

Weekly work summary

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content

- comparison between different algorithm
- revisiting to former simulation result
- modification of energy transfer equation
- ZF/RHF/GAM

comparision between different algorithm

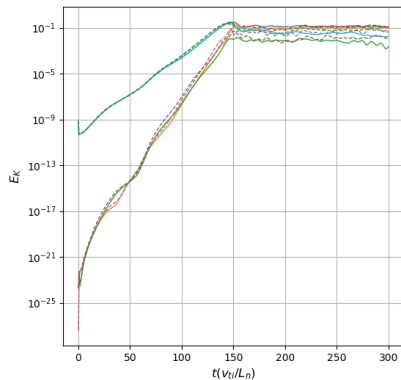


Figure: time evolution of energy in LW/RK/AB

comparision between different algorithm

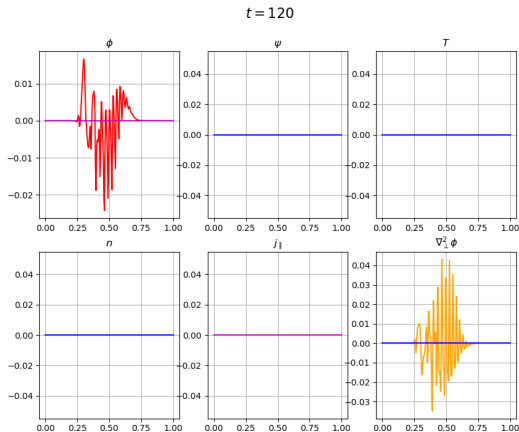


Figure: radial mode structure with AB-256

comparision between different algorithm

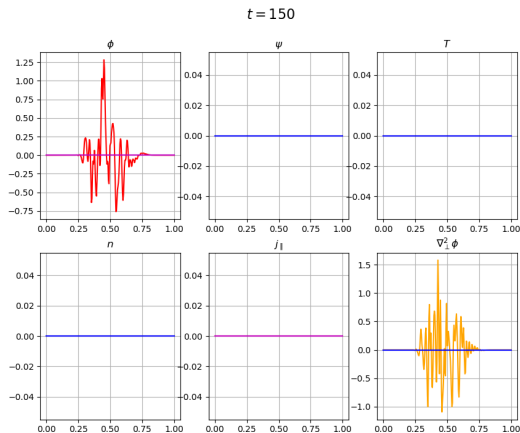


Figure: radial mode structure with LW-256

comparision between different algorithm

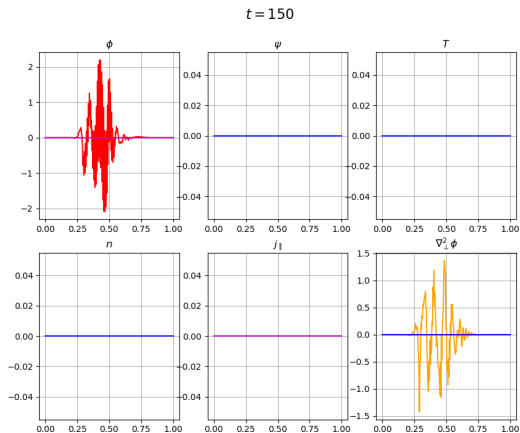


Figure: radial mode structure with AB-256

comparision between different algorithm

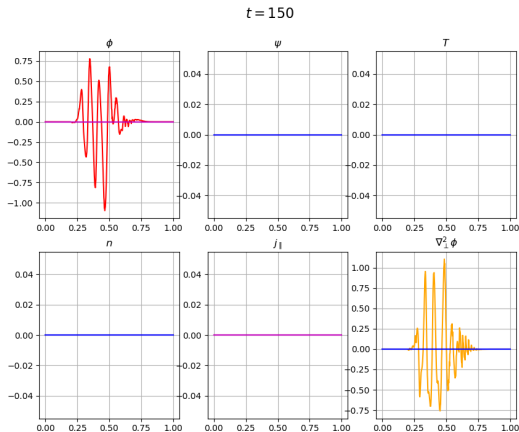
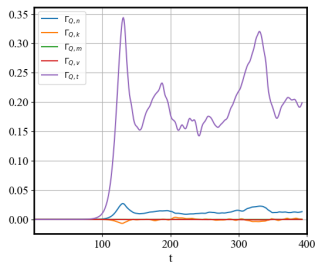
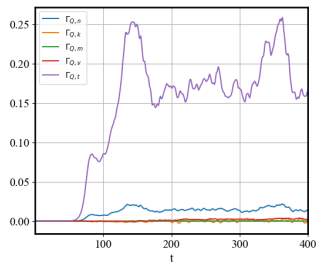


