



Alexandria University
Alexandria Engineering Journal

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Hall and ion slip impacts on unsteady MHD convective rotating flow of heat generating/absorbing second grade fluid

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Received 2 August 2020; revised 8 September 2020; accepted 7 October 2020

KEYWORDS

Hall and ion slip effects;
 MHD;
 Porous medium;
 Rotation;
 Second grade fluids

Abstract We have explored theoretically the Hall and ion slip impacts on an unsteady laminar MHD convective rotating flow of heat generating or absorbing second grade fluid over a semi-infinite vertical moving permeable surface. The non-dimensional equations for the governing flow are solved to the most excellent possible investigative solution using perturbation methodology. The effects of parameters on velocity, temperature and concentration are demonstrated graphically and described in detail. For engineering curiosity, the shear stresses, Nusselt number and Sherwood number are obtained analytically, represented computationally in a tabular format as well as explained with respect to foremost parameters. It is concluded that, the resultant velocity is increased with an increasing in Hall and ion slip parameters throughout fluid region. The thermal and solutal buoyancy forces contribute to the resultant velocity ever-increasing to high. The temperature distribution is trim downs through an increasing in heat source parameter. The concentration is reduced with an increase in the chemical reaction parameter in the entire fluid region. Rotation parameter is to diminish the skin friction, whereas it is augmented through an increase of the Hall and ion slip effects. The rate of mass transfer is increased with increasing chemical reaction parameter.

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1. Introduction

The survey associated to free convective flow movement within the existence of temperature resource has drained substantial concentration of numerous investigators for the duration of last few decades, since of its extensive purpose in astrophysical disciplines and cosmical study etc. Those types of flows engage

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Peer review under responsibility of Faculty of Engineering, Alexandria University.

<https://doi.org/10.1016/j.aej.2020.10.013>

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Nomenclature

(u, v)	the velocity components along the (x, y) directions	P_e	Electron pressure
A	Suction coefficient	B	Magnetic induction vector
B_0	Electro-magnetic induction	J_x, J_y	Current densities along x and y directions
C_p	Specific heat	E	Electric field vector
g	Acceleration due to gravity	J	current density vector
k_1	thermal conductivity		
k	permeability of the porous medium		
Q_0	Heat generation/absorption	<i>Greek symbols</i>	
T	temperature of the fluid	α_1, α_2	the material moduli of second grade fluid
T_w	fluid temperature at the surface (wall)	β	Coefficient of thermal expansion of the fluid
T_∞	fluid temperature in the free stream	β^*	Coefficient of mass expansion of the solid
U_0	Constant velocity	θ	Dimensionless temperature
U_∞	free stream velocity	Ω	Angular velocity
w_0	suction velocity	σ	Electrical conductivity of the fluid
D	molecular diffusivity	ρ	Density of the fluid
K_c	chemical reaction constant	ν	Kinematic viscosity
Kc	chemical reaction parameter	τ	Shear stress
t	time	ω	Frequency of oscillation
q	complex velocity	ω_e	Cyclotron frequency
M	Hartmann number	τ_e	Electron collision time
K	Permeability parameter	β_e	Hall parameter
S	second grade fluid parameter	β_i	ion slip parameter
R	Rotation parameter		
H	Heat generation/absorption parameter	<i>Subscripts</i>	
Gr	thermal Grashof number	w	conditions on the wall
Gm	mass Grashof number	∞	free stream conditions
Pr	Prandtl number	e	Electron charge
Sc	Schmidt number		

in recreation and vital function in chemical engineering, aerospace science and technology etc. The gyratory fluids are extremely imperative for the reason that of its happening in an assortment of expected phenomenon and technological requirements by the Coriolis force. The comprehensive regions of numerous sciences are full of a quantity of momentous and requisite characteristics of rotational fluids. Coriolis force effect is an essential than viscous and nonreactive forces. Moreover, strengths of magnetic and Coriolis are comparable in terrible nature.

The time dependent fluctuating flows have numerous applications in a lot of domains such as chemical engineering, paper manufacturing and many other scientific and industrial fields. Asghar et al. [1] have researched the flow of a non-Newtonian fluid provoked owing toward the fluctuations for a absorbent plate. Choudhury and Das [2] explored the visco-elastic magnetohydrodynamic (MHD) free convective flow in the course of permeable medium in the occurrence of radiation and chemical reaction phenomenon through heat and mass transportation. Deka et al. [3] have researched that a free convective consequences for MHD flow through an infinite perpendicular oscillating surface by constant heat discharge. Das et al. [4] have researched mass transportation effects on free convective MHD flow of a viscous fluid enclosed through a oscillating porous plate in the slip flow managed by heat source. The imperative investigation prepared by Hayat et al. [5] for the flow of a non-Newtonian fluid for a oscillating surface. Manna

et al. [6] addressed results of radiation on time addicted MHD free convective flow over a fluctuating vertical porous plate entrenched in an absorbent medium through oscillating heat flux. Shen et al. [7] researched the Rayleigh-Stokes predicament for a temperature and comprehensive second order fluid through an important fragmentary derivative modeling. Singh and Gupta [8] studied free convective MHD flow of viscous fluid during a permeable medium enclosed along with a fluctuating porous plate in slip flow management through mass transportation. Jhansi Rani and Murthy [9] explored the radiation and absorption consequences on a time dependent convective flow through a semi-infinite, inclined porous plate embedded in a porous medium through the heat and mass transport. Veera Krishna et al. [10–13] researched the MHD flows for an incompressible, electrically conducting fluid in two-dimensional channels. The results of heat radiation on MHD nanofluid flow between two parallel rotating plates are premeditated through Sheikholeslami et al. [14]. Rashid et al. [15] explored a precise modeling for two dimensional stream wise transverse magnetic fluid flows with heat transfer around a porous obstacle. Ellahi et al. [16] addressed the blood flow of Prandtl liquid through tapered and stenosed arteries in permeable walls with magnetic field. Ellahi et al. [17] explored a new hybrid technique supported on pseudo-spectral collocations inside the intellect of least-squares technique is used to scrutinize the MHD flow of non-Newtonian fluid. Oahimire and Olajuwon [18] addressed the effects of Hall current, chem-

ical reaction and heat radiation on heat and mass transportation of MHD flow of a micro-polar liquid through a porous medium.

Recently Veera Krishna et al. [19] explored heat and mass transportation on unsteady MHD fluctuating flow of blood through a porous arteriole. Prakash and Muthamilselvan [20] researched the effect of radiation on transient MHD flow of micropolar liquid connecting absorbent vertical conduit through the boundary conditions of the third type. The contemporaneous effects of coagulation like blood thicken and variable magnetic field on peristaltic flow of Jeffrey nanofluid comprising gyro tactical micro organism during alike ring include and explored by Bhatti et al. [21]. An innovative modeling is proposing by Ellahi et al. [22] to research the effects of nano-ferro-fluid under the influence of small oscillating through expandable gyrating disk. Bhatti et al. [23] reviewed the heat and mass transportation through the transverse magnetic field for the peristaltic flow on two-segment flow (particle-liquid interruption) during a two-dimensional conduit. Hassan et al. [24] explored to examination on nanosized particle profiles performance on mass along with heat transfer flow of ferro-liquid through a gyrating disk by the occurrence of small fluctuating magnetic field. Ellahi et al. [25] researched the peristaltic flow of nanofluid through a perpendicular asymmetric channel. Shirvan et al. [26] explored a two-dimensional numerical simulation and sensibility scrutiny for turbulent heat transport and temperature exchanging efficiency augmentation in a twofold pipe temperature exchanger packed through the porous medium. A numerical exploration on free convection through the surface radiation heat transportation in an inclined porous solar hollow recipient by sources of reaction surface method was explored by Shirvan et al. [27]. Ellahi et al. [28] investigated the predicament of the peristaltic flow for Jeffrey fluid in a non-homogeneous rectangular conduit under the effects by Hall and ion slip. Maqbool et al. [29] discussed the Falkner-Skan boundary layer time independent flow over a smooth stretching sheet. Hayat et al. [30] respected an inquisitive management of MHD flow of micro-polar fluid due to a flexible stretching surface. Homogeneous as well as heterogeneous repercussions are adopted into contemplation. Veera Krishna et al. [31] explored the heat and mass transport on unsteady MHD fluctuating flow of second order fluid in the course of porous medium between two perpendicular plates. The effects of radiation and Hall current for an unsteady MHD free convection flow through a perpendicular channel packed by a porous medium and explored by Veera Krishna et al. [32]. The heat generation or absorption and thermo-diffusion for an unsteady free convection MHD flow of radiating and chemical reacting second grade fluid at an infinite vertical surface through a permeable medium in addition to adopting the Hall effects into description and has been researched by Veera Krishna and Chamkha [33]. Krishna et al. [34] discussed the heat and mass transfer on MHD rotating flow of second grade fluid past an infinite vertical plate embedded in uniform porous medium with Hall effects.

Most recently, Veera Krishna and Chamkha [36] addressed the diffusion-thermo, radiation-absorption, Hall and ion-slip effects on hydromagnetic natural convective rotating flow for nanofluids over a semi infinite absorbent moving plate with invariable heat source. Hall and ion slip effects on unsteady hydro-magnetic convection gyrating flow for nanofluids explored by Veera Krishna and Chamkha [37]. Veera Krishna

et al.[38] discussed the Hall and ion slip impacts on the unsteady hydromagnetic natural convective gyrating flow through a saturated porous medium over an exponential accelerated plate. The combined effects of Hall and ion slip on MHD rotating flow of ciliary propulsion of microscopic organism through porous medium have been studied by Veera Krishna et al.[39]. Veera Krishna and Chamkha [40] investigated the Hall and ion slip effects on the MHD convective flow of elastico-viscous fluid through porous medium between two rigidly rotating parallel plates with time fluctuating sinusoidal pressure gradient. Veera Krishna [41] reported that the Hall and ion slip effects on MHD free convective rotating flow bounded by the semi-infinite vertical porous surface. Veera Krishna [42] discussed the MHD laminar flow of an elastico-viscous electrically conducting Walter's-B fluid through a circular cylinder or a pipe. Veera Krishna et al. [43] discussed the heat and mass transfer on unsteady MHD oscillatory flow of second-grade fluid through a porous medium between two vertical plates, under the influence of fluctuating heat source/sink, and chemical reaction.

With the enthusiasm from all the aforementioned, acknowledged efforts and specialized literature inquiry tolerated that, the scrutiny for the Hall and ion slip impacts on a unsteady laminar MHD convective rotating flow of heat generating or absorbing second grade fluid over a semi-infinite vertical moving permeable surface has not been inspected yet. Therefore, within the present analysis, we have explored theoretically the Hall and ion slip impacts on an unsteady laminar MHD convective rotating flow of heat generating or absorbing second grade fluid over a semi-infinite vertical moving permeable surface. The non-dimensional controlling equations are solved to the most excellent possible investigative solution. The effects of parameters are demonstrated graphically as well as described in detail.

2. Formulation and solution of the problem

We consider the heat and mass transport on an unsteady two dimensional MHD convective flow of a viscous laminar heat generating/absorbing second grade fluid over a semi-infinite vertical moving porous plate embedded in a uniform porous medium and applied to a uniform transverse magnetic field taking Hall and ion slip effects into account. The Cartesian coordinate scheme is chosen such that the x -axis is kept along the wall in the upward direction and the z -axis is occupied perpendicular to this. A uniform magnetic field of strength B_0 is proceeding in the transverse direction to the flow. Initially undisturbed state, both the fluid and plate are in rigid rotation by the uniform angular velocity Ω about the perpendicular to the plate. Also both the fluid and the plate are at respite to invariable temperature and concentration at the surface. The physical model of the investigative problem is as revealed in the Fig. 1.

The constitutive equation for the fluids of second grade is in the following form (Hayat et al. [46]),

$$\mathbf{T} = -p\mathbf{I} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2 \quad (1)$$

where \mathbf{T} is the Cauchy stress tensor, \mathbf{I} is the identity tensor, p is the static fluid pressure, μ is the dynamic viscosity co-efficient, α_1 and α_2 are the normal stress moduli, i.e., α_1 is the elastic coefficient and α_2 is the transverse viscosity coefficient, and the kinematic tensors \mathbf{A}_1 and \mathbf{A}_2 are defined through

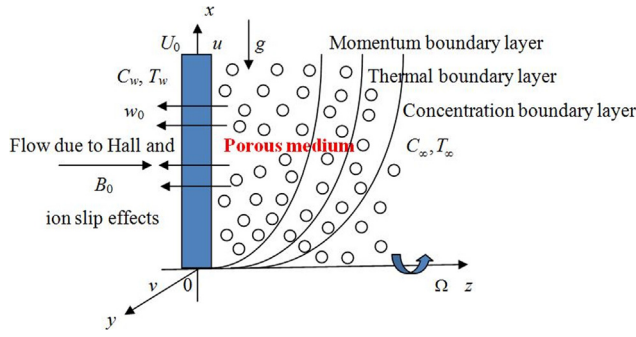


Fig. 1 . Physical model.

$$\begin{aligned} \mathbf{A}_1 &= (\text{grad} \mathbf{V}) + (\text{grad} \mathbf{V})^T, \\ \mathbf{A}_2 &= \frac{d\mathbf{A}_1}{dt} + \mathbf{A}_1(\text{grad} \mathbf{V}) + (\text{grad} \mathbf{V})^T \mathbf{A}_1 \end{aligned} \quad (2)$$

where \mathbf{V} is the velocity vector, grad is the gradient operator and d/dt denotes the material time derivative. Since the fluid is incompressible, it can undergo only isochoric motion and hence, $\text{div} \mathbf{V} = 0$ and the equation of motion is,

$$\rho \frac{d\mathbf{V}}{dt} = \text{div} \mathbf{T} + \rho \mathbf{F} \quad (3)$$

where ρ is the density of the fluid and \mathbf{F} is the body force. If the fluid modeled by the Eq. (2) is to be compatible with thermodynamics, in the sense that all motions of the fluid meet the Clausius-Duhem inequality and the assumption that the specific Helmholtz free energy of the fluid takes its minimum value in equilibrium, then the material moduli must be satisfied as follows (Benharbit and Siddiqui [47]):

$$\mu \geq 0, \quad \alpha_1 \geq 0, \quad \alpha_1 + \alpha_2 = 0 \quad (4)$$

This, then, was shown to give to the theory a rather well behaved and pleasant stability and boundedness structure. It was also shown that if α_1 was taken negative, the remainder of (4) being preserved, then in quite arbitrary flows instability and unboundedness were unavoidable. However, it is well known that for most non-Newtonian fluids of current rheological interest, conclusions (4) are contradicted by experiments.

$$\mu \geq 0, \quad \alpha_1 \leq 0, \quad \alpha_1 + \alpha_2 \neq 0, \quad (5)$$

Which were supposedly obtained by data reduction from experiments for those fluids it is assumed to be constitutively described by (1) as a second grade fluid, and it showed that such values for the material moduli led to anomalous behavior, thus questioning whether the fluid under consideration in the experiments could be described as a second grade fluid.

The unsteady hydro magnetic flow in a rotating system is controlled by the continuity, momentum, energy and concentration equations in the form as,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0 \quad (6)$$

$$\begin{aligned} \rho \left(\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} + 2\Omega \times \mathbf{V} + \Omega \times (\Omega \times \mathbf{r}) \right) \\ = \nabla \cdot \mathbf{T} + \mathbf{J} \times \mathbf{B} \end{aligned} \quad (7)$$

where \mathbf{J} is the current density, \mathbf{B} is the total magnetic field, ∇ is the operator, \mathbf{T} is the Cauchy stress tensor for second grade fluid specified in [45], Ω is the angular velocity as well as r is radial co-ordinate specified through $r^2 = x^2 + y^2$.

The equation of energy can be specified in various manners, such as,

$$\rho \left(\frac{\partial h}{\partial t} + \nabla \cdot (h \mathbf{V}) \right) = - \frac{Dp}{Dt} + \nabla \cdot (k_t \nabla T) + \Phi \quad (8)$$

where h is the unambiguous enthalpy this is related to particular internal energy as $h = e + p/\rho$, T is the total temperature, k_t is the conductivity of thermal energy, and Φ is the dissipation variable portraying the work done versus forces of viscosity, this is irrevocably changed into internal energy. This is specified as

$$\Phi = (\tau \cdot \nabla) \mathbf{V} = \tau_{ij} \frac{\partial V_i}{\partial x_j} \quad (9)$$

The pressure term on the RHS of Eq. (8) is generally abandoned. It is developed the equation of energy and assumed that, the conductive heat transport is controlled as a result of Fourier's law through the conductivity of thermal energy of the fluid. Also, radiative heat transfer and internal heat generation due to a probable chemical or nuclear reaction is deserted.

The equation of mass transfer with chemical reaction is specified by,

$$\frac{\partial C}{\partial t} = D \nabla^2 C - K_c (C - C_\infty) \quad (10)$$

The entire thermo-physical characteristics are assumed to be constant of the momentum equation in linear form; it is estimated in accordance with the Boussinesq approximation. The plate is extending to infinitely hence all the physical variables are function of z and the time t merely. Under these assumptions, the governing equations that portray the physical conditions for the flow with respect to the rotating frame are specified by,

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \quad (11)$$

$$\begin{aligned} \frac{\partial u}{\partial t} + w \frac{\partial u}{\partial z} - 2\Omega v = - \frac{1}{\rho} \frac{\partial p}{\partial x} + v \frac{\partial^2 u}{\partial z^2} + \frac{\alpha}{\rho} \frac{\partial^3 u}{\partial z^2 \partial t} + \frac{B_0 J_y}{\rho} \\ - \frac{v}{k} u + g\beta(T - T_\infty) + g\beta * (C - C_\infty) \end{aligned} \quad (12)$$

$$\frac{\partial v}{\partial t} + w \frac{\partial v}{\partial z} + 2\Omega u = - \frac{1}{\rho} \frac{\partial p}{\partial y} + v \frac{\partial^2 v}{\partial z^2} + \frac{\alpha}{\rho} \frac{\partial^3 v}{\partial z^2 \partial t} - \frac{B_0 J_x}{\rho} - \frac{v}{k} v \quad (13)$$

$$\frac{\partial T}{\partial t} + w \frac{\partial T}{\partial z} = \frac{k_1}{\rho C_p} \frac{\partial^2 T}{\partial z^2} - \frac{Q_0}{\rho C_p} (T_w - T_\infty) \quad (14)$$

$$\frac{\partial C}{\partial t} + w \frac{\partial C}{\partial z} = D \frac{\partial^2 C}{\partial z^2} - K_c (C_w - C_\infty) \quad (15)$$

It is assumed that the permeable surface actuates with a constant velocity in the direction of fluid flow. Also, the temperature and concentrations at the wall and the suction velocity is expeditiously unreliable by means of time.

$$u = U_0, v = 0, T = T_w + \varepsilon(T_w - T_\infty)e^{i\omega t},$$

$$C = C_w + \varepsilon(C_w - C_\infty)e^{i\omega t} \quad \text{at} \quad z = 0 \quad (16)$$

$$u \rightarrow U_\infty, v \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \quad \text{as} \quad z \rightarrow \infty \quad (17)$$

From equation (11), the suction velocity at the surface is moreover a steady or a segment of time. Then, the velocity of suction perpendicular to the plate is assumed in the frame be,

$$w = -w_0(1 + \varepsilon A e^{i\omega t}) \quad (18)$$

where A and ε are optimistic constants, which fulfills the constraint $\varepsilon A \ll 1$ and w_0 is extent of suction velocity and non-zero productive constant. The negative symbol designates the suction be achieve the plate.

The electron-atom collision frequency is assumed to be very high, so that Hall and ion slip currents cannot be neglected. Hence, the Hall and ion slip currents give rise to the velocity in y -direction. When the strength of the magnetic field is very large, the generalized Ohm's law is modified to include the Hall and ion slip effect (Sutton and Sherman [35]),

$$J = \sigma(E + V \times B) - \frac{\omega_e \tau_e}{B_0} (J \times B)$$

$$+ \frac{\omega_e \tau_e \beta_i}{B_0^2} ((J \times B) \times B) \quad (19)$$

Additionally, it is assumed that the Hall parameter $\beta_e = \omega_e \tau_e \sim O(1)$ and the ion slip parameter $\beta_i = \omega_i \tau_i \ll 1$, in Eq. (19), the electron pressure gradient and thermo-electric effects are abandoned, i.e., the electric field $E = 0$ under these assumptions, Eq. (19) condensed to,

$$(1 + \beta_i \beta_e) J_x + \beta_e J_y = \sigma B_0 v \quad (20)$$

$$(1 + \beta_i \beta_e) J_y - \beta_e J_x = -\sigma B_0 u \quad (21)$$

On solving equations (20) and (21), we acquired as,

$$J_x = \sigma B_0 (\alpha_2 u + \alpha_1 v) \quad (22)$$

$$J_y = -\sigma B_0 (\alpha_2 v - \alpha_1 u) \quad (23)$$

where $\alpha_1 = \frac{1 + \beta_e \beta_i}{(1 + \beta_e \beta_i)^2 + \beta_e^2}$ and $\alpha_2 = \frac{\beta_e}{(1 + \beta_e \beta_i)^2 + \beta_e^2}$

Substituting Eqs. (22) and (23) in (13) and (12) respectively, we acquired,

$$\frac{\partial u}{\partial t} + w \frac{\partial u}{\partial z} - 2\Omega v = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial z^2} + \frac{\alpha}{\rho} \frac{\partial^3 u}{\partial z^2 \partial t} + \frac{\sigma B_0^2 (\alpha_2 v - \alpha_1 u)}{\rho}$$

$$- \frac{\nu}{k} u$$

$$+ g\beta(T - T_\infty) + g\beta^*(C - C_\infty) \quad (24)$$

$$\frac{\partial v}{\partial t} + w \frac{\partial v}{\partial z} + 2\Omega u = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \frac{\partial^2 v}{\partial z^2} + \frac{\alpha}{\rho} \frac{\partial^3 v}{\partial z^2 \partial t}$$

$$- \frac{\sigma B_0^2 (\alpha_2 u + \alpha_1 v)}{\rho} - \frac{\nu}{k} v \quad (25)$$

Combining Eqs. (24) and (25), let

$$q = u + iv \text{ and } \xi = x - iy, \quad \text{we acquired that,}$$

$$\frac{\partial q}{\partial t} + w \frac{\partial q}{\partial z} + 2i\Omega q = -\frac{1}{\rho} \frac{\partial p}{\partial \xi} + \nu \frac{\partial^2 q}{\partial z^2} + \frac{\alpha}{\rho} \frac{\partial^3 q}{\partial z^2 \partial t} - \frac{\sigma B_0^2 (\alpha_1 + i\alpha_2)}{\rho} q - \frac{\nu}{k} q$$

$$+ g\beta(T - T_\infty) + g\beta^*(C - C_\infty) \quad (26)$$

Outer surface of the boundary layer, Eq. (26) gives,

$$-\frac{1}{\rho} \frac{\partial p}{\partial \xi} = \frac{dU_\infty}{dt} + \left(\frac{\sigma \beta_0^2}{\rho} + \frac{\nu}{k} \right) U_\infty \quad (27)$$

It is introducing the non-dimensional variables,

$$q^* = \frac{q}{w_0}, w^* = \frac{w}{w_0}, z^* = \frac{w_0 z}{\nu}, U_0^* = \frac{U_0}{w_0}, U_\infty^* = \frac{U_\infty}{w_0},$$

$$t^* = \frac{tw_0^2}{\nu}, \theta = \frac{T - T_\infty}{T_w - T_\infty}, \phi = \frac{C - C_\infty}{C_w - C_\infty},$$

$$M^2 = \frac{\sigma B_0^2 \nu}{\rho w_0^2}, K = \frac{kw_0^2}{\nu^2}, \text{Pr} = \frac{\nu \rho C_p}{k_1} = \frac{\nu}{\alpha}, R = \frac{\Omega \nu}{w_0^2},$$

$$\text{Gr} = \frac{\nu \beta g (T_w - T_\infty)}{w_0^3},$$

$$\text{Gm} = \frac{\nu \beta g^* (C_w - C_\infty)}{w_0^3}, H = \frac{\nu Q_0}{\rho C_p w_0^2}, S = \frac{w_0^2 \alpha_1}{\rho \nu^2},$$

$$\text{Sc} = \frac{\nu}{D}, \text{Kc} = \frac{K_c \nu}{w_0^2}.$$

Making use of the non-dimensional variables, the governing equations are diminished to

$$\frac{\partial q}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial q}{\partial z} = \frac{dU_\infty}{dt} + \frac{\partial^2 q}{\partial z^2} + S \frac{\partial^3 q}{\partial z^2 \partial t} - \lambda q$$

$$+ \text{Gr} \theta + \text{Gm} \phi \quad (28)$$

$$\frac{\partial \theta}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial \theta}{\partial z} = \frac{1}{\text{Pr}} \frac{\partial^2 \theta}{\partial z^2} - H \theta \quad (29)$$

$$\frac{\partial \phi}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial \phi}{\partial z} = \frac{1}{\text{Sc}} \frac{\partial^2 \phi}{\partial z^2} - \text{Kc} \phi \quad (30)$$

The boundary conditions be,

$$q = U_0, \theta = 1 + \varepsilon e^{i\omega t}, \phi = 1 + \varepsilon e^{i\omega t} \quad \text{at} \quad z = 0 \quad (31)$$

$$q = 0, \theta = 0, \phi = 0 \quad \text{as} \quad z \rightarrow \infty \quad (32)$$

By using of perturbation technique ($\varepsilon \ll 1$), the velocity, temperature and concentration are assumed be,

$$q = q_0(z) + \varepsilon e^{i\omega t} q_1(z) + O(\varepsilon^2) \quad (33)$$

$$\theta = \theta_0(z) + \varepsilon e^{i\omega t} \theta_1(z) + O(\varepsilon^2) \quad (34)$$

$$\phi = \phi_0(z) + \varepsilon e^{i\omega t} \phi_1(z) + O(\varepsilon^2) \quad (35)$$

Substituting Eqs. (33), (34) and (35) in Eqs. (28), (29) and (30) respectively, we obtained the equations of zeroth and first order be,

$$\frac{d^2 q_0}{dz^2} + \frac{dq_0}{dz} - \lambda q_0 = -\text{Gr} \theta_0 - \text{Gm} \phi_0 \quad (36)$$

$$\frac{d^2 \theta_0}{dz^2} + \text{Pr} \frac{d\theta_0}{dz} - H \text{Pr} \theta_0 = 0 \quad (37)$$

$$\frac{d^2\phi_0}{dz^2} + \text{Sc} \frac{d\phi_0}{dz} - \text{ScKc}\phi_0 = 0 \quad (38)$$

$$(1 + \text{Si}\omega) \frac{d^2q_1}{dz^2} + \frac{dq_1}{dz} - \lambda q_1 = -\text{Gr}\theta_1 - \text{Gm}\phi_1 - A \frac{dq_0}{dz} - i\omega \quad (39)$$

$$\frac{d^2\theta_1}{dz^2} + \text{Pr} \frac{d\theta_1}{dz} - (i\omega + H)\text{Pr}\theta_1 = -A\text{Pr} \frac{d\theta_0}{dz} \quad (40)$$

$$\frac{d^2\phi_1}{dz^2} + \text{Sc} \frac{d\phi_1}{dz} - (i\omega + \text{Kc})\text{Sc}\phi_1 = -A \text{Sc} \frac{d\phi_0}{dz} \quad (41)$$

Corresponding boundary conditions are,

$$q_0 = U_0, q_1 = 0, \theta_0 = 1, \theta_1 = 1, \phi_0 = 1, \phi_1 = 1 \quad \text{at } z = 0 \quad (42)$$

$$q_0 = 0, q_1 = 0, \theta_0 = 0, \theta_1 = 0, \phi_0 = 0, \phi_1 = 0 \quad \text{at } z \rightarrow \infty \quad (43)$$

Solving Eqs. (36) to (41) with respect to the boundary conditions (42) and (43), we obtained as,

$$q = a_3 e^{-m_5 z} - a_4 e^{-m_3 z} - a_5 e^{-m_1 z} + \varepsilon e^{i\omega t} \{ -(a_6 + a_7 + b_3 + b_4 + a_8 + a_9 + a_{10} + a_{11}) e^{-m_6 z} + a_8 e^{-m_5 z} + a_6 e^{-m_4 z} + (a_7 + a_9) e^{-m_3 z} + b_3 e^{-m_2 z} + (b_4 + a_{10}) e^{-m_1 z} + a_{11} \} \quad (44)$$

$$\theta = e^{-m_3 z} + \varepsilon (a_1 e^{-m_4 z} + a_2 e^{-m_3 z}) e^{i\omega t} \quad (45)$$

$$\phi = e^{-m_1 z} + \varepsilon (b_1 e^{-m_2 z} + b_2 e^{-m_1 z}) e^{i\omega t} \quad (46)$$

Eq. (44) disclosed that, the steady component for the velocity field include three layered nature, where the oscillatory component of the fluid domain demonstrates a multilayered nature. As of Eqs. (45) and (46), it is observed that in case of substantially deliberate movement of the fluid. *i.e.*, As soon as the viscous dissipation expression is deserted, the temperature and concentration profiles are mostly influenced by Prandtl number and source parameter; Schmidt number and chemical reaction parameter of the fluid accordingly. Taking into consideration,

$$q_0 = u_0 + iv_0 \text{ and } q_1 = u_1 + iv_1$$

At present, this is expedient to inscribe the primary and secondary velocity distributions in expressions of the oscillating components, splitting the real and imaginary parts as of the equation (44) and taking only the real parts as that include the substantial importance. The velocity distribution of the flow can be articulated as into oscillating parts,

$$q(z, t) = q_0(z) + \varepsilon q_1(z) e^{i\omega t}$$

$$u + iv = u_0 + iv_0 + \varepsilon u_1 \cos\omega t + i \varepsilon u_1 \sin\omega t + i \varepsilon v_1 \cos\omega t - \varepsilon v_1 \sin\omega t \quad (47)$$

Equating real in addition to imaginary parts,

$$u(z, t) = w_0(u_0(z) + \varepsilon(u_1 \cos\omega t - v_1 \sin\omega t)) \quad (48)$$

$$v(z, t) = w_0(v_0(z) + \varepsilon(u_1 \sin\omega t + v_1 \cos\omega t)) \quad (49)$$

Then the part for the unsteady velocity profiles for

$\omega t = \pi/2$ are specified by

$$u\left(z, \frac{\pi}{2\omega}\right) = w_0(u_0(z) - \varepsilon v_1(z)) \quad (50)$$

$$v\left(z, \frac{\pi}{2\omega}\right) = w_0(v_0(z) + \varepsilon u_1(z)) \quad (51)$$

For engineering curiosity, the non-dimensional skin friction, Nusselt and Sherwood number at the surface of the plate $z = 0$ are specified by,

$$\tau = \left(\frac{dq}{dz}\right)_{z=0}, \quad Nu = -\left(\frac{d\theta}{dz}\right)_{z=0} \quad \text{and} \quad Sh = -\left(\frac{d\phi}{dz}\right)_{z=0} \quad (52)$$

3. Results and discussion

The present problem is explored the Hall and ion slip effects for an unsteady MHD laminar flow of heat generating/absorbing second grade fluid over an infinite vertical moving porous surface. The flow movement presides over by the non-dimensional parameters for the velocity, temperature and concentration profiles are shown in Figs. 2–5, Fig. 6 and Fig. 7 respectively. Tables 1–3 are represented the skin friction, Nusselt and Sherwood number for variations in the governing parameters. For computational intention, we are setting up the values $A = 5$, $\varepsilon = 0.001$, $U_0 = 0.2$, while the parameters being $M = 2$, $K = 0.5$, $R = 1$, $S = 0.5$, $\text{Gr} = 5$, $\text{Gm} = 3$, $\beta_e = 1$, $\beta_i = 0.2$, $\text{Pr} = 0.71$, $H = 1$, $\text{Sc} = 0.22$, $\text{Kc} = 1$, $\omega = \pi/6$, $t = 0.2$ fixed over the range. The fluid velocity is rising and afterwards extremely small distance from the surface achieve to its highest accure and subsequently seek a progressively decline within the fluid velocity to vanish.

The significance of Hartmann number M for velocity components u and v is interpreted from Fig. 2(a). The primary velocity component u lessens and the secondary velocity v augments through a mounting in the Hartmann number M . As likely, the enlargement in M diminishes the resulting velocity. It is since of the classical result, the Lorentz force this emerges owing towards the submission by magnetic domain just before an electrical conducting fluid and provide climb to a resistive type of force. Appropriate to this strength, the movement of the fluid flow in momentum boundary layer thickness reaches to slow down. It is looked from the Fig. 2(b) that, the subsequent velocity components u heightens v condenses through ever growing permeability parameter K . Obviously, the larger values of K , enhances the resulting velocity and consequently enlargement in the thickness of the momentum boundary layer. Lower the permeability causes slighter the fluid speed is observed inside the flow region occupied by the fluid. The Fig. 2(c–d) depicted that the demeanor of the velocity dispensations of u and v through rotation and second grade fluid parameters. It is noticed that, as reinforcement in second grade fluid and rotation parameters, the primary velocity component u reduces throughout the fluid region. While it is noted that, the secondary velocity component v is lessens with a growing in rotation parameter and an improvement with an escalating in second grade fluid parameter in the fluid region. Which results, diminishes the resulting speed and the momentum boundary layer thickness all over the fluid region when an increase in rotation and second grade fluid parameters.

Fig. 3(a–b) disclosed the effects of thermal and solutal Grashof numbers on the fluid velocity components for u and v . The

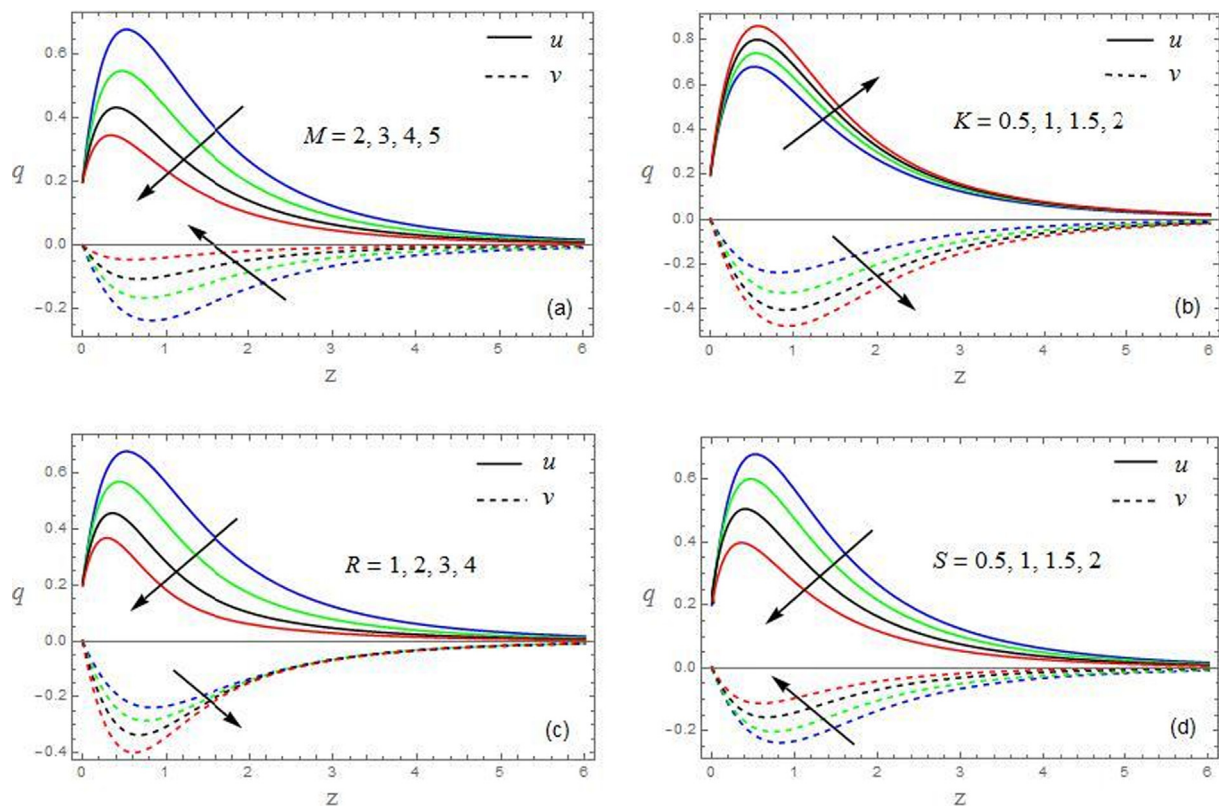


Fig. 2 (a-d) the velocity profiles for u and v against M , K , R and S

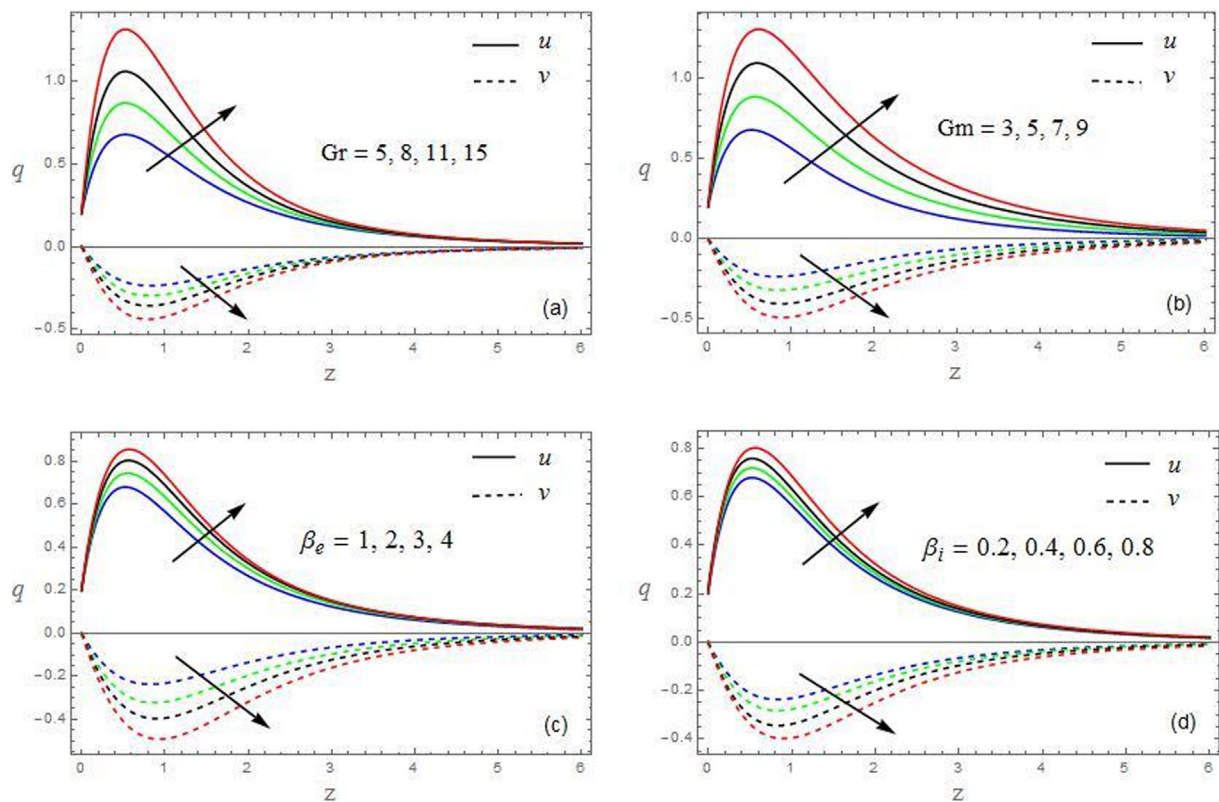


Fig. 3 (a-d) the velocity profiles for u and v against Gr , Gm , β_e and β_i

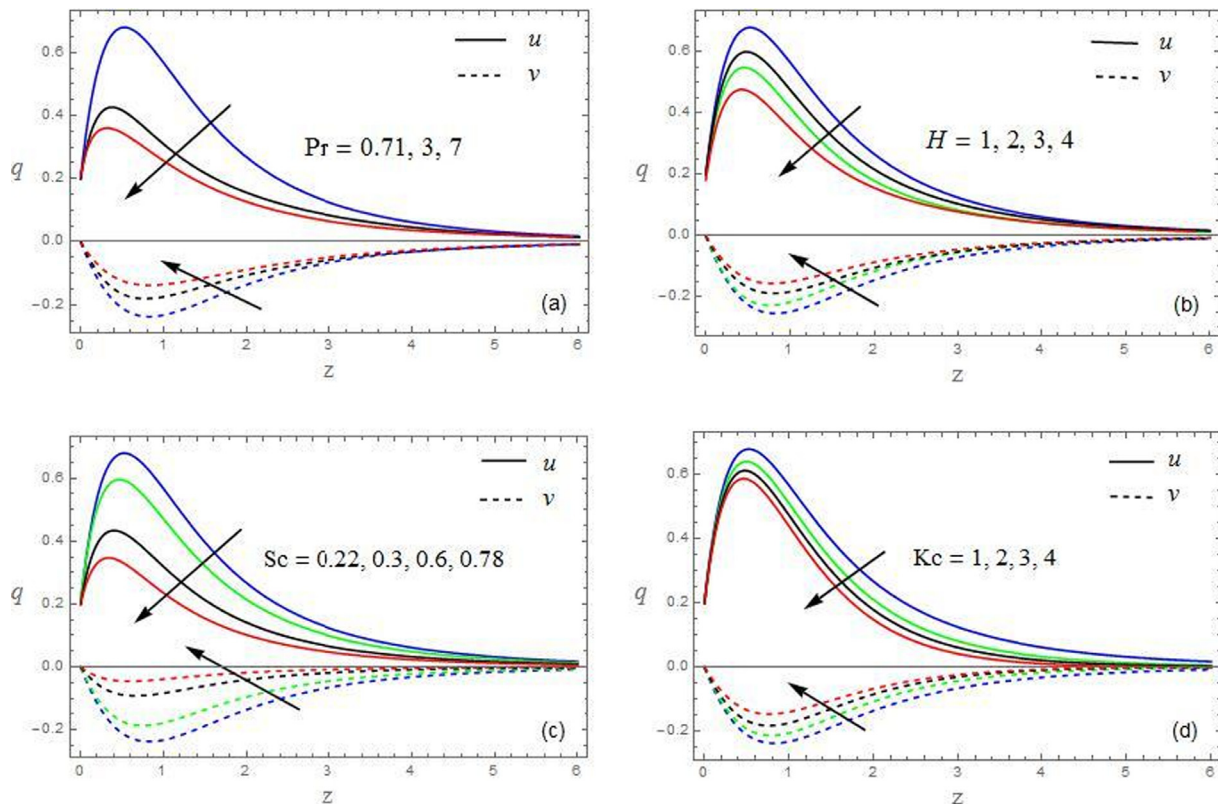


Fig. 4 (a-d) the velocity profiles for u and v against Pr , H , Sc and Kc

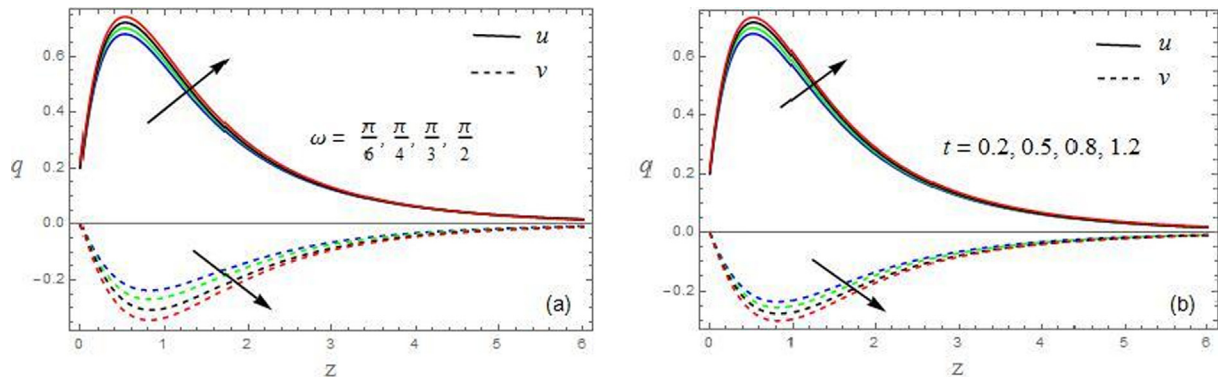


Fig. 5 (a-b) the velocity profiles for u and v against ω and t

primary velocity u improves and secondary velocity v lessens with growing in thermal Grashof number Gr and solutal Grashof number Gm throughout the fluid region. The thermal Grashof number Gr implies the ratio of the thermal buoyancy force to the viscous hydrodynamic force through the boundary layer, at the same time as the mass Grashof number Gm establishes the proportion for the concentration buoyancy force to the viscous hydrodynamic force. The fluid velocity boosts by virtue of the strengthening of heat and solutal buoyancy forces. The velocity distribution enlarges swiftly next to the porous surface and later these declined easily for the viscous stream quantity. The momentum velocity transverse the boundary layer enlarges through a heightening in thermal Grashof number Gr or mass Grashof number Gm . Hence bound-

ary layer thickness augments with an increase in Gr or Gm . Fig. 3(c and d) depicted that the behavior of the velocity dispersions with Hall and ion-slip parameters. The primary velocity u get better and secondary velocity v minimizes with growing in Hall and ion slip parameters β_e and β_i throughout the fluid medium. It is marked that, as a strengthening in Hall and ion-slip parameters, which results, enhances the resulting velocity and the momentum boundary layer thickness throughout the fluid region. The incorporation of Hall parameter lessens the effectual conductivity and therefore descends the magnetic renitent fierceness. Also, the efficient conductivity augments as enlarge in ion-slip parameter, for this reason the attenuation force are lessening consequently velocity heightens.

