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Hall effects on unsteady MHD oscillatory free convective flow of second grade fluid through porous medium between two vertical plates

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The effects of radiation and Hall current on an unsteady magnetohydrodynamic free convective flow in a vertical channel filled with a porous medium have been studied. We consider an incompressible viscous and electrically conducting incompressible viscous second grade fluid bounded by a loosely packed porous medium. The fluid is driven by an oscillating pressure gradient parallel to the channel plates, and the entire flow field is subjected to a uniform inclined magnetic field of strength H_o inclined at an angle of inclination α with the normal to the boundaries in the transverse xy-plane. The temperature of one of the plates varies periodically, and the temperature difference of the plates is high enough to induce the radiative heat transfer. The effects of various parameters on the velocity profiles, the skin friction, temperature field, rate of heat transfer in terms of their amplitude, and phase angles are shown graphically. *Published by AIP Publishing.* https://doi.org/10.1063/1.5010863

NOMENCLATURE

B the magnetic induction vector (Wb),
C_p the specific heat at constant pressure (J kg^{-1} K),
e the electron charge (C),
E the electric field (N/C),
g the acceleration due to gravity (m s^{-2}),
H_o the applied magnetic field (A/m),
$\overline{J}$ the current density (kg m^{-3}),
K the thermal conductivity (W/m K),
k the permeability of the porous medium (Darcy),
p the pressure (N m^{-2}),
p_e the electron pressure(N m^{-2}),
q_r the radiative heat (P/A),
q the complex velocity $u + iw$,
t the time (s),
T the temperature (K),
T_0 the reference temperature that of the left plate (K),
$\overline{U}$ the mean axial velocity (m s^{-1}),
$\overline{V}$ the velocity vector (m s^{-1}),

Greek symbols

σ the electrical conductivity (s m^{-1}),
η_e the density of electron (kg m^{-3}),
μ_e the magnetic permeability (H m^{-1}),
ν the coefficient of kinematic viscosity (m^2 s^{-1}),
ω_e the electron frequency (J),
τ_e the electron collision time (s),
ρ the density (kg m^{-3}),
μ the coefficient of viscosity (N S m^{-2}),
β the coefficient of volume expansion (K^{-1}).

I. INTRODUCTION

The study of the oscillatory flow of an electrically conducting fluid through a porous channel is important in many physiological flows and engineering applications such as magnetohydrodynamic (MHD) generators, arterial blood flow, petroleum engineering, and many more. Several authors have studied the flow and heat transfer in oscillatory fluid problems. To mention just a few, Makinde and Mhone[1] investigated the forced convective MHD oscillatory fluid flow through a channel filled with a porous medium, and analyses were based on the assumption that the plates are impervious. In a related study, Mehmood and Ali[2] investigated the effect of slip on the free convective oscillatory flow through a vertical channel with periodic temperature and dissipative heat. In addition, Chauchan and Kumar[3] studied the steady flow and heat transfer in a composite vertical channel. Palani and Abbas[4] investigated the combined effects of magneto-hydrodynamics and radiation effect on a free convection flow past an impulsively started isothermal vertical plate using the Rosseland approximation. Hussain *et al.*[5] presented the analytical study of the oscillatory second grade fluid flow in the presence of a transverse magnetic field and many more. In all the above studies, the channel walls are assumed to be impervious. This assumption is not valid in studying flows such as blood flow in the miniature level where digested food particles are diffused into the bloodstream through the wall of the blood capillary. Hence, due to several other important suction/injection controlled applications, there have been several studies on the convective heat transfer through a porous channel; for instance,

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Umavathi *et al.*[6] investigated the unsteady flow of viscous fluid through a horizontal composite channel whose half width is filled with a porous medium. Ajibade and Jha[7] presented the effects of suction and injection on hydrodynamics of oscillatory fluid through parallel plates. This problem is extended to heat generating/absorbing fluids by Jha and Ajibade,[8] while in (Jha and Ajibade[9]) the effect of viscous dissipation of the free convective flow with a time dependent boundary condition was investigated. More recently, Adesanya and Makinde[10] investigated the effect of the radiative heat transfer on the pulsatile couple stress fluid flow with the time dependent boundary condition on the heated plate. It is well known that the no-slip condition is not realistic in some flows involving nanochannel and micro-channel and flows over coated plates with hydrophobic substances. In view of this, Adesanya and Gbadeyan[11] studied the flow and heat transfer of the steady non-Newtonian fluid flow noting the fluid slip in the porous channel. Other interesting cases on the hydromagnetic oscillatory fluid flow under different geometries can be found in Refs. 12–20 and the references therein. The effect on suction/injection on the slip flow of oscillatory hydromagnetic fluid through a channel filled with a saturated porous medium has been investigated by Falade *et al.*[21] Recently, VeeraKrishna and Swarnalathamma[24,25] discussed the peristaltic MHD flows. VeeraKrishna and Gangadhar Reddy[26,27] discussed MHD free convective rotating flows.

Keeping the above-mentioned facts, the MHD free convective flow in a vertical channel filled with a porous medium has been studied in the present paper.

II. FORMULATION AND SOLUTION OF THE PROBLEM

Consider an unsteady MHD free convective flow of an electrically conducting, viscous, incompressible second grade fluid through a porous medium bounded between two infinite vertical plates in the presence of the Hall current and thermal radiation. The plates are at a distance d apart. A Cartesian coordinate system with the x-axis oriented vertically upward along the centre line of the channel is introduced. The z-axis is taken perpendicular to the planes of the plates as shown in Fig. 1.

