AITKEN'S DELTA SQUARED PROCESS

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Agenda

- Derivation
- Analytical Analysis
- Graphical Analysis
- Practical Applications
- Comparison with other methods
- Case Study

Problem Statement

- In mathematics, **extrapolation** is the process of estimating, beyond the original observation range, the value of a variable on the basis of its relationship with another variable.

- In numerical analysis, **Aitken's delta-squared process** or **Aitken Extrapolation** is a series acceleration method, used for accelerating the rate of convergence of a sequence.

Methodology

- Aitken's Algorithm
  - Start with a suitable x_1
  - Take $x_2 = g(x_1)$
  - Take $x_3 = g(x_2)$
  - Take $x_4 = x_3 - (x_3-x_2)^2/(x_3-2x_2+x_1)$
  - Take $x_5 = g(x_4)$
  - Take $x_6 = g(x_5)$
  - Take $x_7 = x_6 - (x_6-x_5)^2/(x_6-2x_5+x_4)$

Continue this process to the required accuracy

Methodology

- Practical Applications
  - *Aitkens Method and Rectification of Circle*
  - *Calculating Largest Eigen Values*
  - *Numerical Inversion Transforms of Laplace*
  - *Accelerating Page Rank Computations*
  - *Conjugate Gradient Acceleration for EM Algorithm*

Comparison Table

Minimum Polynomial Extrapolation	***Shanks Transformation***	***Quadratic Extrapolation***	***Steffensen’s Method***
The minimal polynomial extrapolation (MPE) is exceptionally viable procedures that have been utilized as a part of quickening the converging of vector arrangements	In Acceleration technique we take a sequence (sn) and an accelerated sequence is produced. It is offer necessary in computational sciences to get a limit sequence of objects vector spaces.	The extrapolation technique is basically estimating beyond the original observation range, the value of a variable on the basis of its Uncertainty and estimations.	The primary preferred standpoint of Steffensen's strategy is that it has quadratic convergence like Newton's technique

Error Analysis

- Error of extrapolation only increases quadratically as you move away from average of x values when data is known to be linear
- It can be shown that any extrapolation using an interpolating linear function has no statistical significance it is like using a single point to estimate a mean: you cannot say anything about the error associated with your estimator.

Results

- Aitkens process is to used to accelerate the convergence greatly
- It has a wide variety of applications in CS domain
- It is used for rectification of many sequences e.g. circle
- It is very efficient than many other methods like quasi newton and EM extrapolation

Report Demo

Aitkens Delta Squared Process

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This report presents the idea and usefulness of convergence accelerators ideally in a basic way clear to any reader with insignificant scientific and designing aptitudes. Aitken's Δ2 strategy is utilized to accelerate convergence of sequences, e.g. sequences acquired from iterative strategies. Source code is given as an approach to skirt the potential formula confusion with a specific end goal to enable the reader to actualize and utilize the Aitken joining quickening agent instantly subsequent to perusing this report. It also explains the practical applications and comparison of Aitken's Δ2 process with other methods for better understanding

Keywords—**Aitken's Δ2 strategy, convergence accelerators,**

I. Introduction

A. Extrapolation

In arithmetic, extrapolation is the way toward evaluating, past the first perception extend, the estimation of a variable based on its association with another variable. It is like interjection, which produces appraises between known perceptions, yet extrapolation is liable to more noteworthy vulnerability and a higher danger of creating futile outcomes. Extrapolation may likewise mean augmentation of a strategy, accepting comparable strategies will be relevant. Extrapolation may likewise apply to human experience to extend, broaden, or grow known involvement into a zone not referred to or beforehand experienced to touch base at a (typically approximated) information of the obscure (e.g. a driver extrapolates street conditions past his sight while driving). The extrapolation technique can be connected in the inside recreation issue.