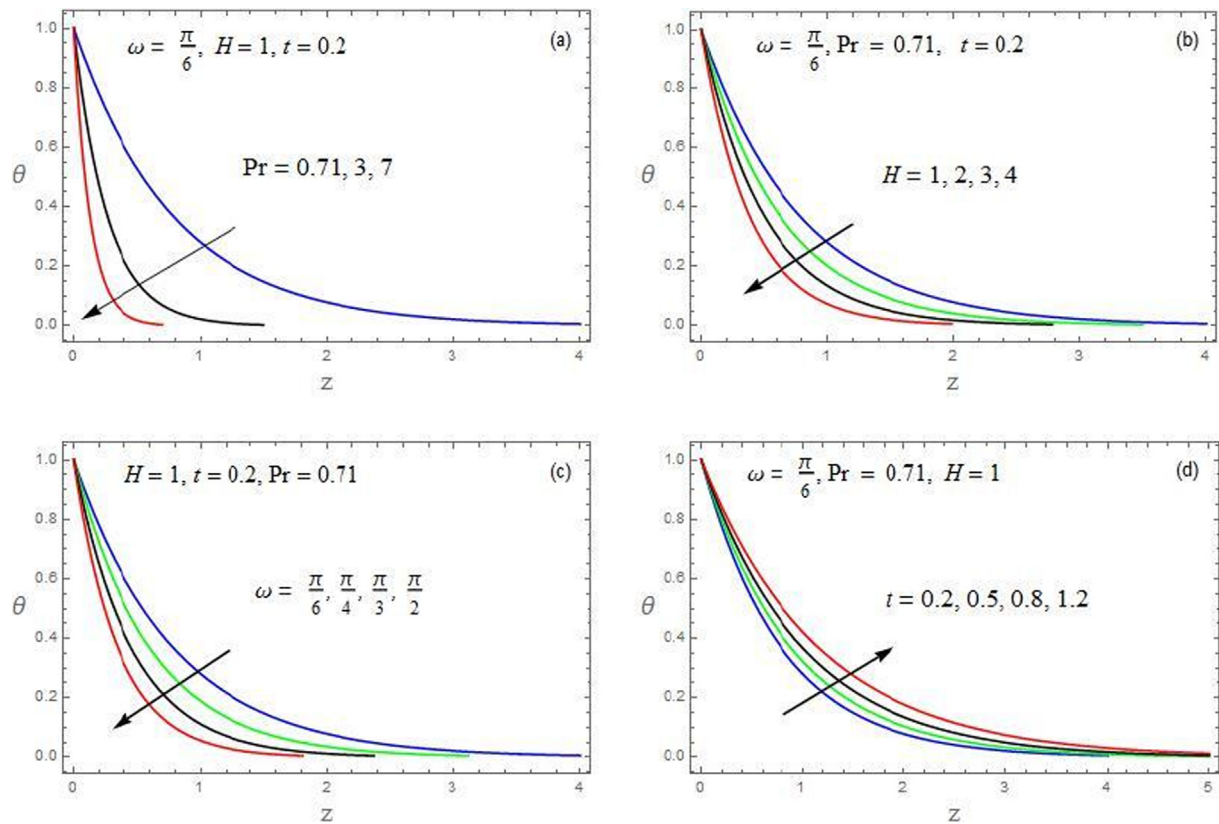


Fig. 6 (a-d) the temperature profiles against Pr , H , ω and t

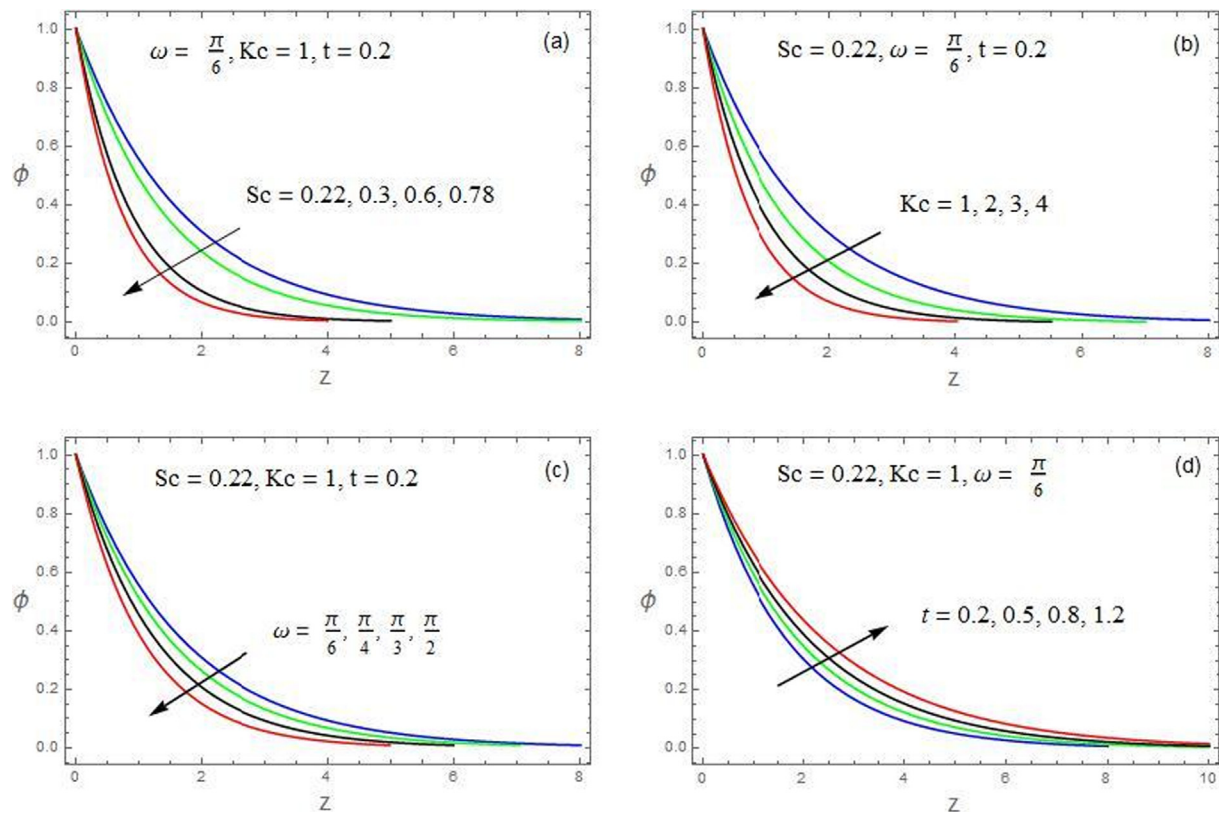


Fig. 7 (a-d) the concentration profiles against Sc , Kc , ω and t

Table 1 Skin friction.

M	K	R	S	Gr	Gm	β_e	β_i	Pr	H	Sc	Kc	ω	t	τ
2	0.5	1	0.5	5	3	1	0.2	0.71	1	0.22	1	$\pi/6$	0.2	2.65346
3														2.11169
4														1.61113
	1.0													3.01485
	1.5													3.15427
		2												2.41967
		3												2.20354
			1.0											2.65483
			1.5											2.65607
				8										3.70049
				12										5.09887
					5									3.56789
					7									4.48258
						2								3.02643
						3								3.19029
							0.4							2.67833
							0.6							2.70996
								3.0						1.78643
								7.0						1.37621
									2					2.45166
									3					2.32424
										0.30				2.57918
										0.60				2.38207
											2			2.54430
											3			2.47063
												$\pi/4$		2.70461
												$\pi/3$		2.75472
													0.6	2.65283
													1.0	2.65255

Table 2 Nusselt number.

Pr	H	ω	t	Nu
0.71	1	$\pi/6$	0.1	1.27303
3				3.80735
7				7.92661
	2			1.60224
	3			1.86103
		$\pi/4$		1.27297
		$\pi/3$		1.27290
			0.2	1.27297
			0.3	1.27288

The amendments of velocity profiles for u and v by the Prandtl number are designed in Fig. 4(a). This is perceived that the enlargement of Pr made the fluid flow slowing down for primary velocity, where as it enhances for secondary velocity. Therefore, the resultant velocity is reduced with an increasing in Pr throughout the region occupied by the fluid. Actually, it is warranted due to the quantity of information so as to the fluid through the highest Prandtl number has elevated viscosity this made as the fluid has substantial thickness. Fig. 4(b) portrayed the effect of heat source parameter on velocity components u and v across the boundary layer. The primary component u lessens and v augments with an escalating in heat source parameter. Raising the values of heat source parameter

trend to decreased the primary velocity component and therefore it dominates to decrease the rate of the momentum boundary layer thickness. It is noticed from the Fig. 4(c) with the purpose of the velocity through the different values of Schmidt number Sc. On enlarges in values of Schmidt number Sc be inclined to decreasing of the velocity and hence reduced the momentum boundary layer thickness. Similar performance is scrutinized with ever-increasing chemical reaction parameter Kc (Fig. 4(d)). Heavier diffusing species, thus, through the higher value of Schmidt number and increasing rate of chemical reaction sources a lessening in the resultant velocity. Furthermore, severest species through disparaging reaction reasons deceleration within the velocity distribution.

On the other hand, Fig. 5 (a and b) demonstrated the effects of the frequency of oscillations and time t in the primary and secondary velocity components. This can be observed from the figures that, the primary velocity profiles are increasing with an increasing in the frequency of oscillations and time, whereas the secondary velocity profiles are reduced with an enlargement in the frequency of oscillations and time throughout the fluid region. This is fascinating to note that, the resultant velocity and momentum boundary layer thickness are diminished with an escalating in frequency of oscillations and also which are augmented through an increasing of time frequently.

Fig. 6(a-d) exhibited the disparities of temperature profiles with different values of Prandtl number Pr, heat source parameter H , frequency of oscillations ω and variation of time t . The

Table 3 Sherwood number.

Sc	Kc	ω	t	Sh
0.22	1	$\pi/6$	0.1	0.593019
0.3				0.719535
0.6				1.133790
	2			0.783794
	3			0.931358
		$\pi/4$		0.593001
		$\pi/3$		0.592977
			0.2	0.592994
			0.3	0.592956

temperature distribution lessens with escalating Prandtl number Pr intact the fluid region. The elucidation of that performance conceals through the information to Prandtl number. It was described as the proportion of momentum diffusivity to thermal diffusivity. Hence, at less important quantities of Prandtl fluids acquired large thermal conductivity this elicits the thermal diffusion gone from the heating surface additionally accelerate and quicker contrasted to bigger quantities of Prandtl number. Consequently the thickness of the thermal boundary layer is slow down by an increasing in Prandtl number. The similar tendency is notified for temperature distribution through an increasing in heat source parameter H . As intended that, an enlargement in heat source parameter reduce the temperature distribution. In sight of the information that, heat resource parameter indicates the qualified role of heat conducting transport to heat absorption transport. Hence the

thickness of the thermal boundary layer is decreased. The temperature and thermal boundary layer thickness lessens with an increasing in frequency of oscillation. Eventually, the consequence of non-dimensional time on temperature depictions is delineated within the boundary layer. This has been obvious that temperature is a growing function of time.

The concentration depictions with esteem to the parameters the Schmidt number Sc , chemical reaction parameter Kc , the frequency of oscillation ω and time t are depicted in Fig. 7(a-d). We perceived that, from Fig. 7(a) the concentration diminishes through an increase in Schmidt number Sc throughout the fluid region. It is picked the Schmidt number esteems as $Sc = 0.22, 0.3, 0.6, 0.78$ this corresponding to hydrogen H_2 , Helium He , water H_2O - vapour, and ammonia NH_3 respectively. Schmidt number Sc is a dimensionless number defined as the ratio of momentum diffusivity (kinematic viscosity) and mass diffusivity, and is used to characterize fluid flows in which there are simultaneous momentum and mass diffusion convection processes and concentration boundary layers. This was obtained that an increase in the value of Schmidt number induced the absorption of particles and therefore the concentration boundary layer thickness to diminish extensively. Analogous behaviour is identified with an increasing in chemical reaction parameter Kc (Fig. 7(b)). An enlargement in the chemical reaction parameter Kc declines the concentration distribution quickly. Because, the quantity of solute particles is experiencing by chemical reaction as chemical reaction parameter enlarges, this leads to diminish in concentration distribution. Therefore, the chemical reaction lessens the