Figure: radial mode structure with AB-512

revisiting to former simulation result



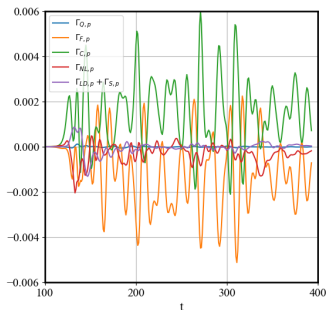
(a) $\beta = 0.1\%$



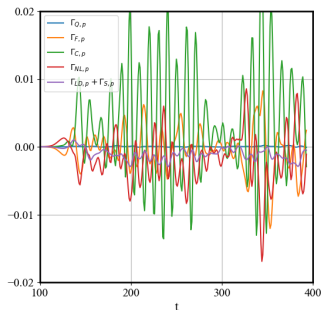
(b) $\beta = 1.0\%$

Figure: Temporal evolution of radial transport flux

revisiting to former simulation result



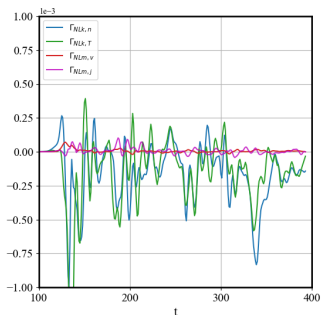
(a) inner region



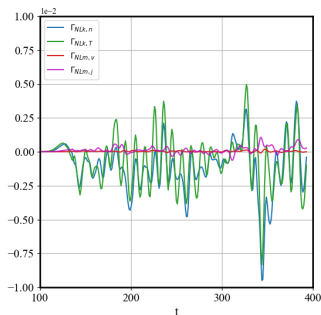
(b) outer region

Figure: Temporal evolution of $p_{1,0}$ energy drives with $\beta = 0.1\%$

revisiting to former simulation result



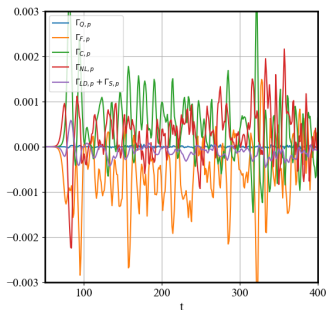
(a) inner region



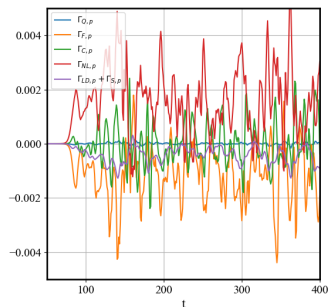
(b) outer region

Figure: Temporal evolution of the component of $\Gamma_{NL,p}$

revisiting to former simulation result



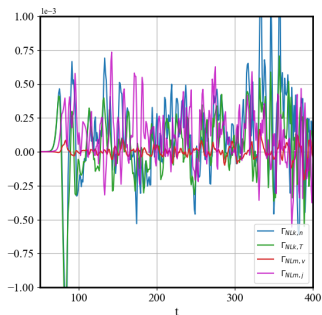
(a) inner region



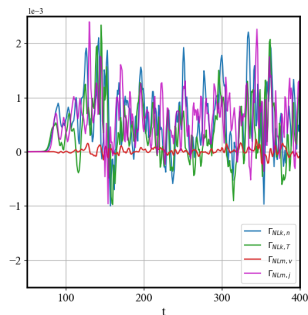
(b) outer region

Figure: Temporal evolution of $p_{1,0}$ energy drives with $\beta = 0.1\%$

revisiting to former simulation result



(a) inner region



(b) outer region

Figure: Temporal evolution of the component of $\Gamma_{NL,p}$

modification of energy transfer equation

$$\langle f \rangle = \frac{\int f dV}{\int dV} = \frac{\int f r dr d\theta d\zeta}{\int r dr d\theta d\zeta} \quad (1)$$

for zonal flow energy $E_k = \frac{1}{2} \langle v_E^2 \rangle$:

$$\frac{\partial}{\partial t} E_k = \Gamma_{NLk,k} + \Gamma_{NLm,k} + \Gamma_{C,k} \quad (2)$$

for pressure $E_p = \frac{1}{2} \langle p^2 \rangle$:

$$p = p_i + p_e = n_0 T + (1 + \tau) T_0 n \quad (3)$$

$$\frac{\partial}{\partial t} E_p = \Gamma_{Q,p} + \Gamma_{F,p} + \Gamma_{C,p} + \Gamma_{LD,p} + \Gamma_{NL,p} + \Gamma_{S,p} \quad (4)$$

modification of energy transfer equation

flux surface average

$$\langle f \rangle = \frac{\int f dS}{\int dS} = \frac{\int f d\theta d\zeta}{\int d\theta d\zeta} \quad (5)$$

