Weekly work summary

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content

- simulation of electromagnetic 5-field landau fluid using spectral method
  - Fourier transform and relevant treatment in the simulation

Spectral Method(Fourier transform)

$$
\begin{aligned}
forward: f(m) &= \frac{1}{N}\sum_{n=0}^{N-1} f(n)e^{-i2\pi mn/N} \\
backward: f(n) &= \sum_{m=0}^{N-1} f(m)e^{i2\pi mn/N}
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
f(k) &= \frac{1}{N}\sum_{n=0}^{N-1} f(x)e^{-i2\pi mx/L_x} \sim \frac{1}{N}\sum_{n=0}^{N-1} f(x)e^{-ik_x x} \\
f(x) &= \sum_{m=0}^{N-1} f(k)e^{i2\pi mx/L_x} \sim \sum_{m=0}^{N-1} f(k)e^{ik_x x}
\end{aligned}
\tag{2}
$$

for linear terms $L(x)$:

$$\mathcal{F}(L(x)) = L_k(k) \tag{3}$$

for nonlinear terms $A(x)B(x)$:

$$\begin{aligned} \mathcal{F}(A(x)B(x)) &= \mathcal{F}[\mathcal{F}^{-1}(A_k(k)) \cdot \mathcal{F}^{-1}(B_k(k))] \\ &\sim A_k(k) * B_k(k) \end{aligned} \tag{4}$$

if $1/N$ in forward process,

$$\mathcal{F}(A(x)B(x)) = A_k(k) * B_k(k)$$

if $1/N$ in backward process,

$$\mathcal{F}(A(x)B(x)) = \frac{1}{N} A_k(k) * B_k(k)$$

1D Fourier Transform using FFTW3

$$f(x) = 1 + \sin(5x) + 2\sin(10x), x \in [0, 2\pi]$$
$$i.e. f(n) = 1 + \sin(5\frac{2\pi n}{N}) + 2\sin(10\frac{2\pi n}{N}), n \in [0, N] \quad (5)$$

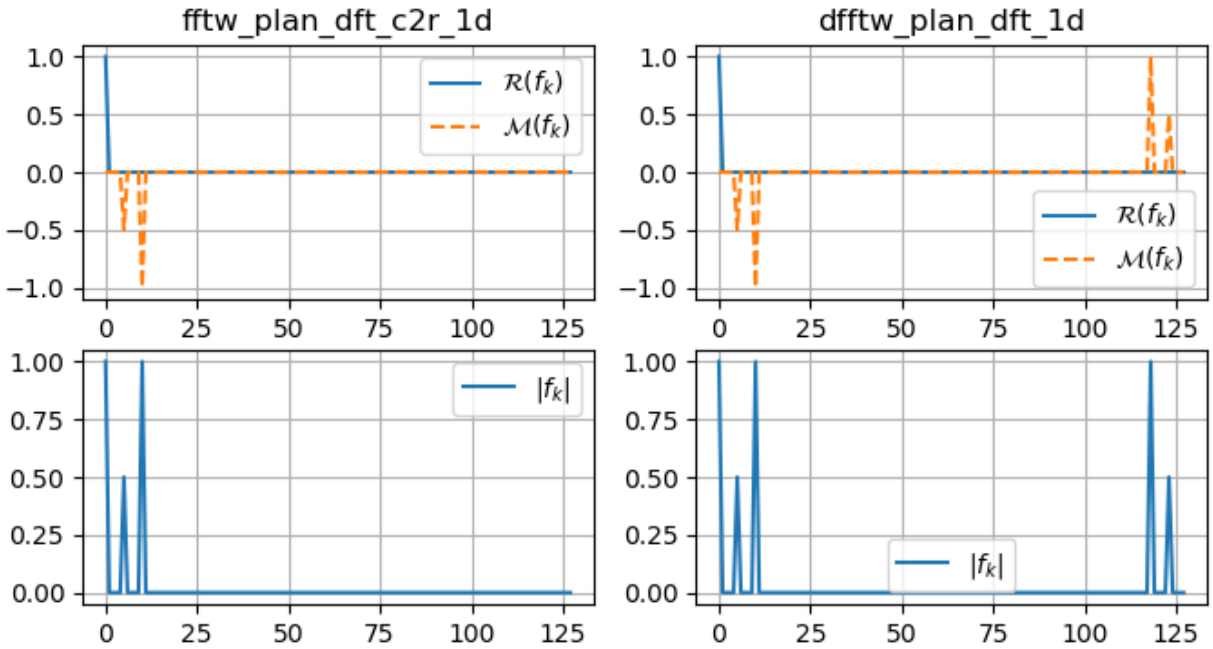


Figure:

1D Fourier Transform using FFTW3

$$m \in [0, 1..N/2-1, -N/2, .. -1]$$
$$k_x \in [0, 1\frac{2\pi}{L_x} ... -1\frac{2\pi}{L_x}] \tag{6}$$

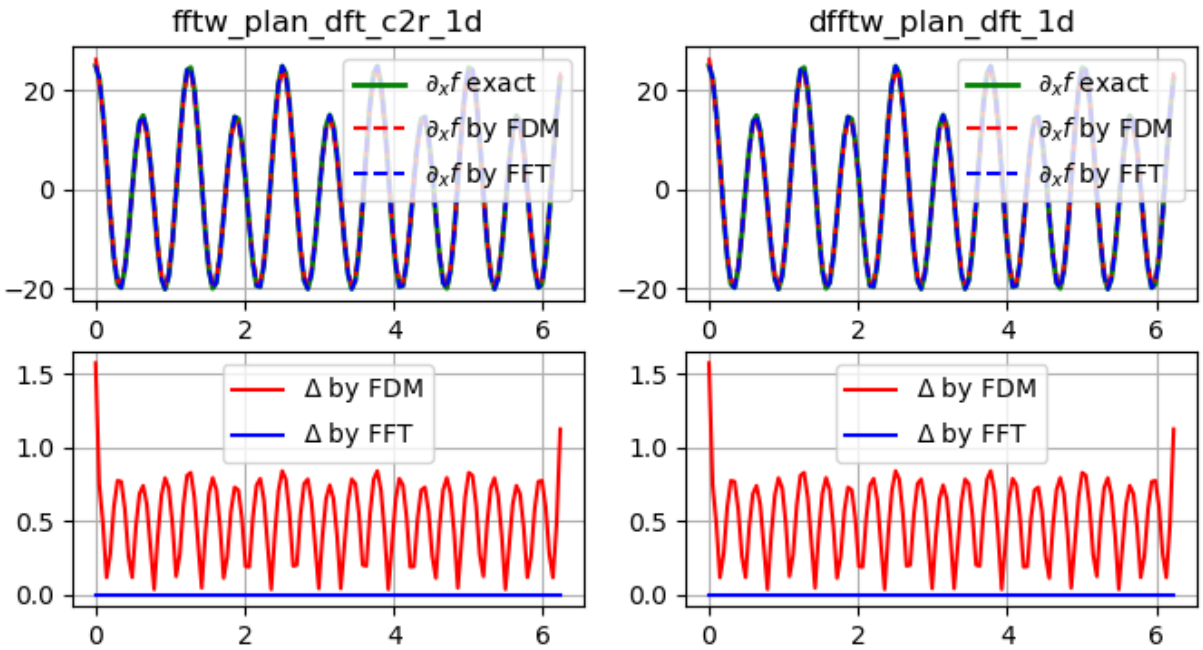


Figure:

2D Parallel Fourier Transform using P3DFFT

2D FFT,

$$f(m,n) = \frac{1}{MN}\sum_{y=0}^{M-1}[\sum_{z=0}^{N-1} f(\theta,\phi)e^{-i2\pi nz/N}]e^{-i2\pi my/M}$$

$$f(y,z) = \sum_{m=0}^{M-1}[\sum_{n=0}^{N-1} f(m,n)e^{i2\pi nz/N}]e^{i2\pi my/M} = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} f(m,n)e^{im\theta+n\zeta} \tag{7}$$

in P3DFFT output,

$$\begin{aligned} m &\in [0, 1..M/2-1, -M/2, ..-1] \\ n &\in [0, 1, 2, ..N/2-1] \end{aligned} \tag{8}$$