There exist in Mathematics and Computer Science an expansive number of iterative calculations whose objective is for the most part to achieve an answer for problem inside a specific tolerance inside a given number of iterations. Iterating implies going over an example of steps and methods that can at times be unpredictable and now and again take a generous measure of time even for quick present day PCs. Examples of basic iterative algorithms are root solvers,
matrix inversion, linear equation solvers, and integrators. Each of these has profound implications in spaces, for example, Engineering, all in all; Computer Graphics; Video Games where runtime execution is basic; and furthermore shockingly in Internet web crawlers for some features.
What if there is possibility that there existed strategies to achieve a similar outcome quicker with less iteration?. This is the reason for meeting convergence accelerators.

B. Aitkens Delta Squared Process

In numerical analysis, Aitken's delta-squared process or Aitken Extrapolation is a series acceleration method, utilized for accelerating the rate of convergence of a sequence. It is named after Alexander Aitken, who presented this strategy in 1926
Its initial form was known to Seki Kōwa (end of seventeenth century) and was found for amendment of the circle, i.e. the count of π.
It is most valuable for accelerating the convergence of a sequence that is converging linearly.

C. *Example Calculations*

1.Find the infinite sum of value $\pi/4$

n	term	x = partial sum	Ax
0	1	1	0.79166667
1	−0.33333333	0.66666667	0.78333333
2	0.2	0.86666667	0.78630952
3	−0.14285714	0.72380952	0.78492063
4	0.11111111	0.83492063	0.78567821
5	-9.0909091×10^{-2}	0.74401154	0.78522034
6	7.6923077×10^{-2}	0.82093462	0.78551795
7	-6.6666667×10^{-2}	0.75426795	--
8	5.8823529×10^{-2}	0.81309148	--

In this case, Aitken's technique is connected to a sublinearly merging arrangement, quickening union significantly. It is still sublinear, yet significantly quicker than the first convergence. the primary Ax esteem, whose calculation required the initial three x esteems, is nearer as far as possible than the eighth x esteem.

2. The value of $\sqrt{2}$ can be calculated using the initial value of a_0 and iterating the following

$$a_{n+1} = (a_n + (2 / a_n)) / 2$$
starting with $a_0 = 1$

n	x = iterated value	Ax
0	1	1.4285714
1	1.5	1.4141414
2	1.4166667	1.4142136
3	1.4142157	--
4	1.4142136	--

It is important here that Aitken's strategy does not spare two cycle steps; calculation of the initial three Ax esteems required the initial five x esteems. Likewise, the second Ax esteem is strongly mediocre compared to the fourth x esteem, for the most part because of the way that Aitken's procedure accept straight, as opposed to quadratic, merging

II. Graphical Introduction to Aitken's Formula and Derivation

Suppose you have a convergent sequence $(x_n)_{n \in N}$ iteratively characterized by $x_{n+1} = f(x_n)$, and that this sequence converges to $\bar{x}$ with $f(\bar{x}) = \bar{x}$.

Given an initial point x_0, you can build the sequence $(x_n)_{n \in N}$ graphically. Figure 1 demonstrates to continue. Place x_0 on the x-axis, discover the point of curve $y = f (x)$ that is vertically lined up with x_0, the y estimation of this point is $f (x_0)$. Since by definition $x_1 = f (x_0)$ we now have discovered the estimation of x_1. To put x_1 graphically on the x-axis, discover the point of curve $y = x$ that is horizontally aligned with $f (x_0)$ and venture this point vertically on the x-axis. Repeating the similar technique will give you x_2 , then x_3 , and you can keep on getting the same number of terms of the sequence $(x_n)_{n \in N}$ as required.

At the present time, we are considering the different root discovering calculations in Numerical Analysis (separation, Newton's, and so on.)
Such calculations yield a succession of numbers, which (ideally) meet to an answer: p_0, p_1, p_2, p_3,

Obviously, each point in the grouping is acquired by estimations and, if there were an approach to join these focuses to get a succession that merges speedier (while not adding much to the calculation unpredictability), there is some advantage. Also, indeed, it is conceivable to utilize the grouping focuses, do the arrangement control and utilize the controlled focuses in the root discovering calculation itself (e. g. Steffensen's strategy).

In this post, I'll discuss Aitken's technique and how one can concoct cases that demonstrate that the strategy can