

Energy transfer in ITG/KBM system

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1 model equation

$$\frac{dn}{dt} = a \frac{dn_{eq}}{dr} \nabla_{\theta} \phi - n_{eq} \nabla_{\parallel} v + \nabla_{\parallel} j + \omega_d (n_{eq} \phi - p_e) + D_n \nabla_{\perp}^2 n \quad (1)$$

$$\frac{d\nabla_{\perp}^2 \phi}{dt} = -a T_{eq} \left(\frac{1}{n} \frac{dn_{eq}}{dr} + \frac{1}{T_{eq}} \frac{dT_{eq}}{dr} \right) \nabla_{\theta} \nabla_{\perp}^2 \phi + \frac{1}{n_{eq}} \nabla_{\parallel} j - \omega_d \left(T_i + \frac{T_{eq}}{n_{eq}} n + \frac{p_e}{n_{eq}} \right) + D_U \nabla_{\perp}^4 \phi \quad (2)$$

$$\frac{dv}{dt} = -\nabla_{\parallel} T_i - 2 \frac{T_{eq}}{n_{eq}} \nabla_{\parallel} n - \beta a T_{eq} \left(\frac{2}{n_{eq}} \frac{dn_{eq}}{dr} + \frac{1}{T_{eq}} \frac{dT_{eq}}{dr} \right) \nabla_{\theta} A + D_v \nabla_{\perp}^2 v \quad (3)$$

$$\beta \frac{\partial A}{\partial t} = -\nabla_{\parallel} \phi + \frac{T_{eq}}{n_{eq}} \nabla_{\parallel} n + \beta a T_{eq} \frac{1}{n_{eq}} \frac{dn_{eq}}{dr} \nabla_{\theta} A + \sqrt{\frac{\pi m_e}{2 m_i}} |\nabla_{\parallel}| \left(v - \frac{j}{n_{eq}} \right) - \eta j \quad (4)$$

$$\frac{dT_i}{dt} = a \frac{\partial T_{eq}}{\partial r} \nabla_{\theta} \phi - (\Gamma - 1) T_{eq} \nabla_{\parallel} v + T_{eq} \omega_d \left((\Gamma - 1) \phi + (2\Gamma - 1) T_i + (\Gamma - 1) \frac{T_{eq}}{n_{eq}} n \right) - (\Gamma - 1) \sqrt{\frac{8 T_{eq}}{\pi}} |\nabla_{\parallel}| T_i + D_T \nabla_{\perp}^2 T_i \quad (5)$$

2 energy balance equation

energy in system

$$\begin{aligned} E_k &= \frac{1}{2} \langle |\nabla_{\perp} \phi|^2 \rangle \\ E_m &= \frac{1}{2} \langle \frac{\beta}{n_{eq}} |\nabla_{\perp} \psi|^2 \rangle \\ E_n &= \frac{1}{2} \langle \tau \frac{T_{eq}}{n_{eq}^2} |n|^2 \rangle \\ E_v &= \frac{1}{2} \langle \frac{\tau}{\tau + 1} |v|^2 \rangle \\ E_t &= \frac{1}{2} \langle \frac{\tau}{(\Gamma - 1)(\tau + 1) T_{eq}} |T|^2 \rangle \end{aligned} \quad (6)$$

where

$$\langle f \rangle = \frac{\int f dV}{\int dV} = \frac{\int f r dr d\theta d\zeta}{\int r dr d\theta d\zeta} \quad (7)$$

total energy

$$E_{total} = E_k + E_m + E_n + E_v + E_t \quad (8)$$

time dependence of energy

$$\frac{d}{dt} E_i = \Gamma_F + \Gamma_Q + \Gamma_C + \Gamma_{LD} + \Gamma_{NL} + \Gamma_S \quad (9)$$