for zonal flow

$$\frac{\partial}{\partial t} \langle v_E \rangle = -\frac{1}{r^2} \frac{\partial}{\partial r} r^2 \langle v_\theta v_r \rangle + \frac{\beta}{n_{eq}} \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \langle B_\theta B_r \rangle - \frac{2\epsilon}{n_{eq}} \langle p \sin \theta \rangle \quad (6)$$

and in the code, we use the alternative expression,

$$\frac{\partial}{\partial t} \langle v_E \rangle = \frac{1}{r} \langle \phi \frac{\partial \Omega}{\partial \theta} \rangle + \frac{\beta}{n_{eq}} \frac{1}{r} \langle A \frac{\partial A}{\partial \theta} \rangle - \frac{2\epsilon}{n_{eq}} \langle p \sin \theta \rangle \quad (7)$$

with the Fourier expansion,

$$\frac{\partial}{\partial t} \langle v_E \rangle = \sum_{m1+m2=0} \phi_{m1} \Omega_{\theta, m2} + \sum_{m1+m2=0} A_{m1} j_{\theta, m2} - \frac{i\epsilon}{n_{eq}} (p_{1,0} - p_{-1,0}) \quad (8)$$

modification of energy transfer equation

as for equation for the evolution of $\langle p \sin \theta \rangle$

$$\frac{\partial}{\partial t} \langle p \sin \theta \rangle = \langle (R_Q + R_F + R_C + R_{NL} + R_{LD} + R_S) \sin \theta \rangle \quad (9)$$

$$\begin{aligned} \langle R_Q \sin \theta \rangle &= a[(1 + \tau) T_{eq} \frac{\partial n_{eq}}{\partial r} + n_{eq} \frac{\partial T_{eq}}{\partial r}] \langle \phi \cos \theta \rangle \\ &\quad + (1 + \tau) \beta T_{eq} \frac{\partial j_0}{\partial r} \langle A \cos \theta \rangle \\ \langle R_F \sin \theta \rangle &= (\Gamma + \tau) n_{eq} T_{eq} \frac{\epsilon}{q} \langle v \cos \theta \rangle - (1 + \tau) T_{eq} \frac{\epsilon}{q} \langle j \cos \theta \rangle \\ \langle R_C \sin \theta \rangle &= \epsilon(\Gamma + \tau) p_{eq} \langle v_\theta \rangle \\ &\quad + \epsilon(2\Gamma - 1) p_{eq} \langle \frac{\partial T}{\partial r} \rangle + \epsilon((\Gamma - 1) - \tau(\tau + 1)) T_{eq}^2 \langle \frac{\partial n}{\partial r} \rangle \\ \langle R_{LD} \sin \theta \rangle &= -(\Gamma - 1) \sqrt{\frac{8 T_{eq}}{\pi}} n_{eq} \frac{\epsilon}{q} \langle T \sin \theta \rangle \end{aligned} \quad (10)$$

Residual zonal flow

- gyrofluid simulation [Beer,Waltz]: a total collisionless decay of poloidal rotation
- gyrokinetic analysis by [RH 1998]: linear collisionless kinetic mechanisms do not damp the zonal flows completely
- verification by various gyrokinetic codes
- considering shaping effect [Xiao 2006]
- modification of zonal flow closures in gyrofluid model[Mandell,Sugama]

Residual zonal flow

Considering the polarization drift in plasma,

$$\mathbf{v}_{pj} = \frac{m_j}{e_j B^2} \frac{d\mathbf{E}}{dt} \quad (11)$$

this gives rise to the polarization current,

$$\mathbf{j}^{cl} = \sum_j e_j n_j \mathbf{v}_{pj} = \epsilon_0 \epsilon^{cl} \frac{d\mathbf{E}}{dt} \quad (12)$$

where $\epsilon^{cl} = m_i n_i / \epsilon_0 B^2 = (\omega_{pi} / \Omega_i)^2 = (k_{Di} \rho_i)^2 \gg 1$

Residual zonal flow

And in tokamak, considering the toroidal effect, we should include the trapped and passing particles. Some fraction of charged particles ($f_t \sim \sqrt{\epsilon}$, $\epsilon = r/R$) are trapped by the magnetic mirror and have a radial excursion by

$$\Delta_t = \sqrt{\epsilon} \rho_{pi} = \frac{q \rho_i}{\sqrt{\epsilon}} \quad (13)$$

and as for passing particle the excursion is

$$\Delta_p = q \rho_i \quad (14)$$

Residual zonal flow

So during this trapped partial orbit motion, we also have the similar polarization effect if we have radial electric field E_r ,

$$j^{nc} = \epsilon_0 \epsilon^{nc} \frac{dE_r}{dt} \quad (15)$$

$$\epsilon^{nc} = \sqrt{\epsilon} k_{Di}^2 \Delta_t^2 = \frac{q^2}{\sqrt{\epsilon}} \epsilon^{cl} \quad (16)$$

and Hinton-Rosenbluth give an expression for the polarization as,

$$\epsilon = \epsilon^{cl} + \epsilon^{nc} = \frac{\omega_{pi}^2}{\Omega_i^2} \left(1 + \frac{1.6 q^2}{\sqrt{\epsilon}} \right) \quad (17)$$

The factor 1.6 comes from detailed kinetic calculation including passing particle contribution.

