

Task/Eq 的介绍和使用

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1 定义

1.1 坐标系

1. 柱坐标系 (R, ϕ, Z)
2. 环坐标系 (r, θ, ζ)

$$\zeta = -\phi, \nabla\zeta = -\frac{1}{R}\mathbf{e}_\phi$$

1.2 微分方程

$$\nabla \times (\nabla\zeta \times \nabla f) = [R^2 \nabla \cdot (\frac{1}{R^2} \nabla f)] \nabla\zeta$$

1.3 平衡磁场

$$\mathbf{B} = \frac{1}{2\pi} [I_\theta \nabla\zeta + \nabla\zeta \times \psi_\theta]$$
$$\mathbf{j} = \frac{1}{\mu_0} \nabla \times \mathbf{B}, \mathbf{j} = \frac{1}{2\pi\mu_0} [R^2 \nabla \cdot \frac{1}{R^2} \nabla \psi_\theta \nabla\zeta - \nabla\zeta \times \nabla I_\theta]$$

1.4 磁面平均

1.5 环向磁通和环向电流

1.6 磁面函数

1.7 Grad-Shafranov 方程

根据磁流体力学平衡 $\mathbf{j} \times \mathbf{B} = \nabla P$, 我们可以通过

$$\begin{aligned} \mathbf{j} \times \mathbf{B} &= \frac{1}{4\pi^2\mu_0} [R^2 \nabla \cdot \frac{1}{R^2} \nabla \psi_\theta + I_\theta \frac{dI_\theta}{d\psi_\theta}] \nabla\zeta \times (\nabla\zeta \times (\nabla\zeta \times \nabla \psi_\theta)) \\ &= -\frac{1}{4\pi^2\mu_0} [\nabla \cdot \frac{1}{R^2} \nabla \psi_\theta + \frac{I_\theta}{R^2} \frac{dI_\theta}{d\psi_\theta}] \nabla \psi_\theta \\ \nabla P &= \frac{dP}{d\psi_\theta} \nabla \psi_\theta \end{aligned}$$

得到

$$\nabla \cdot \frac{1}{R^2} \nabla \psi_\theta = -4\pi^2\mu_0 \frac{dP}{d\psi_\theta} - \frac{I_\theta}{R^2} \frac{dI_\theta}{d\psi_\theta}$$