Task/Eq 的介绍和使用

任广智

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1 定义

1.1 坐标系

1. 柱坐标系 (R, ϕ, Z)
2. 环坐标系 (r, θ, ζ)

$$\zeta = -\phi, \nabla\zeta = -\frac{1}{R}\boldsymbol{e_\phi}$$

1.2 微分方程

$$\nabla \times (\nabla\zeta \times \nabla f) = [R^2\nabla \cdot (\frac{1}{R^2}\nabla f)]\nabla\zeta$$

1.3 平衡磁场

$$\boldsymbol{B} = \frac{1}{2\pi}[I_\theta\nabla\zeta + \nabla\zeta \times \psi_\theta]$$

$$\boldsymbol{j} = \frac{1}{\mu_0}\nabla \times \boldsymbol{B}, \boldsymbol{j} = \frac{1}{2\pi\mu_0}[R^2\nabla \cdot \frac{1}{R^2}\nabla\psi_\theta\nabla\zeta - \nabla\zeta \times \nabla I_\theta]$$

1.4 磁面平均

1.5 环向磁通和环向电流

1.6 磁面函数

1.7 Grad-Shafranov 方程

根据磁流体力学平衡 $\boldsymbol{j} \times \boldsymbol{B} = \nabla P$，我们可以通过

$$\begin{aligned}\boldsymbol{j} \times \boldsymbol{B} &= \frac{1}{4\pi^2\mu_0}[R^2\nabla \cdot \frac{1}{R^2}\nabla\psi_\theta + I_\theta\frac{dI_\theta}{d\psi_\theta}]\nabla\zeta \times (\nabla\zeta \times (\nabla\zeta \times \nabla\psi_\theta)) \\ &= -\frac{1}{4\pi^2\mu_0}[\nabla \cdot \frac{1}{R^2}\nabla\psi_\theta + \frac{I_\theta}{R^2}\frac{dI_\theta}{d\psi_\theta}]\nabla\psi_\theta\end{aligned}$$

$$\nabla P = \frac{dP}{d\psi_\theta}\nabla\psi_\theta$$

得到

$$\nabla \cdot \frac{1}{R^2}\nabla\psi_\theta = -4\pi^2\mu_0\frac{dP}{d\psi_\theta} - \frac{I_\theta}{R^2}\frac{dI_\theta}{d\psi_\theta}$$