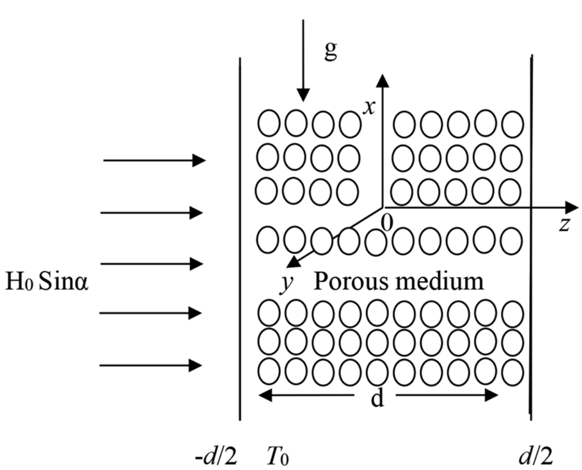

FIG. 1. Physical configuration of the problem.

We choose a Cartesian system O(x, y, z) such that the boundary walls are at $z = -d/2$ and $z = d/2$ and are assumed to be parallel to the xy-plane. The steady flow through the porous medium is governed by Brinkman's equations. At the interface, the fluid satisfies the continuity condition of velocity and stress. The boundary plates are assumed to be parallel to the xy-plane and the magnetic field of strength H_o inclined at an angle of inclination α to the z-axis in the transverse xz-plane. The component along the z-direction induces a secondary flow in that direction while its x-components change perturbation to the axial flow. The steady hydromagnetic equations governing the incompressible fluid under the influence of a uniform inclined magnetic field of strength H_o inclined at an angle of inclination α with reference to a frame are

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial z^2} + \frac{\alpha_1}{\rho}\frac{\partial^3 u}{\partial z^2 \partial t} - \frac{\mu_e J_z H_0 \,\mathrm{Sin}\,\alpha}{\rho} - \frac{\nu}{k}u + g\beta T, \quad (1)$$

$$\frac{\partial w}{\partial t} = \nu\frac{\partial^2 w}{\partial z^2} + \frac{\alpha_1}{\rho}\frac{\partial^3 w}{\partial z^2 \partial t} + \frac{\mu_e J_x H_0 \,\mathrm{Sin}\,\alpha}{\rho} - \frac{\nu}{k}w. \quad (2)$$

When the strength of the magnetic field is very large, the generalized Ohm's law is modified to include the Hall current so that

$$J + \frac{\omega_e \tau_e}{H_0} J \times H = \sigma(E + \mu_e q \times H). \quad (3)$$

In Eq. (3), the electron pressure gradient, the ion-slip, and thermo-electric effects are neglected. We also assume that the electric field $E = 0$ under assumptions reduces to

$$J_x - m J_z \,\mathrm{Sin}\,\alpha = -\sigma \mu_e H_0 w \,\mathrm{Sin}\,\alpha, \quad (4)$$

$$J_z + m J_x \,\mathrm{Sin}\,\alpha = -\sigma \mu_e H_0 u \,\mathrm{Sin}\,\alpha, \quad (5)$$

where $m = \omega_e \tau_e$ is the Hall parameter.

On solving Eqs. (4) and (5), we obtain

$$J_x = \frac{\sigma \mu_e H_0 \,\mathrm{Sin}\,\alpha}{1 + m^2 \,\mathrm{Sin}^2\,\alpha}(um \,\mathrm{Sin}\,\alpha - w), \quad (6)$$

$$J_z = \frac{\sigma \mu_e H_0 \,\mathrm{Sin}\,\alpha}{1 + m^2 \,\mathrm{Sin}^2\,\alpha}(u + wm \,\mathrm{Sin}\,\alpha). \quad (7)$$

Using Eqs. (6) and (7), the equations of the motion with reference to the frame are given by

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial z^2} + \frac{\alpha_1}{\rho}\frac{\partial^3 u}{\partial z^2 \partial t} - \frac{\sigma {\mu_e}^2 {H_0}^2 \,\mathrm{Sin}\,\alpha}{\rho(1 + m^2 \,\mathrm{Sin}^2\,\alpha)} \times (u + wm \,\mathrm{Sin}\,\alpha) - \frac{\nu}{k}u + g\beta T, \quad (8)$$

$$\frac{\partial w}{\partial t} = \nu\frac{\partial^2 w}{\partial z^2} + \frac{\alpha_1}{\rho}\frac{\partial^3 w}{\partial z^2 \partial t} + \frac{\sigma {\mu_e}^2 {H_0}^2 \,\mathrm{Sin}\,\alpha}{\rho(1 + m^2 \,\mathrm{Sin}^2\,\alpha)} \times (um \,\mathrm{Sin}\,\alpha - w) - \frac{\nu}{k}w, \quad (9)$$

$$\rho C_p \frac{\partial T}{\partial t} = K\frac{\partial^2 T}{\partial z^2} - \frac{\partial q_r}{\partial z}. \quad (10)$$

The boundary conditions for the problem are

$$u = w = T = 0, \quad z = -\frac{d}{2}, \quad (11)$$

$$u = w = 0, T = T_w \,\mathrm{Cos}\,\omega t, \quad z = \frac{d}{2}, \quad (12)$$

where T_w is the mean temperature of the plate at $z = d/2$ and ω is the frequency of oscillations. Following Cogley *et al.*,[22] the last term in the energy Eq. (10),

$$\frac{\partial q_r}{\partial z} = 4\alpha_2^2(T - T_0)$$

stands for the radiative heat flux that modifies to

$$\frac{\partial q_r}{\partial z} = 4\alpha_2^2 T. \tag{13}$$

We have considered the reference temperature $T_0 = 0$, where α_2 is the mean radiation absorption co-efficient.

