Generative modeling 2

CSE 849 Deep Learning
Spring 2025

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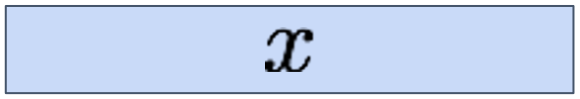
Slides are revised based on 598 University of Michigan by Justin Johnson

Variational Autoencoders

Train by maximizing the
variational lower bound

$$E_{z\sim q_\phi(z|x)}[\log p_\theta(x|z)] - D_{KL}\left(q_\phi(z|x), p(z)\right)$$

Input Data



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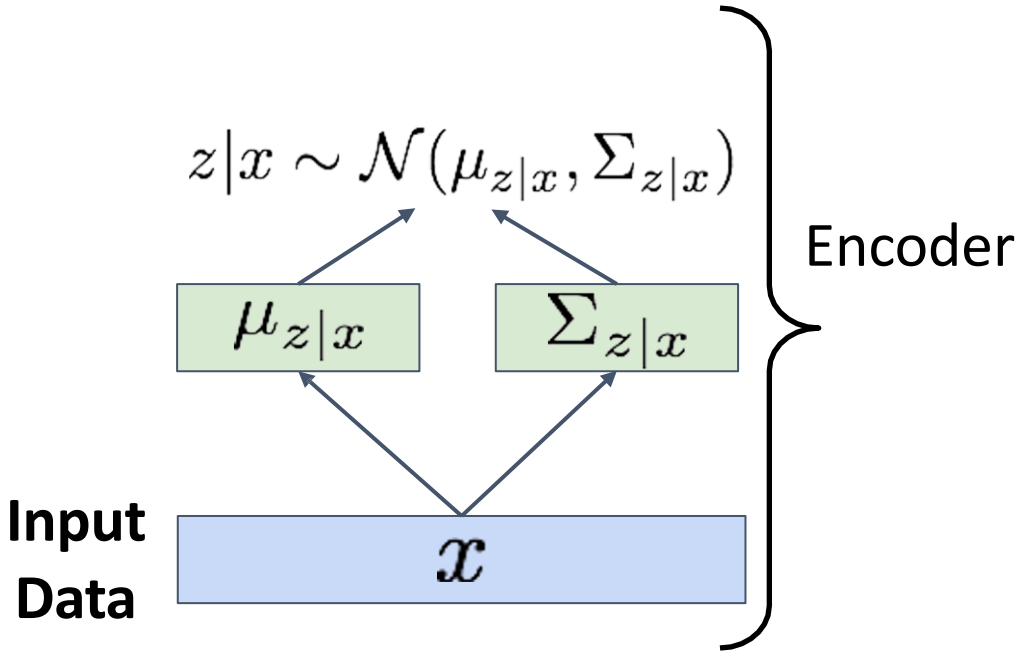
1. Run input data through **encoder** to get a distribution over latent codes



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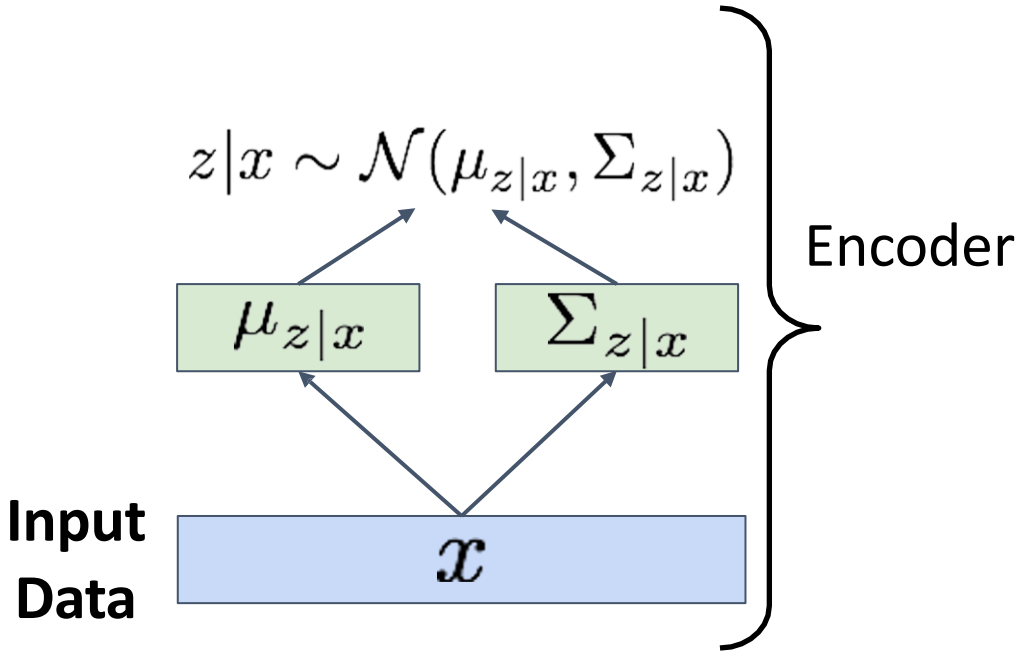
1. Run input data through **encoder** to get a distribution over latent codes
2. **Encoder output should match the prior p(z)**



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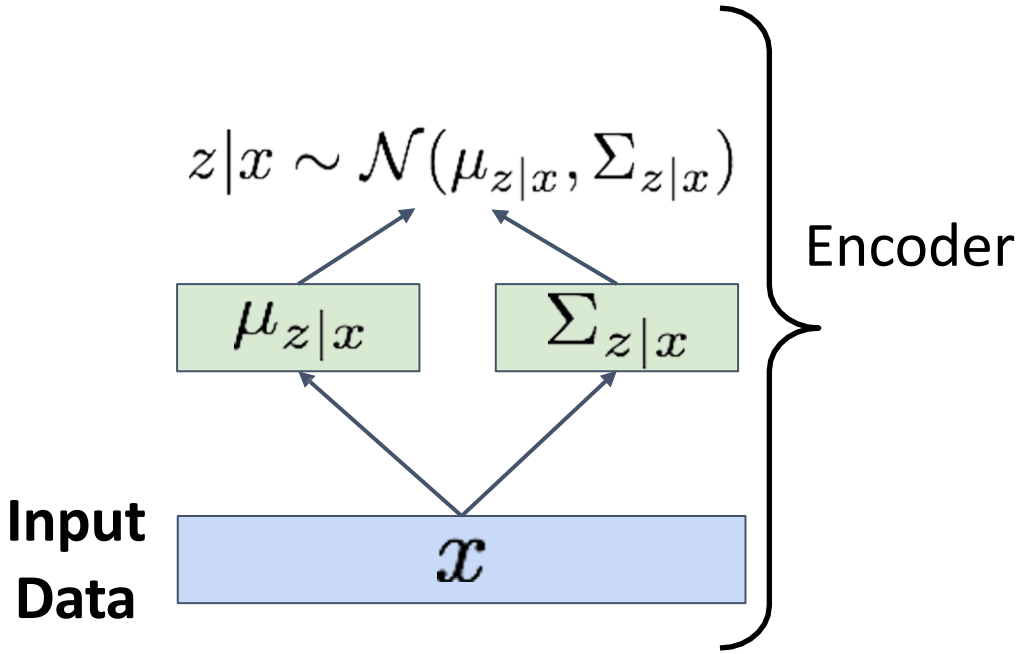
$$-D_{KL}\left(q_\phi(z|x), p(z)\right) = \int_Z q_\phi(z|x)\log\frac{p(z)}{q_\phi(z|x)}dz$$

$$= \int_Z N\left(z;\mu_{z|x},\Sigma_{z|x}\right)\log\frac{N(z;0,I)}{N\left(z;\mu_{z|x},\Sigma_{z|x}\right)}dz$$

$$= \frac{1}{2}\sum_{j=1}^{J}\left(1+\log\left(\left(\Sigma_{z|x}\right)_j^2\right)-\left(\mu_{z|x}\right)_j^2-\left(\Sigma_{z|x}\right)_j^2\right)$$

Closed form solution when q_ϕ is diagonal Gaussian and p is unit Gaussian (Assume z has dimension J)



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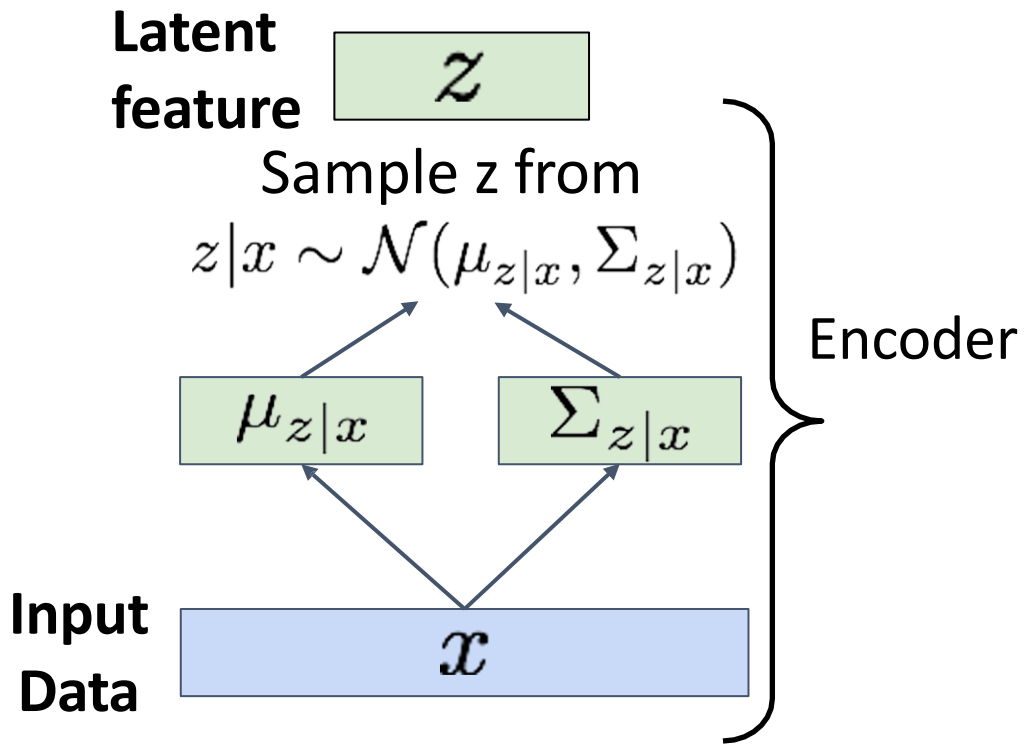
2. **Encoder output should match the prior p(z)**
3. Sample latent feature z from encoder output



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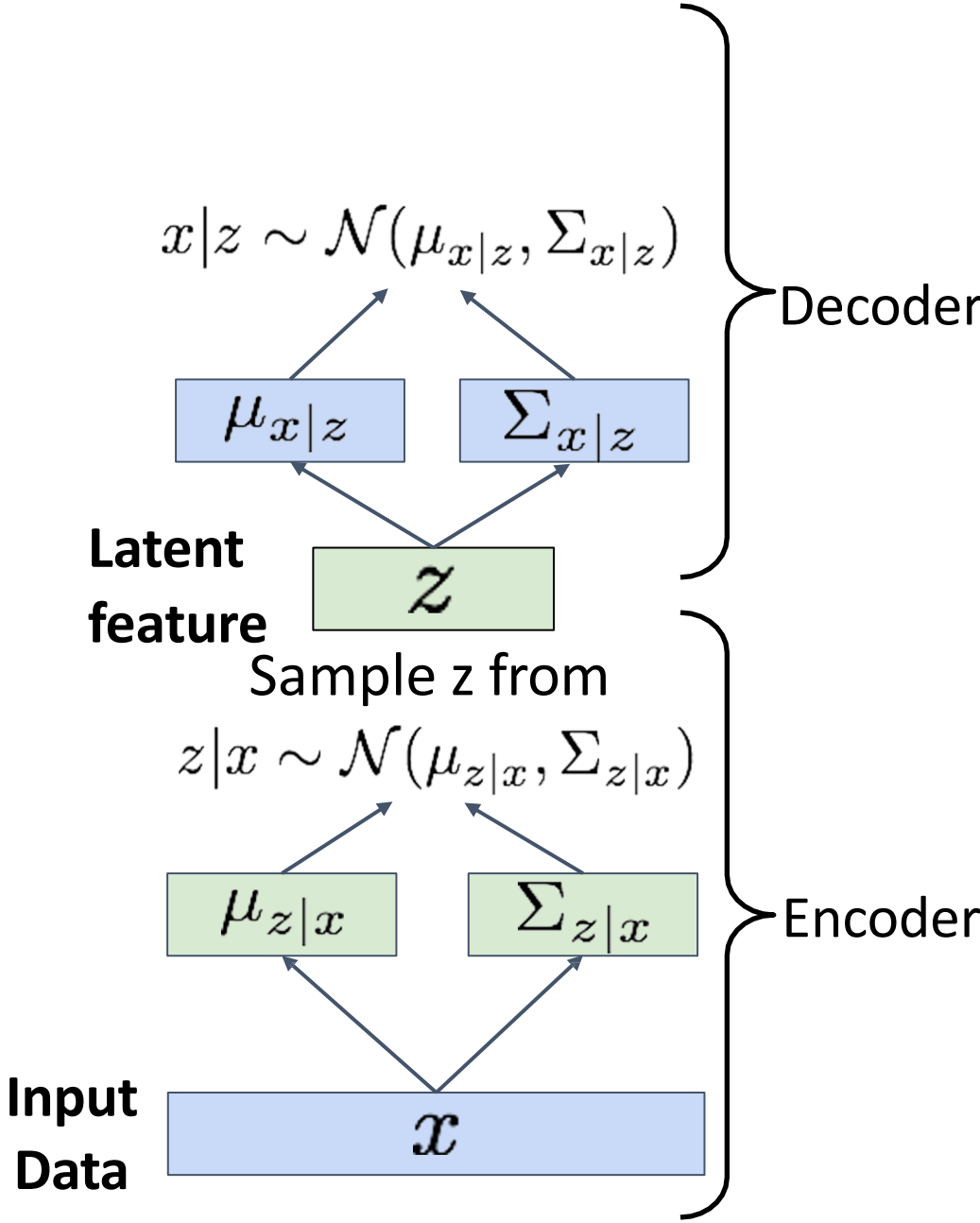
1. Run input data through **encoder** to get a distribution over latent codes
2. **Encoder output should match the prior p(z)**
3. Sample latent feature z from encoder output
4. Run sampled z through **decoder** to get a distribution over data samples



