zonal flow/geodesic acoustic mode

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1 zonal flow

2 residula flow

2.1 history

- gyrofluid simulation [Beer,Waltz]: a total collisionless decay of poloidal rotation

- gyrokinetic analysis by [RH 1998]: linear collisionless kinetic mechanisms do not damp the zonal flows completely

- verification by various gyrokinetic codes

- considering shaping effect [Xiao 2006]

- modification of zonal flow closures in gyrofluid model[Mandell,Sugama]

2.2 description

Considering the polarization drift in plasma,

$$v_{pj} = \frac{m_j}{e_j B^2} \frac{d\boldsymbol{E}}{dt} \tag{1}$$

this gives rise to the polarization current,

$$j^{cl} = \sum_j e_j n_j \boldsymbol{v}_{pj} = \epsilon_0 \epsilon^{cl} \frac{d\boldsymbol{E}}{dt} \tag{2}$$

where $\epsilon^{cl} = m_i n_i / \epsilon_0 B^2 = (\omega_{pi}/\Omega_i)^2 = (k_{Di}\rho_i)^2 >> 1$. This polarization current originates from delayed ion gyro motion from time varying electric field and ϵ^{cl} is called classical polarization, which is a low frenquency dielectric constant perpendicular to the magnetic field.

And in tokamak, considering the toroidal effect, we should include the traped and passing particals. Some fraction of charged particals($f_t \sim \sqrt{\epsilon}, \epsilon = r/R$) are trapped by the magnetic mirror and have a radial excursion by

$$\Delta_t = \sqrt{\epsilon}\rho_{pi} = \frac{q\rho_i}{\sqrt{\epsilon}} \tag{3}$$

and as for passing partical the excursion is

$$\Delta_p = q\rho_i \tag{4}$$

So during this trapped partial orbit motion, we alse have the similar polarization effect if we have radial electric field E_r,

$$j^{nc} = \epsilon_0 \epsilon^{nc} \frac{dE_r}{dt} \tag{5}$$

$$\epsilon^{nc} = \sqrt{\epsilon}k_{Di}^2 \Delta_t^2 = \frac{q^2}{\sqrt{\epsilon}}\epsilon^{cl} \tag{6}$$

and Hinton-Rosenbluth give a expression for the polarization as,

$$\epsilon = \epsilon^{cl} + \epsilon^{nc} = \frac{\omega_{pi}^2}{\Omega_i^2}(1 + \frac{1.6q^2}{\sqrt{\epsilon}}) \tag{7}$$

The factor 1.6 comes from detialed kinetic calculation including passing partical contribution.

Then we consider the continuity equation of the polarization current,

$$\frac{\partial \rho_p}{\partial t} + \nabla \cdot \boldsymbol{j}_p = S \tag{8}$$

S is the external source density.Taking the flux surface average and Fourier expansion in space,

$$\frac{\partial}{\partial t} < \rho_p(\boldsymbol{k}) > + < i\boldsymbol{k}_\perp \cdot \boldsymbol{j}_p(\boldsymbol{k}) >=< S(\boldsymbol{k}) > \tag{9}$$

$$\epsilon_0 \epsilon_p < k_\perp^2 > \Phi_k = < \rho_p > - \int < S(\boldsymbol{k}) > dt \tag{10}$$

Consider an initial source perturbation $< S(\boldsymbol{k}) >= \delta_k(0)\delta(t)$. when the time scale is in a few gyro motion and much shorter than the bounce time of trapped partical.we have,

$$\epsilon_0 \epsilon^{cl} \Phi_k(t = +0) = -e_i \delta n_k(0) \tag{11}$$

when the time scale is longer than the bounce time of trapped particals, the electrostatic potential is further shielded by the addition of the neoclassical polarization, then we have,

$$\epsilon_0(\epsilon^{cl} + \epsilon^{nc})\Phi_k(t = +\infty) = -e_i \delta n_k(0) \tag{12}$$

Therefore, the ratio of the long term zonal flow potential to the initial zonal flow potential is given by,

$$\frac{\Phi_k(t = \infty)}{\Phi_k(t = 0)} = \frac{\epsilon^{cl}}{\epsilon^{cl} + \epsilon^{nc}} \tag{13}$$

Using the Hinton formula, we have,

$$\frac{\Phi_k(t = \infty)}{\Phi_k(t = 0)} = \frac{1}{1 + 1.6q^2/\sqrt{\epsilon}} \tag{14}$$

[reference]

1. Kikuchi ebook

2. Diamond PPCF 2005

3. Hinton PRL 1998

4. Idomura ...

3 Geodesic acoustic mode

3.1 history

- first prediction by [Winsor 1968]

- Fully forgotten between 1968 and 1996

- dispersion relation, frequency, radial structure and propagation

- close relation to alfven eigen mode

- experimental observations of GAM, H1-heliac,[Shats PRL 2002]; DIII- D, [McKee PoP 2003]

3.2 description

3.2.1 Winsor1968

The starting equations are the follows:

$$\rho_0 \frac{\partial \boldsymbol{v}}{\partial t} = \frac{1}{c}\boldsymbol{J} \times \boldsymbol{B} - \nabla p$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho_0 \boldsymbol{v} = 0$$

$$\nabla \phi = \frac{1}{c}\boldsymbol{b} \times \boldsymbol{B} \tag{15}$$

$$\nabla \cdot \boldsymbol{J} = 0$$

$$\rho_0^{-\gamma} - \gamma p_0 \rho_0^{-\gamma-1}\frac{\partial \rho}{\partial t} + \boldsymbol{v} \cdot \nabla(p_0 \rho_0^{-\gamma}) = 0$$

finally, we can deduce the dispersion relation:

$$\omega^2 \int |\rho|^2 \mathcal{J}dS = \frac{\gamma p_0}{\rho_0}(|\int \rho_0 \frac{\boldsymbol{B} \times \nabla\psi \cdot \nabla B^2}{B^4}\mathcal{J}dS|^2 / \int \frac{|\nabla\psi|^2}{B^2}\mathcal{J}dS + \int \frac{|\boldsymbol{B} \cdot \nabla\rho|^2}{B^2}\mathcal{J}dS) \tag{16}$$

The first term is due to motion in the $\boldsymbol{B} \times \nabla\psi$ direction.It is associated with geodesic curvature, i.e., the surface component of the magnetic field line curvature. And the second **ordinary sound propagation propagating along the field lines**.

The physical mechanism of geodesic acoustic mode is explained as follows. An electric field pertubation E_ψ causes a flow $\boldsymbol{v}_\perp = \boldsymbol{E} \times \boldsymbol{B}/B^2$ and since the compressibility, the flow causes a density accumulation proportional to $-\nabla \cdot \boldsymbol{v}_\perp = \boldsymbol{E} \times \boldsymbol{B} \cdot \nabla B^2/B^4$.This n generates a current $\boldsymbol{J} = \boldsymbol{B} \times \nabla n/B^2$ which transports charge across the magnetic surface and acts to recerse E_ψ

In the limit of circular cross section with large aspect ratio($r \ll R$), the dispersion relation becomes,

$$\omega^2 = \frac{2\gamma p_0}{\rho_0 R^2}[1 + 1/(2q^2)] = 2\frac{C_s^2}{R^2}(1 + 1/(2q^2)) \tag{17}$$

with the defination of sound wave velocity in neutral gas $C_s = (\gamma p_0/\rho_0)^{1/2}$.

3.2.2 GAM in 5-field landau fluid model

[reference]

1. Diamond PPCF 2005

2. Winsor POF 1968