We introduce the following non-dimensional variables and parameters:

$$z^* = \frac{z}{d}, x = \frac{x}{d}, u^* = \frac{u}{U}, v^* = \frac{v}{U}, q^* = \frac{q}{U}, t^* = \frac{tU}{d},$$
$$\omega^* = \frac{\omega d}{U} p^* = \frac{p}{\rho U^2}, T^* = \frac{T}{T_w}.$$

Making use of non-dimensional variables, the governing equations reduces to (dropping asterisks)

$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{1}{\mathrm{Re}}\frac{\partial^2 u}{\partial z^2} + S\frac{\partial^3 u}{\partial z^2 \partial t} - \frac{M^2 \mathrm{Sin}^2\,\alpha}{\mathrm{Re}(1+m^2\,\mathrm{Sin}^2\,\alpha)}$$
$$\times (u + wm\,\mathrm{Sin}\,\alpha) - \frac{D^{-1}}{\mathrm{Re}}u + \frac{\mathrm{Gr}}{\mathrm{Re}}T, \tag{14}$$
$$\frac{\partial w}{\partial t} = \frac{1}{\mathrm{Re}}\frac{\partial^2 w}{\partial z^2} + S\frac{\partial^3 w}{\partial z^2 \partial t} + \frac{M^2 \mathrm{Sin}^2\,\alpha}{\mathrm{Re}(1+m^2\,\mathrm{Sin}^2\,\alpha)}$$
$$\times (um\,\mathrm{Sin}\,\alpha - w) - \frac{D^{-1}}{\mathrm{Re}}w, \tag{15}$$
$$\frac{\partial T}{\partial t} = \frac{1}{\mathrm{Pe}}\frac{\partial^2 T}{\partial z^2} - \frac{\mathrm{R}^2}{\mathrm{Pe}}\frac{\partial q_r}{\partial z}, \tag{16}$$

where $\mathrm{Re} = \frac{Ud}{\nu}$ is the Reynolds number, $S = \frac{\alpha_1}{\rho d^2}$ is the second grade fluid parameter, $D = \frac{k}{d^2}$ is the permeability parameter (Darcy parameter), $\mathrm{Gr} = \frac{g\beta d^2 T_w}{\nu U}$ is the thermal Grashof number, $\mathrm{Pe} = \frac{\rho C_p d U}{K}$ is the Peclet number, and $R = \frac{2\alpha_2 d}{\sqrt{K}}$ is the radiation parameter.

The corresponding transformed boundary conditions are

$$u = w = T = 0, \quad z = -\frac{1}{2}, \tag{17}$$
$$u = w = 0, T = \mathrm{Cos}\,\omega t, \quad z = \frac{1}{2}. \tag{18}$$

We shall assume that the fluid flows only under the influence of a non-dimensional pressure gradient oscillating in the direction of x-axis only which is of the form

$$\frac{\partial p}{\partial x} = P\,\mathrm{Cos}\,\omega t. \tag{19}$$

In order to combine Eqs. (14) and (15) into single equation, we introduce a complex function $q = u + iw$ VeeraKrishna *et al.*,[27] and using Eq. (19), we obtain

$$\mathrm{Re}\frac{\partial q}{\partial t} = -P\,\mathrm{Cos}\,\omega t + \frac{\partial^2 q}{\partial z^2} + \mathrm{Re}S\frac{\partial^3 q}{\partial z^2 \partial t}$$
$$-\left(\frac{M^2\,\mathrm{Sin}^2\,\alpha}{1 - im\,\mathrm{Sin}\,\alpha} + i\omega\mathrm{Re} + D^{-1}\right)q + \mathrm{Gr}T. \tag{20}$$

The boundary conditions in the complex form are

$$q = T = 0, \quad z = -\frac{1}{2}, \tag{21}$$
$$q = 0, T = e^{i\omega t}, \quad z = \frac{1}{2}. \tag{22}$$

In order to solve Eqs. (16) and (20) making use of boundary conditions (21) and (22), we assume in the complex form the solution of the problem as

$$q(z,t) = q_0(z)e^{i\omega t}, \quad T(z,t) = \theta_0(z)e^{i\omega t}, \quad -\frac{\partial p}{\partial x} = Pe^{i\omega t}. \tag{23}$$

Substituting Eq. (23) in Eqs. (16) and (20), we get

$$\frac{d^2 q_0}{dz^2} - \lambda^2 q_0 = \frac{-P\mathrm{Re} + \mathrm{Gr}\,\theta_0}{1 + i\omega\mathrm{Re}\,S} \tag{24}$$

and

$$\frac{d^2 q_0}{dz^2} - \xi^2\theta_0 = 0, \tag{25}$$

where $\lambda^2 = \frac{\frac{M^2\,\mathrm{Sin}^2\,\alpha}{1-im\,\mathrm{Sin}\,\alpha} + i\omega\mathrm{Re} + D^{-1}}{1 + i\omega\mathrm{Re}S}$ and $\xi^2 = i\omega\mathrm{Pe} + R^2$.