2D Parallel Fourier Transform using P3DFFT

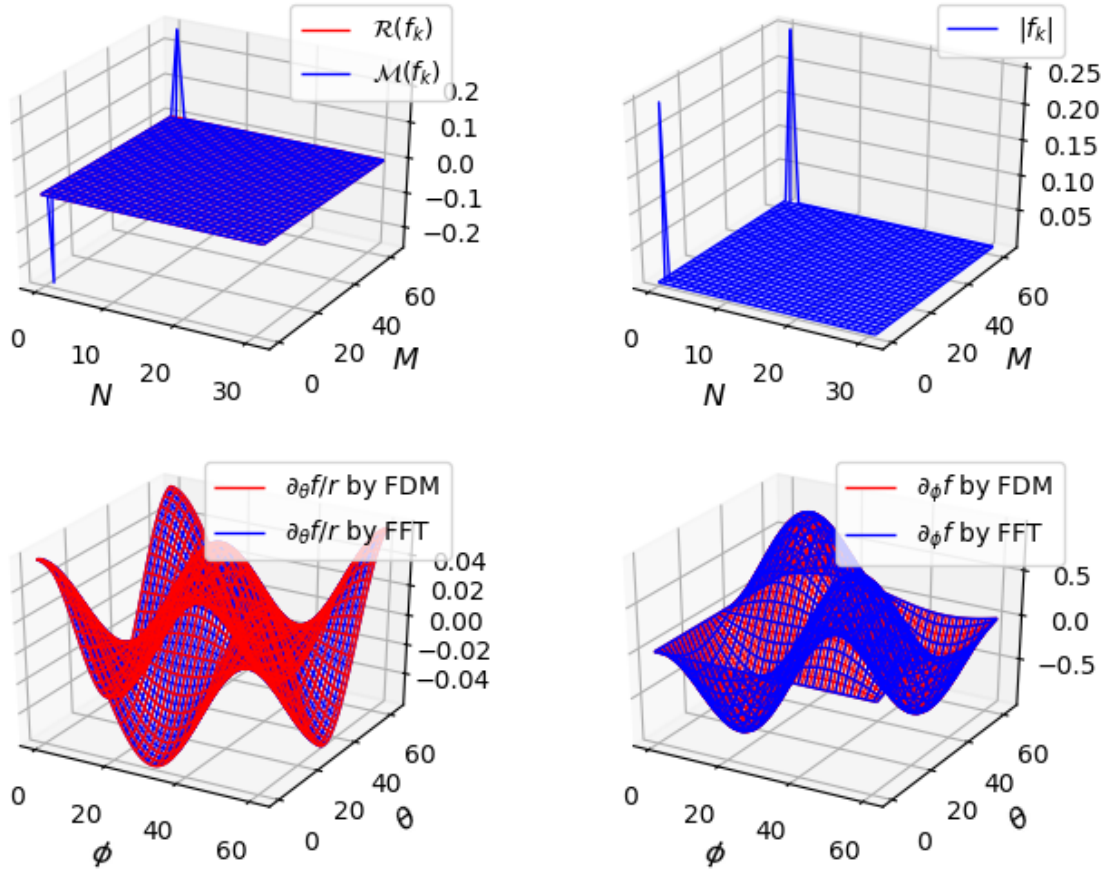


Figure:

Result Comparision of poisson bracket calculation

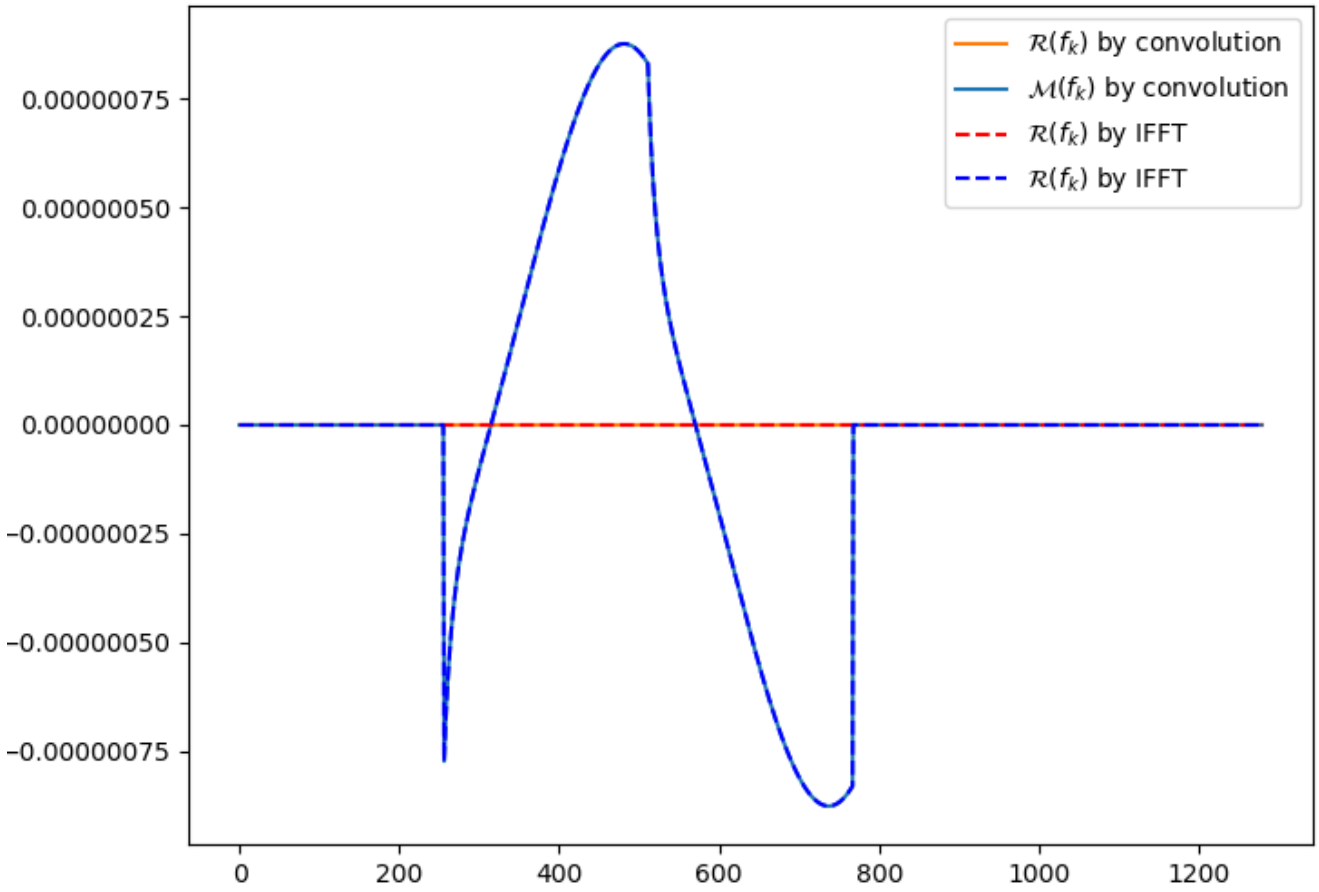
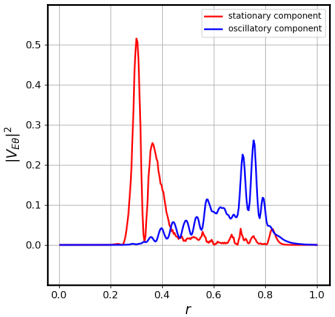
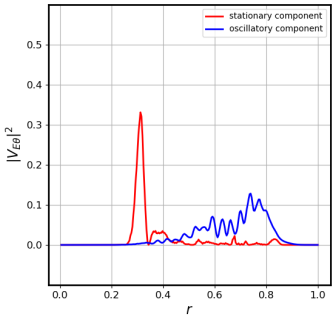


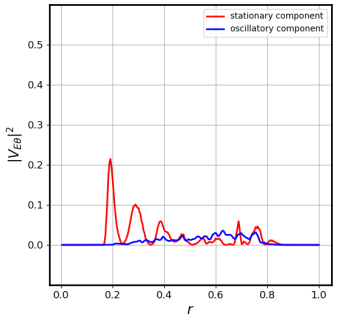
Figure: $[n, \phi]$ in different method for poisson bracket calculation



