Project Title:

Black-Scholes Model

By:

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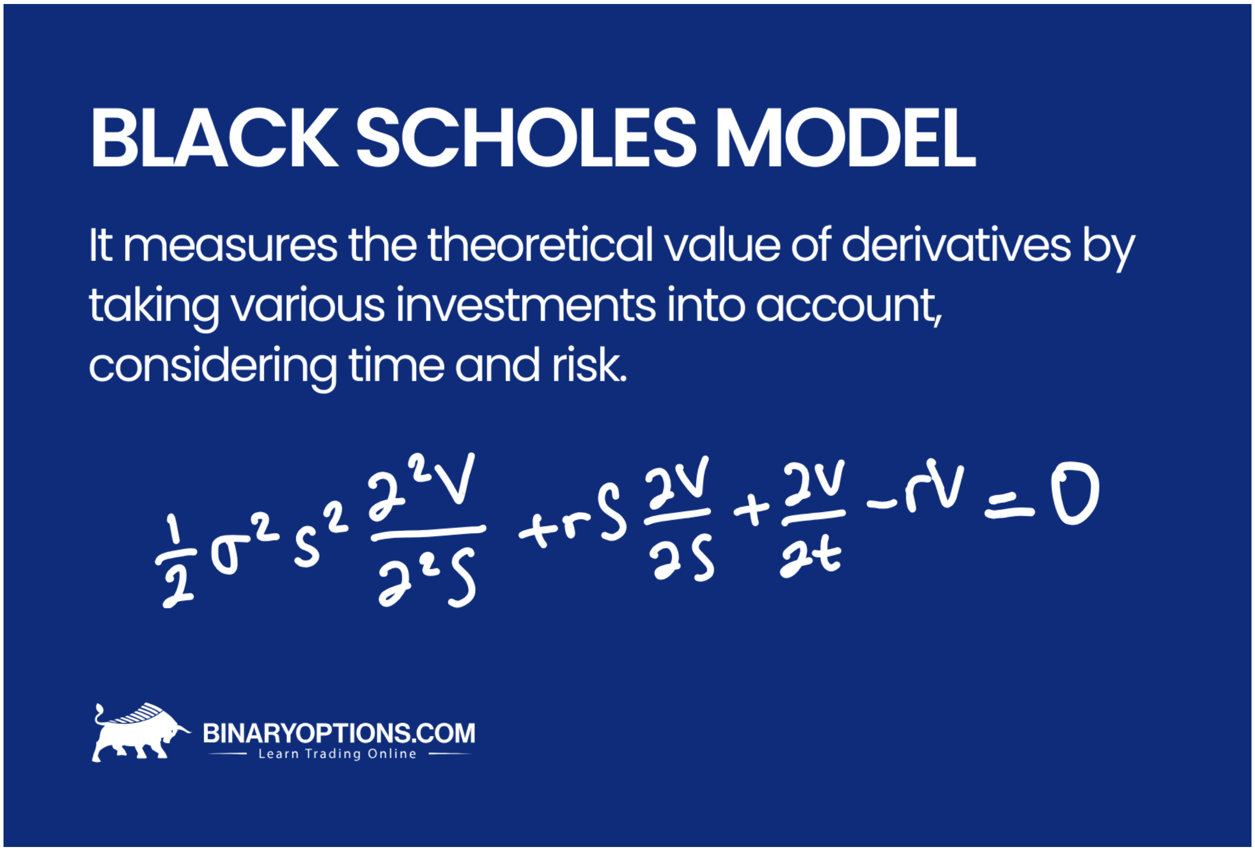


Introduction:

The Black-Scholes Model is a mathematical formula developed by **Fischer Black**, **Myron Scholes**, and **Robert Merton** in the early **1970s** for pricing European-style options. It provides a theoretical framework to determine the fair market value of options based on factors such as the current stock price, strike price, time to expiration, risk-free interest rate, and volatility. The model has been instrumental in options pricing and risk management, although it comes with assumptions and limitations.

Abstract:

The Black-Scholes Model, created by Fischer Black, Myron Scholes, and Robert Merton in the early 1970s, is a groundbreaking mathematical framework for pricing European-style options. It revolutionized financial markets by providing a formula to estimate the fair market value of options based on key variables such as the current stock price, strike price, time to expiration, risk-free interest rate, and volatility. The model assumes efficient markets and continuous trading, presenting

formulas for both call and put option pricing. While widely used in finance, the Black-Scholes Model has limitations, such as assuming constant volatility and interest rates, and serves as a foundational tool for understanding option pricing and risk management.

Terms in Black-Scholes model:

$$C = N(d_1)S_t - N(d_2)Ke^{-rt}$$

$$\text{where } d_1 = \frac{\ln \frac{S_t}{K} + (r + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}$$

$$\text{and } d_2 = d_1 - \sigma\sqrt{t}$$

C = call option price
N = CDF of the normal distribution
S_t = spot price of an asset
K = strike price
r = risk-free interest rate
t = time to maturity
σ = volatility of the asset

The Black-Scholes Model incorporates several key terms and variables that play crucial roles in option pricing. Here are the primary terms used in the Black-Scholes Model:

- **S_0 (Stock Price):** The current market price of the underlying asset (e.g., a stock).
- **K (Strike Price):** The predetermined price at which the option holder can buy (for a call option) or sell (for a put option) the underlying asset.
- **T (Time to Expiration):** The remaining time until the option expires. It is usually expressed in years.
- **r (Risk-Free Interest Rate):** The interest rate that assumes no risk of financial loss. It is typically the rate of return on a risk-free investment, such as government bonds.