The boundary conditions given in Eqs. (21) and (23) become

$$q_0 = \theta_0 = 0, \quad z = -\frac{1}{2}, \tag{26}$$
$$q_0 = 0, \theta_0 = 1, \quad z = \frac{1}{2}. \tag{27}$$

The ordinary differential equations (24) and (25) are solved under the boundary conditions given in Eqs. (26) and (27) for the velocity and temperature fields. The solution of the problem is obtained as

$$q(z,t) = \left(\frac{P\mathrm{Re}}{\lambda^2(1+i\omega\mathrm{Re}S)}\left(1 - \frac{\mathrm{Cosh}\,\lambda z}{\mathrm{Cosh}(\lambda/2)}\right) + \frac{\mathrm{Gr}}{(\lambda^2-\xi^2)(1+i\omega\mathrm{Re}S)}\left(\frac{\mathrm{Sinh}\,\lambda\left(z+\frac{1}{2}\right)}{\mathrm{Sinh}\,\lambda} - \frac{\mathrm{Sinh}\,\xi\left(z+\frac{1}{2}\right)}{\mathrm{Sinh}\,\xi}\right)\right\} e^{i\omega t}, \tag{28}$$

$$T(z,t) = \frac{\mathrm{Sinh}\,\xi\left(z+\frac{1}{2}\right)}{\mathrm{Sinh}\,\xi}e^{i\omega t}. \tag{29}$$

Now from the velocity field, we can obtain the skin-friction at the left plate in terms of its amplitude and phase angle as

$$\tau_L = \left(\frac{\partial q}{\partial z}\right)_{z=-1/2} = \left(\frac{\partial q_0}{\partial z}\right)_{z=-1/2} e^{i\omega t} = |q|\ \mathrm{Cos}(\omega t + \varphi),$$

where $|q| = \sqrt{(R.P.of\ q)^2 + (Im.P.of\ q)^2}$ and $\varphi = \mathrm{Tan}^{-1}\left(\frac{R.P.\ of\ q}{Im.\ P.\ of\ q}\right)$,

$$(R.P.\ of\ q) + i(Im.\ P.\ of\ q) = \frac{P\,\mathrm{Re}}{\lambda^2}\ \mathrm{Tanh}\left(\frac{\lambda}{2}\right) + \frac{\mathrm{Gr}}{\lambda^2 - \xi^2}\left[\frac{\lambda}{\mathrm{Sinh}\,\lambda} - \frac{\xi}{\mathrm{Sinh}\,\xi}\right]. \tag{30}$$

From the temperature field, the rate of heat transfer Nu (Nusselt number) at the left plate in terms of its amplitude and phase angle is obtained,

$$Nu = \left(\frac{\partial T}{\partial z}\right)_{z=-1/2} = \left(\frac{\partial \theta_0}{\partial z}\right)_{z=-1/2} e^{i\omega t} = |H|\ \mathrm{Cos}(\omega t + \psi), \tag{31}$$

where $|H| = \sqrt{(R.\ P.\ of\ H)^2 + (Im.\ P.\ of\ H)^2}$,

$$\psi = \mathrm{Tan}^{-1}\left(\frac{R.P.\ of\ H}{Im.P.\ of\ H}\right),$$

and

$$(R.\ P.\ of\ H) + i(Im.\ P.\ of\ H) = \frac{\xi}{\mathrm{Sinh}\,\xi}. \tag{32}$$

III. RESULTS AND DISCUSSION

We consider an incompressible viscous and electrically conducting second grade fluid in a parallel plate channel bounded by a loosely packed porous medium. The fluid is driven by the oscillating pressure gradient parallel to the channel plates, and the entire flow field is subjected to a uniform inclined magnetic field of strength H_o inclined at an angle of inclination α with the normal to the boundaries in the transverse xy-plane. The temperature of one of the plates varies periodically, and the temperature difference of the plates is high enough to induce the radiative heat transfer. The complete expressions for the velocity, $q(z)$, and temperature, $T(z)$, profiles as well as the skin friction, τ, and the heat transfer rate, Nu, are given in Eqs. (28) and (31). In order to understand the physical situation of the problem and hence the manifestations of the effects of the material parameters entering into the solution of problem, to study the effects of these different parameters appearing in the governing flow problem, we have carried out computational results for the velocity field, skin-friction, temperature field, and Nusselt number in terms of its amplitude and the phase. The computational results are presented in Figs. 2–12 for the velocity profiles (fixing $\alpha = \pi/6$) and Fig. 13 for temperature profiles. Tables I and II represents the shear stresses and the rate of heat transfer in terms of

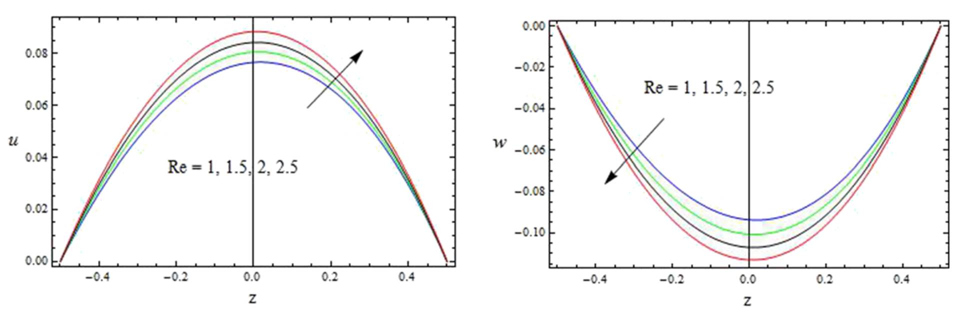

FIG. 2. The velocity profile for u and w against Re with $P = 5$, Pe = 0.7, Gr = 1, $D = 1$, $R = 1$, $M = 2$, $m = 1$, $S = 1$, $\omega = 5$, $t = 0.1$.