radial transport items

$$\begin{aligned}
\Gamma_{Q,n} &= \langle \tau a \frac{T_{eq}}{n_{eq}^2} n \nabla_\theta \phi \rangle + \langle \beta \tau \frac{T_{eq}}{n_{eq}^2} \frac{\partial j_0}{\partial r} n \nabla_\theta A \rangle \\
\Gamma_{Q,v} &= - \langle \frac{\tau}{\tau+1} \beta a T_{eq} (\frac{2}{n_{eq}} \frac{dn_{eq}}{dr} + \frac{1}{T_{eq}} \frac{dT_{eq}}{dr}) v \nabla_\theta A \rangle \\
\Gamma_{Q,t} &= \langle \frac{\tau}{(\Gamma-1)(\tau+1)T_{eq}} a \frac{\partial T_{eq}}{\partial r} T \nabla_\theta \phi \rangle \\
\Gamma_{Q,k} &= \langle -a T_{eq} (\frac{1}{n} \frac{dn_{eq}}{dr} + \frac{1}{T_{eq}} \frac{dT_{eq}}{dr}) \phi \nabla_\theta \nabla_\perp^2 \phi \rangle + \langle \frac{\beta}{n_{eq}} \frac{\partial j_0}{\partial r} \phi \nabla_\theta A \rangle \\
\Gamma_{Q,m} &= \langle \frac{\beta}{n_{eq}} a T_{eq} \frac{1}{n_{eq}} \frac{dn_{eq}}{dr} j \nabla_\theta A \rangle
\end{aligned} \tag{10}$$

parallel transport items

$$\Gamma_{F,k} = \langle \rangle \tag{11}$$

curvature items

$$\Gamma_{C,k} = \langle \rangle \tag{12}$$

landau damping items

$$\begin{aligned}
D_m &= \langle \frac{1}{n_{eq}} \sqrt{\frac{\pi \tau m_e}{2 m_i}} j \nabla_\parallel | (v - \frac{j}{n_{eq}}) \rangle \\
D_t &= - \langle \frac{\tau}{(\tau+1)T_{eq}} \sqrt{\frac{8T_{eq}}{\pi}} T |\nabla_\parallel| T \rangle
\end{aligned} \tag{13}$$

diffusion items

$$\begin{aligned}
\Gamma_{S,k} &= -D_U \langle |\nabla_\perp^2 \phi|^2 \rangle \\
\Gamma_{S,m} &= -\frac{1}{n_{eq}} D_m \langle |j|^2 \rangle \\
\Gamma_{S,n} &= \tau \frac{T_{eq}}{n_{eq}^2} D_n \langle |\nabla_\perp n|^2 \rangle \\
\Gamma_{S,v} &= -\frac{\tau}{\tau+1} D_v \langle |\nabla_\perp v|^2 \rangle \\
\Gamma_{S,t} &= -\frac{\tau}{(\Gamma-1)(\tau+1)T_{eq}} D_t \langle |\nabla_\perp T|^2 \rangle
\end{aligned} \tag{14}$$

time dependence of energy

$$\frac{\partial}{\partial t} E_{total} = \Sigma_i [\frac{1}{2} (\Gamma_{Q,i} + \Gamma_{C,i} + \Gamma_{LD,i}) + \Gamma_{S,i}] \tag{15}$$

3 turbulence

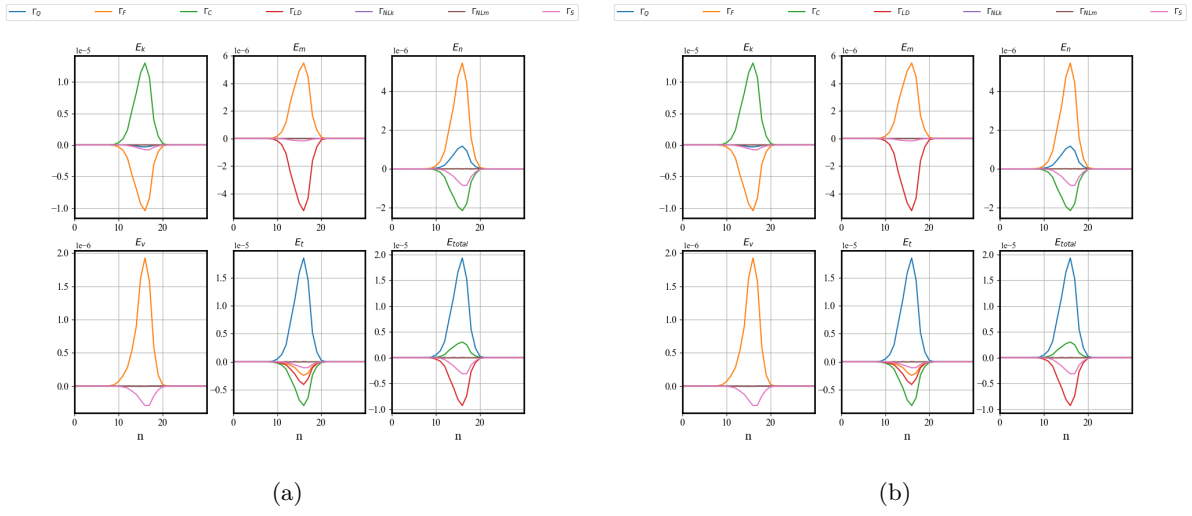


Figure 1: (a),(b)

