7 shows that the primary velocity increases with time until it reaches its steady state for both cases. However, the primary velocity is seen to attain its steady state quicker in the case when $K = 0$ than when $K = 1$. The secondary velocity bahaves in an opposite manner towards time as seen in Fig. 8.

The influence of the governing parameters on the skin friction are presented in Figs. 9–12. The skin friction on the moving plate ($z = 0$) decreases with increases in both the Hall and ion-slip parameters for fixed values of $E = 1$, $M = 35.0$, and $t = 0.1$ when $K = 0$ as seen in Fig. 9(a), while the opposite behaviour is noticed in Fig. 9(b) when $K = 1$. At the stationary plate ($z = 1$), we noticed that when the magnetic field is fixed relative to the fluid ($K = 0$) in Fig. 10(a), the skin friction tends to zero as both the Hall and ion-slip parameter are decreased. However, the influence of the Hall and ion-slip parameter on the skin friction on the stationary plate when ($K = 1$) is the same as when ($K = 0$) on the moving plate. Figures 11 and 12 reflect that the skin friction decreases with time on both porous plates when magnetic field is fixed with respect to the moving porous plate. Moreover, the skin friction decreases with time at the moving porous plate when the magnetic field is fixed to the fluid while the role of time is just contrast at the stationary plate.

4. Conclusion

The combined effect of Hall and ion-slip currents on unsteady MHD Couette flows of a viscous incompressible electrically conducting fluid in a rotating system when one of the plate is set into impulsive motion is considered. It is observed that the effect of both the Hall and ion-slip parameters is to increase the primary velocity and reduce the secondary velocity. The resultant skin friction on the moving plate decreases with increase in both the Hall and ion-slip parameters when the magnetic field is fixed relative to the fluid, while the opposite behaviour is noticed the magnetic field is fixed relative to the moving plate. On the stationary plate however, the skin friction increases with both the Hall and ion-slip currents when ($K = 0$), an opposite behaviour is observed when ($K = 1$).

Acknowledgement

The authors are grateful to the Physical Society of Japan for financial support in publication.

Appendix

List of constants used to define the velocity and the skin friction:

$$\lambda_1 = \frac{M^2 K(\alpha + \mathrm{i}\gamma)}{m},$$

$$f_1(x_1, x_2, x_3) = \frac{1}{2}\left[\exp(x_2\sqrt{x_3})\,\mathrm{erfc}\left(\frac{x_2}{2\sqrt{x_1}} + \sqrt{x_1 x_3}\right) + \exp(-x_2\sqrt{x_3})\,\mathrm{erfc}\left(\frac{x_2}{2\sqrt{x_1}} - \sqrt{x_1 x_3}\right)\right],$$

$$f_2(x_1, x_2) = \mathrm{erfc}\left(\frac{x_2}{2\sqrt{x_1}}\right),$$

$$f_3(x_1,x_2,x_3) = \frac{1}{2}\left[\sqrt{x_3}\exp(x_2\sqrt{x_3})\,\mathrm{erfc}\left(\frac{x_2}{2\sqrt{x_1}} + \sqrt{x_1x_3}\right)\right.$$
$$\left. - \sqrt{x_3}\exp(-x_2\sqrt{x_3})\,\mathrm{erfc}\left(\frac{x_2}{2\sqrt{x_1}} - \sqrt{x_1x_3}\right) - \frac{2}{\sqrt{\pi x_1}}\exp\left(\frac{-x_2^2}{4} - x_1x_3\right)\right],$$
$$f_4(x_1,x_2) = -\frac{1}{\sqrt{\pi x_1}}\exp\left(-\frac{x_2^2}{4x_1}\right).$$

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