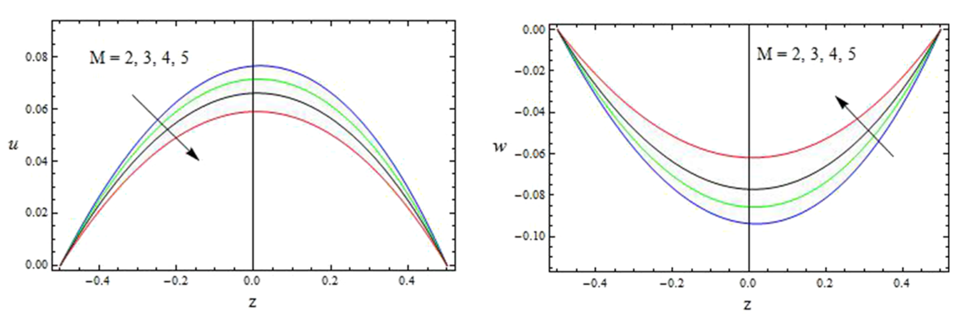

FIG. 3. The velocity profile for u and w against M with $P = 5$, Pe = 0.7, Re = 1, Gr = 1, $D = 1$, $R = 1$, $m = 1$, $S = 1$, $\omega = 5$, $t = 0.1$.

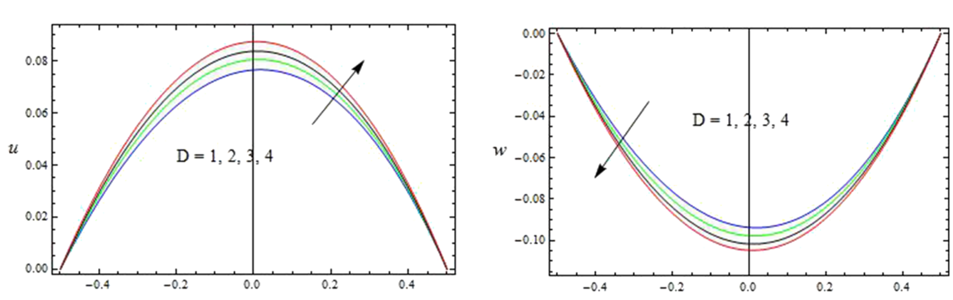

FIG. 4. The velocity profile for u and w against D with $P = 5$, Pe = 0.7, Re = 1, Gr = 1, $R = 1$, $M = 2$, $m = 1$, $S = 1$, $\omega = 5$, $t = 0.1$.

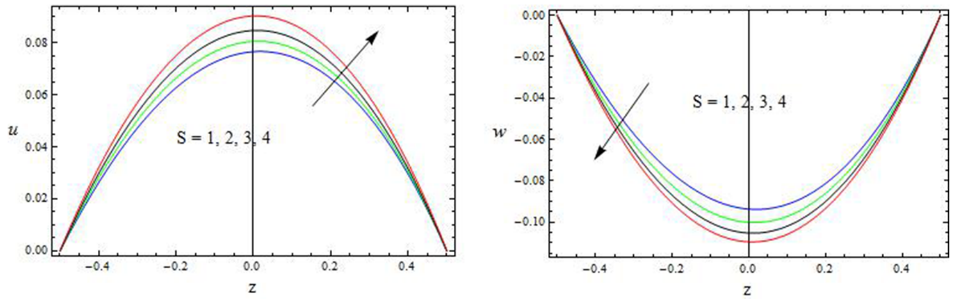

FIG. 5. The velocity profile for u and w against S with $P = 5$, Pe = 0.7, Re = 1, Gr = 1, $D = 1$, $R = 1$, $m = 1$, $M = 2$, $\omega = 5$, $t = 0.1$.

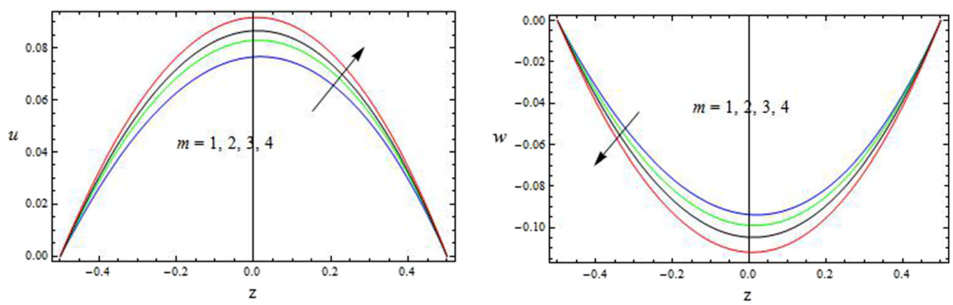

FIG. 6. The velocity profile for u and w against m with $P = 5$, Pe = 0.7, Re = 1, Gr = 1, $D = 1$, $R = 1$, $M = 2$, $S = 1$, $\omega = 5$, $t = 0.1$.

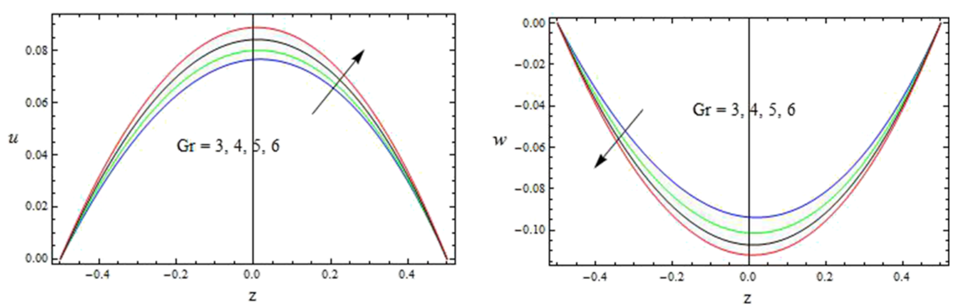

FIG. 7. The velocity profile for u and w against Gr with $P = 5$, Pe = 0.7, Re = 1, $D = 1$, $R = 1$, $M = 5$, $m = 1$, $S = 1$, $\omega = 5$, $t = 0.1$.

