$$\frac{1}{\varepsilon}(1+\Delta)f''' - \Delta\Xi_{2}' + G_{r}\theta + G_{m}\phi + \frac{1}{\varepsilon^{2}}ff'' - \left(K + \frac{M\alpha_{e}}{\alpha_{e}^{2} + \beta_{e}^{2}}\right)f' + \left(R - \frac{M\beta_{e}}{\alpha_{e}^{2} + \beta_{e}^{2}}\right)g - \Gamma f'^{2} = 0 \tag{1}$$

$$\frac{1}{\varepsilon}(1+\Delta)g'' + \Delta\Xi_{1}' + \frac{1}{\varepsilon^{2}}fg' - \left(K + \frac{M\alpha_{e}}{(\alpha_{e}^{2} + \beta_{e}^{2})}\right)g - \left(R - \frac{M\beta_{e}}{(\alpha_{e}^{2} + \beta_{e}^{2})}\right)f' - \Gamma g^{2} = 0 \tag{2}$$

$$\Delta\Xi_{1}'' + \Xi_{1}f' + f\Xi_{1}' = 0 \tag{3}$$

$$\Delta\Xi_{2}'' + \Xi_{2}f' + f\Xi_{2}' = 0 \tag{4}$$

$$\theta'' + (1+\Delta)P_{r}E_{c}[f'''^{2} + g'^{2}] + \frac{E_{c}MP_{r}}{(\alpha_{e}^{2} + \beta_{e}^{2})}[f'^{2} + g^{2}] + P_{r}f\theta' - S_{T}P_{r}f' = 0 \tag{5}$$

Corresponding boundary conditions

 $\emptyset'' + fS_c\emptyset' - S_T^*S_cf' + S_0S_c\theta'' - R_cS_c\emptyset = 0$ 

$$f' = 1, f = f_w, \qquad g = 0, \Xi_1 = -s_1 g', \qquad \Xi_2 = s_1 f'', \theta = 1 - \frac{1}{2} S_T, \phi = 1 - \frac{1}{2} S_T^* \quad at \quad \eta = 0$$

$$f' = 0, g = 0, \Xi_1 = 0, \Xi_2 = 0, \qquad \theta = 0, \phi = 0 \qquad at \quad \eta \to \infty$$
(7)

(6)

Again

$$f''' = \frac{\varepsilon \Delta}{1 + \Delta} \Xi_2' - \frac{\varepsilon}{1 + \Delta} G_r \theta - \frac{\varepsilon}{1 + \Delta} G_m \phi - \frac{1}{1 + \Delta} \frac{1}{\varepsilon} f f'' + \frac{\varepsilon}{1 + \Delta} \left( K + \frac{M \alpha_e}{(\alpha_e^2 + \beta_e^2)} \right) f' - \frac{\varepsilon}{1 + \Delta} \left( R - \frac{M \beta_e}{(\alpha_e^2 + \beta_e^2)} \right) g$$

$$+ \frac{\varepsilon}{1 + \Delta} \Gamma f'^2$$
(8)

$$g'' = -\frac{\varepsilon \Delta}{1 + \Delta} \Xi_1' - \frac{1}{1 + \Delta} \frac{1}{\varepsilon} f g' + \frac{\varepsilon}{1 + \Delta} \left( K + \frac{M\alpha_e}{(\alpha_e^2 + \beta_e^2)} \right) g + \frac{\varepsilon}{1 + \Delta} \left( R - \frac{M\beta_e}{(\alpha_e^2 + \beta_e^2)} \right) f' + \frac{\varepsilon}{1 + \Delta} \Gamma g^2$$

$$(9)$$

$$\Xi_{1}^{"} = -\frac{1}{\Lambda}\Xi_{1}f' - \frac{1}{\Lambda}f\Xi_{1}^{'} \qquad (10)$$

$$\Xi_{2}^{"} = -\frac{1}{\Lambda} + \Xi_{2}f' - \frac{1}{\Lambda}f\Xi_{2}^{'} \qquad (11)$$

$$\theta'' = -(1+\Delta)P_{r}E_{c}[f''^{2} + g'^{2}] - \frac{E_{c}MP_{r}}{(\alpha_{e}^{2} + \beta_{e}^{2})}[f'^{2} + g^{2}] - P_{r}f\theta' + S_{T}P_{r}f' \qquad (12)$$

$$\theta'' = -fS_{c}\theta' + S_{T}^{*}S_{c}f' - S_{0}S_{c}\theta'' + R_{c}S_{c}\theta \qquad (13)$$

Corresponding boundary conditions

$$f' = 1$$
,  $f = f_w$ ,  $g = 0$ ,  $\Xi_1 = -s_1 g'$ ,  $\Xi_2 = s_1 f''$ ,  $\theta = 1 - \frac{1}{2} S_T$ ,  $\phi = 1 - \frac{1}{2} S_T^*$  at  $\eta = 0$   
 $f' = 0$ ,  $g = 0$ ,  $\Xi_1 = 0$ ,  $\Xi_2 = 0$ ,  $\theta = 0$ ,  $\phi = 0$  at  $\eta \to \infty$  (14)

## Consider the value of the parameters

$$\begin{split} & \text{M=}0.5 \\ & \alpha_e = 1 + \beta_e \beta_i = 2.1 \\ & \beta_i = 0.7 \\ & \beta_e = 0.4 \\ & \text{R=}0.6 \\ & \text{Gr=}10.0 \\ & \text{Gm=}5.0 \\ & \text{Pr=}0.71 \\ & \text{Ec=}0.01 \\ & s_1 = 0.5 \\ & s_0 = 1.0 \\ & \text{K=}0.5 \\ & s_0 = 1.0 \\ & \text{K=}0.5 \\ & \text{Sc=}0.6 \\ & \Delta = 0.5 \\ & \Gamma = 0.5 \\ & \varepsilon = 0.6 \\ & s_T = 0.5 \\ & s_T = 0.5 \\ & s_T = 0.5 \\ & \Lambda = 3.0 \end{split}$$