Residual zonal flow

Then we consider the continuity equation of the polarization current,

$$\frac{\partial \rho_p}{\partial t} + \nabla \cdot \mathbf{j}_p = S \quad (18)$$

S is the external source density. Taking the flux surface average and Fourier expansion in space,

$$\frac{\partial}{\partial t} \langle \rho_p(\mathbf{k}) \rangle + \langle i\mathbf{k}_\perp \cdot \mathbf{j}_p(\mathbf{k}) \rangle = \langle S(\mathbf{k}) \rangle \quad (19)$$

$$\epsilon_0 \epsilon_p \langle k_\perp^2 \rangle \Phi_k = \langle \rho_p \rangle - \int \langle S(\mathbf{k}) \rangle dt \quad (20)$$

Residula zonal flow

Consider an initial source perturbation $\langle S(\mathbf{k}) \rangle = \delta_k(0)\delta(t)$. when the time scale is in a few gyro motion and much shorter than the bounce time of trapped partical.we have,

$$\epsilon_0 \epsilon^{cl} \Phi_k(t = +0) = -e_i \delta n_k(0) \quad (21)$$

when the time scale is longer than the bounce time of trapped particals, the electrostatic potential is further shielded by the addition of the neoclassical polarization, then we have,

$$\epsilon_0 (\epsilon^{cl} + \epsilon^{nc}) \Phi_k(t = +\infty) = -e_i \delta n_k(0) \quad (22)$$

Therefore, the ratio of the long term zonal flow potential to the initial zonal flow potential is given by,

$$\frac{\Phi_k(t = \infty)}{\Phi_k(t = 0)} = \frac{\epsilon^{cl}}{\epsilon^{cl} + \epsilon^{nc}} \quad (23)$$

Using the Hinton formula, we have,

$$\frac{\Phi_k(t = \infty)}{\Phi_k(t = 0)} = \frac{1}{1 + 1.6q^2/\sqrt{\epsilon}} \quad (24)$$

Geodesic acoustic mode

- first prediction by [Winsor 1968]
- Fully forgotten between 1968 and 1996
- dispersion relation, frequency, radial structure and propagation
- close relation to alfvén eigen mode
- experimental observations of GAM, H1-heliac, [Shats PRL 2002]; DIII-D, [McKee PoP 2003]

Geodesic acoustic mode

The starting equations are the follows:

$$\begin{aligned}\rho_0 \frac{\partial \mathbf{v}}{\partial t} &= \frac{1}{c} \mathbf{J} \times \mathbf{B} - \nabla p \\ \frac{\partial \rho}{\partial t} + \nabla \cdot \rho_0 \mathbf{v} &= 0 \\ \nabla \phi &= \frac{1}{c} \mathbf{b} \times \mathbf{B} \\ \nabla \cdot \mathbf{J} &= 0 \\ \rho_0^{-\gamma} - \gamma p_0 \rho_0^{-\gamma-1} \frac{\partial \rho}{\partial t} + \mathbf{v} \cdot \nabla (p_0 \rho_0^{-\gamma}) &= 0\end{aligned}\tag{25}$$

Geodesic acoustic mode

finally, we can deduce the dispersion relation:

$$\omega^2 \int |\rho|^2 \mathcal{J} dS = \frac{\gamma p_0}{\rho_0} \left(\left| \int \rho_0 \frac{\mathbf{B} \times \nabla \psi \cdot \nabla B^2}{B^4} \mathcal{J} dS \right|^2 / \int \frac{|\nabla \psi|^2}{B^2} \mathcal{J} dS + \int \frac{|\mathbf{B} \cdot \nabla \rho|^2}{B^2} \mathcal{J} dS \right) \quad (26)$$

The first term is due to motion in the $\mathbf{B} \times \nabla \psi$ direction. It is associated with geodesic curvature, i.e., the surface component of the magnetic field line curvature. And the second **ordinary sound propagation propagating along the field lines**.

Geodesic acoustic mode

In the limit of circular cross section with large aspect ratio ($r \ll R$), the dispersion relation becomes,

$$\omega^2 = \frac{2\gamma p_0}{\rho_0 R^2} [1 + 1/(2q^2)] = 2 \frac{C_s^2}{R^2} (1 + 1/(2q^2)) \quad (27)$$

with the definition of sound wave velocity in neutral gas

$$C_s = (\gamma p_0 / \rho_0)^{1/2}.$$

question: relation between RHF and GAM