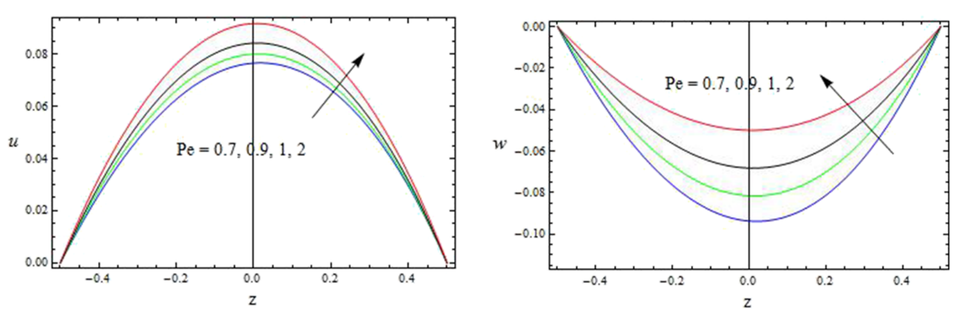

FIG. 8. The velocity profile for u and w against Pe with $P = 5$, Re = 1, Gr = 1, $D = 1$, $R = 1$, $M = 2$, $m = 1$, $S = 1$, $\omega = 5$, $t = 0.1$.

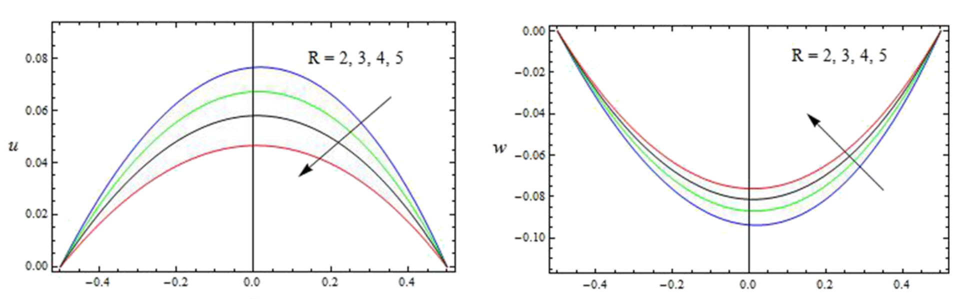

FIG. 9. The velocity profile for u and w against R with $P = 5$, Pe = 0.7, Re = 1, Gr = 1, $D = 1$, $M = 2$, $m = 1$, $S = 1$, $\omega = 5$, $t = 0.1$.

amplitude and phase angle with respect to different governing parameters at $z = -1/2$.

Figure 2 depicts the variation of velocity profiles under the influence of the Reynolds parameter Re. The magnitudes of the velocity components u and w increase with increasing Reynolds number. It is evident from that that increasing value of Re leads to the increase in resultant velocity. It is interesting to note that from Fig. 3 both the magnitudes of velocity components u and w decrease with the increase in intensity of the magnetic field. This is because of the reason that the

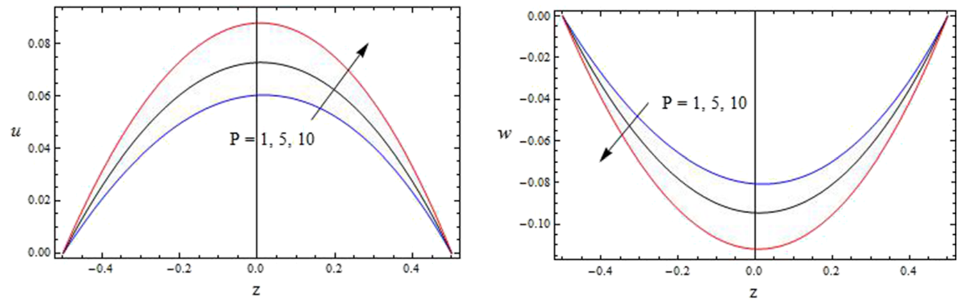

FIG. 10. The velocity profile for u and w against P with Pe = 0.7, Re = 1, Gr = 1, D = 1, R = 1, M = 2, m = 1, S = 1, ω = 5, t = 0.1.

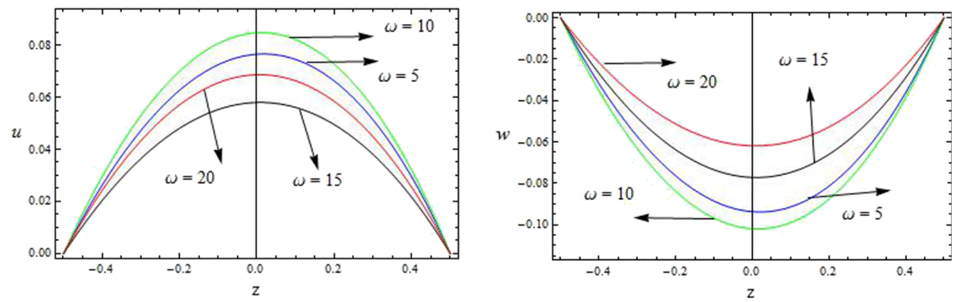

FIG. 11. The velocity profile for u and w against ω with Pe = 0.7, Re = 1, Gr = 1, D = 1, R = 1, M = 2, m = 1, S = 1, P = 5, t = 0.1.