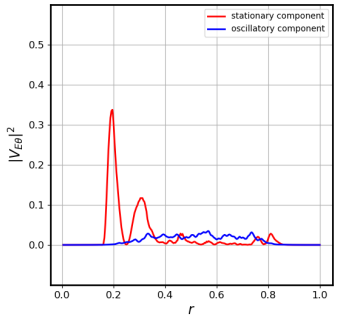
(a) $\beta = 0.1\%$



(b) $\beta = 0.3\%$

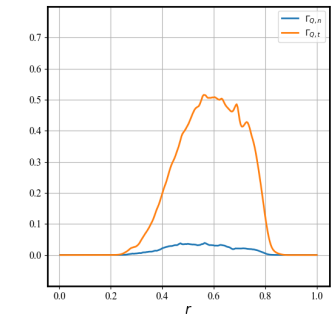


(c) $\beta = 1.0\%$

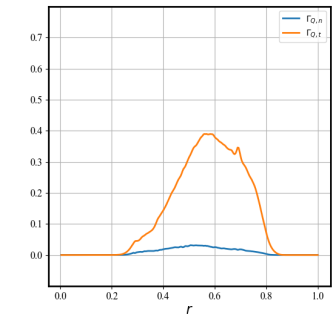


(d) $\beta = 1.2\%$

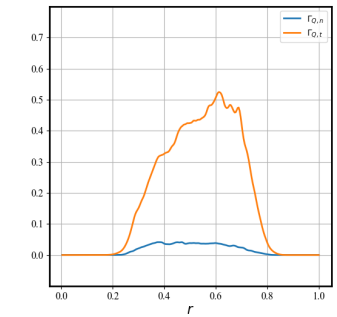
Figure: Time averaged zonal flow profile



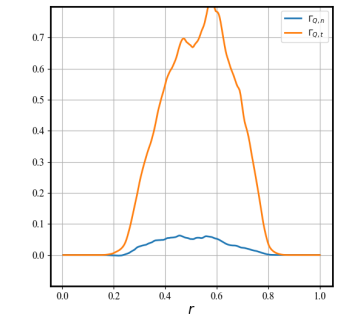
(a) $\beta = 0.1\%$



(b) $\beta = 0.3\%$



(c) $\beta = 1.0\%$



(d) $\beta = 1.2\%$

Figure: Time averaged transport flux profile

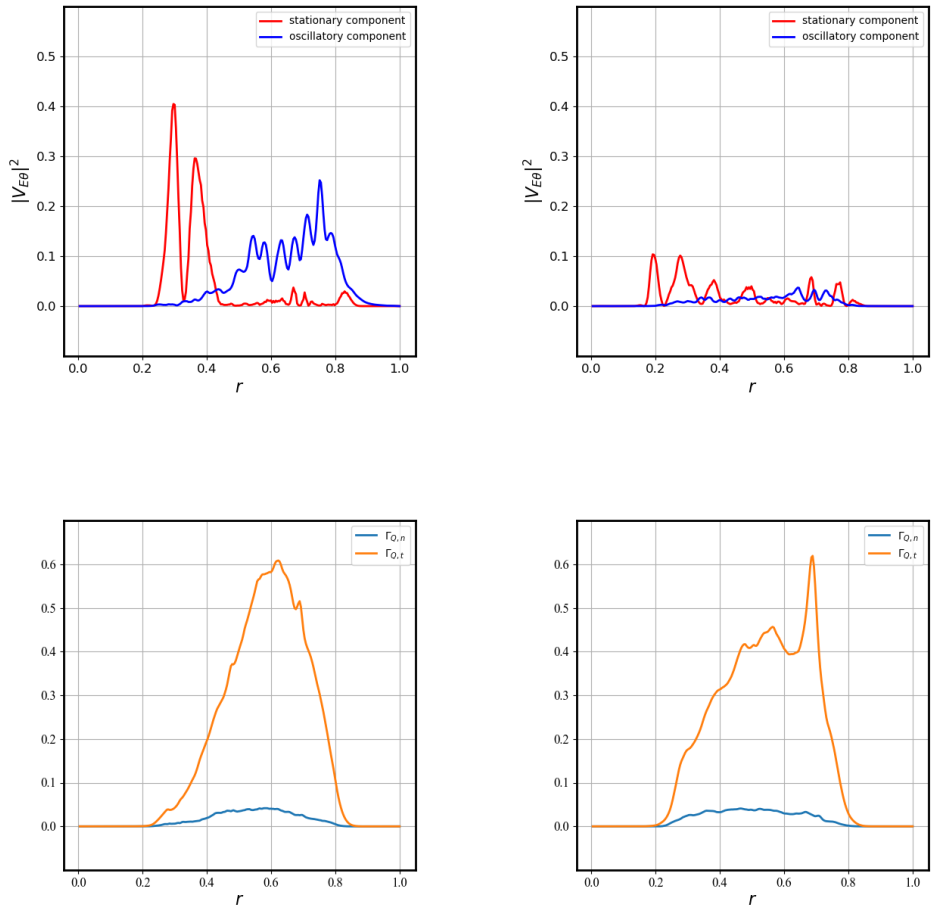

stationary component
oscillatory component
$|V_{E\theta}|^2$
r
$\Gamma_{Q,n}$
$\Gamma_{Q,t}$

Figure: