zonal flow/geodesic acoustic mode

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1 closure for fluid moments model

1.1 Hammett-Perkins closure

First consider the simplest case of linear one-dimensional electrostatic waves.Here's the Vlasov equations with cold ions,

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + \frac{eE}{m}\frac{\partial f}{\partial v} = 0 \tag{1}$$

Consider the linear response $f = f_0(v) + \tilde{f}(x, v, t)$ to a small driving electric field $E = -\partial\tilde{\phi}/\partial x$ and linearize the equation with $\tilde{f}(x, v, t) \sim \exp(ikx - i\omega t)$, the exact linear response is

$$\tilde{n} = \int \tilde{f} dv \equiv -n_0 \frac{e\tilde{\phi}}{T_0} R(\zeta) = \frac{e\tilde{\phi}}{T_0} k v_t^2 \int \frac{\partial f_0/\partial v}{kv - \omega} \tag{2}$$

where $R(\zeta)$ is the normalized reponse function and $\zeta = \omega/(|k|\sqrt{2}v_t)$ is a normalized frequency, $T_0 = mv_t^2 = m\int f_0 v^2 dv/\int f_0 dv$

2 zonal flow

3 residula flow

3.1 history

- gyrofluid simulation [Beer,Waltz]: a total collisionless decay of poloidal rotation
- gyrokinetic analysis by [RH 1998]: linear collisionless kinetic mechanisms do not damp the zonal flows completely
- verification by various gyrokinetic codes
- considering shaping effect [Xiao 2006]
- modification of zonal flow closures in gyrofluid model[Mandell,Sugama]

3.2 description

Considering the polarization drift in plasma,

$$v_{pj} = \frac{m_j}{e_j B^2} \frac{d\boldsymbol{E}}{dt} \tag{3}$$

this gives rise to the polarization current,

$$j^{cl} = \sum_j e_j n_j \boldsymbol{v}_{pj} = \epsilon_0 \epsilon^{cl} \frac{d\boldsymbol{E}}{dt} \tag{4}$$

where $\epsilon^{cl} = m_i n_i/\epsilon_0 B^2 = (\omega_{pi}/\Omega_i)^2 = (k_{Di}\rho_i)^2 >> 1$. This polarization current originates from delayed ion gyro motion from time varying electric field and ϵ^{cl} is called classical polarization, which is a low frenquency dielectric constant perpendicular to the magnetic field.

And in tokamak, considering the toroidal effect, we should include the traped and passing particals. Some fraction of charged particals($f_t \sim \sqrt{\epsilon}, \epsilon = r/R$) are trapped by the magnetic mirror and have a radial excursion by

$$\Delta_t = \sqrt{\epsilon}\rho_{pi} = \frac{q\rho_i}{\sqrt{\epsilon}} \tag{5}$$

and as for passing partical the excursion is

$$\Delta_p = q\rho_i \tag{6}$$

So during this trapped partial orbit motion, we alse have the similar polarization effect if we have radial electric field E_r,

$$j^{nc} = \epsilon_0 \epsilon^{nc} \frac{dE_r}{dt} \tag{7}$$

$$\epsilon^{nc} = \sqrt{\epsilon} k_{Di}^2 \Delta_t^2 = \frac{q^2}{\sqrt{\epsilon}} \epsilon^{cl} \tag{8}$$

and Hinton-Rosenbluth give a expression for the polarization as,

$$\epsilon = \epsilon^{cl} + \epsilon^{nc} = \frac{\omega_{pi}^2}{\Omega_i^2}(1 + \frac{1.6q^2}{\sqrt{\epsilon}}) \tag{9}$$

The factor 1.6 comes from detialed kinetic calculation including passing partical contribution.

Then we consider the continuity equation of the polarization current,

$$\frac{\partial \rho_p}{\partial t} + \nabla \cdot \boldsymbol{j}_p = S \tag{10}$$

S is the external source density.Taking the flux surface average and Fourier expansion in space,

$$\frac{\partial}{\partial t} < \rho_p(\boldsymbol{k}) > + < i\boldsymbol{k}_\perp \cdot \boldsymbol{j}_p(\boldsymbol{k}) > = < S(\boldsymbol{k}) > \tag{11}$$

$$\epsilon_0\epsilon_p < k_\perp^2 > \Phi_k =< \rho_p > - \int < S(\boldsymbol{k}) > dt \tag{12}$$