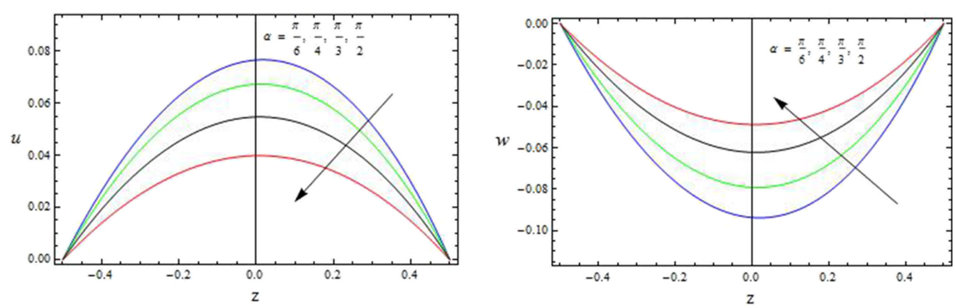

FIG. 12. The velocity profile for u and w against α with Pe = 0.7, Re = 1, Gr = 1, D = 1, R = 1, M = 2, m = 1, S = 1, ω = 5, P = 5, t = 0.1.

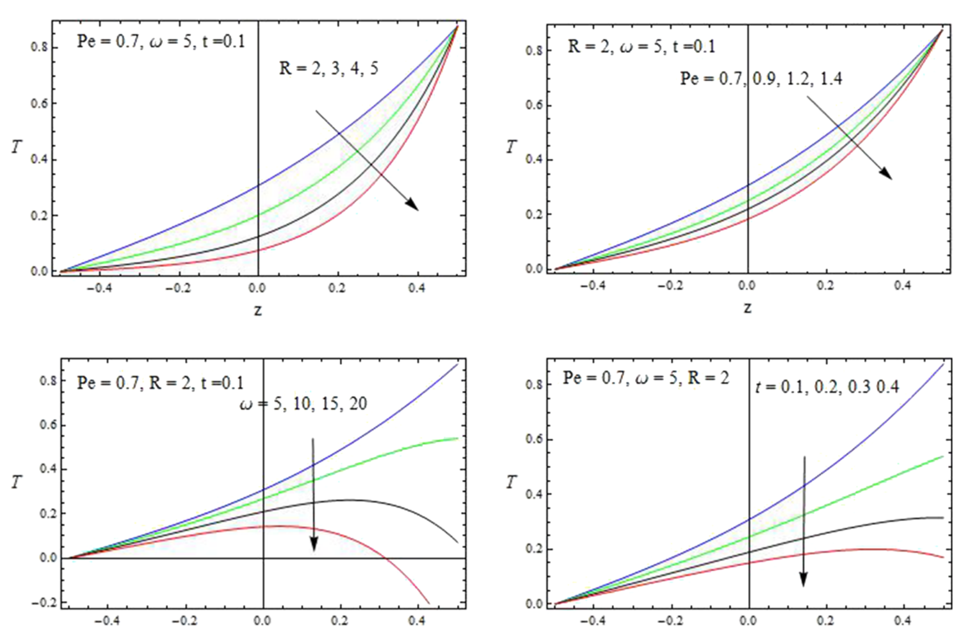

FIG. 13. The temperature profiles for T against R, Pe, ω, and t.

effect of an inclined magnetic field on an electrically conducting fluid gives rise to a resistive type force (called the Lorentz force) similar to drag force and upon increasing the values of M increases the drag force which has tendency to slow down the motion of the fluid. The resultant velocity also reduces with increase in the intensity of the magnetic field. The magnitude of the velocity components u and w increases with the increase in permeability of the porous medium (D), or second grade fluid parameter S is observed from Figs. 4 and 5. The lower the permeability of the porous medium, the lesser the

TABLE I. Skin friction (τ_L) at the left plate. Bold face values represent the variation of particular parameter being other parameters fixed.

Re	M	m	S	D	Gr	Pe	R	P	ω	α	Amplitude $\lvert q \rvert$	Phase angle (ϕ)
1	**2**	**1**	**1**	**1**	**3**	**0.7**	**1**	**5**	**5**	$\pi/6$	1.119 42	−0.630 318
1.5											1.369 56	−0.675 564
2											1.580 66	−0.700 398
	3										1.064 29	−0.562 733
	4										0.989 46	−0.490 428
		2									1.123 31	−0.656 557
		3									1.129 91	−0.672 529
			2								1.092 24	−0.630010
			3								1.083 12	−0.629 286
				2							1.113 31	−0.672 557
				3							1.136 80	−0.687 117
					4						1.137 97	−0.639394
					5						1.156 62	−0.648 179
						0.9					1.115 67	−0.633 482
						1.2					1.109 58	−0.637 124
							5				2.497 74	−0.749 193
							10				3.532 78	−0.766 803
								8			1.757 86	−0.619 742
								10			2.183 54	−0.616 126
									10		0.804 56	−0.701 169
									15		0.656 21	−0.722 159
										$\pi/4$	1.063 81	−0.602 550
										$\pi/3$	1.009 47	−0.593 862

fluid speed is in the entire fluid region. It is expected physically also because the resistance posed by the porous medium to the decelerated flow due to the inclined magnetic field reduces with decreasing permeability D which leads to decrease in the velocity. The resultant velocity also increases with increase in D. The variation of the velocity profiles with Hall parameter m is shown in Fig. 6. The magnitudes of the velocity components u and w and the resultant velocity increase with the increase in Hall parameter m throughout the channel, and there is no significant effect of Hall parameter m on both the velocity components with the effect of the inclined magnetic field. The variations of the velocity profiles with the Grashof number Gr are shown in Fig. 7. The magnitude of the velocity components u and w enhances with the increasing Grashof number Gr. The maximum of the velocity profiles shifts toward the right half of the channel due to the greater buoyancy force in this part of the channel due to the presence of hotter plate. In the right half, there lies a hot plate at $z = 1/2$ and heat is transferred from the hot plate to the fluid and consequently the buoyancy force enhances the flow velocity further. In the left half of the channel, the transfer of heat takes place from the fluid to the cooler plate at $z = -1/2$. Thus, the effect of the Grashof number on the resultant velocity is reversed, i.e., velocity decreases with increasing Gr. The velocity profiles with the Peclet number Pe are shown in Fig. 8. The magnitude of the velocity components u enhances and w decreases with the increasing Peclet number Pe. We noticed that with increasing Peclet number Pe the resultant velocity decreases. The variation of velocity profile with radiation parameter R is shown in Fig. 9. The magnitude of velocity components u and w decreases with increase in Radiation parameter R. In the left half of the channel, the effect of R on velocity is insignificant while in the right half of the channel velocity decreases with increase of R. It is evident from Fig. 10 that the velocity components u and w enhance with increase in pressure gradient P. The increasing pressure gradient P leads to the increase of resultant velocity. The velocity profiles with the frequency of oscillation ω are shown in Fig. 11. The magnitude of the velocity component u enhances first, gradually decreases, and then experiences enhancement as observed with increase in the frequency of oscillation ω. Likewise the behavior of the velocity component w experiences enhancement and then gradually decreases throughout the fluid region with increase in the frequency of oscillation ω. The resultant velocity decreases with increasing frequency of oscillations ω. Generally the frequency of oscillation increases; there is some disturbance in the particles of the fluid particles, and hence the velocity is automatically reduced in the fluid region.

TABLE II. Rate of heat transfer (Nu).

R	Pe	ω	t	Amplitude ($\lvert H \rvert$)	Phase angle (ψ)
2	0.7	5	0.1	0.532 335	−0.464 794
3				0.293 364	−0.389 559
4				0.144 828	−0.327 514
	0.9			0.520 735	−0.593 349
	1.2			0.499 652	−0.780 681
		10		0.365 671	−1.251 970
		15		0.313 108	−1.299 511
			0.3	0.315 004	−0.378 854
			0.5	0.124 458	−0.310 522

TABLE III. Comparison of results for velocity Pe = 0.7, Re = 1, R = 1, P = 5, ω = 5, t = 0.1, z = 0.2.

M	m	D	Gr	Previous results of Singh and Pathak,[23] u	Present results, u $\alpha \to 0$, S = 0
2	1	0.5	3	0.521 341	0.521 313
3				0.478 852	0.478 855
4				0.369 922	0.369 985
	2			0.652 258 2	0.652 289
	3			0.785 595	0.785 548
		1		0.560 422	0.560 452
		2		0.581 828	0.581 842
			4	0.561 545	0.561 544
			5	0.601 752	0.601 775

From Fig. 12, we observed that the magnitude of the velocity components u and w reduces with increasing angle of inclination α. The results are verified with those of Singh and Pathak[23] in Table III.

The temperature profiles are shown in Fig. 13. The temperature decreases with the increase in radiation parameter R, the Peclet number Pe, and time. The temperature enhances initially and then gradually decreases with increase in the frequency of oscillations ω. We notice that the flow of heat transfer is reversed with the increase in Peclet number Pe.

The skin-friction at the plate $z = -1/2$ is obtained in terms of its amplitude and the phase angle ϕ. The amplitude $|q|$ and phase angle ϕ are presented in Table I. Since all the values of phase angle ϕ presented in Table I are negative, therefore, there is always a phase lag. The amplitude $|q|$ and the magnitude of the phase angle ϕ increase with increase in Reynolds number Re, Darcy parameter D, Radiation parameter R, Grashof number Gr and the Hall parameter m, where as decreases with increasing Hartmann number M, second grade fluid parameter S and the angle of inclination α. The amplitude $|q|$ reduces and the magnitude of phase angle ϕ enhances with increasing Peclet number Pe and frequency of oscillation ω. The reversal behavior is observed with increasing pressure P.

The rate of heat transfer (Nusselt number Nu) in terms of the amplitude and phase angle has been evaluated in Table II. We noticed that the amplitude $|H|$ decreases with the increase in R, Pe, ω, and t. It is noticed that the magnitude of the phase angle ψ decreases with the increase in R and t, and increases with increase in R and ω.

IV. CONCLUSIONS

The effects of radiation and the Hall current on the unsteady MHD free convective flow in a vertical channel filled with a porous medium have been studied. The conclusions are made as the following. The velocity component for the primary flow enhances with an increase in permeability parameter, Hall parameter, and Grashof number and reduces with an increase in the intensity of the magnetic field and radiation parameter. The velocity component for the secondary flow enhances with increase in permeability parameter and Hall parameter and reduces with increase in Hartmann number, Grashof number, and radiation parameter. The temperature reduces with increase in radiation parameter or Peclet number, while it enhances initially and then gradually reduces with increase in frequency of oscillation.

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