2 运行

3 输出

```
    integer fileUnit,NRG,NZG,NPS,NSG,NTG,NRV

    real(kind=8) RR,BB,RIP
    ! RR     : Plasma major radius
    ! BB     : Magnetic field at center
    ! RIP    : Plasma current
    integer NRGMAX,NZGMAX
    ! NRGMAX: Number of horizontal mesh points in R-Z plane
    ! NZGMAX: Number of vertical mesh points in R-Z plane
    real(kind=8),dimension(:),allocatable :: RG,ZG
    real(kind=8),dimension(:,:),allocatable :: PSIRZ

    integer NPSMAX
    ! NPSMAX: Number of flux surfaces
    real(kind=8),dimension(:),allocatable :: PSIPS,PPPS,TTPS,TEPS,OMPS

    integer NSGMAX,NTGMAX,NUGMAX,NRMAX,NTHMAX,NSUMAX,NRVMAX,NTVMAX
    ! NSGMAX: Number of radial mesh points for Grad-Shafranov eq.
    ! NTGMAX: Number of poloidal mesh points for Grad-Shafranov eq.
    ! NUGMAX: Number of radial mesh points for flux-average quantities
    ! NPSMAX: Number of flux surfaces
    ! NRMAX : Number of radial mesh points for flux coordinates
    ! NTHMAX: Number of poloidal mesh points for flux coordinates
    ! NSUMAX: Number of boundary points
    ! NRVMAX: Number of radial mesh of surface average
    ! NTVMAX: Number of poloidal mesh for surface average
    real(kind=8),dimension(:,:),allocatable :: PSI,DELPSI,HJT
    real(kind=8),dimension(:),allocatable :: PSIPNV,PSIPV,PSITV,QPV,TTV
    real(kind=8) RAXIS,ZAXIS,PSITA,PSIPA,PSI0


    real(kind=8) RA,RKAP,RDLT,RB,FRBIN
    ! RA     : Plasma minor radius
    ! RKAP   : Plasma shape elongation
    ! RDLT   : Plasma shape triangularity
    ! RB     : Wall minor radius
    ! FRBIN  : (RB_inside-RA)/(RB_outside-RA)

    real(kind=8) PJ0,PJ1,PJ2,PROFJ0,PROFJ1,PROFJ2
    ! PJ0    : Current density at R=RR (main component) : Fixed to 1
    ! PJ1    : Current density at R=RR (sub component)       (arb)
    ! PJ2    : Current density at R=RR (sub component)       (arb)
    ! PROFJ0: Current density profile parameter
    ! PROFJ1: Current density profile parameter
    ! PROFJ2: Current density profile parameter

    real(kind=8) PP0,PP1,PP2,PROFP0,PROFP1,PROFP2
    ! PP0    : Plasma pressure (main component)              (MPa)
    ! PP1    : Plasma pressure (sub component)               (MPa)
    ! PP2    : Plasma pressure (increment within ITB)        (MPa)
    ! PROFP0: Pressure profile parameter
    ! PROFP1: Pressure profile parameter
    ! PROFP2: Pressure profile parameter

    real(kind=8) PT0,PT1,PT2,PROFT0,PROFT1,PROFT2
    ! PT0    : Plasma temperature (main component)           (keV)
    ! PT1    : Plasma temperature (sub component)            (keV)
    ! PT2    : Plasma temperature (increment within ITB)     (keV)
    ! PTSEQ  : Plasma temperature (at surface)               (keV)
    ! PROFTP0: Temperature profile parameter
    ! PROFTP1: Temperature profile parameter
```

```
62      ! PROFTP2: Temperature profile parameter
63
64      real(kind=8) PV0,PV1,PV2,PROFV0,PROFV1,PROFV2
65      ! PV0    : Toroidal rotation (main component)              (m/s)
66      ! PV1    : Toroidal rotation (sub component)               (m/s)
67      ! PV2    : Toroidal rotation (increment within ITB)        (m/s)
68      ! PROFV0: Velocity profile parameter
69      ! PROFV1: Velocity profile parameter
70      ! PROFV2: Velocity profile parameter
71
72      real(kind=8) PROFR0,PROFR1,PROFR2
73      ! PROFR0: Profile parameter
74      ! PROFR1: Profile parameter
75      ! PROFR2: Profile parameter
76
77      real(kind=8),dimension(:,:),allocatable :: HJTRZ
78
79      open(Newunit=fileUnit, file="data_test", form='unformatted')
80
81      read(fileUnit) RR,BB,RIP
82      read(fileUnit) NRGMAX,NZGMAX
83
84      allocate( RG(NRGMAX), ZG(NZGMAX) )
85      allocate( PSIRZ(NRGMAX,NZGMAX) )
86
87      read(fileUnit) (RG(NRG),NRG=1,NRGMAX)
88      read(fileUnit) (ZG(NZG),NZG=1,NZGMAX)
89      read(fileUnit) ((PSIRZ(NRG,NZG),NRG=1,NRGMAX),NZG=1,NZGMAX)
90
91      read(fileUnit) NPSMAX
92
93      allocate( PSIPS(NPSMAX), PPPS(NPSMAX), TTPS(NPSMAX), TEPS(NPSMAX), OMPS(NPSMAX) )
94
95      read(fileUnit) (PSIPS(NPS),NPS=1,NPSMAX)
96      read(fileUnit) (PPPS(NPS),NPS=1,NPSMAX)
97      read(fileUnit) (TTPS(NPS),NPS=1,NPSMAX)
98      read(fileUnit) (TEPS(NPS),NPS=1,NPSMAX)
99      read(fileUnit) (OMPS(NPS),NPS=1,NPSMAX)
100
101     read(fileUnit) NSGMAX,NTGMAX,NUGMAX,NRMAX,NTHMAX,NSUMAX,NRVMAX,NTVMAX
102
103     allocate( PSI(NSGMAX,NTGMAX),DELPSI(NSGMAX,NTGMAX),HJT(NSGMAX,NTGMAX) )
104
105     read(fileUnit) ((PSI(NSG,NTG),NSG=1,NSGMAX),NTG=1,NTGMAX)
106     read(fileUnit) ((DELPSI(NSG,NTG),NSG=1,NSGMAX),NTG=1,NTGMAX)
107     read(fileUnit) ((HJT(NSG,NTG),NSG=1,NSGMAX),NTG=1,NTGMAX)
108
109     read(fileUnit) RAXIS,ZAXIS,PSITA,PSIPA,PSI0
110
111     allocate( PSIPNV(NRVMAX),PSIPV(NRVMAX),PSITV(NRVMAX),QPV(NRVMAX),TTV(NRVMAX) )
112     read(fileUnit) (PSIPNV(NRV),NRV=1,NRVMAX)
113     read(fileUnit) (PSIPV(NRV),NRV=1,NRVMAX)
114     read(fileUnit) (PSITV(NRV),NRV=1,NRVMAX)
115     read(fileUnit) (QPV(NRV),NRV=1,NRVMAX)
116     read(fileUnit) (TTV(NRV),NRV=1,NRVMAX)
117
118     read(fileUnit) RA,RKAP,RDLT,RB,FRBIN
119     read(fileUnit) PJ0,PJ1,PJ2,PROFJ0,PROFJ1,PROFJ2
120     read(fileUnit) PP0,PP1,PP2,PROFP0,PROFP1,PROFP2
121     read(fileUnit) PT0,PT1,PT2,PROFT0,PROFT1,PROFT2
122     read(fileUnit) PV0,PV1,PV2,PROFV0,PROFV1,PROFV2
123     read(fileUnit) PROFR0,PROFR1,PROFR2
124
125     allocate(HJTRZ(NRGMAX,NZGMAX))
126
127     read(fileUnit) ((HJTRZ(NRG,NZG),NRG=1,NRGMAX),NZG=1,NZGMAX)
```

```
128
129       close(fileUnit)
130
```