Consider an initial source perturbation $< S(\boldsymbol{k}) >= \delta_k(0)\delta(t)$. when the time scale is in a few gyro motion and much shorter than the bounce time of trapped partical.we have,

$$\epsilon_0\epsilon^{cl}\Phi_k(t = +0) = -e_i\delta n_k(0) \tag{13}$$

when the time scale is longer than the bounce time of trapped particals, the electrostatic potential is further shielded by the addition of the neoclassical polarization, then we have,

$$\epsilon_0(\epsilon^{cl} + \epsilon^{nc})\Phi_k(t = +\infty) = -e_i\delta n_k(0) \tag{14}$$

Therefore, the ratio of the long term zonal flow potential to the initial zonal flow potential is given by,

$$\frac{\Phi_k(t = \infty)}{\Phi_k(t = 0)} = \frac{\epsilon^{cl}}{\epsilon^{cl} + \epsilon^{nc}} \tag{15}$$

Using the Hinton formula, we have,

$$\frac{\Phi_k(t = \infty)}{\Phi_k(t = 0)} = \frac{1}{1 + 1.6q^2/\sqrt{\epsilon}} \tag{16}$$

[reference]

1. Kikuchi ebook
2. Diamond PPCF 2005
3. Hinton PRL 1998
4. Idomura ...

4 Geodesic acoustic mode

4.1 history

- first prediction by [Winsor 1968]
- Fully forgotten between 1968 and 1996
- dispersion relation, frequency, radial structure and propagation
- close relation to alfven eigen mode
- experimental observations of GAM, H1-heliac,[Shats PRL 2002]; DIII- D, [McKee PoP 2003]

4.2 description

4.2.1 Winsor1968

The starting equations are the follows:

$$\begin{aligned}
&\rho_0 \frac{\partial \boldsymbol{v}}{\partial t} = \frac{1}{c} \boldsymbol{J} \times \boldsymbol{B} - \nabla p \\
&\frac{\partial \rho}{\partial t} + \nabla \cdot \rho_0 \boldsymbol{v} = 0 \\
&\nabla \phi = \frac{1}{c} \boldsymbol{b} \times \boldsymbol{B} \\
&\nabla \cdot \boldsymbol{J} = 0 \\
&\rho_0^{-\gamma} - \gamma p_0 \rho_0^{-\gamma-1} \frac{\partial \rho}{\partial t} + \boldsymbol{v} \cdot \nabla (p_0 \rho_0^{-\gamma}) = 0
\end{aligned} \tag{17}$$

finally, we can deduce the dispersion relation:

$$\omega^2 \int |\rho|^2 \mathcal{J} dS = \frac{\gamma p_0}{\rho_0} (|\int \rho_0 \frac{\boldsymbol{B} \times \nabla \psi \cdot \nabla B^2}{B^4} \mathcal{J} dS|^2 / \int \frac{|\nabla \psi|^2}{B^2} \mathcal{J} dS + \int \frac{|\boldsymbol{B} \cdot \nabla \rho|^2}{B^2} \mathcal{J} dS) \tag{18}$$

The first term is due to motion in the $\boldsymbol{B} \times \nabla \psi$direction.It is associated with geodesic curvature, i.e., the surface component of the magnetic field line curvature. And the second **ordinary sound propagation propagating along the field lines**.

The physical mechanism of geodesic acoustic mode is explained as follows. An electric field pertubation E_ψ causes a flow $\boldsymbol{v}_\perp = \boldsymbol{E} \times \boldsymbol{B}/B^2$ and since the compressibility, the flow causes a density accumulation proportional to $-\nabla \cdot \boldsymbol{v}_\perp = \boldsymbol{E} \times \boldsymbol{B} \cdot \nabla B^2 / B^4$.This n generates a current $\boldsymbol{J} = \boldsymbol{B} \times \nabla n / B^2$ which transports charge across the magnetic surface and acts to recerse E_ψ

In the limit of circular cross section with large aspect ratio($r \ll R$), the dispersion relation becomes,

$$\omega^2 = \frac{2\gamma p_0}{\rho_0 R^2} [1 + 1/(2q^2)] = 2 \frac{C_s^2}{R^2} (1 + 1/(2q^2)) \tag{19}$$

with the defination of sound wave velocity in neutral gas $C_s = (\gamma p_0 / \rho_0)^{1/2}$.

4.2.2 GAM in 5-field landau fluid model

[reference]

1. Diamond PPCF 2005
2. Winsor POF 1968