Task/Eq 的介绍和使用

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1 定义

- 1.1 坐标系
 - 1. 柱坐标系 (R, ϕ, Z)
 - 2. 环坐标系 (r, θ, ζ)

$$\zeta = -\phi,
abla \zeta = -rac{1}{R} oldsymbol{e_{\phi}}$$

1.2 微分方程

$$\nabla \times (\nabla \zeta \times \nabla f) = [R^2 \nabla \cdot (\frac{1}{R^2} \nabla f)] \nabla \zeta$$

1.3 平衡磁场

$$m{B} = rac{1}{2\pi} [I_{ heta}
abla \zeta +
abla \zeta imes \psi_{ heta}]$$
 $m{j} = rac{1}{\mu_0}
abla imes m{B}, m{j} = rac{1}{2\pi\mu_0} [R^2
abla \cdot rac{1}{R^2}
abla \psi_{ heta}
abla \zeta -
abla \zeta imes
abla I_{ heta}]$

- 1.4 磁面平均
- 1.5 环向磁通和环向电流
- 1.6 磁面函数
- 1.7 Grad-Shafranov 方程

根据磁流体力学平衡 $\mathbf{j} \times \mathbf{B} = \nabla P$, 我们可以通过

$$\begin{split} \boldsymbol{j} \times \boldsymbol{B} &= \frac{1}{4\pi^2 \mu_0} [R^2 \nabla \cdot \frac{1}{R^2} \nabla \psi_\theta + I_\theta \frac{dI_\theta}{d\psi_\theta}] \nabla \zeta \times (\nabla \zeta \times (\nabla \zeta \times \nabla \psi_\theta)) \\ &= -\frac{1}{4\pi^2 \mu_0} [\nabla \cdot \frac{1}{R^2} \nabla \psi_\theta + \frac{I_\theta}{R^2} \frac{dI_\theta}{d\psi_\theta}] \nabla \psi_\theta \\ &\nabla P &= \frac{dP}{d\psi_\theta} \nabla \psi_\theta \end{split}$$

得到

$$\nabla \cdot \frac{1}{R^2} \nabla \psi_{\theta} = -4\pi^2 \mu_0 \frac{dP}{d\psi_{\theta}} - \frac{I_{\theta}}{R^2} \frac{dI_{\theta}}{d\psi_{\theta}}$$