4 EQMESH

(σ, θ) 网格的划分

- NSGMAX: σ 径向网格数, N_σ
- NTGMAX: θ 极向网格数, N_σ
- DSG: $\Delta\sigma$, DTG: $\Delta\theta$
- SIGM(1:NSGMAX): $\sigma_{i+1/2}$
- SIGG(1:NSGMAX+1): σ_i
- THGM(1:NTGMAX): $\theta_{i+1/2}$
- THGG(1:NTGMAX+1): θ_i

5 EQPSIN

初始分布

- RAXIS=RR: $R_{axis} = R_0$
- ZAXIS=0: $Z_{axis} = 0$
- (MDLEQF=4,9)

$$\psi_{\zeta a} = \pi\kappa a^2 B_0$$
$$\psi_{\theta a} = 2\psi_{\zeta a}/q_a$$
$$\psi_0 = -\psi_{\theta a}$$

  (MDLEQF=0-3,5-8)

$$\cdots$$

```
1       IF (MOD(MDLEQF,5).EQ.4)  THEN
2             PSITA=PI*RKAP*RA**2*BB
3             PSIPA=PSITA*2/QA
4             PSI0=-PSIPA
5       ELSE
6             PSI0=-0.5D0*RMU0*RIP*1.D6*RR
7             PSIPA=-PSI0
8             PSITA=PI*RKAP*RA**2*BB
9       ENDIF
10
```

- NRVMAX: 磁面数

$$\hat{\psi_\theta}(\rho) = \rho^2$$
$$\psi_\theta(\rho) = \psi_{\theta a}\rho^2$$
$$\psi_\zeta(\rho) = \psi_{\zeta a}\rho^2$$
$$q(\rho) = \psi_{\zeta a}/\psi_{\theta a}$$
$$I_\theta(\rho) = 2\pi R_0 B_0$$

```
      DRHO=1.D0/(NRVMAX-1)
      DO NRV=1,NRVMAX
         RHOL=(NRV-1)*DRHO
         PSIPNV(NRV)=RHOL*RHOL
         PSIPV(NRV)=PSIPA*RHOL*RHOL
         PSITV(NRV)=PSITA*RHOL*RHOL
         QPV(NRV)=PSITA/PSIPA
         TTV(NRV)=2.D0*PI*RR*BB
      ENDDO

```

$$\psi(\sigma,\theta) = \psi_0(1-\sigma^2)$$

$$\delta\psi(\sigma,\theta) = 0$$

```
      DO NSG=1,NSGMAX
         DO NTG=1,NTGMAX
            PSI(NTG,NSG)=PSI0*(1.D0-SIGM(NSG)*SIGM(NSG))
            DELPSI(NTG,NSG)=0.D0
            HJT(NTG,NSG)=0.D0
         ENDDO
      ENDDO

```

6 EQDEFB

- RHOM(1:NTGMAX): $\rho(\theta_{j+1/2})$
- RHOG(1:NTGMAX+1): $\rho(\theta_j)$
- RMG(1:NSGMAX,1;NTGMAX+1): $R_{i+1/2,j}$
- RGM(1:NSGMAX+1,1;NTGMAX): $R_{i,j+1/2}$
- RMM(1:NSGMAX,1;NTGMAX): $R_{i+1/2,j+1/2}$
- ZMM(1:NSGMAX,1;NTGMAX): $Z_{i+1/2,j+1/2}$