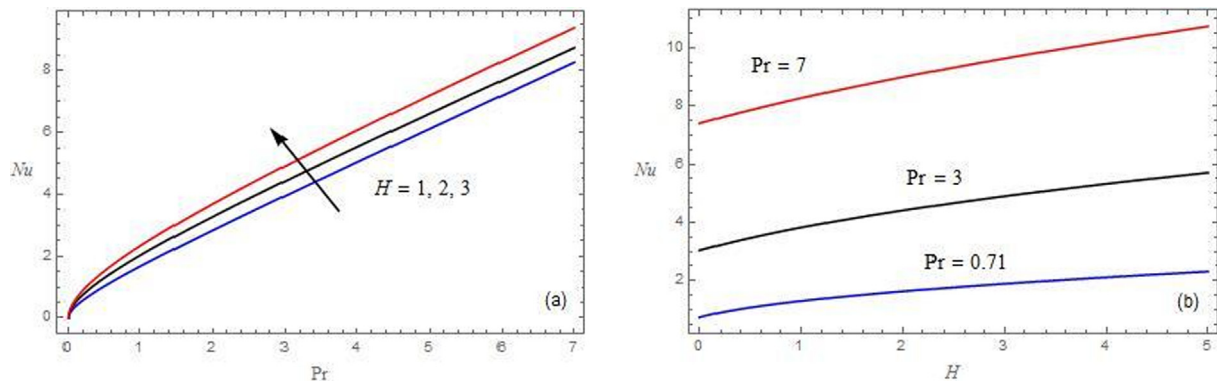


Fig. 8 (a-b) Nusselt number against H and Pr

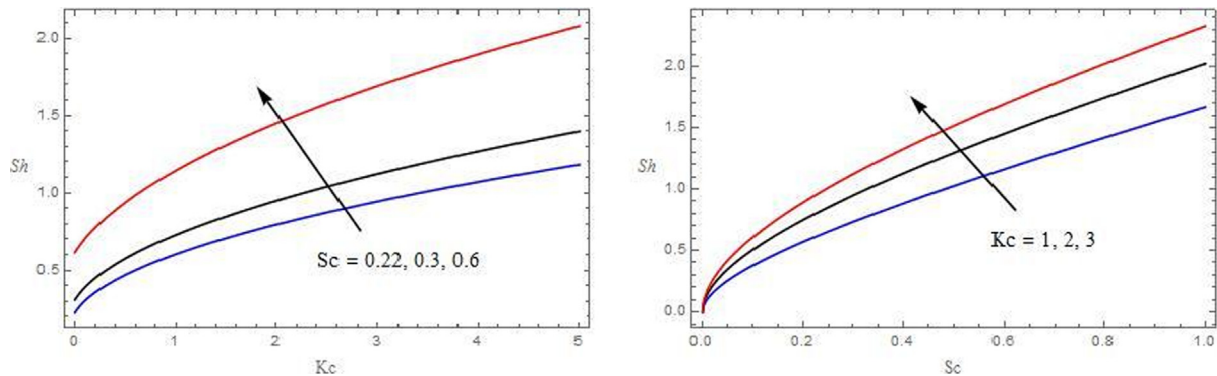


Fig. 9 (a-b) Sherwood number against Sc and Kc

Table 4 Comparison of results for primary velocity $S = R = \beta_e = \beta_i = 0$, $Pr = 0.71$, $H = 1$, $Sc = 0.22$, $Kc = 1$, $\omega = \pi/6$, $t = 0.2$.

M	K	Gr	Gm	Previous results rani and murthy [44]	Present results
2	0.5	5	3	0.587748	0.587746
3				0.478995	0.478992
4				0.366897	0.366894
	1.0			0.688747	0.688740
	1.5			0.802214	0.802211
		8		0.758849	0.758844
		12		0.922547	0.922541
			5	0.682214	0.682209
			7	0.854474	0.854469

concentration boundary layer thickness substantially. The concentration distribution diminishes with an increase in frequency of oscillations as well as accentuates with heightening in time criteria during the fluid region (Fig. 7(c-d)).

Table 1 is characterized the magnitudes of skin friction. An enlargement of Hartmann number goes ahead to diminishes in skin friction. Because, elastic feature in visco-elastic second grade fluid reduced the frictional drag. Analogous behavior is scrutinized through an increase in rotation parameter, Prandtl number, heat source parameter, Schmidt number, chemical reaction parameter and time. Additionally, an enlargement in permeability parameter or second grade fluid parameter goes ahead to bring to bear superior skin friction in enormity on the boundary of the surface, while the analogous demeanor is scrutinized for the same when an augment in thermal Grashof number, mass Grashof number, frequency of oscillation, Hall and ion slip parameters at the boundary of the surface. Since from the Table 2, an increasing in the Prandtl number or heat source parameter goes ahead to an enhancement in Nusselt number. Also it is reduced with an increasing in frequency of oscillations or time. The same variation is also observed in the Fig. 8(a-b). From the Table 3, a growing in Schmidt number or chemical reaction parameter goes ahead to a strengthening in Sherwood number. This is also lessens with a mounting in frequency of fluctuation or time. The similar variation is also observed in the Fig. 9(a-b). The present results are excellent conformity with the previous results of Rani and Murthy et al.[44] (Table 4).

4. Conclusions

We have explored theoretically the Hall and ion slip impacts on an unsteady laminar MHD convective rotating flow of heat generating or absorbing second grade fluid over a semi-infinite vertical moving permeable surface. The non-dimensional governing equations are solved to the most excellent possible investigative solution. The effects of parameters are demonstrated graphically and described in detail. The determinations are completed as the succeeding. The resultant velocity is reduced with an increase in the strength of magnetic field, rotation or second grade fluid parameters, while it is boost up through an increasing in the permeability of porous medium. If the pore size of the porous medium diminishes, then the velocity is patterned to be declining. The resultant velocity is increased through an enlargement in Hall and ion slip parameters throughout fluid region. The thermal and solutal buoyancy forces contribute to the resultant velocity ever-

increasing to high. Temperature distribution is trim downs through an increasing in heat source parameter and frequency of oscillation. The concentration is reduced with an increase in the chemical reaction parameter and frequency of oscillations in the entire fluid region. Rotation effect is to diminish the skin friction, whereas it is augmented through an increase of the Hall and ion slip effects. The viscous-ness of a fluid controls over conduction after that the rate of heat transport augments comprehensively. The Schmidt number and chemical reaction parameter enhance the rate of mass transfer extensively.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A.

$$\lambda = M^2(\alpha_1 + i\alpha_2) + 2iR + (1/K),$$

$$m_1 = \frac{Sc + \sqrt{Sc^2 + 4ScKc}}{2}$$

$$m_2 = \frac{Sc + \sqrt{Sc^2 + 4Sc(i\omega + Kc)}}{2}$$

$$m_3 = \frac{Pr + \sqrt{Pr^2 + 4PrH}}{2}$$

$$m_4 = \frac{Pr + \sqrt{Pr^2 + 4Pr(i\omega + H)}}{2},$$

$$m_5 = \frac{1 + \sqrt{1 + 4\lambda}}{2}$$

$$m_6 = \frac{1 + \sqrt{1 + 4\lambda(1 + Si\omega)}}{2},$$

$$b_1 = 1 - \frac{A Sc m_1}{m_1^2 - Sc m_1 - (i\omega + Kc) Sc}$$

$$b_2 = \frac{A Sc m_1}{m_1^2 - Sc m_1 - (i\omega + Kc) Sc},$$

$$a_1 = 1 - \frac{A Pr m_3}{m_3^2 - Pr m_3 - (i\omega + H) Pr}$$

$$a_2 = \frac{A Pr m_3}{m_3^2 - Pr m_3 - (i\omega + H) Pr},$$

$$\begin{aligned}
 a_3 &= U_0 + \frac{Gr}{m_3^2 - m_3 - \lambda} + \frac{Gm}{m_1^2 - m_1 - \lambda}, \\
 a_4 &= \frac{Gr}{m_3^2 - m_3 - \lambda}, \\
 a_5 &= \frac{Gm}{m_1^2 - m_1 - \lambda}, \\
 a_6 &= \frac{-Gr a_1}{(1 + Si\omega)m_4^2 - m_4 - \lambda}, \\
 a_7 &= \frac{-Gr a_2}{(1 + Si\omega)m_3^2 - m_3 - \lambda}, \\
 b_3 &= \frac{-Gm b_1}{(1 + Si\omega)m_2^2 - m_2 - \lambda}, \\
 b_4 &= \frac{-Gm b_2}{(1 + Si\omega)m_1^2 - m_1 - \lambda}, \\
 a_8 &= \frac{-A m_5 a_3}{(1 + Si\omega)m_5^2 - m_5 - \lambda}, \\
 a_9 &= \frac{-A m_3 a_4}{(1 + Si\omega)m_3^2 - m_3 - \lambda}, \\
 a_{10} &= \frac{-A m_1 a_5}{(1 + Si\omega)m_1^2 - m_1 - \lambda}, \\
 a_{11} &= \frac{i\omega}{\lambda}.
 \end{aligned}$$

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