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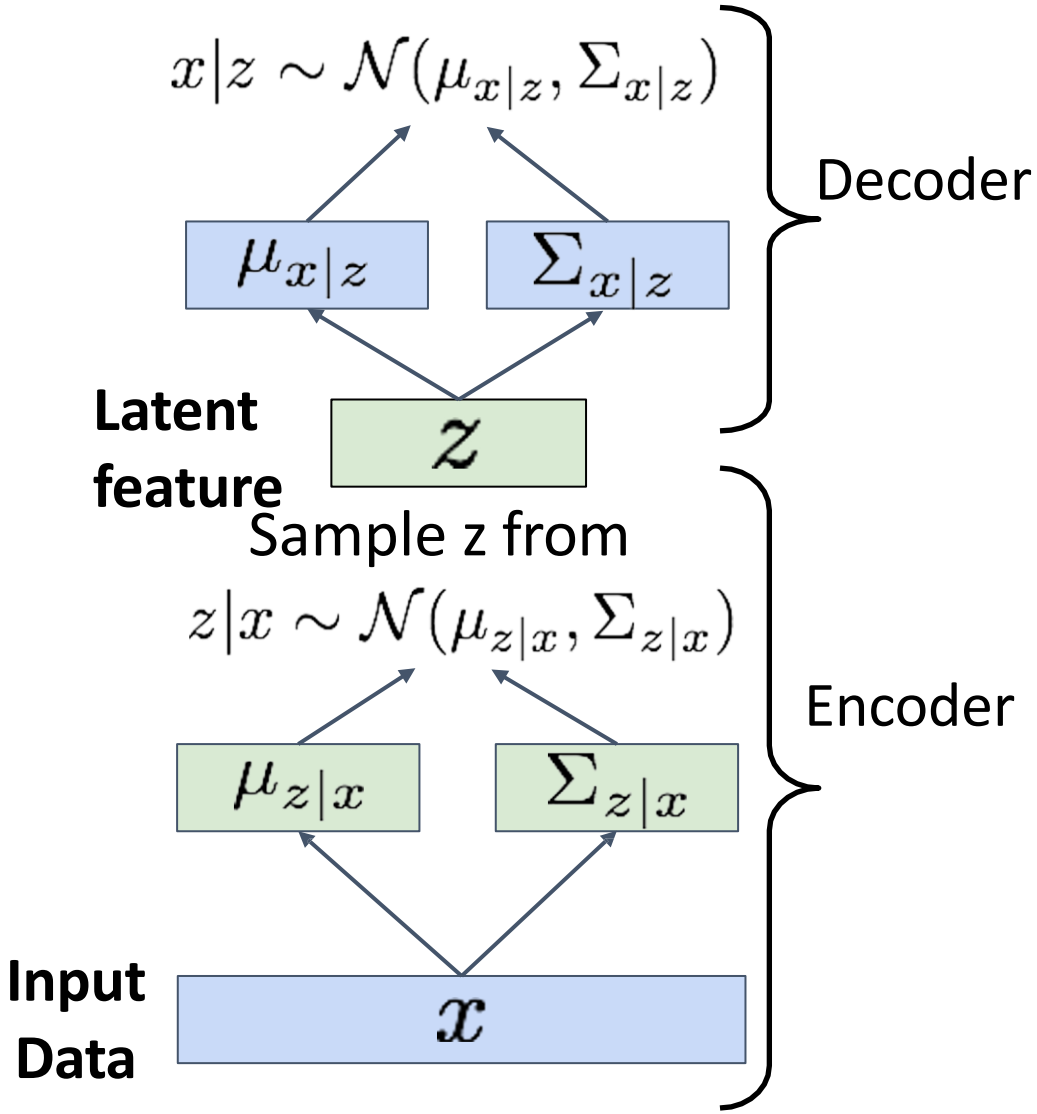
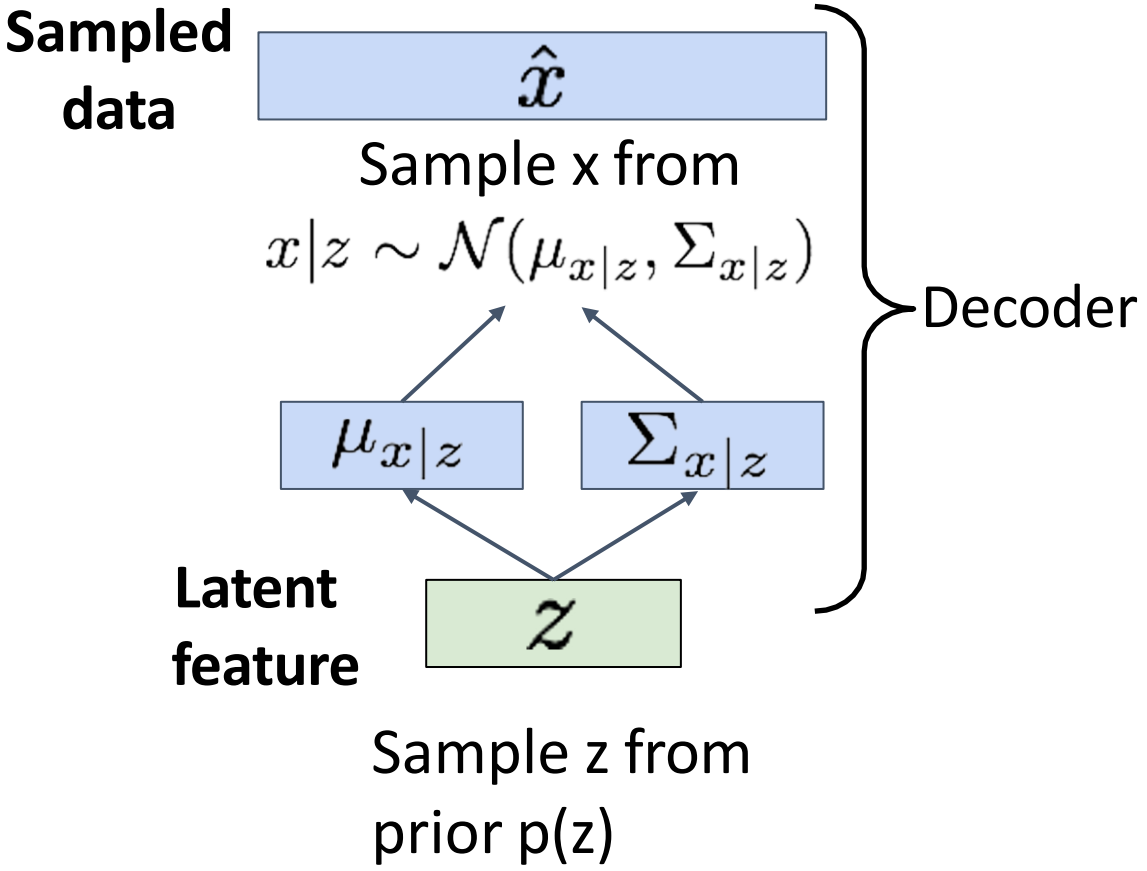
1. Run input data through **encoder** to get a distribution over latent codes
2. **Encoder output should match the prior p(z)**
3. Sample feature z from encoder output
4. Run sampled z through **decoder** to get a distribution over data samples
5. **Original input data should be likely under the distribution output from (4)**



Variational Autoencoders: Generating Data

After training we can generate new data!

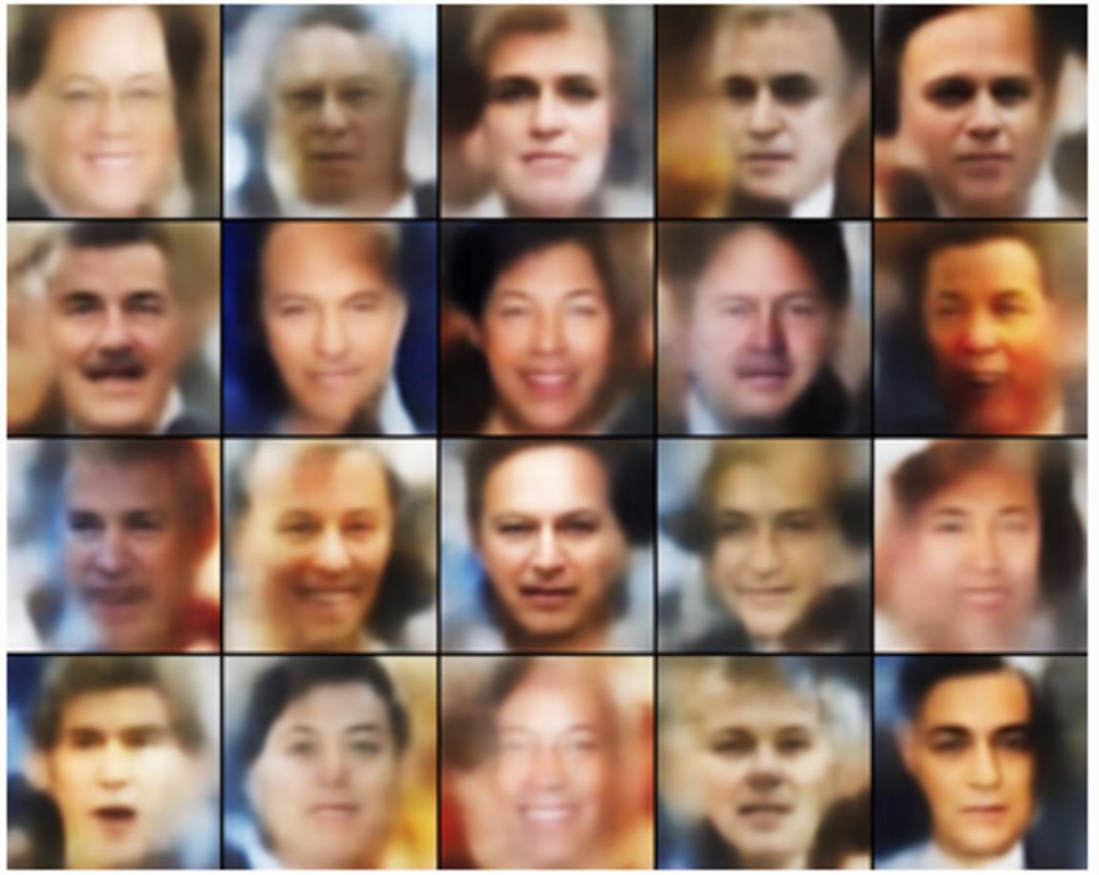
1. Sample z from prior p(z)
2. Run sampled z through decoder to get distribution over data x
3. Sample from distribution in (2) to generate data



Variational Autoencoders: Generating Data

32x32 CIFAR-10

Labeled Faces in the Wild

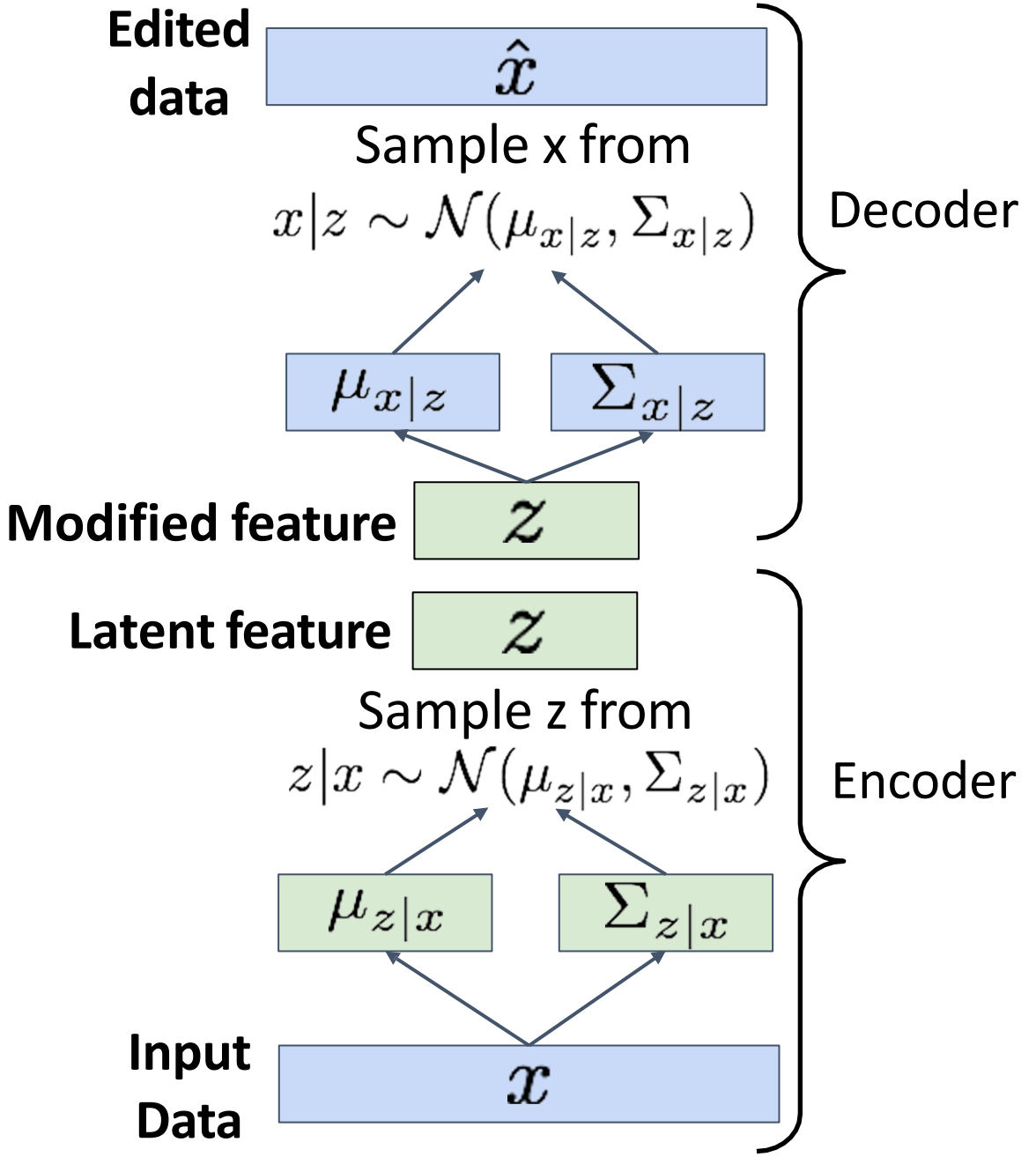


Figures from (L) Dirk Kingma et al. 2016; (R) Anders Larsen et al. 2017.

Variational Autoencoders

After training we can **edit images**

1. Run input data through **encoder** to get a distribution over latent codes
2. Sample feature z from encoder output
3. Modify some dimensions of sampled feature
4. Run modified z through **decoder** to get a distribution over data samples
5. Sample new data from (4)



