能量密度:

$$\mathcal{E} = \frac{1}{2}|f|^2 = \frac{1}{2}ff^* \tag{1}$$

各种能量密度定义:

$$\begin{aligned}
\mathcal{E}_k &= \frac{1}{2}\nabla_\perp\phi^* \cdot \nabla_\perp\phi \\
\mathcal{E}_m &= \frac{1}{2}\nabla_\perp A^* \cdot \nabla_\perp A \\
\mathcal{E}_n &= \frac{1}{2}n^* n \\
\mathcal{E}_v &= \frac{1}{2}v^* v \\
\mathcal{E}_t &= \frac{1}{2}T^* T
\end{aligned} \tag{2}$$

考虑平行压缩项:

$$\begin{aligned}
&\partial_t\mathcal{E}_k \times 2 = -\frac{1}{n_{eq}}(\phi^*\nabla_\parallel j + \phi\nabla_\parallel j^*) \\
&\partial_t\frac{\beta}{n_{eq}}\mathcal{E}_m \times 2 = -\frac{1}{n_{eq}}(j^*\nabla_\parallel\phi + j\nabla_\parallel\phi^*) + \tau\frac{T_{eq}}{n_{eq}^2}(j^*\nabla_\parallel n + j\nabla_\parallel n^*) \\
&\partial_t\tau\frac{T_{eq}}{n_{eq}^2}\mathcal{E}_n \times 2 = \tau\frac{T_{eq}}{n_{eq}^2}(n^*\nabla_\parallel j + n\nabla_\parallel j^*) - \tau\frac{T_{eq}}{n_{eq}}(n^*\nabla_\parallel v + n\nabla_\parallel v^*) \\
&\partial_t\frac{\tau}{\tau+1}\mathcal{E}_v \times 2 = -\frac{\tau}{\tau+1}(v^*\nabla_\parallel T + v\nabla_\parallel T^*) - \tau\frac{T_{eq}}{n_{eq}}(v^*\nabla_\parallel n + v\nabla_\parallel n^*) \\
&\partial_t\frac{\tau}{(\Gamma-1)(\tau+1)T_{eq}}\mathcal{E}_t \times 2 = -\frac{\tau}{\tau+1}(T^*\nabla_\parallel v + T\nabla_\parallel v^*)
\end{aligned} \tag{3}$$

空间平均:

$$< f >= \frac{\int f dV}{\int dV} = \frac{\int f \ rdrd\theta d\zeta}{\int \ rdrd\theta d\zeta} \tag{4}$$

对于谱方法:

$$< f >= \frac{\int f(r) \ rdr}{r^2/2} \tag{5}$$

各种能量定义:

$$\begin{aligned}
E_k &=< \mathcal{E}_k > \\
E_m &=< \frac{\beta}{n_{eq}}\mathcal{E}_m > \\
E_n &=< \tau\frac{T_{eq}}{n_{eq}^2}\mathcal{E}_n > \\
E_v &=< \frac{\tau}{\tau+1}\mathcal{E}_v > \\
E_t &=< \frac{\tau}{(\Gamma-1)(\tau+1)T_{eq}}\mathcal{E}_t > \\
E_{total} &= E_k + E_m + E_n + E_v + E_t
\end{aligned} \tag{6}$$

由于:

$$\begin{aligned}
< \nabla_\parallel(AB) > &= \int \vec{b}\cdot\nabla(AB) \ dV/\int dV \\
&= \int \nabla\cdot(AB\vec{b}) \ dV/\int dV \\
&= \int (AB\vec{b}) \ d\vec{S}/\int dV = 0
\end{aligned} \tag{7}$$

则只考虑平行压缩项时，有:

$$\partial_t E_{total} = 0 \tag{8}$$

考虑对流项 (*):

$$\begin{aligned} < f^*[\phi, f] + f[\phi, f^*] > &= < f[f^*, \phi] + f^*[f, \phi] > \\ &= - < f[\phi, f^*] + f^*[\phi, f] > \end{aligned} \tag{9}$$