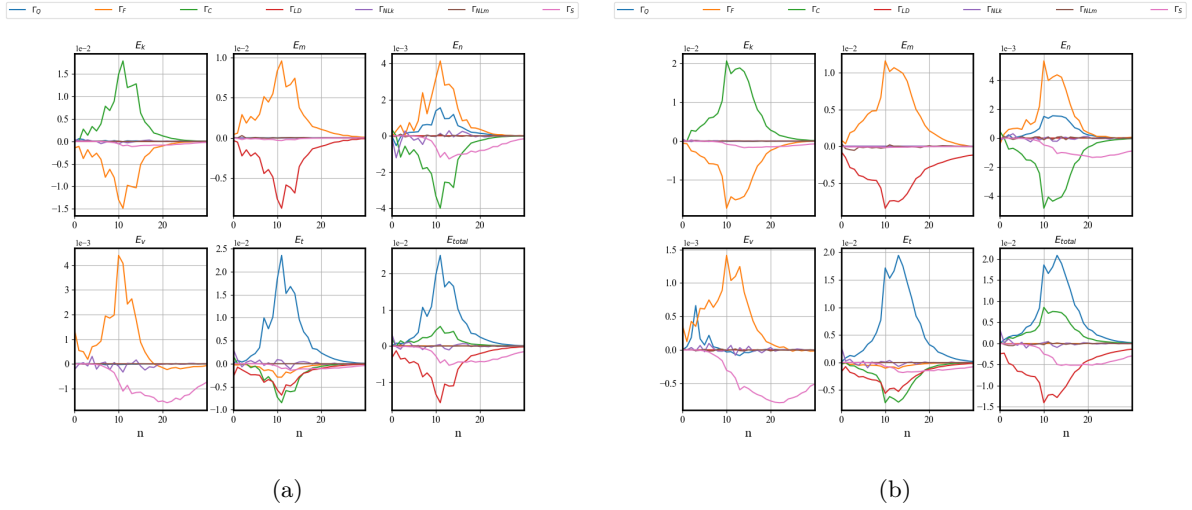


Figure 2: (a),(b)

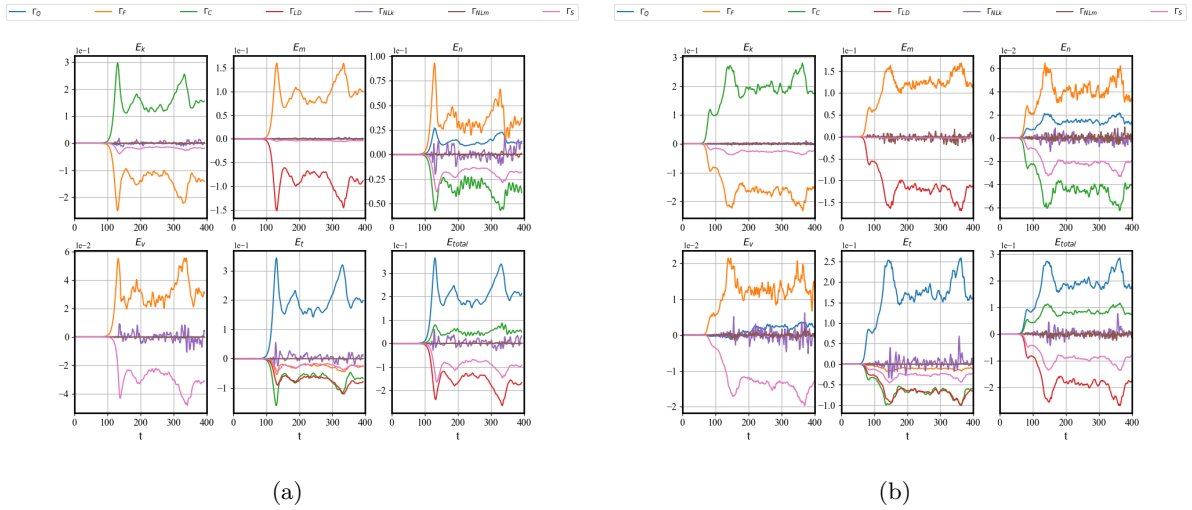


Figure 3: (a),(b)

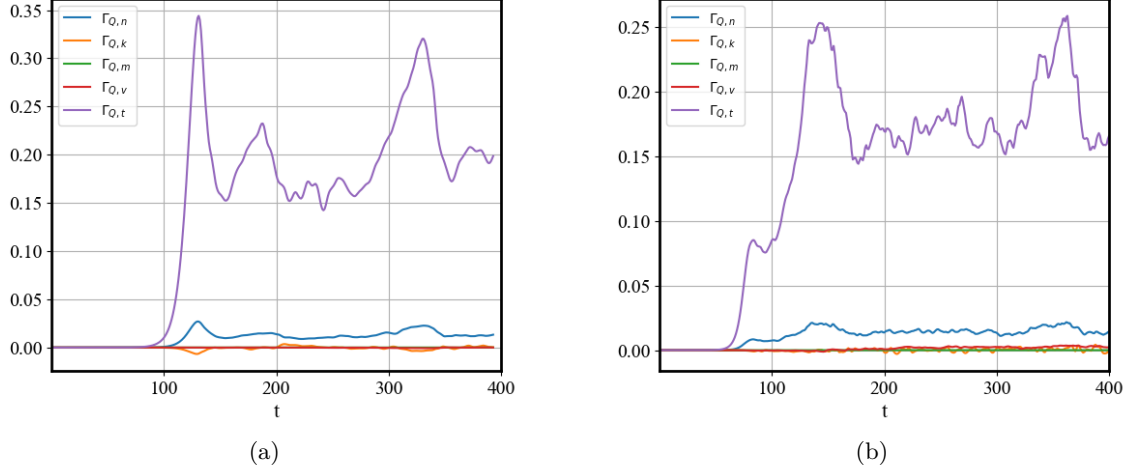


Figure 4: (a),(b)

4 zonal flow character

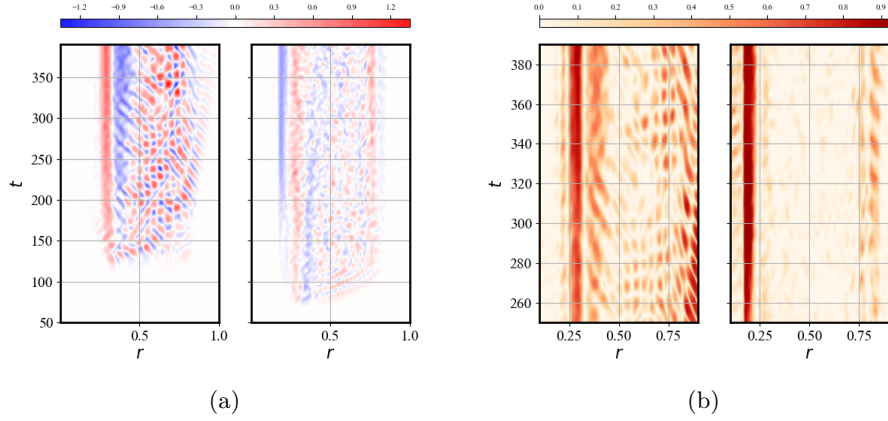


Figure 5: (a)zonal flow as a function of radius and time,(b) $E_{k,ZF}/E_{k,all}$ as a function of radius and time

5 energy drive of $E_{k,m=0,n=0}$ and $E_{p,m=1,n=0}$

for zonal flow energy:

$$\frac{\partial}{\partial t} E_k = \Gamma_{NLk,k} + \Gamma_{NLm,k} + \Gamma_{C,k} \quad (16)$$

for pressure:

$$p = p_i + p_e = n_0 T + (1 + \tau) T_0 n \quad (17)$$

$$\frac{\partial}{\partial t} E_p = \Gamma_{Q,p} + \Gamma_{F,p} + \Gamma_{C,p} + \Gamma_{LD,p} + \Gamma_{NL,p} + \Gamma_{S,p} \quad (18)$$

$$\begin{aligned}
\Gamma_{Q,p} &= \langle a[(1+\tau)T_{eq} \frac{\partial n_{eq}}{\partial r} + n_{eq} \frac{\partial T_{eq}}{\partial r}] p \nabla_{\theta} \phi \rangle + \langle (1+\tau)\beta T_{eq} \frac{\partial j_0}{\partial r} p \nabla_{\theta} A \rangle \\
\Gamma_{F,p} &= - \langle (\Gamma + \tau) n_{eq} T_{eq} p \nabla_{\parallel} v \rangle + \langle (1+\tau) T_{eq} p \nabla_{\parallel} j \rangle \\
\Gamma_{LD,p} &= - \langle (\Gamma - 1) \sqrt{\frac{8T_{eq}}{\pi}} n_{eq} p |\nabla_{\parallel}| T \rangle \\
\Gamma_{C,p} &= \langle p \omega_d ((\Gamma + \tau) n_{eq} T_{eq} \phi) \rangle + \langle p \omega_d ((2\Gamma - 1) n_{eq} T_{eq} T) \rangle + \langle p \omega_d (((\Gamma - 1) - \tau(\tau + 1)) T_{eq}^2 n) \rangle \\
\Gamma_{NL,p} &= - \langle p((1+\tau) T_{eq} [\phi, n] + n_{eq} [\phi, T]) \rangle + \langle p(\Gamma + \tau) n_{eq} T_{eq} \beta [A, v] \rangle - \langle p(1+\tau) T_{eq} \beta [A, j] \rangle \\
\Gamma_{S,p} &= \langle p[(1+\tau) T_{eq} D_n \nabla_{\perp}^2 n + n_{eq} D_T \nabla_{\perp}^2 T] \rangle
\end{aligned} \tag{19}$$

5.1 case of $\beta = 0.1\%$

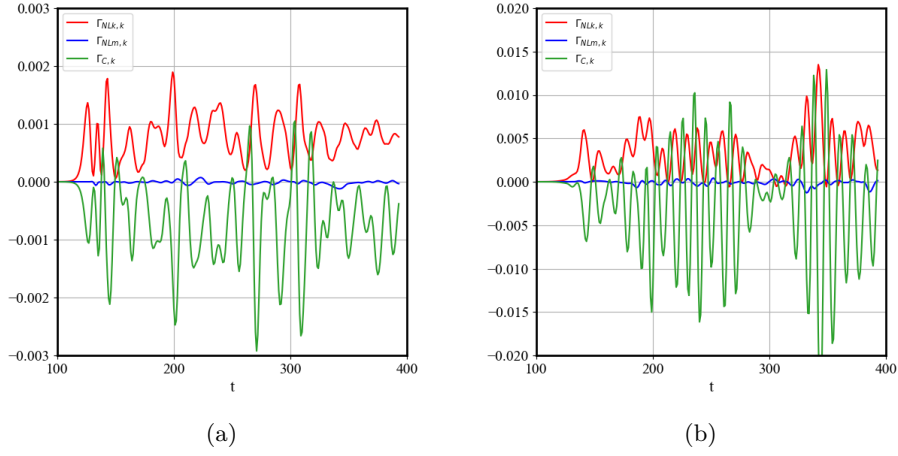


Figure 6: (a),(b)

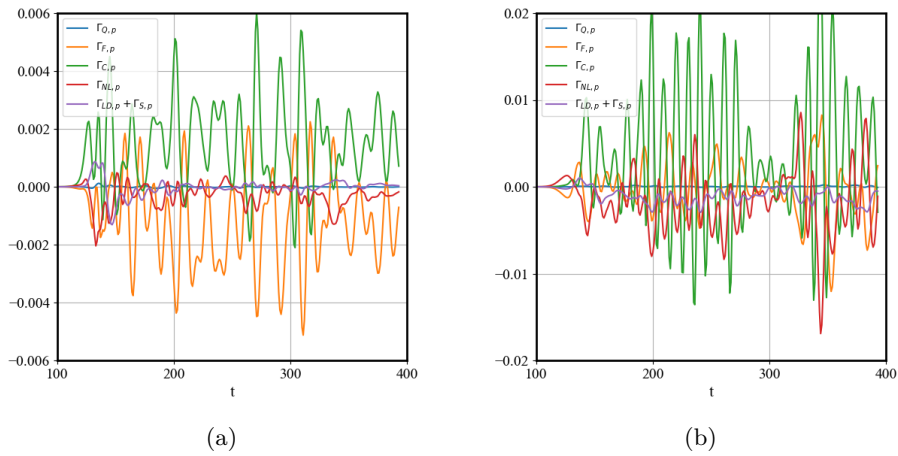
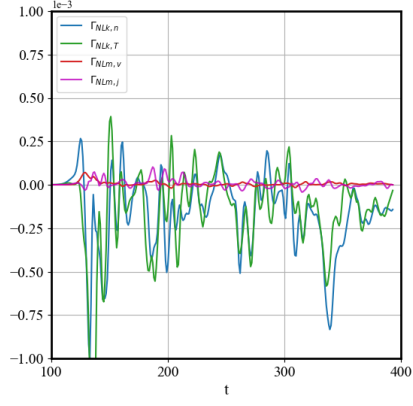
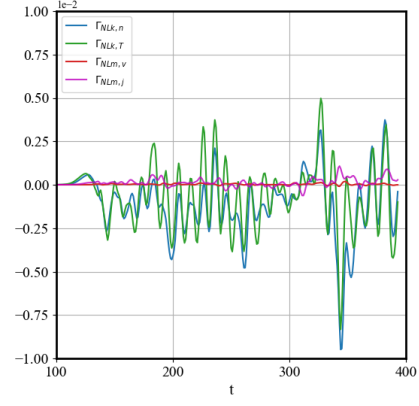


Figure 7: (a),(b)



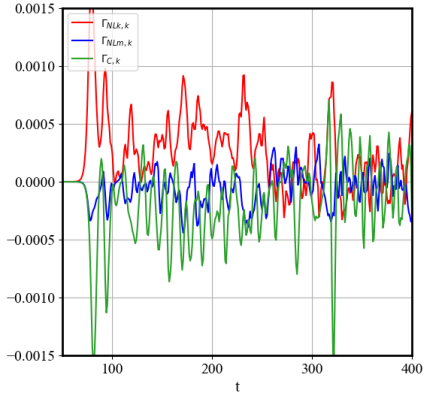
(a)



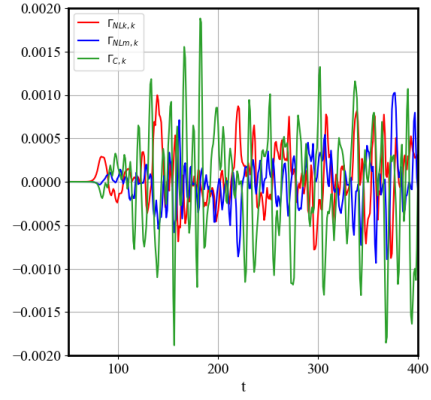
(b)

Figure 8: (a),(b)

5.2 case of $\beta = 1.0\%$

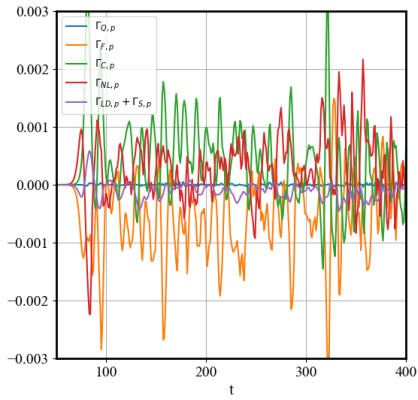


(a)

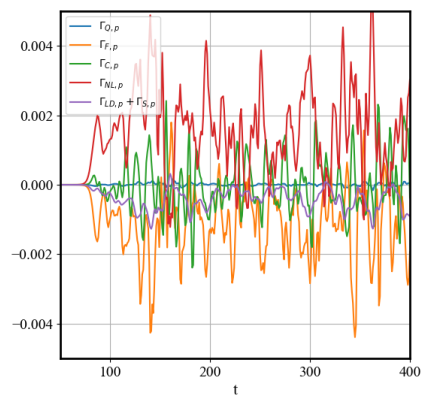


(b)

Figure 9: (a),(b)

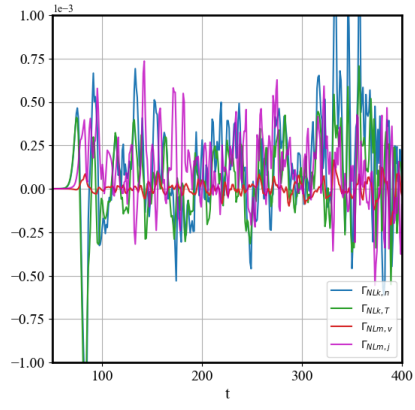


(a)

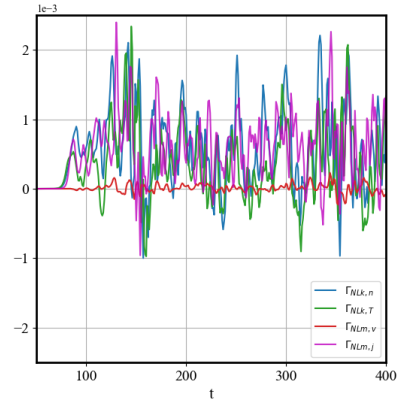


(b)

Figure 10: (a),(b)



(a)



(b)

Figure 11: (a),(b)