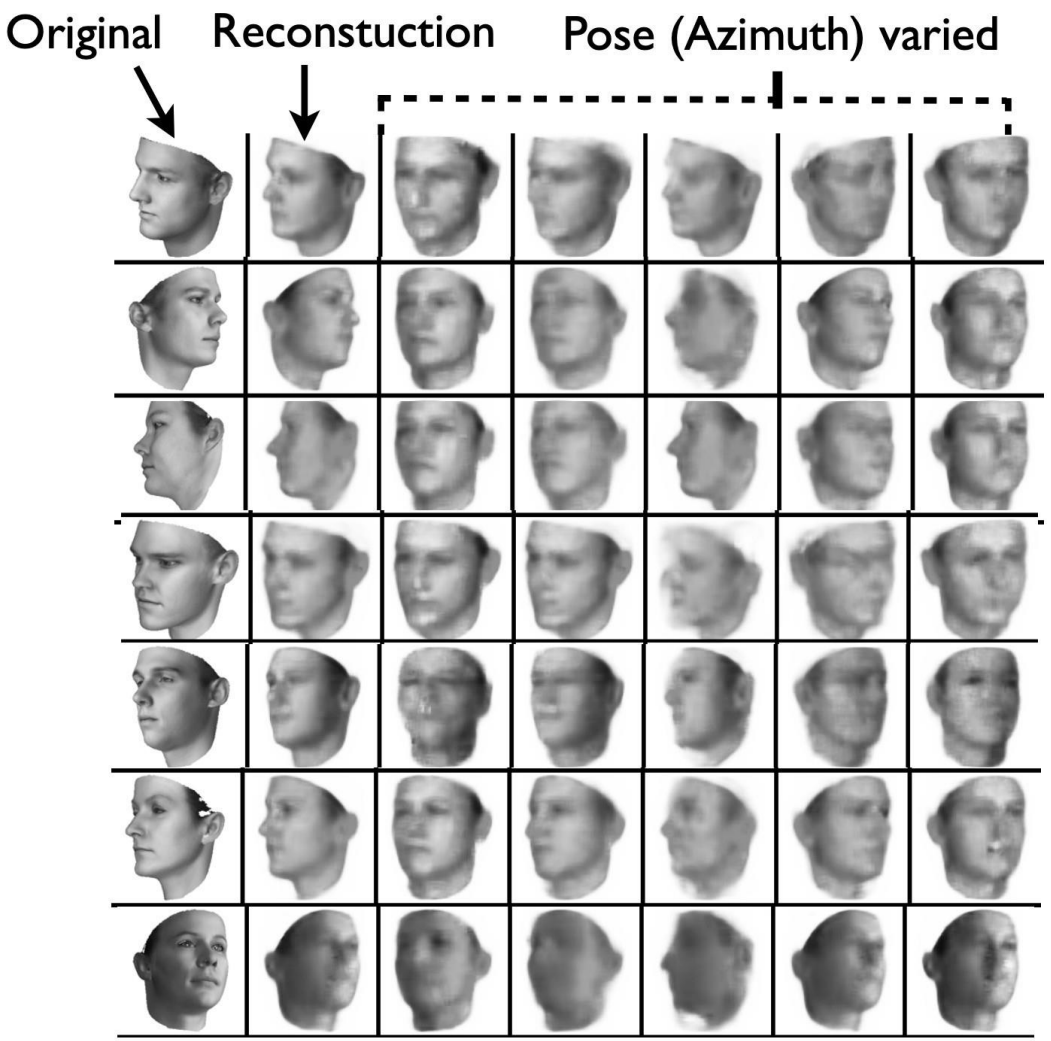
Variational Autoencoders

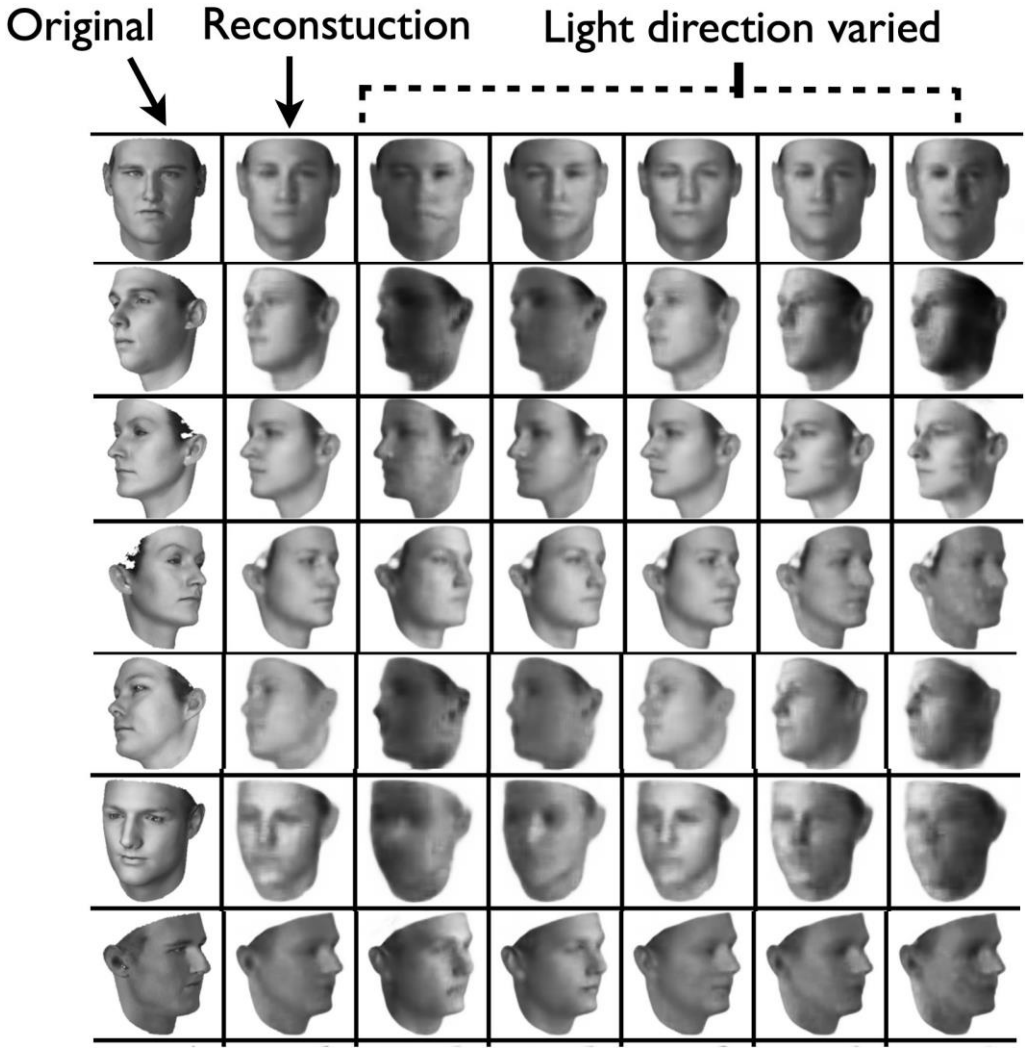
The diagonal prior on p(z) causes dimensions of z to be independent

“Disentangling factors of variation”

Kingma and Welling, Auto-Encoding Variational Beyes, ICLR 2014

Variational Autoencoders: Image Editing





Kulkarni et al, “Deep Convolutional Inverse Graphics Networks”, NeurIPS 2014

Generative Models So Far:

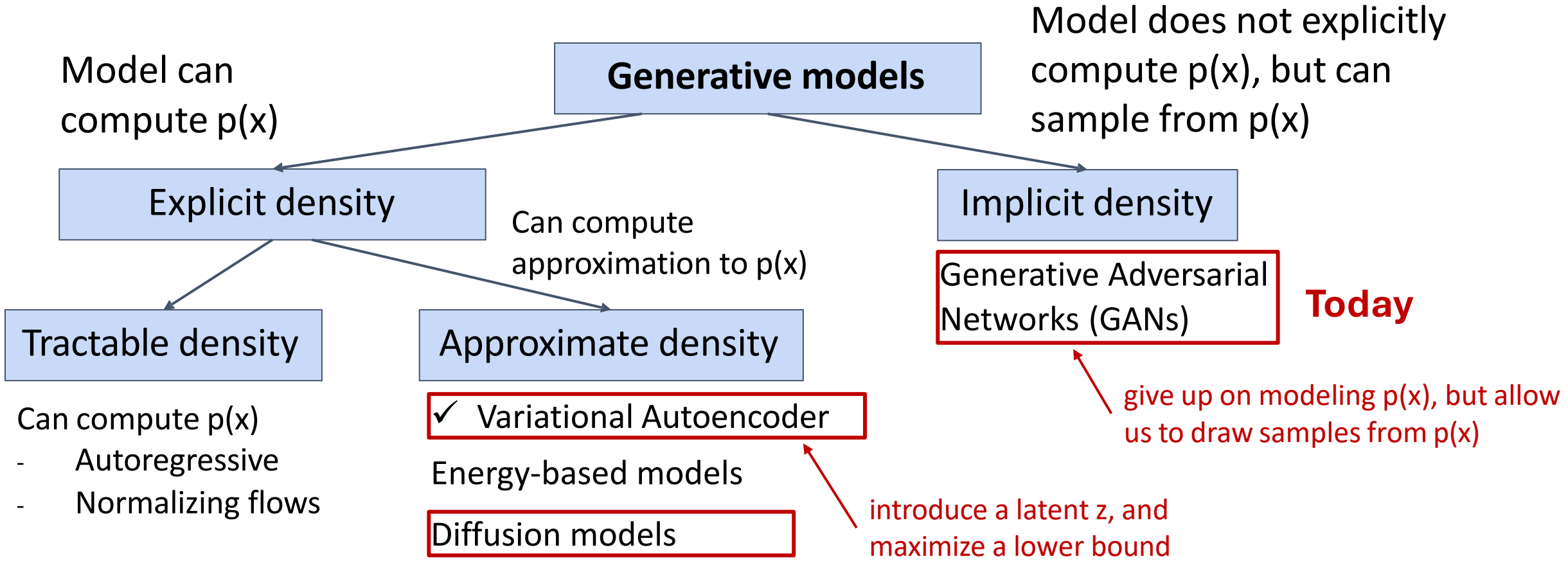


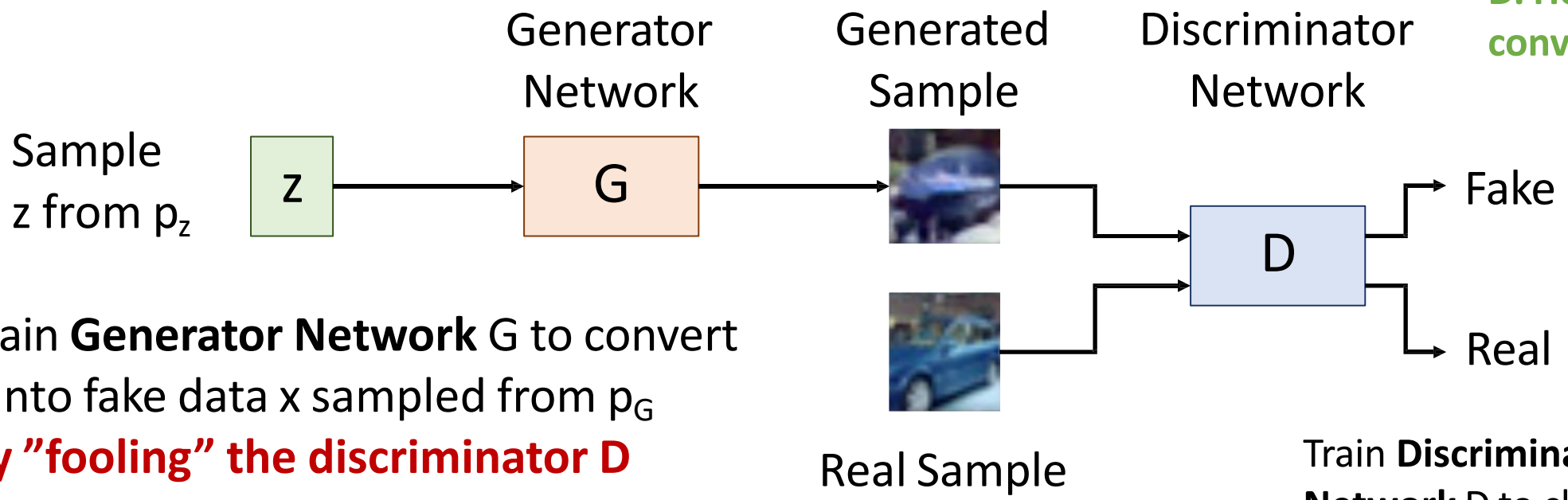
Figure is revised based on Ian Goodfellow, Tutorial on Generative Adversarial Networks, 2017.

Generative Adversarial Networks

Setup: Assume we have data x_i drawn from distribution $p_{data}(x)$. Want to sample from p_{data}.

Idea: Introduce a latent variable z with simple prior p(z).
Sample $z \sim p(z)$ and pass to a **Generator Network** x = G(z)
Then x is a sample from the **Generator distribution** p_G. Want $p_G = p_{data}$!

Jointly train G and D. Hopefully p_G converges to p_{data}!



Train **Generator Network** G to convert z into fake data x sampled from p_G
by "fooling" the discriminator D

Train **Discriminator Network** D to classify data as real or fake (1/0)

Goodfellow et al, "Generative Adversarial Nets", NeurIPS 2014

Generative Adversarial Networks: Training Objective

Jointly train generator G and discriminator D with a **minimax game**

$$\min_G \max_D \left(E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} \left[\log \left(1 - D\left(G(z)\right) \right) \right] \right)$$

Goodfellow et al, “Generative Adversarial Nets”, NeurIPS 2014

Generative Adversarial Networks: Training Objective

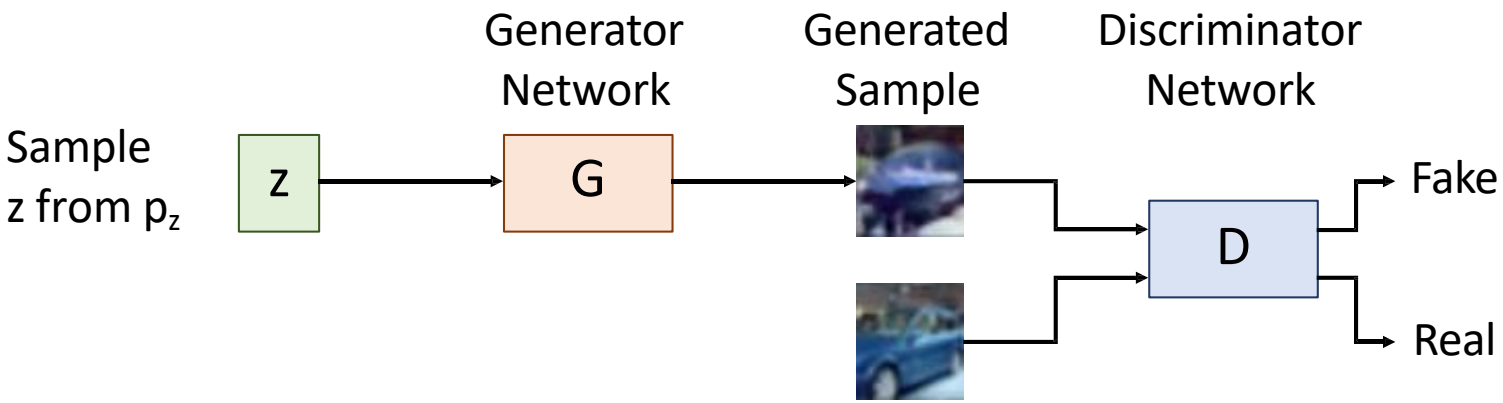
Jointly train generator G and discriminator D with a **minimax game**

Discriminator wants
D(x) = 1 for real data

Discriminator wants
D(x) = 0 for fake data

$$\min_{G} \max_{D} \left(E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} \left[\log \left(1 - D(G(z)) \right) \right] \right)$$

Generator wants
D(x) = 1 for fake data



Goodfellow et al, "Generative Adversarial Nets", NeurIPS 2014

Generative Adversarial Networks: Training Objective

Jointly train generator G and discriminator D with a **minimax game**

Train G and D using alternating gradient updates

$$\min_{G} \max_{D} \left(E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} \left[\log \left(1 - D(G(z)) \right) \right] \right)$$

$$= \min_{G} \max_{D} V(G, D)$$

We are not minimizing any overall loss.

No training curves to look at.

For t in 1, … T:

1. (Update D) $D = D + \alpha_D \frac{\partial V}{\partial D}$
2. (Update G) $G = G - \alpha_G \frac{\partial V}{\partial G}$

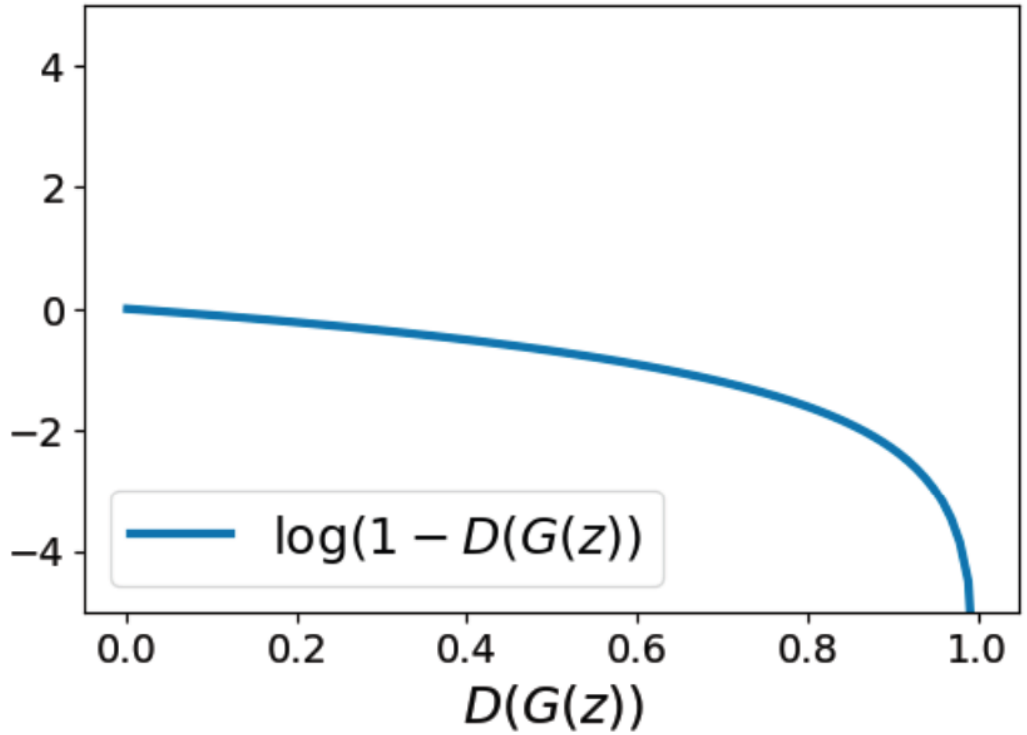
Goodfellow et al, “Generative Adversarial Nets”, NeurIPS 2014

Generative Adversarial Networks: Training Objective

Jointly train generator G and discriminator D with a **minimax game**

$$\min_{G} \max_{D} \left(E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} \left[\log \left(1 - D(G(z)) \right) \right] \right)$$

At start of training, generator is very bad and discriminator can easily tell apart real/fake, so D(G(z)) close to 0