则有:

$$< f^*[\phi, f] + f[\phi, f^*] >= 0 \tag{10}$$

考虑径向输运:

$$\begin{aligned} Q_n &= < \tau \frac{T_{eq}}{n_{eq}} \frac{a}{L_n} (n^* \nabla_\theta \phi + n \nabla_\theta \phi^*) > \\ &\quad + < \beta \frac{T_{eq}}{n_{eq}^2} \frac{\partial j_0}{\partial r} (n^* \nabla_\theta A + n \nabla_\theta A^*) > \\ Q_v &= - < \frac{\tau}{\tau + 1} \beta T_{eq} \frac{a}{L_n} (1 + \eta_i + \tau)(v^* \nabla_\theta A + v \nabla_\theta A^*) > \\ Q_t &= < \frac{\tau \eta_i}{(\Gamma - 1)(\tau + 1)} \frac{a}{L_n} (T^* \nabla_\theta \phi + T \nabla_\theta \phi^*) > \\ Q_m &= < \frac{\beta}{n_{eq}} \tau T_{eq} \frac{a}{L_n} (j^* \nabla_\theta A + j \nabla_\theta A^*) > \\ Q_k &= < T_{eq} (1 + \eta_i) \frac{a}{L_n} (\phi^* \nabla_\theta \nabla_\perp^2 \phi + \phi \nabla_\theta \nabla_\perp^2 \phi^*) > \\ &\quad + < \frac{\beta}{n_{eq}} \frac{\partial j_0}{\partial r} (\phi^* \nabla_\theta A + \phi \nabla_\theta A^*) > \end{aligned} \tag{11}$$

考虑耗散项:

$$\begin{aligned} S_n &= -\tau \frac{T_{eq}}{n_{eq}^2} \mu_n < |\nabla_\perp n|^2 > \times 2 \\ S_v &= -\frac{\tau}{\tau + 1} \mu_v < |\nabla_\perp v|^2 > \times 2 \\ S_t &= -\frac{\tau}{(\Gamma - 1)(\tau + 1) T_{eq}} \mu_t < |\nabla_\perp T|^2 > \times 2 \\ S_m &= -\frac{1}{n_{eq}} \mu_m < j^* j > \times 2 \\ S_k &= -\mu_k < |\nabla_\perp^2 \phi|^2 > \times 2 \end{aligned} \tag{12}$$

考虑曲率项:

$$C_k, C_n, C_t \tag{13}$$

考虑朗道阻尼项:

$$\begin{aligned} D_m &= < \frac{1}{n_{eq}} \sqrt{\frac{\pi \tau}{2} \frac{m_e}{m_i}} [j^* |\nabla_\parallel| (v - \frac{j}{n_{eq}}) + j |\nabla_\parallel| (v^* - \frac{j^*}{n_{eq}})] > \\ D_t &= - < \frac{\tau}{(\tau + 1) T_{eq}} \sqrt{\frac{8 T_{eq}}{\pi}} (T^* |\nabla_\parallel| T + T |\nabla_\parallel| T^*) > \end{aligned} \tag{14}$$

则有:

$$\partial_t E_{total} \times 2 = \Sigma_i Q_i + \Sigma_i S_i + \Sigma_i C_i + \Sigma_i D_i \tag{15}$$