- **σ (Volatility):** A measure of the stock price's variability over time. It reflects the degree of uncertainty or risk associated with the underlying asset.
- **N(d_1) and N(d_2):** Cumulative distribution functions of the standard normal distribution. These functions are used in the Black-Scholes formulas to calculate probabilities associated with the option pricing.
- **C (Call Option Price):** The theoretical price of a European call option, representing the cost of buying the option to purchase the underlying asset at the strike price.
- **P (Put Option Price):** The theoretical price of a European put option, representing the cost of buying the option to sell the underlying asset at the strike price.

The Black-Scholes Model provides formulas for calculating the call and put option prices (C and P) based on these terms. The model is widely used in financial markets for option pricing and risk management, but it has assumptions and limitations that may impact its accuracy in certain market conditions. Traders and analysts often adjust the model to account for real-world complexities and market dynamics.

Scenario:

Imagine an investor, Alice, is interested in purchasing a European call option on a stock. The current stock price (S_0) is $100, the strike price (K) is $105, the time to expiration (T) is 3 months (or 0.25 years), the risk-free interest rate (r) is 3%, and the stock's volatility (σ) is 20%.

Using the Black-Scholes Model:

Calculate (d_1) and (d_2):

$(d_1)=\ln((S_0/K)+(r+\sigma^2/2)T/\sigma\sqrt{T}$

$(d_2) = (d_1)- \sigma\sqrt{T}$

Plugging in the values, we calculate (d_1) and (d_2):

Calculate N(d_1) and N(d_2):

Using the cumulative distribution function of the standard normal distribution, find the probabilities N(d_1) and N(d_2):

Calculate Call Option Price (C):

C= (S_0) ·N(d 1)−K·e^{-rt} ·N(d 2)

Substituting the values, we calculate the call option price.

Results:

(d_1)=-0.4124

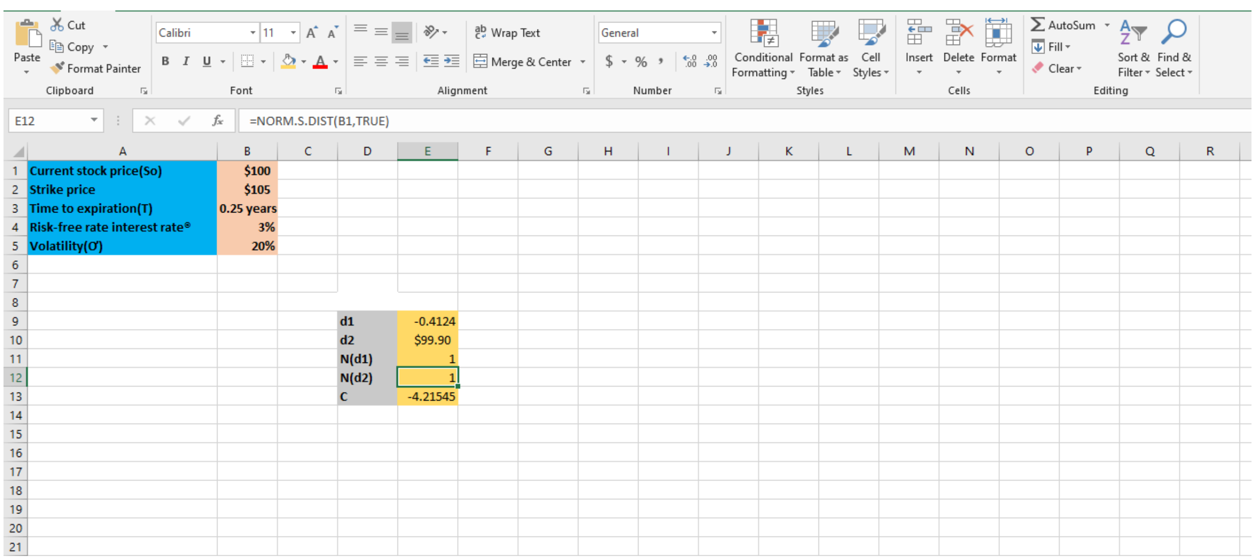
(d_2)=$99.90

N(d_1)=1

N(d_2)=1

Call Option Price (C) ≈ $-4.21545

Interpretation:



E12 =NORM.S.DIST(B1,TRUE)

	A	B	C	D	E
1	Current stock price(So)	$100			
2	Strike price	$105			
3	Time to expiration(T)	0.25 years			
4	Risk-free rate interest rate*	3%			
5	Volatility(σ)	20%			
9				d1	-0.4124
10				d2	$99.90
11				N(d1)	1
12				N(d2)	1
13				C	-4.21545

In this scenario, the Black-Scholes Model estimates the theoretical price of the European call option to be approximately $-4.21545. This means that, based on the specified parameters, Alice would expect to pay around $-4.21545 for the option, allowing her the right to buy the stock at $105 within the next three months.

It's important to note that this is a simplified example, and real-world scenarios may involve additional factors and considerations. Traders often use the Black-Scholes Model as a starting point and adjust it based on market conditions and other relevant information.

Conclusion:

The Black-Scholes Model has had a profound impact on the field of financial derivatives and has become a cornerstone for options pricing and risk management. In conclusion, several key points can be highlighted regarding the Black-Scholes Model:

Revolutionary Contribution: The Black-Scholes Model revolutionized the way financial professionals approach options pricing. Introduced in 1973, it provided a groundbreaking framework for determining the theoretical value of European-style options.

Educational and Practical Tool: The Black-Scholes Model serves as a valuable educational tool, helping students and professionals understand the principles of options pricing. It is also a practical tool for traders and investors, providing a starting point for evaluating option values and risks.

In conclusion, while the Black-Scholes Model has played a pivotal role in shaping financial markets, it is essential for market participants to recognize its assumptions and limitations. Ongoing research and the development of more sophisticated models continue to refine our understanding of options pricing and risk management in dynamic financial environments.

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