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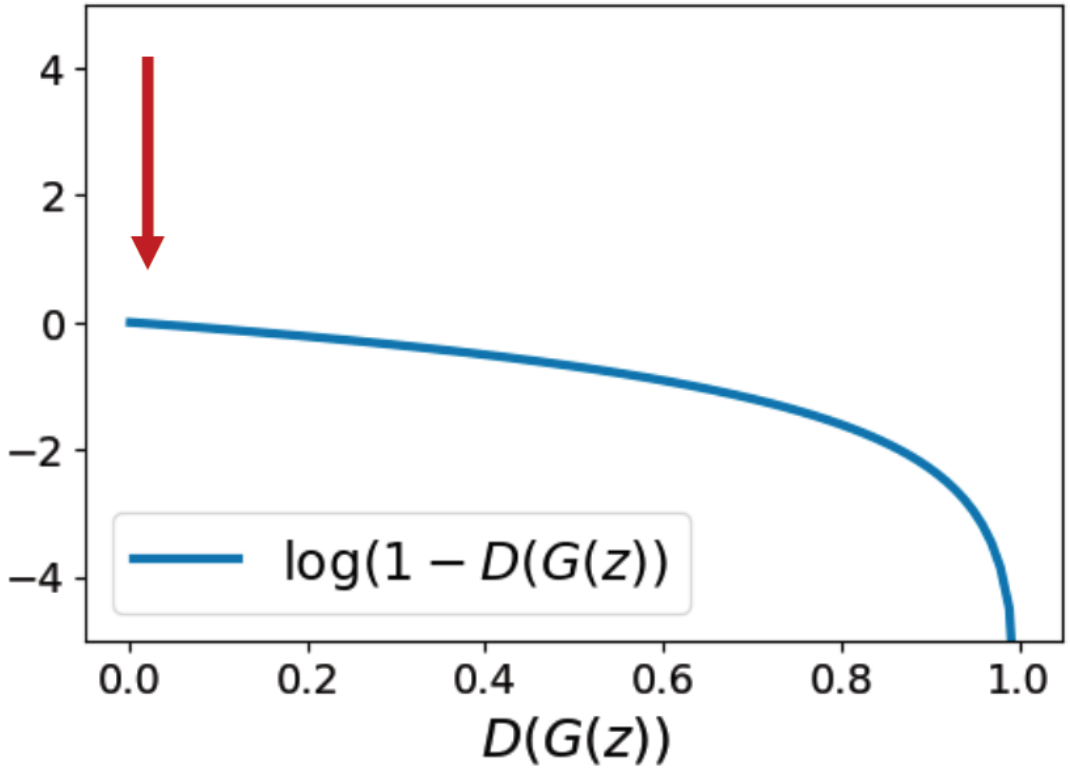
Problem: Vanishing gradients for G



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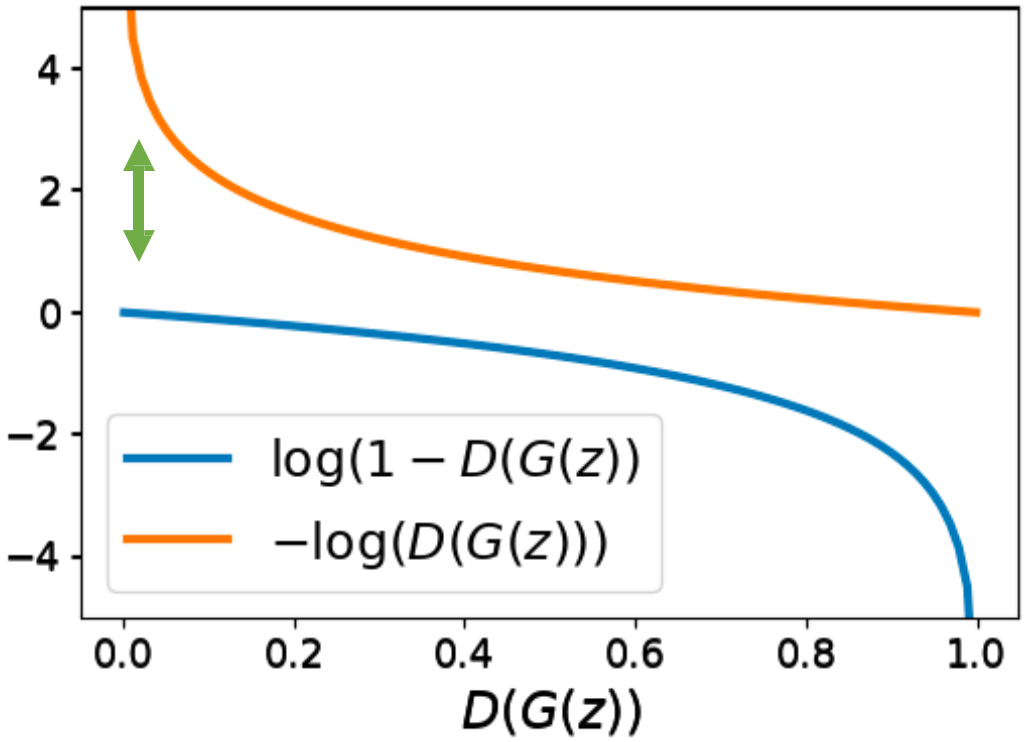
Jointly train generator G and discriminator D with a **minimax game**

$$\min_{G} \max_{D} \left(E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} \left[\log \left(1 - D(G(z)) \right) \right] \right)$$

At start of training, generator is very bad and discriminator can easily tell apart real/fake, so D(G(z)) close to 0

Problem: Vanishing gradients for G

Solution: Right now G is trained to minimize log(1-D(G(z)). Instead, train G to minimize –log(D(G(z)). Then G gets strong gradients at start of training!



Goodfellow et al, “Generative Adversarial Nets”, NeurIPS 2014

Generative Adversarial Networks: Optimality

Jointly train generator G and discriminator D with a **minimax game**

Why is this particular objective a good idea?

$$\min_{G} \max_{D} \left(E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} \left[\log \left(1 - D(G(z)) \right) \right] \right)$$

This minimax game achieves its global minimum when $p_G = p_{data}$!

Goodfellow et al, “Generative Adversarial Nets”, NeurIPS 2014

Generative Adversarial Networks: Optimality

$$\min_{G} \max_{D} \left(E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} \left[\log \left(1 - D(G(z)) \right) \right] \right)$$

(Our objective so far)

Generative Adversarial Networks: Optimality

$$\min_G \max_D \left(E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} \left[\log \left(1 - D(G(z)) \right) \right] \right)$$

$$= \min_G \max_D \left(E_{x \sim p_{data}} [\log D(x)] + E_{x \sim p_G} [\log (1 - D(x))] \right)$$

(Change of variables on second term)

Generative Adversarial Networks: Optimality

$$\min_G \max_D \left(E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} \left[\log \left(1 - D(G(z)) \right) \right] \right)$$

$$= \min_G \max_D \left(E_{x \sim p_{data}} [\log D(x)] + E_{x \sim p_G} [\log (1 - D(x))] \right)$$

$$= \min_G \max_D \int_X \left(p_{data}(x) \log D(x) + p_G(x) \log (1 - D(x)) \right) dx$$

(Definition of expectation)

Generative Adversarial Networks: Optimality

$$\min_G \max_D \left(E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} \left[\log \left(1 - D(G(z)) \right) \right] \right)$$

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$$= \min_G \int_X \max_D \left(p_{data}(x) \log D(x) + p_G(x) \log (1 - D(x)) \right) dx$$

(Push $\max_D$ inside integral)

Generative Adversarial Networks: Optimality

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$$= \min_G \int_X \max_D \left(p_{data}(x) \log D(x) + p_G(x) \log (1 - D(x)) \right) dx$$

$$f(y) = a \log y + b \log (1 - y)$$

(Side computation to compute max)

Generative Adversarial Networks: Optimality

$$\min_G \max_D \left(E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} \left[\log \left(1 - D(G(z)) \right) \right] \right)$$

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$$f(y) = a \log y + b \log (1 - y)$$

$$f'(y) = \frac{a}{y} - \frac{b}{1 - y}$$

Generative Adversarial Networks: Optimality

$$\min_G \max_D \left(E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} \left[\log \left(1 - D(G(z)) \right) \right] \right)$$

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$$= \min_G \int_X \max_D \left(p_{data}(x) \log D(x) + p_G(x) \log (1 - D(x)) \right) dx$$

$$f(y) = a \log y + b \log (1 - y) \qquad f'(y) = 0 \iff y = \frac{a}{a+b} \text{ (local max)}$$

$$f'(y) = \frac{a}{y} - \frac{b}{1 - y}$$

Optimal Discriminator: $D_G^*(x) = \frac{p_{data}(x)}{p_{data}(x) + p_G(x)}$

Generative Adversarial Networks: Optimality

$$\min_G \max_D \left(E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} \left[\log \left(1 - D(G(z)) \right) \right] \right)$$

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Optimal Discriminator: $D_G^*(x) = \frac{p_{data}(x)}{p_{data}(x) + p_G(x)}$

Generative Adversarial Networks: Optimality

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$$= \min_G \int_X \left(p_{data}(x) \log \frac{p_{data}(x)}{p_{data}(x) + p_G(x)} + p_G(x) \log \frac{p_G(x)}{p_{data}(x) + p_G(x)} \right) dx$$

Optimal Discriminator: $D_G^*(x) = \frac{p_{data}(x)}{p_{data}(x) + p_G(x)}$

Generative Adversarial Networks: Optimality

$$\min_G \max_D \left(E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} \left[\log \left(1 - D(G(z)) \right) \right] \right)$$

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$$\min_G \max_D \left(E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} \left[\log \left(1 - D(G(z)) \right) \right] \right)$$

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(Definition of expectation)

Generative Adversarial Networks: Optimality

$$\min_G \max_D \left(E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} \left[\log \left(1 - D(G(z)) \right) \right] \right)$$

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$$= \min_G \left(E_{x \sim p_{data}} \left[\log \frac{2}{2} \frac{p_{data}(x)}{p_{data}(x) + p_G(x)} \right] + E_{x \sim p_G} \left[\log \frac{2}{2} \frac{p_G(x)}{p_{data}(x) + p_G(x)} \right] \right)$$

(Multiply by a constant)

Generative Adversarial Networks: Optimality

$$\min_G \max_D \left(E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} \left[\log \left(1 - D(G(z)) \right) \right] \right)$$

$$= \min_G \int_X \left(p_{data}(x) \log \frac{p_{data}(x)}{p_{data}(x) + p_G(x)} + p_G(x) \log \frac{p_G(x)}{p_{data}(x) + p_G(x)} \right) dx$$

$$= \min_G \left(E_{x \sim p_{data}} \left[\log \frac{2}{2} \frac{p_{data}(x)}{p_{data}(x) + p_G(x)} \right] + E_{x \sim p_G} \left[\log \frac{2}{2} \frac{p_G(x)}{p_{data}(x) + p_G(x)} \right] \right)$$

$$= \min_G \left(E_{x \sim p_{data}} \left[\log \frac{2 * p_{data}(x)}{p_{data}(x) + p_G(x)} \right] + E_{x \sim p_G} \left[\log \frac{2 * p_G(x)}{p_{data}(x) + p_G(x)} \right] - \log 4 \right)$$

Generative Adversarial Networks: Optimality

$$\min_{G} \max_{D} \left(E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} \left[\log \left(1 - D(G(z)) \right) \right] \right)$$

$$= \min_{G} \left(E_{x \sim p_{data}} \left[\log \frac{2 * p_{data}(x)}{p_{data}(x) + p_G(x)} \right] + E_{x \sim p_G} \left[\log \frac{2 * p_G(x)}{p_{data}(x) + p_G(x)} \right] - \log 4 \right)$$

Generative Adversarial Networks: Optimality

$$\min_G \max_D \left(E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} \left[\log \left(1 - D(G(z)) \right) \right] \right)$$

$$= \min_G \left(E_{x \sim p_{data}} \left[\log \frac{2 * p_{data}(x)}{p_{data}(x) + p_G(x)} \right] + E_{x \sim p_G} \left[\log \frac{2 * p_G(x)}{p_{data}(x) + p_G(x)} \right] - \log 4 \right)$$

Kullback-Leibler Divergence:

$$KL(p, q) = E_{x \sim p} \left[\log \frac{p(x)}{q(x)} \right]$$

Generative Adversarial Networks: Optimality

$$\min_G \max_D \left(E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} \left[\log \left(1 - D(G(z)) \right) \right] \right)$$

$$= \min_G \left(E_{x \sim p_{data}} \left[\log \frac{2 * p_{data}(x)}{p_{data}(x) + p_G(x)} \right] + E_{x \sim p_G} \left[\log \frac{2 * p_G(x)}{p_{data}(x) + p_G(x)} \right] - \log 4 \right)$$

$$= \min_G \left(KL \left(p_{data}, \frac{p_{data} + p_G}{2} \right) + KL \left(p_G, \frac{p_{data} + p_G}{2} \right) - \log 4 \right)$$

Kullback-Leibler Divergence:

$$KL(p, q) = E_{x \sim p} \left[\log \frac{p(x)}{q(x)} \right]$$

Generative Adversarial Networks: Optimality

$$\min_G \max_D \left(E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} \left[\log \left(1 - D(G(z)) \right) \right] \right)$$

$$= \min_G \left(E_{x \sim p_{data}} \left[\log \frac{2 * p_{data}(x)}{p_{data}(x) + p_G(x)} \right] + E_{x \sim p_G} \left[\log \frac{2 * p_G(x)}{p_{data}(x) + p_G(x)} \right] - \log 4 \right)$$

$$= \min_G \left(KL \left(p_{data}, \frac{p_{data} + p_G}{2} \right) + KL \left(p_G, \frac{p_{data} + p_G}{2} \right) - \log 4 \right)$$

Jensen-Shannon Divergence:

$$JSD(p, q) = \frac{1}{2} KL \left(p, \frac{p + q}{2} \right) + \frac{1}{2} KL \left(q, \frac{p + q}{2} \right)$$

Generative Adversarial Networks: Optimality

$$\min_G \max_D \left(E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} \left[\log \left(1 - D(G(z)) \right) \right] \right)$$

$$= \min_G \left(E_{x \sim p_{data}} \left[\log \frac{2 * p_{data}(x)}{p_{data}(x) + p_G(x)} \right] + E_{x \sim p_G} \left[\log \frac{2 * p_G(x)}{p_{data}(x) + p_G(x)} \right] - \log 4 \right)$$

$$= \min_G \left(KL \left(p_{data}, \frac{p_{data} + p_G}{2} \right) + KL \left(p_G, \frac{p_{data} + p_G}{2} \right) - \log 4 \right)$$

$$= \min_G (2 * JSD(p_{data}, p_G) - \log 4)$$

Jensen-Shannon Divergence:

$$JSD(p, q) = \frac{1}{2} KL \left(p, \frac{p + q}{2} \right) + \frac{1}{2} KL \left(q, \frac{p + q}{2} \right)$$

Generative Adversarial Networks: Optimality

$$\min_G \max_D \left(E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} \left[\log \left(1 - D(G(z)) \right) \right] \right)$$

$$= \min_G \left(E_{x \sim p_{data}} \left[\log \frac{2 * p_{data}(x)}{p_{data}(x) + p_G(x)} \right] + E_{x \sim p_G} \left[\log \frac{2 * p_G(x)}{p_{data}(x) + p_G(x)} \right] - \log 4 \right)$$

$$= \min_G \left(KL \left(p_{data}, \frac{p_{data} + p_G}{2} \right) + KL \left(p_G, \frac{p_{data} + p_G}{2} \right) - \log 4 \right)$$

$$= \min_G (2 * JSD(p_{data}, p_G) - \log 4)$$

JSD is always nonnegative, and zero only when the two distributions are equal!
Thus $p_{data} = p_G$ is the global min

Jensen-Shannon Divergence:

$$JSD(p, q) = \frac{1}{2} KL \left(p, \frac{p + q}{2} \right) + \frac{1}{2} KL \left(q, \frac{p + q}{2} \right)$$

Generative Adversarial Networks: Optimality

$$\min_{G}\max_{D}\left(E_{x\sim p_{data}}[\log D(x)]+E_{z\sim p(z)}\left[\log\left(1-D(G(z))\right)\right]\right)$$

$$=\min_{G}(2*JSD(p_{data},p_{G})-\log 4)$$

Generative Adversarial Networks: Optimality

$$\min_G \max_D \left(E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} \left[\log \left(1 - D(G(z)) \right) \right] \right)$$

$$= \min_G (2 * JSD(p_{data}, p_G) - \log 4)$$

Summary: The global minimum of the minimax game happens when:

1. $D_G^*(x) = \frac{p_{data}(x)}{p_{data}(x) + p_G(x)}$ (Optimal discriminator for any G)
2. $p_G(x) = p_{data}(x)$ (Optimal generator for optimal D)

Generative Adversarial Networks: Optimality

$$\min_G \max_D \left(E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} \left[\log \left(1 - D(G(z)) \right) \right] \right)$$

$$= \min_G (2 * JSD(p_{data}, p_G) - \log 4)$$

Summary: The global minimum of the minimax game happens when:

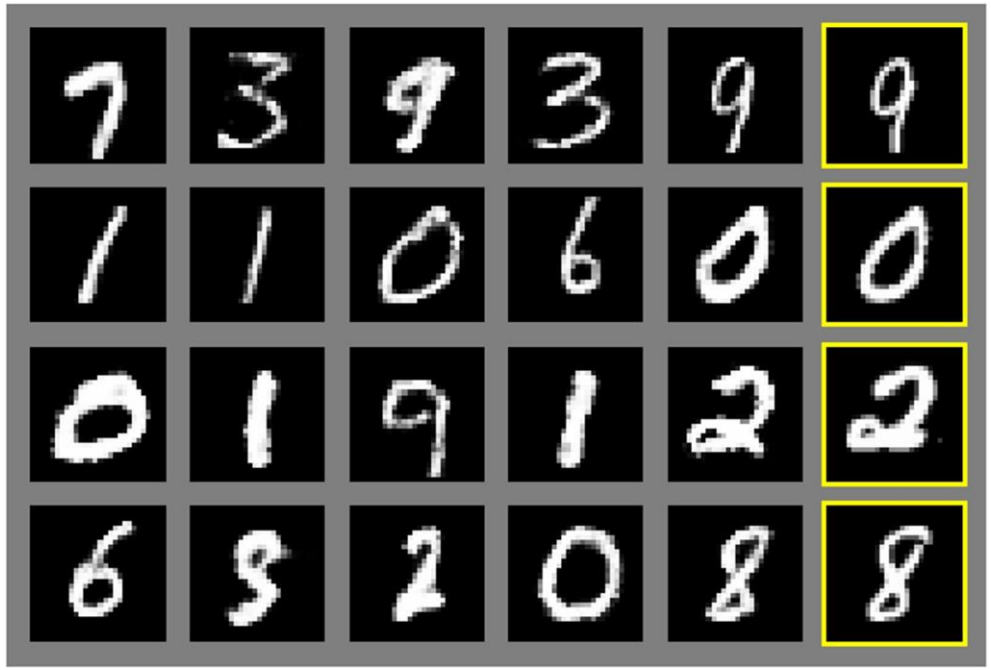
1. $D_G^*(x) = \frac{p_{data}(x)}{p_{data}(x) + p_G(x)}$ (Optimal discriminator for any G)
2. $p_G(x) = p_{data}(x)$ (Optimal generator for optimal D)

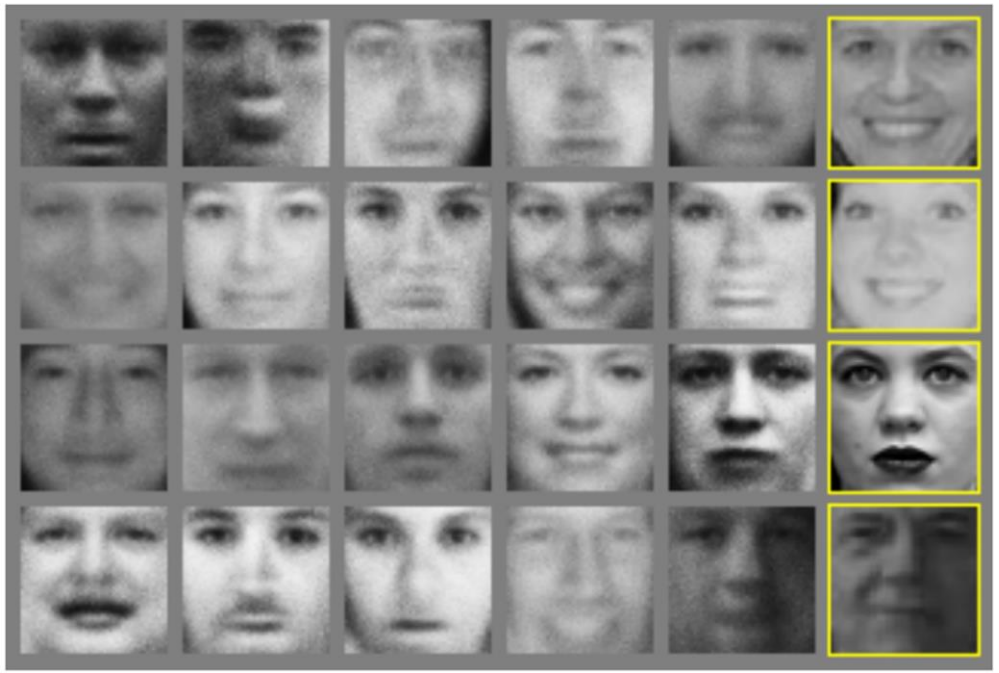
Caveats:

1. G and D are neural nets with fixed architecture. We don't know whether they can actually represent the optimal D and G.
2. This tells us nothing about convergence to the optimal solution

Generative Adversarial Networks: Results

Generated samples

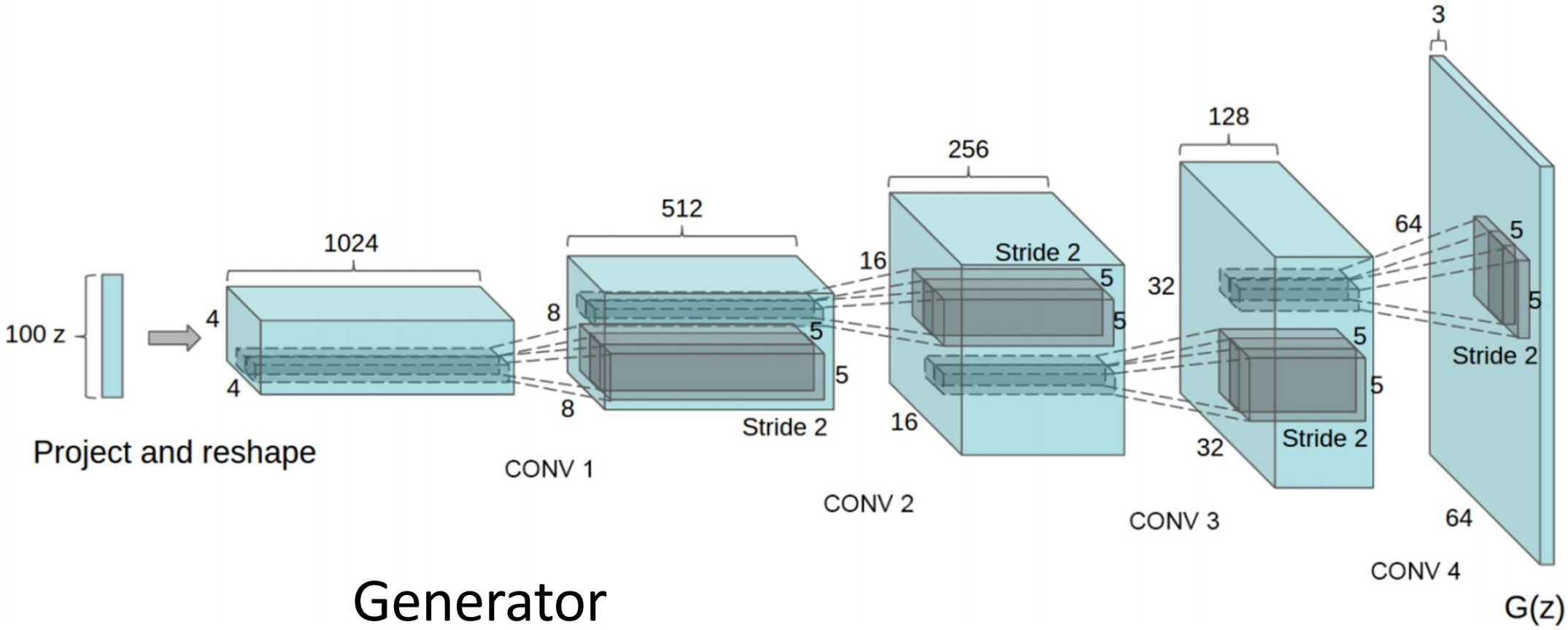




Nearest neighbor from training set

Goodfellow et al, “Generative Adversarial Nets”, NeurIPS 2014

Generative Adversarial Networks: DC-GAN



Generator

Radford et al, “Unsupervised Representation Learning with Deep Convolutional Generative Adversarial Networks”, ICLR 2016

Generative Adversarial Networks: DC-GAN

Samples from the model look much better!

Radford et al, ICLR 2016

Generative Adversarial Networks: Interpolation

Interpolating between points in latent z space

Radford et al, ICLR 2016

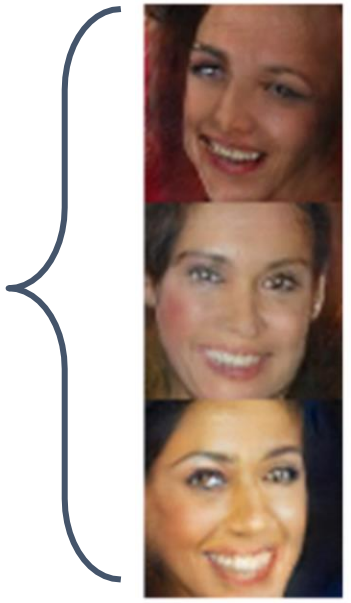
Generative Adversarial Networks: Vector Math

Smiling woman

Neutral woman

Neutral man

Samples from the model



Average Z vectors, do arithmetic

Smiling Man

Radford et al, ICLR 2016

Generative Adversarial Networks: Vector Math

Man with glasses

Man w/o glasses

Woman w/o glasses

Samples from the model

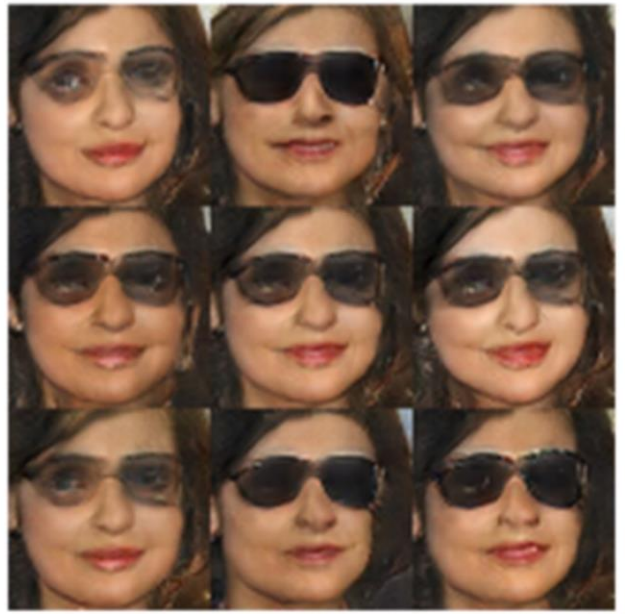
Average Z vectors, do arithmetic

−

+

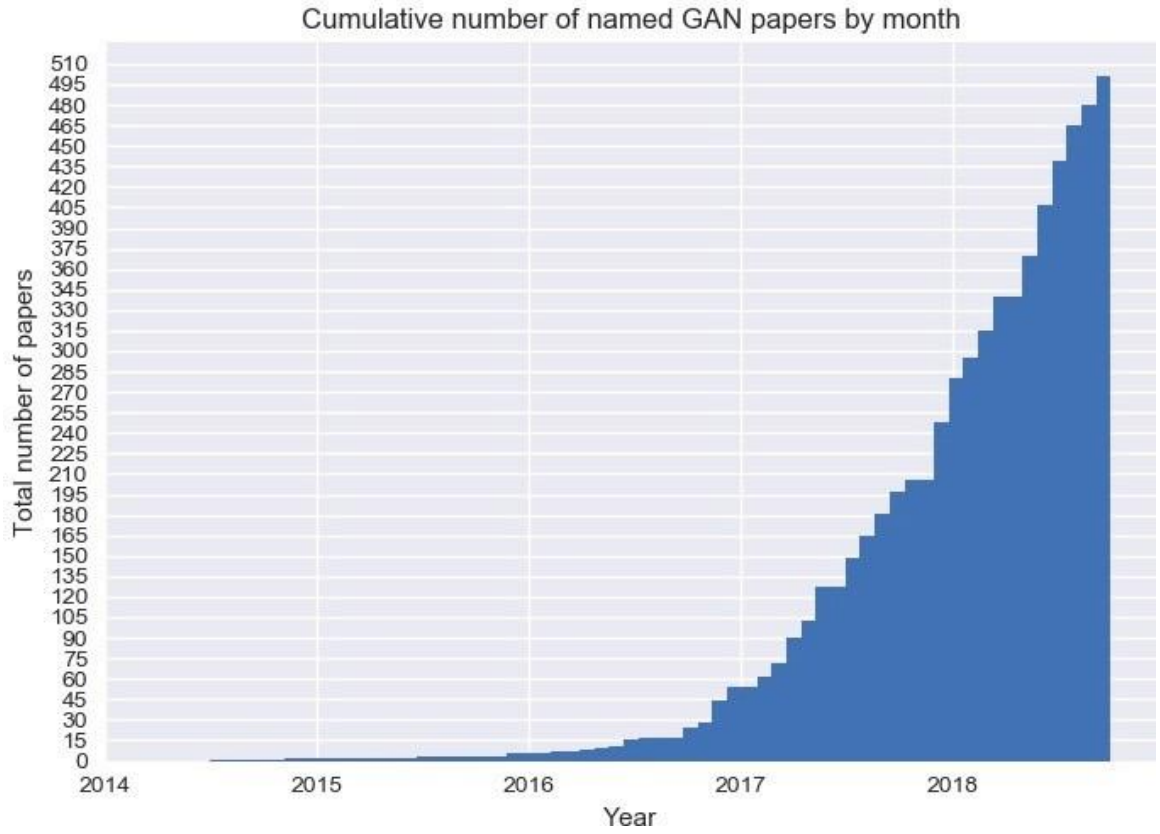
=

Woman with glasses



Radford et al, ICLR 2016

2017 to present: Explosion of GANs



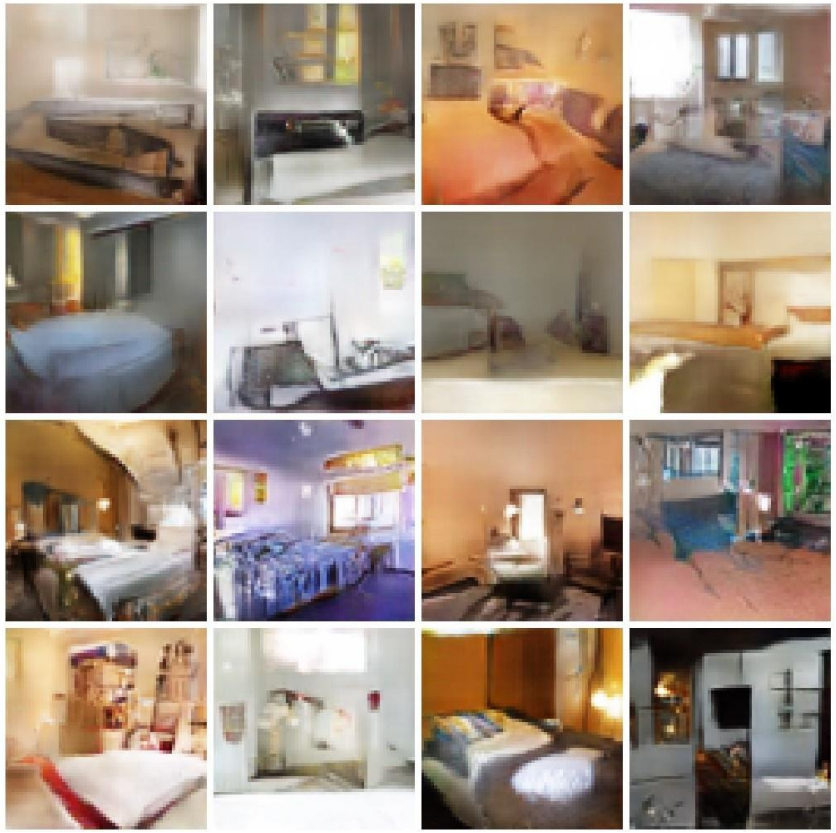
https://github.com/hindupuravinash/the-gan-zoo

- 3D-ED-GAN - Shape Inpainting using 3D Generative Adversarial Network and Recurrent Convolutional Networks
- 3D-GAN - Learning a Probabilistic Latent Space of Object Shapes via 3D Generative-Adversarial Modeling (github)
- 3D-IWGAN - Improved Adversarial Systems for 3D Object Generation and Reconstruction (github)
- 3D-PhysNet - 3D-PhysNet: Learning the Intuitive Physics of Non-Rigid Object Deformations
- 3D-RecGAN - 3D Object Reconstruction from a Single Depth View with Adversarial Learning (github)
- ABC-GAN - ABC-GAN: Adaptive Blur and Control for improved training stability of Generative Adversarial Networks (github)
- ABC-GAN - GANs for LIFE: Generative Adversarial Networks for Likelihood Free Inference
- AC-GAN - Conditional Image Synthesis With Auxiliary Classifier GANs
- acGAN - Face Aging With Conditional Generative Adversarial Networks
- ACGAN - Coverless Information Hiding Based on Generative adversarial networks
- acGAN - On-line Adaptative Curriculum Learning for GANs
- ACtuAL - ACtuAL: Actor-Critic Under Adversarial Learning
- AdaGAN - AdaGAN: Boosting Generative Models
- Adaptive GAN - Customizing an Adversarial Example Generator with Class-Conditional GANs
- AdvEntuRe - AdvEntuRe: Adversarial Training for Textual Entailment with Knowledge-Guided Examples
- AdvGAN - Generating adversarial examples with adversarial networks
- AE-GAN - AE-GAN: adversarial eliminating with GAN
- AE-OT - Latent Space Optimal Transport for Generative Models
- AEGAN - Learning Inverse Mapping by Autoencoder based Generative Adversarial Nets
- AF-DCGAN - AF-DCGAN: Amplitude Feature Deep Convolutional GAN for Fingerprint Construction in Indoor Localization System
- AffGAN - Amortised MAP Inference for Image Super-resolution
- AIM - Generating Informative and Diverse Conversational Responses via Adversarial Information Maximization
- AL-CGAN - Learning to Generate Images of Outdoor Scenes from Attributes and Semantic Layouts
- ALI - Adversarially Learned Inference (github)
- AlignGAN - AlignGAN: Learning to Align Cross-Domain Images with Conditional Generative Adversarial Networks
- AlphaGAN - AlphaGAN: Generative adversarial networks for natural image matting
- AM-GAN - Activation Maximization Generative Adversarial Nets
- AmbientGAN - AmbientGAN: Generative models from lossy measurements (github)
- AMC-GAN - Video Prediction with Appearance and Motion Conditions
- AnoGAN - Unsupervised Anomaly Detection with Generative Adversarial Networks to Guide Marker Discovery
- APD - Adversarial Distillation of Bayesian Neural Network Posteriors
- APE-GAN - APE-GAN: Adversarial Perturbation Elimination with GAN
- ARAE - Adversarially Regularized Autoencoders for Generating Discrete Structures (github)
- ARDA - Adversarial Representation Learning for Domain Adaptation
- ARIGAN - ARIGAN: Synthetic Arabidopsis Plants using Generative Adversarial Network
- ArtGAN - ArtGAN: Artwork Synthesis with Conditional Categorial GANs
- ASDL-GAN - Automatic Steganographic Distortion Learning Using a Generative Adversarial Network
- ATA-GAN - Attention-Aware Generative Adversarial Networks (ATA-GANs)
- Attention-GAN - Attention-GAN for Object Transfiguration in Wild Images
- AttGAN - Arbitrary Facial Attribute Editing: Only Change What You Want (github)
- AttnGAN - AttnGAN: Fine-Grained Text to Image Generation with Attentional Generative Adversarial Networks (github)
- AVID - AVID: Adversarial Visual Irregularity Detection
- B-DCGAN - B-DCGAN:Evaluation of Binarized DCGAN for FPGA
- b-GAN - Generative Adversarial Nets from a Density Ratio Estimation Perspective
- BAGAN - BAGAN: Data Augmentation with Balancing GAN
- Bayesian GAN - Deep and Hierarchical Implicit Models
- Bayesian GAN - Bayesian GAN (github)
- BCGAN - Bayesian Conditional Generative Adverserial Networks
- BCGAN - Bidirectional Conditional Generative Adversarial networks
- BEAM - Boltzmann Encoded Adversarial Machines
- BEGAN - BEGAN: Boundary Equilibrium Generative Adversarial Networks
- BEGAN-CS - Escaping from Collapsing Modes in a Constrained Space
- Bellman GAN - Distributional Multivariate Policy Evaluation and Exploration with the Bellman

- BGAN - Binary Generative Adversarial Networks for Image Retrieval (github)
- Bi-GAN - Autonomously and Simultaneously Refining Deep Neural Network Parameters by a Bi-Generative Adversarial Network Aided Genetic Algorithm
- BicycleGAN - Toward Multimodal Image-to-Image Translation (github)
- BiGAN - Adversarial Feature Learning
- BinGAN - BinGAN: Learning Compact Binary Descriptors with a Regularized GAN
- BourGAN - BourGAN: Generative Networks with Metric Embeddings
- BranchGAN - Branched Generative Adversarial Networks for Multi-Scale Image Manifold Learning
- BRE - Improving GAN Training via Binarized Representation Entropy (BRE) Regularization (github)
- BridgeGAN - Generative Adversarial Frontal View to Bird View Synthesis
- BS-GAN - Boundary-Seeking Generative Adversarial Networks
- BubGAN - BubGAN: Bubble Generative Adversarial Networks for Synthesizing Realistic Bubbly Flow Images
- BWGAN - Banach Wasserstein GAN
- C-GAN - Face Aging with Contextual Generative Adversarial Nets
- C-RNN-GAN - C-RNN-GAN: Continuous recurrent neural networks with adversarial training (github)
- CA-GAN - Composition-aided Sketch-realistic Portrait Generation
- CaloGAN - CaloGAN: Simulating 3D High Energy Particle Showers in Multi-Layer Electromagnetic Calorimeters with Generative Adversarial Networks (github)
- CAN - CAN: Creative Adversarial Networks, Generating Art by Learning About Styles and Deviating from Style Norms
- CapsGAN - CapsGAN: Using Dynamic Routing for Generative Adversarial Networks
- CapsuleGAN - CapsuleGAN: Generative Adversarial Capsule Network
- CatGAN - Unsupervised and Semi-supervised Learning with Categorical Generative Adversarial Networks
- CatGAN - CatGAN: Coupled Adversarial Transfer for Domain Generation
- CausalGAN - CausalGAN: Learning Causal Implicit Generative Models with Adversarial Training
- CC-GAN - Semi-Supervised Learning with Context-Conditional Generative Adversarial Networks (github)
- cd-GAN - Conditional Image-to-Image Translation
- CDcGAN - Simultaneously Color-Depth Super-Resolution with Conditional Generative Adversarial Network
- CE-GAN - Deep Learning for Imbalance Data Classification using Class Expert Generative Adversarial Network
- CFG-GAN - Composite Functional Gradient Learning of Generative Adversarial Models
- CGAN - Conditional Generative Adversarial Nets
- CGAN - Controllable Generative Adversarial Network
- Chekhov GAN - An Online Learning Approach to Generative Adversarial Networks
- ciGAN - Conditional Infilling GANs for Data Augmentation in Mammogram Classification
- CinCGAN - Unsupervised Image Super-Resolution using Cycle-in-Cycle Generative Adversarial Networks
- CipherGAN - Unsupervised Cipher Cracking Using Discrete GANs
- ClusterGAN - ClusterGAN : Latent Space Clustering in Generative Adversarial Networks
- CM-GAN - CM-GANs: Cross-modal Generative Adversarial Networks for Common Representation Learning
- CoAtt-GAN - Are You Talking to Me? Reasoned Visual Dialog Generation through Adversarial Learning
- CoGAN - Coupled Generative Adversarial Networks
- ComboGAN - ComboGAN: Unrestrained Scalability for Image Domain Translation (github)
- ConceptGAN - Learning Compositional Visual Concepts with Mutual Consistency
- Conditional cycleGAN - Conditional CycleGAN for Attribute Guided Face Image Generation
- constrast-GAN - Generative Semantic Manipulation with Contrasting GAN
- Context-RNN-GAN - Contextual RNN-GANs for Abstract Reasoning Diagram Generation
- CorrGAN - Correlated discrete data generation using adversarial training
- Coulomb GAN - Coulomb GANs: Provably Optimal Nash Equilibria via Potential Fields
- Cover-GAN - Generative Steganography with Kerckhoffs' Principle based on Generative Adversarial Networks
- cowboy - Defending Against Adversarial Attacks by Leveraging an Entire GAN
- CR-GAN - CR-GAN: Learning Complete Representations for Multi-view Generation
- Cramèr GAN - The Cramer Distance as a Solution to Biased Wasserstein Gradients
- Cross-GAN - Crossing Generative Adversarial Networks for Cross-View Person Re-identification
- crVAE-GAN - Channel-Recurrent Variational Autoencoders
- CS-GAN - Improving Neural Machine Translation with Conditional Sequence Generative Adversarial Nets
- CSG - Speech-Driven Expressive Talking Lips with Conditional Sequential Generative Adversarial Networks
- CT-GAN - CT-GAN: Conditional Transformation Generative Adversarial Network for Image Attribute Modification
- CVAE-GAN - CVAE-GAN: Fine-Grained Image Generation through Asymmetric Training
- CycleGAN - Unpaired Image-to-Image Translation using Cycle-Consistent Adversarial Networks

GAN Improvements: Improved Loss Functions

Wasserstein GAN (WGAN)



Arjovsky, Chintala, and Bouttou, “Wasserstein GAN”, 2017

WGAN with Gradient Penalty (WGAN-GP)

Gulrajani et al, “Improved Training of Wasserstein GANs”, NeurIPS 2017

GAN Improvements: Higher Resolution

256 x 256 bedrooms

1024 x 1024 faces

Karras et al, “Progressive Growing of GANs for Improved Quality, Stability, and Variation”, ICLR 2018

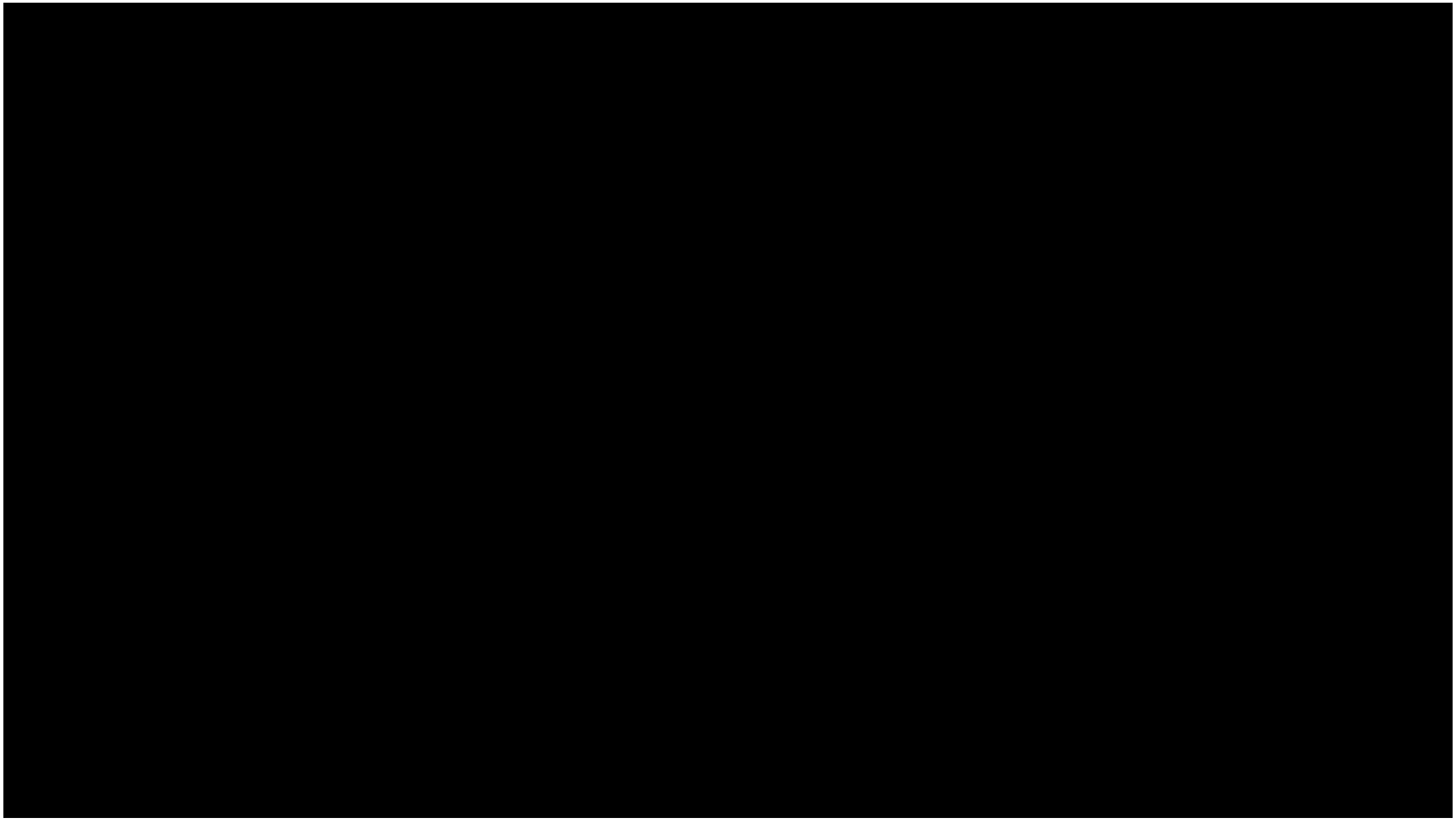
GAN Improvements: Higher Resolution

512 x 384 cars

1024 x 1024 faces

Images are licensed under CC BY-NC 4.0

Karras et al, “A Style-Based Generator Architecture for Generative Adversarial Networks”, CVPR 2019



Karras et al, “A Style-Based Generator Architecture for Generative Adversarial Networks”, CVPR 2019

Video is licensed under CC BY-NC 4.0.

Source: https://drive.google.com/drive/folders/1NFO7_vH0t98J13ckJYFd7kuaTkyeRJ86

StyleGAN2

Karras et al, “Analyzing and Improving the Image Quality of StyleGAN”, CVPR 2020

Conditional GANs

Recall: Conditional Generative Models learn p(x|y) instead of p(x)
Make generator and discriminator both take label y as an additional input!

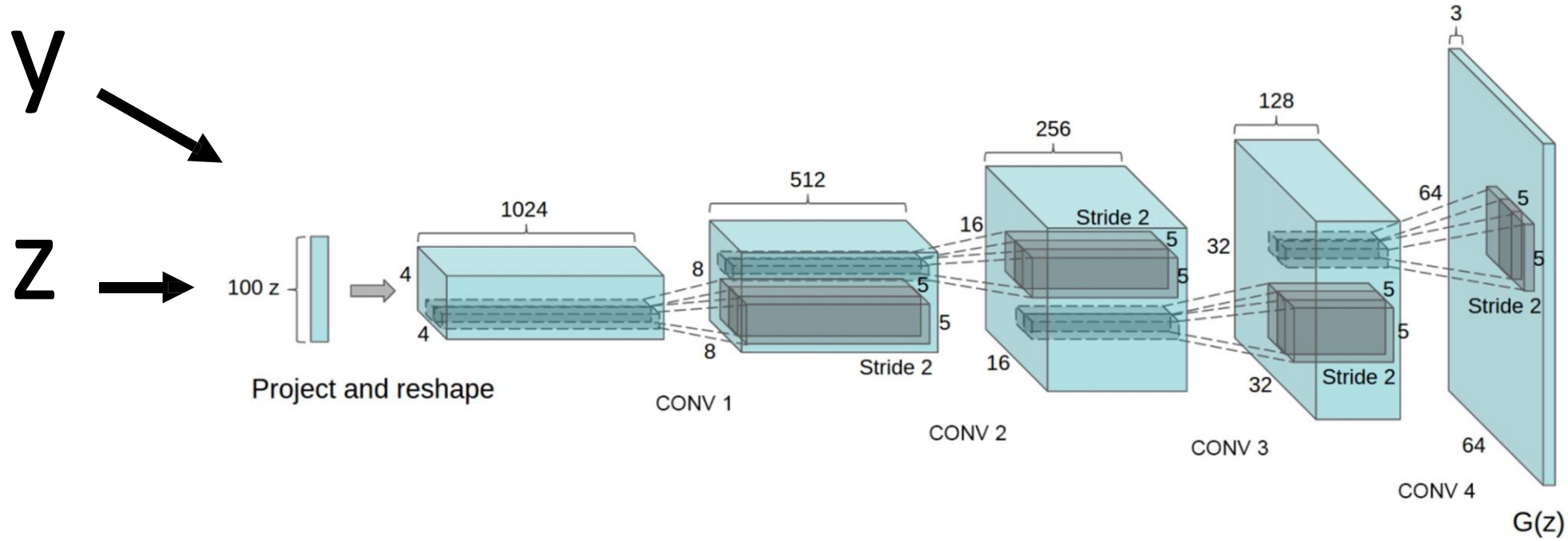


Figure credit: Radford et al, “Unsupervised Representation Learning with Deep Convolutional Generative Adversarial Networks”, ICLR 2016

Conditional GANs: Conditional Batch Normalization

Batch Normalization

$$\mu_j = \frac{1}{N}\sum_{i=1}^{N} x_{i,j}$$

$$\sigma_j^2 = \frac{1}{N}\sum_{i=1}^{N}(x_{i,j} - \mu_j)^2$$

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \epsilon}}$$

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$

Learn a separate scale and shift for each different label y

Conditional Batch Normalization

$$\mu_j = \frac{1}{N}\sum_{i=1}^{N} x_{i,j}$$

$$\sigma_j^2 = \frac{1}{N}\sum_{i=1}^{N}(x_{i,j} - \mu_j)^2$$

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \epsilon}}$$

$$y_{i,j} = \boldsymbol{\gamma_j^y} \hat{x}_{i,j} + \boldsymbol{\beta_j^y}$$

Dumoulin et al, "A learned representation for artistic style", ICLR 2017

Conditional GANs: Spectral Normalization

Miyato et al, “Spectral Normalization for Generative Adversarial Networks”, ICLR 2018

128x128 images on ImageNet

Conditional GANs: Self-Attention

128x128 images on ImageNet

Zhang et al, “Self-Attention Generative Adversarial Networks”, ICML 2019

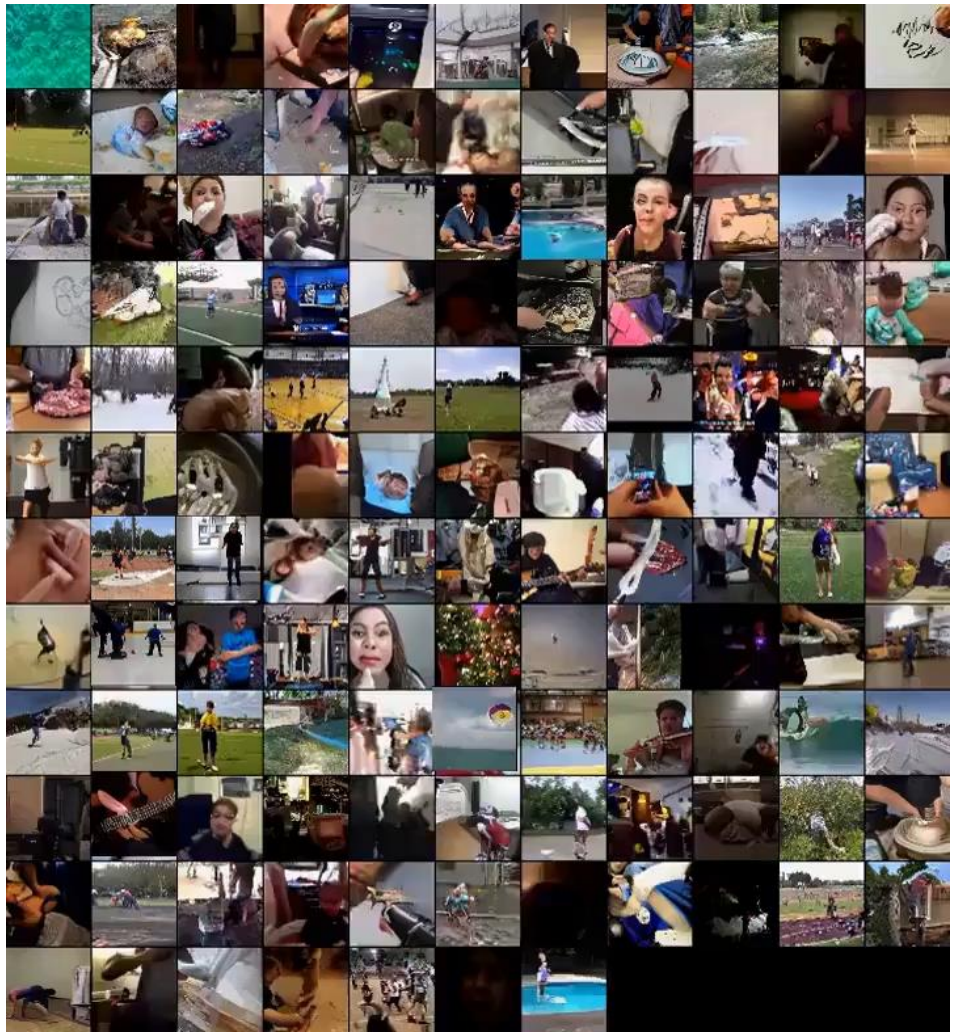
Conditional GANs: BigGAN

512x512 images on ImageNet

Brock et al, “Large Scale GAN Training for High Fidelity Natural Image Synthesis”, ICLR 2019

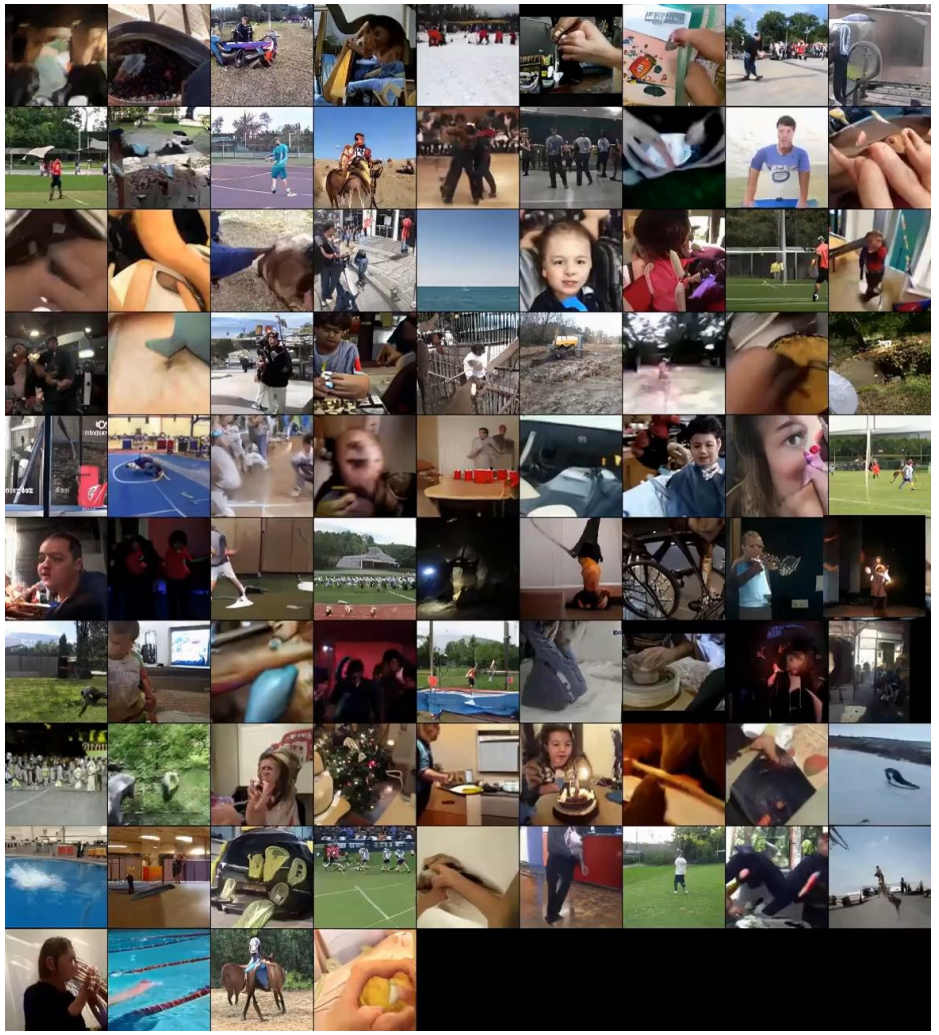
Generating Videos with GANs

Clark et al, “Adversarial Video Generation on Complex Datasets”, arXiv 2019



64x64 images, 48 frames

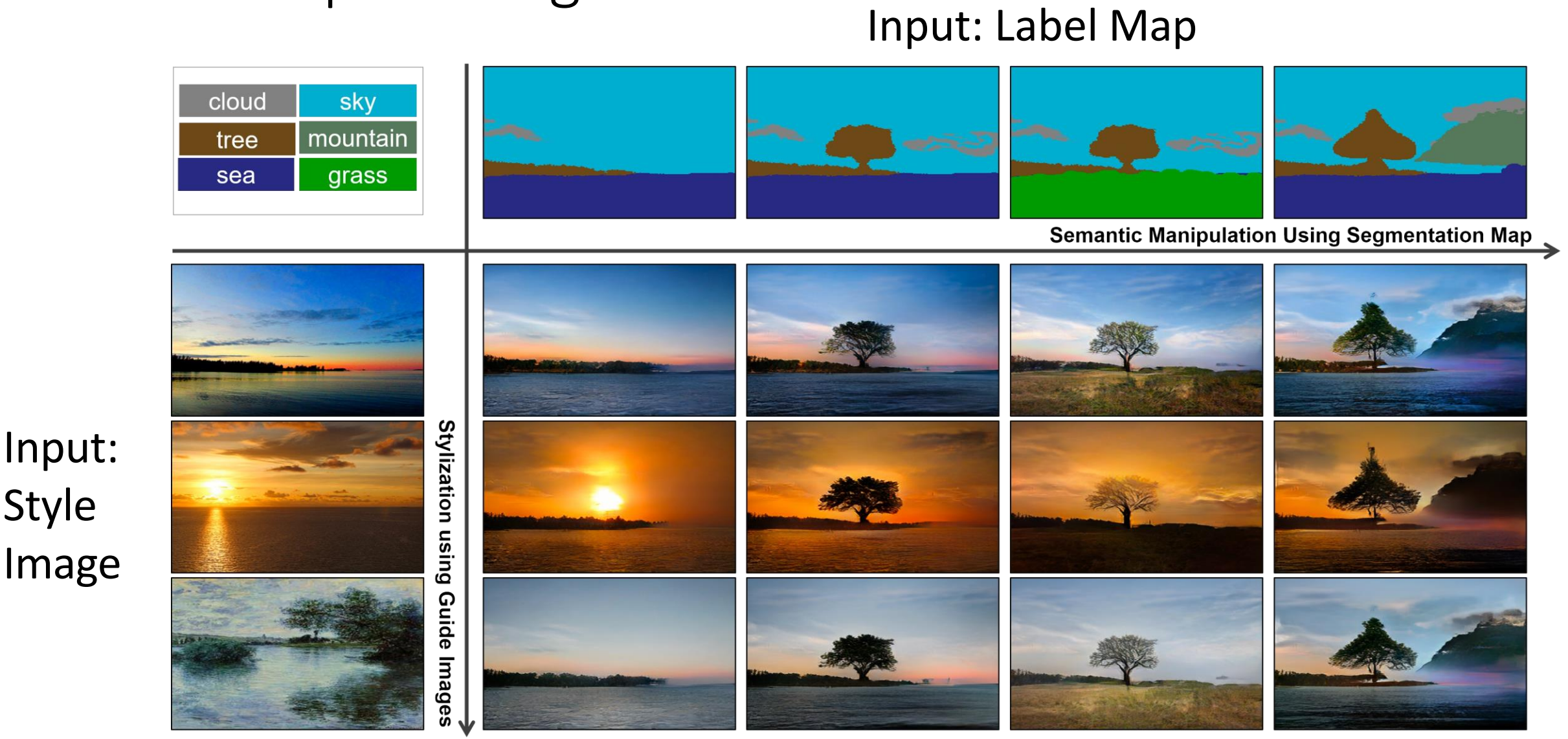
https://drive.google.com/file/d/1FjOQYdUuxPXvS8yeOhXdPQMapUQaklLi/view



128x128 images, 12 frames

https://drive.google.com/file/d/165Yxuvvu3viOy-39LhhSDGtczbWphj_i/view

Label Map to Image



Park et al, “Semantic Image Synthesis with Spatially-Adaptive Normalization”, CVPR 2019

Conditioning on more than labels! Text to Image

This bird is red and brown in color, with a stubby beak

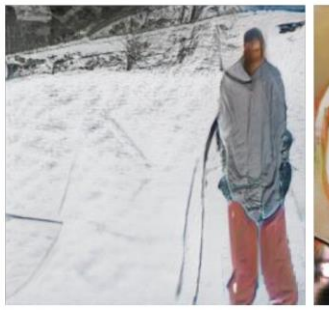
The bird is short and stubby with yellow on its body

A bird with a medium orange bill white body gray wings and webbed feet

This small black bird has a short, slightly curved bill and long legs

A picture of a very clean living room

A group of people on skis stand in the snow



Eggs fruit candy nuts and meat served on white dish

A street sign on a stoplight pole in the middle of a day

Zhang et al, “StackGAN++: Realistic Image Synthesis with Stacked Generative Adversarial Networks.”, TPAMI 2018
Zhang et al, “StackGAN: Text to Photo-realistic Image Synthesis with Stacked Generative Adversarial Networks.”, ICCV 2017
Reed et al, “Generative Adversarial Text-to-Image Synthesis”, ICML 2016

Text to Image: DALL-E

Step 1: Train VQ-VAE (discrete grid of latent codes)

Step 2: Train autoregressive Transformer model to predict sequence of latent codes (Giant model on 250M image/text pairs)

Step 3: Given text prompt, sample new image codes; pass through VQ-VAE decoder to generate images

an illustration of a baby hedgehog in a christmas sweater walking a dog

Ramesh et al, “Zero-Shot Text-to-Image Generation”, ICML 2021

Text to Image: DALL-E

Step 1: Train VQ-VAE (discrete grid of latent codes)

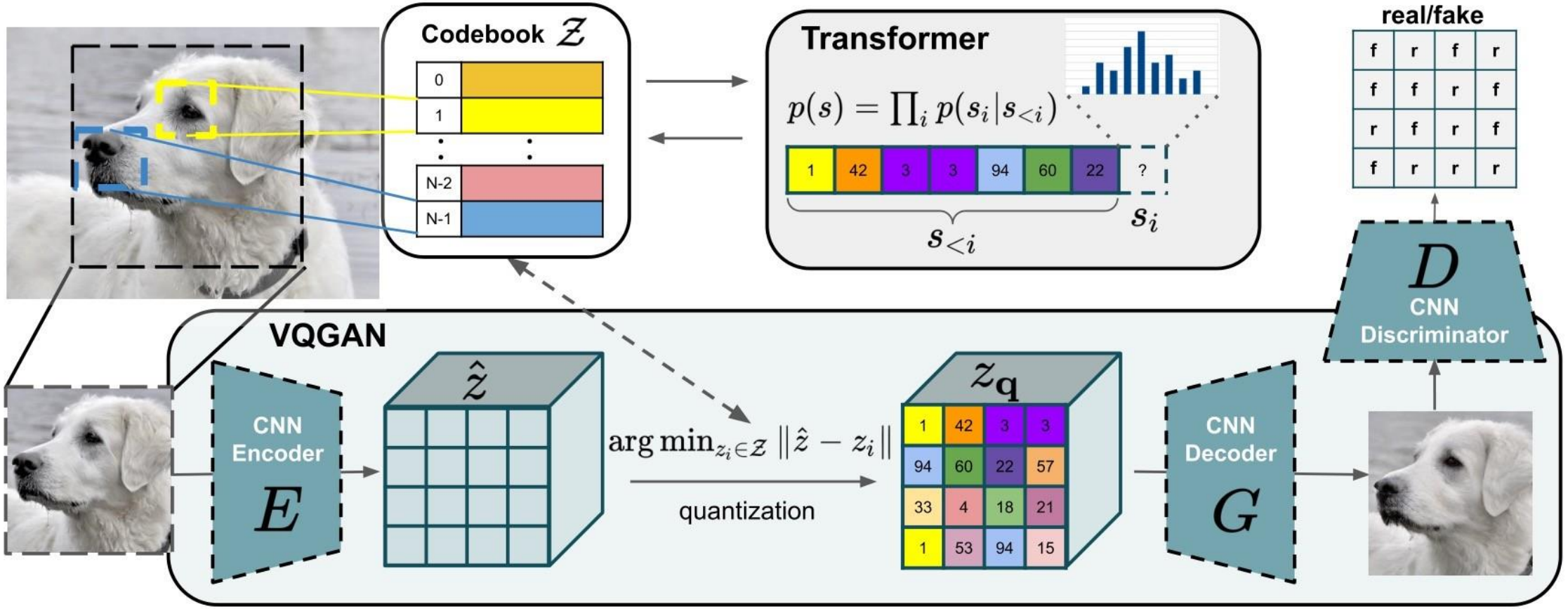
Step 2: Train autoregressive Transformer model to predict sequence of latent codes (Giant model on 250M image/text pairs)

Step 3: Given text prompt, sample new image codes; pass through VQ-VAE decoder to generate images

a neon sign that reads "backprop". a neon sign that reads "backprop". backprop neon sign

Ramesh et al, "Zero-Shot Text-to-Image Generation", ICML 2021

VQ-GAN



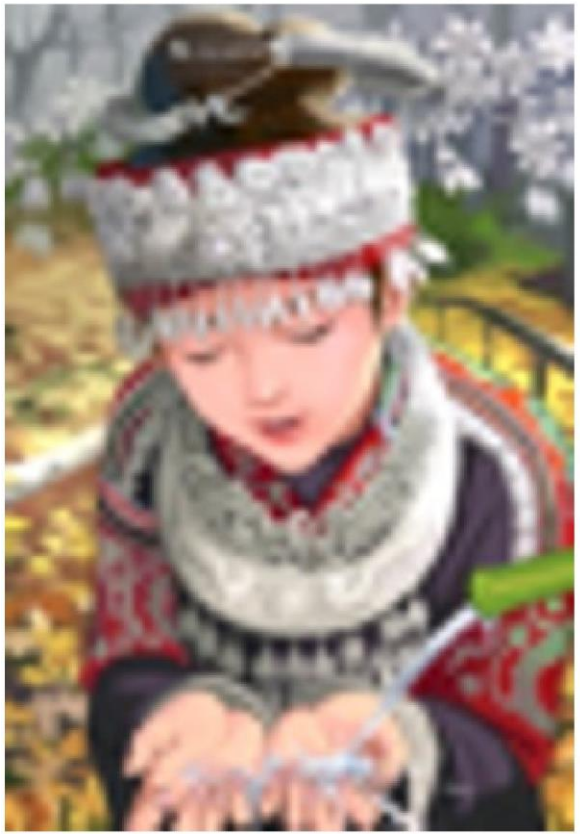
Esser et al, “Taming Transformers for High-Resolution Image Synthesis”, CVPR 2021

VQ-GAN (Semantic Segmentation to Image)

Esser et al, “Taming Transformers for High-Resolution Image Synthesis”, CVPR 2021

Image Super-Resolution: Low-Res to High-Res

bicubic
(21.59dB/0.6423)



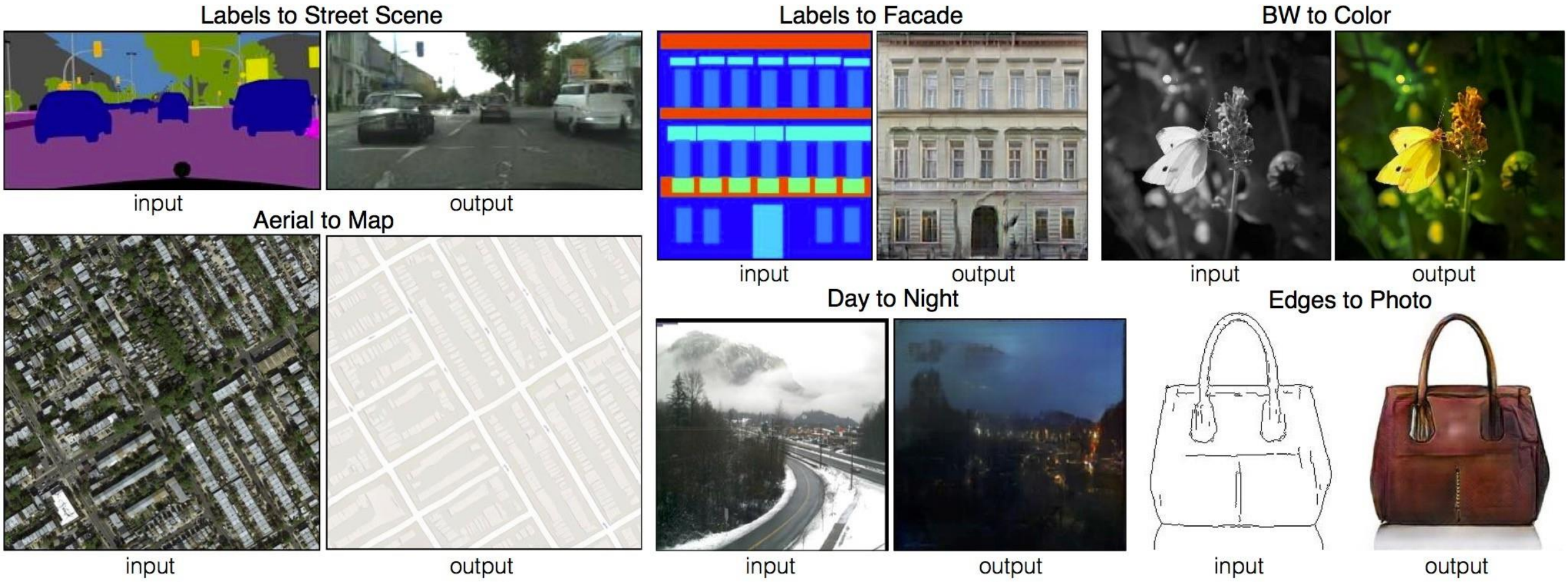
SRResNet
(23.53dB/0.7832)

SRGAN
(21.15dB/0.6868)

original

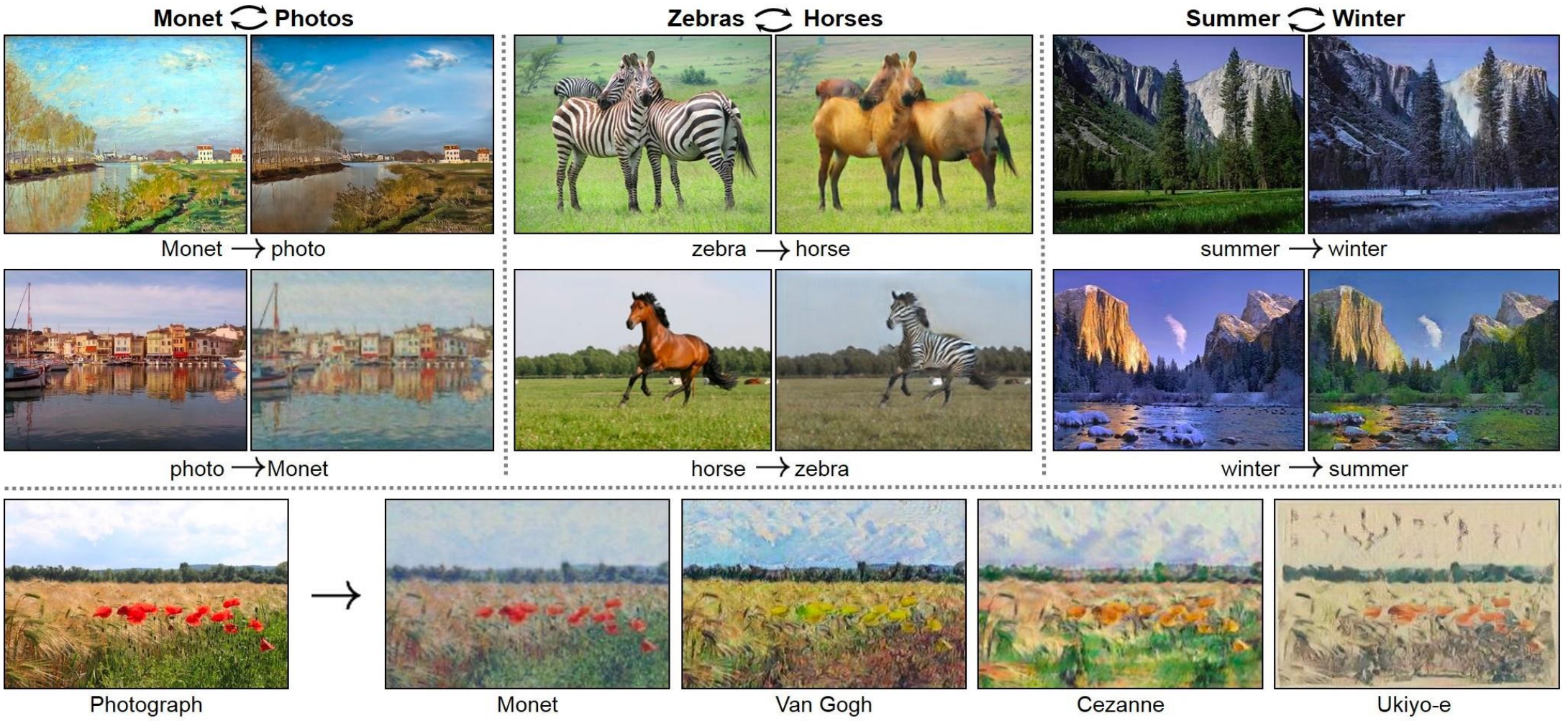
Ledig et al, “Photo-Realistic Single Image Super-Resolution Using a Generative Adversarial Network”, CVPR 2017

Image-to-Image Translation: Pix2Pix



Isola et al, “Image-to-Image Translation with Conditional Adversarial Nets”, CVPR 2017

Unpaired Image-to-Image Translation: CycleGAN



Zhu et al, “Unpaired Image-to-Image Translation using Cycle-Consistent Adversarial Networks”, ICCV 2017

Unpaired Image-to-Image Translation: CycleGAN

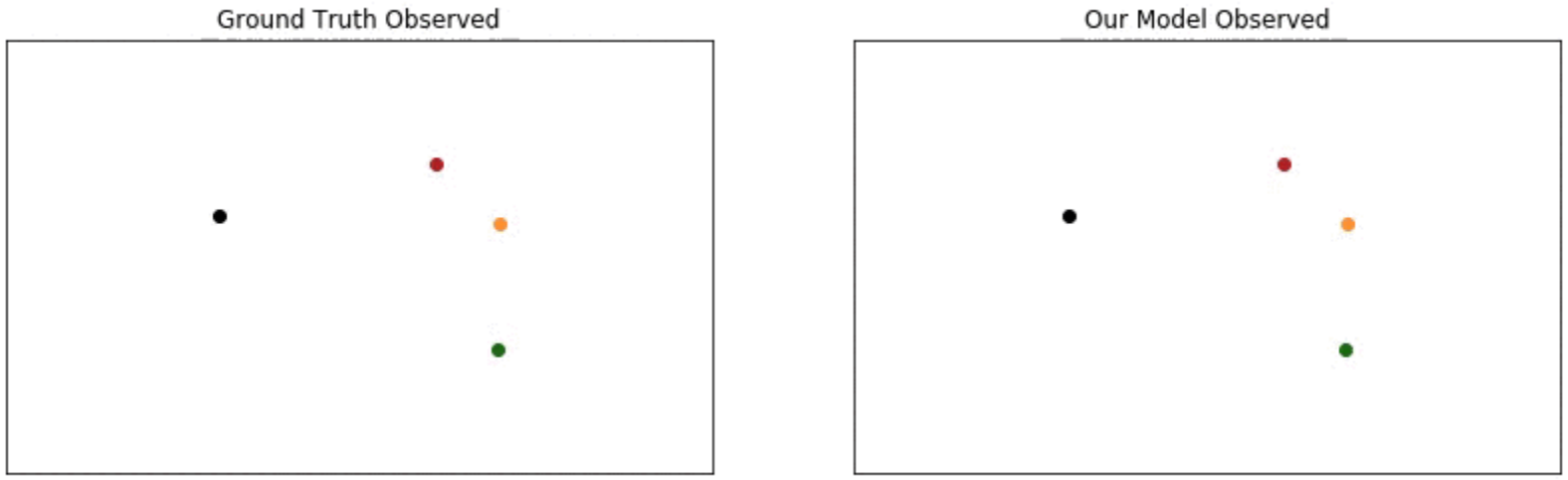
Input Video: Horse

Output Video: Zebra

Zhu et al, “Unpaired Image-to-Image Translation using Cycle-Consistent Adversarial Networks”, ICCV 2017

https://www.youtube.com/watch?v=9reHvktowLY

GANs: Not just for images! Trajectory Prediction

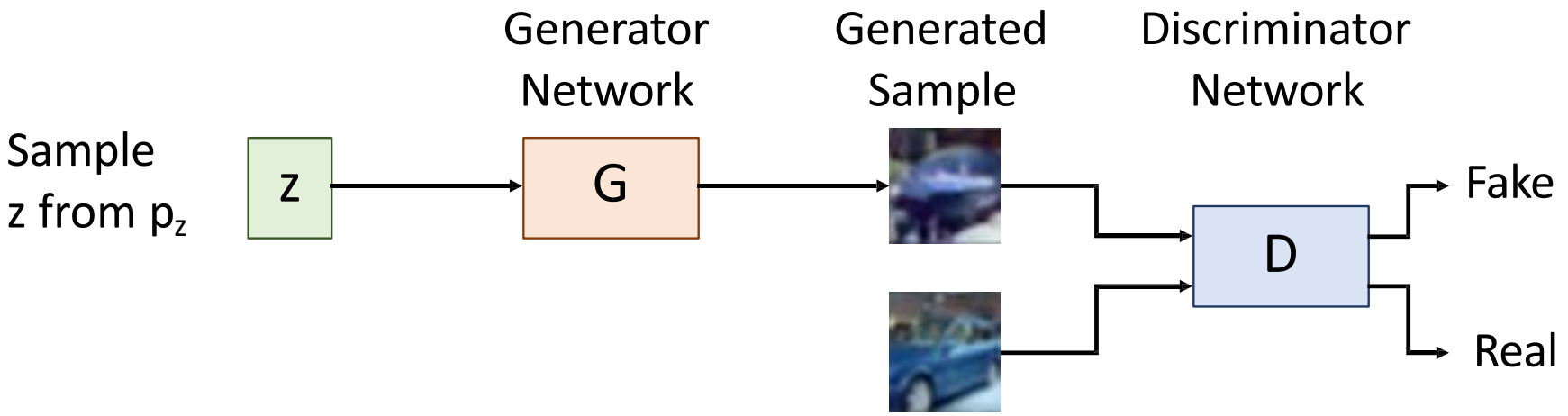


Gupta, **Johnson**, Li, Savarese, Alahi, “Social GAN: Socially Acceptable Trajectories with Generative Adversarial Networks”, CVPR 2018

https://github.com/agrimgupta92/sgan

GAN Summary

Jointly train two networks:
Discriminator: Classify data as real or fake
Generator: Generate data that fools the discriminator



Under some assumptions, generator converges to true data distribution
Many applications
Very active area of research