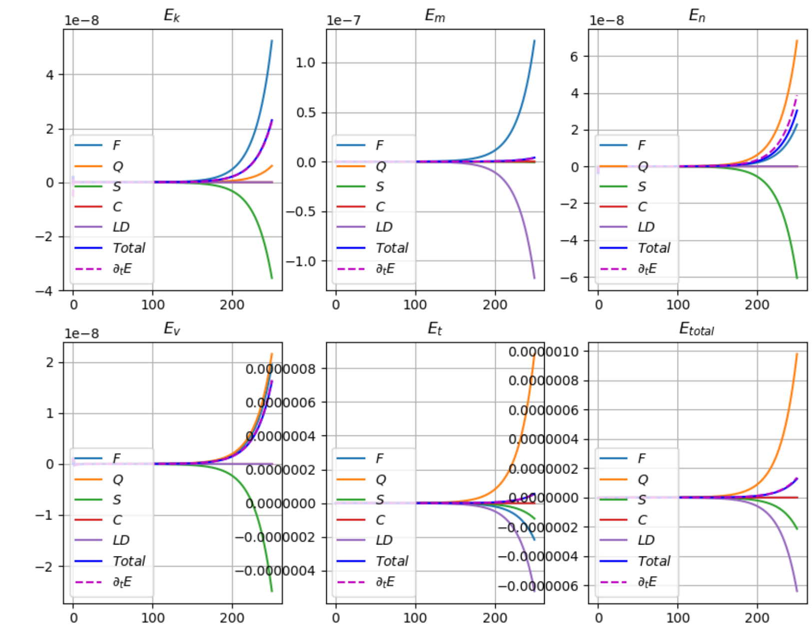

Figure 1: 无曲率项，线性

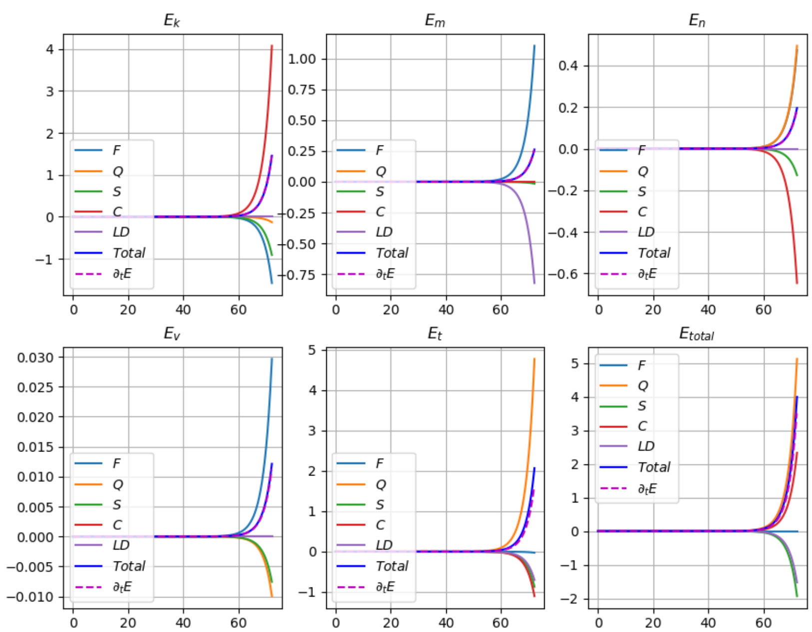

Figure 2: 含曲率项，线性

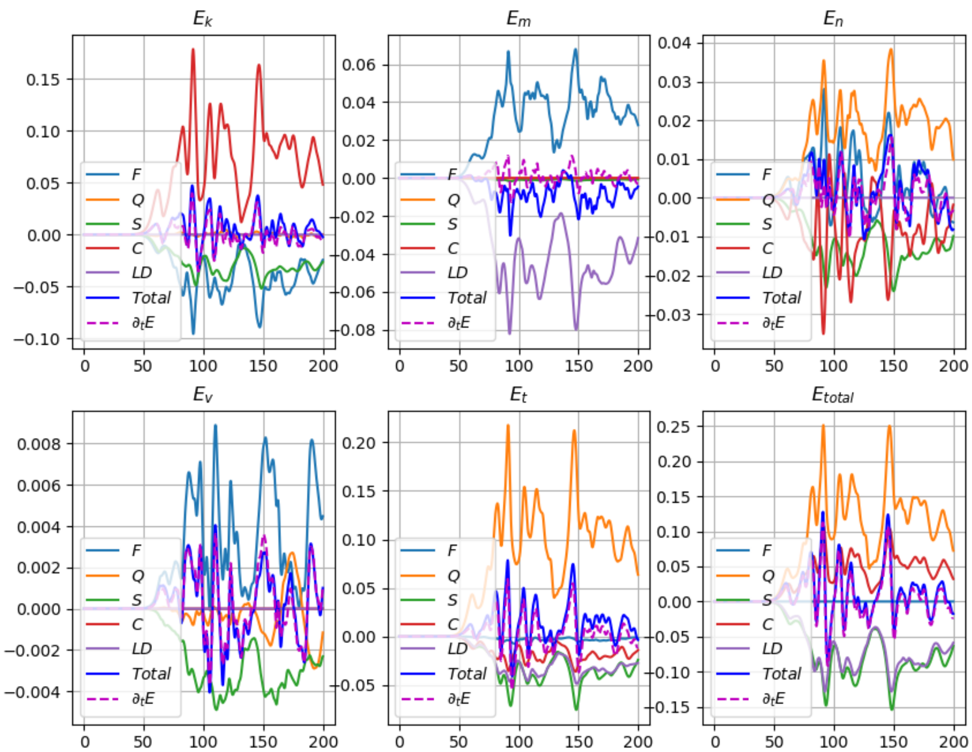

Figure 3: 含曲率项，非线性