Weekly work summary

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content

- ZF/RHF/GAM
- references on electromagnetic turbulence
- modification of energy transfer equation
- revisiting to former simulation result
- comparision between different algorithm

Residual zonal flow

- gyrofluid simulation [Beer,Waltz]: a total collisionless decay of poloidal rotation
- gyrokinetic analysis by [RH 1998]: linear collisionless kinetic mechanisms do not damp the zonal flows completely
- verification by various gyrokinetic codes
- considering shaping effect [Xiao 2006]
- modification of zonal flow closures in gyrofluid model[Mandell,Sugama]

Residual zonal flow

Considering the polarization drift in plasma,

$$v_{pj} = \frac{m_j}{e_j B^2} \frac{d\boldsymbol{E}}{dt} \tag{1}$$

this gives rise to the polarization current,

$$j^{cl} = \sum_j e_j n_j \boldsymbol{v}_{pj} = \epsilon_0 \epsilon^{cl} \frac{d\boldsymbol{E}}{dt} \tag{2}$$

where $\epsilon^{cl} = m_i n_i / \epsilon_0 B^2 = (\omega_{pi}/\Omega_i)^2 = (k_{Di}\rho_i)^2 >> 1$

Residula zonal flow

And in tokamak, considering the toroidal effect, we should include the traped and passing particals. Some fraction of charged particals($f_t \sim \sqrt{\epsilon}, \epsilon = r/R$) are trapped by the magnetic mirror and have a radial excursion by

$$\Delta_t = \sqrt{\epsilon}\rho_{pi} = \frac{q\rho_i}{\sqrt{\epsilon}} \tag{3}$$

and as for passing partical the excursion is

$$\Delta_p = q\rho_i \tag{4}$$

Residula zonal flow

So during this trapped partial orbit motion, we also have the similar polarization effect if we have radial electric field E_r,

$$j^{nc} = \epsilon_0 \epsilon^{nc} \frac{dE_r}{dt} \tag{5}$$

$$\epsilon^{nc} = \sqrt{\epsilon} k_{Di}^2 \Delta_t^2 = \frac{q^2}{\sqrt{\epsilon}} \epsilon^{cl} \tag{6}$$

and Hinton-Rosenbluth give a expression for the polarization as,

$$\epsilon = \epsilon^{cl} + \epsilon^{nc} = \frac{\omega_{pi}^2}{\Omega_i^2}(1 + \frac{1.6q^2}{\sqrt{\epsilon}}) \tag{7}$$

The factor 1.6 comes from detialed kinetic calculation including passing partical contribution.

Residula zonal flow

Then we consider the continuity equation of the polarization current,

$$\frac{\partial \rho_p}{\partial t} + \nabla \cdot \boldsymbol{j}_p = S \tag{8}$$

S is the external source density. Taking the flux surface average and Fourier expansion in space,

$$\frac{\partial}{\partial t} < \rho_p(\boldsymbol{k}) > + < i\boldsymbol{k}_\perp \cdot \boldsymbol{j}_p(\boldsymbol{k}) > = < S(\boldsymbol{k}) > \tag{9}$$

$$\epsilon_0 \epsilon_p < k_\perp^2 > \Phi_k = < \rho_p > - \int < S(\boldsymbol{k}) > dt \tag{10}$$

Residula zonal flow

Consider an initial source perturbation $<S(\boldsymbol{k})> = \delta_k(0)\delta(t)$. when the time scale is in a few gyro motion and much shorter than the bounce time of trapped partical.we have,

$$\epsilon_0 \epsilon^{cl} \Phi_k(t=+0) = -e_i \delta n_k(0) \tag{11}$$

when the time scale is longer than the bounce time of trapped particals, the electrostatic potential is further shielded by the addition of the neoclassical polarization, then we have,

$$\epsilon_0 (\epsilon^{cl} + \epsilon^{nc}) \Phi_k(t=+\infty) = -e_i \delta n_k(0) \tag{12}$$

Therefore, the ratio of the long term zonal flow potential to the initial zonal flow potential is given by,

$$\frac{\Phi_k(t=\infty)}{\Phi_k(t=0)} = \frac{\epsilon^{cl}}{\epsilon^{cl} + \epsilon^{nc}} \tag{13}$$

Using the Hinton formula, we have,

$$\frac{\Phi_k(t=\infty)}{\Phi_k(t=0)} = \frac{1}{1 + 1.6 q^2/\sqrt{\epsilon}} \tag{14}$$

Geodesic acoustic mode

- first prediction by [Winsor 1968]
- Fully forgotten between 1968 and 1996
- dispersion relation, frequency, radial structure and propagation
- close relation to alfven eigen mode
- experimental observations of GAM, H1-heliac,[Shats PRL 2002]; DIII- D, [McKee PoP 2003]

Geodesic acoustic mode

The starting equations are the follows:

$$\rho_0 \frac{\partial \boldsymbol{v}}{\partial t} = \frac{1}{c} \boldsymbol{J} \times \boldsymbol{B} - \nabla p$$
$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho_0 \boldsymbol{v} = 0$$
$$\nabla \phi = \frac{1}{c} \boldsymbol{b} \times \boldsymbol{B} \tag{15}$$
$$\nabla \cdot \boldsymbol{J} = 0$$
$$\rho_0^{-\gamma} - \gamma p_0 \rho_0^{-\gamma-1} \frac{\partial \rho}{\partial t} + \boldsymbol{v} \cdot \nabla(p_0 \rho_0^{-\gamma}) = 0$$

Geodesic acoustic mode

finally, we can deduce the dispersion relation:

$$
\omega^2 \int |\rho|^2 \mathcal{J} \, dS =
$$
$$
\frac{\gamma p_0}{\rho_0} (|\int \rho_0 \frac{\boldsymbol{B} \times \nabla \psi \cdot \nabla B^2}{B^4} \mathcal{J} \, dS|^2 / \int \frac{|\nabla \psi|^2}{B^2} \mathcal{J} \, dS + \int \frac{|\boldsymbol{B} \cdot \nabla \rho|^2}{B^2} \mathcal{J} \, dS)
$$

$$(16)$$

The first term is due to motion in the $\boldsymbol{B} \times \nabla \psi$ direction. It is associated with geodesic curvature, i.e., the surface component of the magnetic field line curvature. And the second **ordinary sound propagation propagating along the field lines**.

Geodesic acoustic mode

In the limit of circular cross section with large aspect ratio($r \ll R$), the dispersion relation becomes,

$$\omega^2 = \frac{2\gamma p_0}{\rho_0 R^2}[1 + 1/(2q^2)] = 2\frac{C_s^2}{R^2}(1 + 1/(2q^2)) \tag{17}$$

with the defination of sound wave velocity in neutral gas $C_s = (\gamma p_0/\rho_0)^{1/2}$.

question: relation between RHF and GAM

electromagnetic turbulence

[scott][naliun][may..]

modification of energy transfer equation

$$<f> = \frac{\int f dV}{\int dV} = \frac{\int f\, r dr d\theta d\zeta}{\int r dr d\theta d\zeta} \tag{18}$$

for zonal flow energy $E_k = \frac{1}{2} <v_E^2>$:

$$\frac{\partial}{\partial t} E_k = \Gamma_{NLk,k} + \Gamma_{NLm,k} + \Gamma_{C,k} \tag{19}$$

for pressure $E_p = \frac{1}{2} <p^2>$:

$$p = p_i + p_e = n_0 T + (1+\tau) T_0 n \tag{20}$$

$$\frac{\partial}{\partial t} E_p = \Gamma_{Q,p} + \Gamma_{F,p} + \Gamma_{C,p} + \Gamma_{LD,p} + \Gamma_{NL,p} + \Gamma_{S,p} \tag{21}$$

modification of energy transfer equation

flux surface average

$$<f> = \frac{\int f dS}{\int dS} = \frac{\int f \, d\theta d\zeta}{\int \, d\theta d\zeta} \tag{22}$$

for zonal flow

$$\frac{\partial}{\partial t} < v_E > = -\frac{1}{r^2}\frac{\partial}{\partial r}r^2 < v_\theta v_r > + \frac{\beta}{n_{eq}}\frac{1}{r^2}\frac{\partial}{\partial r}r^2 < B_\theta B_r > - \frac{2\epsilon}{n_{eq}} < p\sin\theta > \tag{23}$$

and in the code, we use the alternative expression,

$$\frac{\partial}{\partial t} < v_E > = \frac{1}{r} < \phi\frac{\partial\Omega}{\partial\theta} > + \frac{\beta}{n_{eq}}\frac{1}{r} < A\frac{\partial A}{\partial\theta} > - \frac{2\epsilon}{n_{eq}} < p\sin\theta > \tag{24}$$

with the Fourier expansion,

$$\frac{\partial}{\partial t} < v_E > = \sum_{m1+m2=0} \phi_{m1}\Omega_{\theta,m2} + \sum_{m1+m2=0} A_{m1}j_{\theta,m2} - \frac{i\epsilon}{n_{eq}}(p_{1,0} - p_{-1,0}) \tag{25}$$

modification of energy transfer equation

as for equation for thr evolution of $< p \sin \theta >$

$$\frac{\partial}{\partial t} < p \sin \theta > = < (R_Q + R_F + R_C + R_{NL} + R_{LD} + R_S) \sin \theta > \quad (26)$$

$$< R_Q \sin \theta > = a[(1 + \tau) T_{eq} \frac{\partial n_{eq}}{\partial r} + n_{eq} \frac{\partial T_{eq}}{\partial r}] < \phi \cos \theta >$$
$$+ (1 + \tau) \beta T_{eq} \frac{\partial j_0}{\partial r} < A \cos \theta >$$
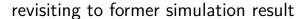$$< R_F \sin \theta > = (\Gamma + \tau) n_{eq} T_{eq} \frac{\epsilon}{q} < v \cos \theta > -(1 + \tau) T_{eq} \frac{\epsilon}{q} < j \cos \theta >$$
$$< R_C \sin \theta > = \epsilon(\Gamma + \tau) p_{eq} < v_\theta >$$
$$+ \epsilon(2\Gamma - 1) p_{eq} < \frac{\partial T}{\partial r} > + \epsilon((\Gamma - 1) - \tau(\tau + 1)) T_{eq}^2 < \frac{\partial n}{\partial r} >$$
$$< R_{LD} \sin \theta > = -(\Gamma - 1) \sqrt{\frac{8 T_{eq}}{\pi}} n_{eq} \frac{\epsilon}{q} < T \sin \theta >$$

$$(27)$$

revisiting to former simulation result

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comparision between different algorithm