Weekly work summary

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content

- comparision between different algorithm
- revisiting to former simulation result
- modification of energy transfer equation
- ZF/RHF/GAM

comparision between different algorithm

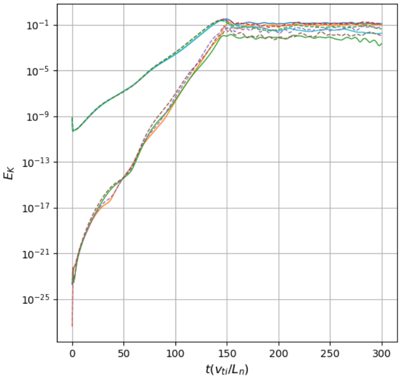


Figure: time evolution of energy in LW/RK/AB

comparision between different algorithm

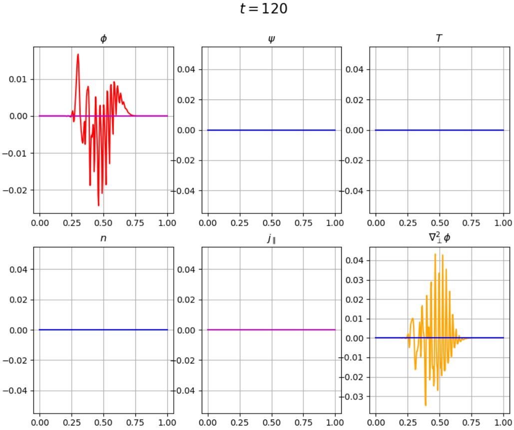


Figure: radial mode structure with AB-256

comparision between different algorithm

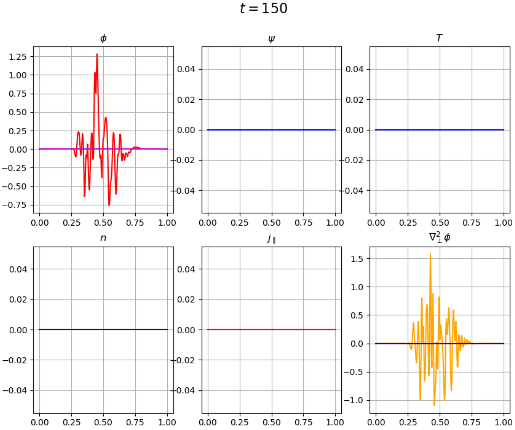


Figure: radial mode structure with LW-256

comparision between different algorithm

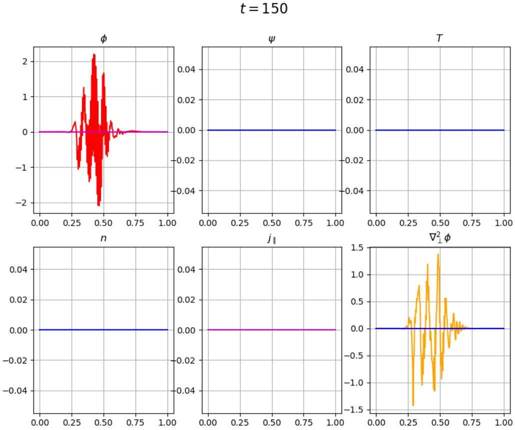


Figure: radial mode structure with AB-256

comparision between different algorithm

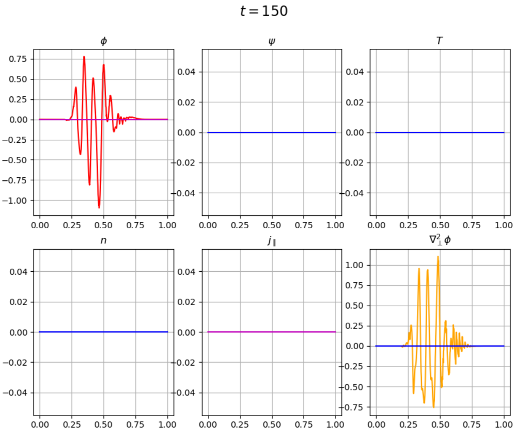
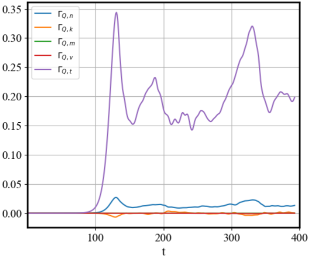
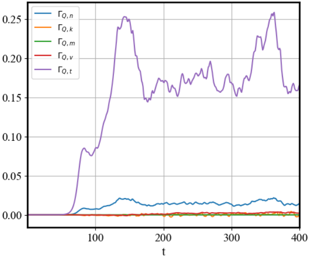


Figure: radial mode structure with AB-512

revisiting to former simulation result



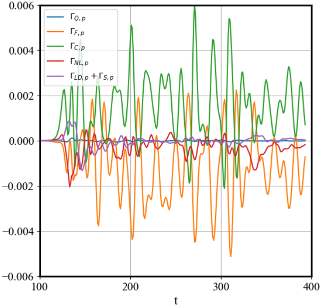
(a) $\beta = 0.1\%$



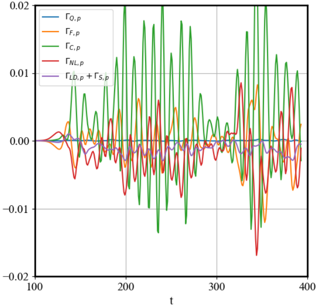
(b) $\beta = 1.0\%$

Figure: Temporal evolution of radial tansport flux

revisiting to former simulation result



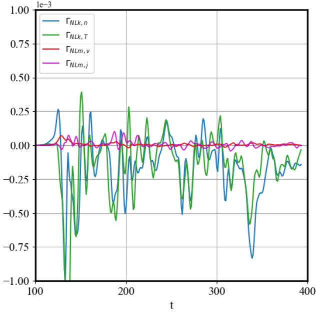
(a) inner region



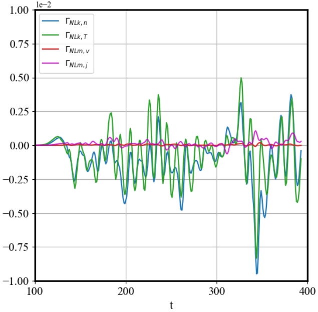
(b) outer region

Figure: Temporal evolution of $p_{1,0}$ energy drives with $\beta = 0.1\%$

revisiting to former simulation result



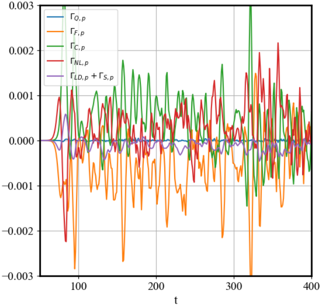
(a) inner region



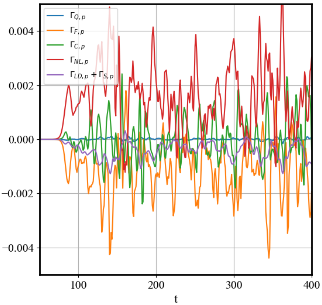
(b) outer region

Figure: Temporal evolution of the component of $\Gamma_{NL,p}$

revisiting to former simulation result



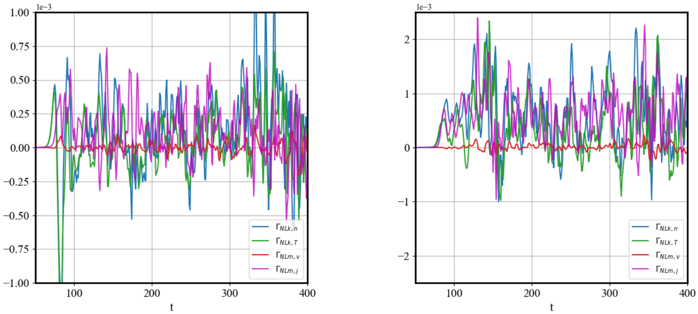
(a) inner region



(b) outer region

Figure: Temporal evolution of $p_{1,0}$ energy drives with $\beta = 0.1\%$

revisiting to former simulation result



(a) inner region (b) outer region

Figure: Temporal evolution of the component of $\Gamma_{NL,p}$

modification of energy transfer equation

$$< f >= \frac{\int f dV}{\int dV} = \frac{\int f \, r dr d\theta d\zeta}{\int \; r dr d\theta d\zeta} \tag{1}$$

for zonal flow energy $E_k = \frac{1}{2} < v_E^2 >$:

$$\frac{\partial}{\partial t} E_k = \Gamma_{NLk,k} + \Gamma_{NLm,k} + \Gamma_{C,k} \tag{2}$$

for pressure $E_p = \frac{1}{2} < p^2 >$:

$$p = p_i + p_e = n_0 T + (1 + \tau) T_0 n \tag{3}$$

$$\frac{\partial}{\partial t} E_p = \Gamma_{Q,p} + \Gamma_{F,p} + \Gamma_{C,p} + \Gamma_{LD,p} + \Gamma_{NL,p} + \Gamma_{S,p} \tag{4}$$

modification of energy transfer equation

flux surface average

$$< f >= \frac{\int f dS}{\int dS} = \frac{\int f \, d\theta d\zeta}{\int \, d\theta d\zeta} \tag{5}$$

for zonal flow

$$\frac{\partial}{\partial t} < v_E >= -\frac{1}{r^2}\frac{\partial}{\partial r} r^2 < v_\theta v_r > + \frac{\beta}{n_{eq}}\frac{1}{r^2}\frac{\partial}{\partial r} r^2 < B_\theta B_r > - \frac{2\epsilon}{n_{eq}} < p \sin\theta > \tag{6}$$

and in the code, we use the alternative expression,

$$\frac{\partial}{\partial t} < v_E >= \frac{1}{r} < \phi \frac{\partial \Omega}{\partial \theta} > + \frac{\beta}{n_{eq}}\frac{1}{r} < A \frac{\partial A}{\partial \theta} > - \frac{2\epsilon}{n_{eq}} < p \sin\theta > \tag{7}$$

with the Fourier expansion,

$$\frac{\partial}{\partial t} < v_E >= \sum_{m1+m2=0} \phi_{m1} \Omega_{\theta,m2} + \sum_{m1+m2=0} A_{m1} j_{\theta,m2} - \frac{i\epsilon}{n_{eq}}(p_{1,0} - p_{-1,0}) \tag{8}$$

modification of energy transfer equation

as for equation for thr evolution of $< p\sin\theta >$

$$\frac{\partial}{\partial t} < p\sin\theta > = < (R_Q + R_F + R_C + R_{NL} + R_{LD} + R_S)\sin\theta > \tag{9}$$

$$\begin{aligned}
< R_Q\sin\theta > &= a[(1+\tau)T_{eq}\frac{\partial n_{eq}}{\partial r} + n_{eq}\frac{\partial T_{eq}}{\partial r}] < \phi\cos\theta > \\
&+ (1+\tau)\beta T_{eq}\frac{\partial j_0}{\partial r} < A\cos\theta > \\
< R_F\sin\theta > &= (\Gamma+\tau)n_{eq}T_{eq}\frac{\epsilon}{q} < v\cos\theta > -(1+\tau)T_{eq}\frac{\epsilon}{q} < j\cos\theta > \\
< R_C\sin\theta > &= \epsilon(\Gamma+\tau)p_{eq} < v_\theta > \\
&+ \epsilon(2\Gamma-1)p_{eq} < \frac{\partial T}{\partial r} > +\epsilon((\Gamma-1)-\tau(\tau+1))T_{eq}^2 < \frac{\partial n}{\partial r} > \\
< R_{LD}\sin\theta > &= -(\Gamma-1)\sqrt{\frac{8T_{eq}}{\pi}}n_{eq}\frac{\epsilon}{q} < T\sin\theta >
\end{aligned} \tag{10}$$

Residual zonal flow

- gyrofluid simulation [Beer,Waltz]: a total collisionless decay of poloidal rotation
- gyrokinetic analysis by [RH 1998]: linear collisionless kinetic mechanisms do not damp the zonal flows completely
- verification by various gyrokinetic codes
- considering shaping effect [Xiao 2006]
- modification of zonal flow closures in gyrofluid model[Mandell,Sugama]

Residual zonal flow

Considering the polarization drift in plasma,

$$v_{pj} = \frac{m_j}{e_j B^2} \frac{d\boldsymbol{E}}{dt} \tag{11}$$

this gives rise to the polarization current,

$$j^{cl} = \sum_j e_j n_j \boldsymbol{v}_{pj} = \epsilon_0 \epsilon^{cl} \frac{d\boldsymbol{E}}{dt} \tag{12}$$

where $\epsilon^{cl} = m_i n_i / \epsilon_0 B^2 = (\omega_{pi}/\Omega_i)^2 = (k_{Di}\rho_i)^2 >> 1$

Residula zonal flow

And in tokamak, considering the toroidal effect, we should include the traped and passing particals. Some fraction of charged particals($f_t \sim \sqrt{\epsilon}, \epsilon = r/R$) are trapped by the magnetic mirror and have a radial excursion by

$$\Delta_t = \sqrt{\epsilon}\rho_{pi} = \frac{q\rho_i}{\sqrt{\epsilon}} \tag{13}$$

and as for passing partical the excursion is

$$\Delta_p = q\rho_i \tag{14}$$

Residula zonal flow

So during this trapped partial orbit motion, we alse have the similar polarization effect if we have radial electric field E_r,

$$j^{nc} = \epsilon_0 \epsilon^{nc} \frac{dE_r}{dt} \tag{15}$$

$$\epsilon^{nc} = \sqrt{\epsilon} k_{Di}^2 \Delta_t^2 = \frac{q^2}{\sqrt{\epsilon}} \epsilon^{cl} \tag{16}$$

and Hinton-Rosenbluth give a expression for the polarization as,

$$\epsilon = \epsilon^{cl} + \epsilon^{nc} = \frac{\omega_{pi}^2}{\Omega_i^2} (1 + \frac{1.6q^2}{\sqrt{\epsilon}}) \tag{17}$$

The factor 1.6 comes from detialed kinetic calculation including passing partical contribution.

Residula zonal flow

Then we consider the continuity equation of the polarization current,

$$\frac{\partial \rho_p}{\partial t} + \nabla \cdot \boldsymbol{j}_p = S \tag{18}$$

S is the external source density.Taking the flux surface average and Fourier expansion in space,

$$\frac{\partial}{\partial t} < \rho_p(\boldsymbol{k}) > + < i\boldsymbol{k}_\perp \cdot \boldsymbol{j}_p(\boldsymbol{k}) > = < S(\boldsymbol{k}) > \tag{19}$$

$$\epsilon_0 \epsilon_p < k_\perp^2 > \Phi_k = < \rho_p > - \int < S(\boldsymbol{k}) > dt \tag{20}$$

Residula zonal flow

Consider an initial source perturbation $< S(\boldsymbol{k}) >= \delta_k(0)\delta(t)$. when the time scale is in a few gyro motion and much shorter than the bounce time of trapped partical.we have,

$$\epsilon_0 \epsilon^{cl} \Phi_k(t = +0) = -e_i \delta n_k(0) \tag{21}$$

when the time scale is longer than the bounce time of trapped particals, the electrostatic potential is further shielded by the addition of the neoclassical polarization, then we have,

$$\epsilon_0(\epsilon^{cl} + \epsilon^{nc}) \Phi_k(t = +\infty) = -e_i \delta n_k(0) \tag{22}$$

Therefore, the ratio of the long term zonal flow potential to the initial zonal flow potential is given by,

$$\frac{\Phi_k(t = \infty)}{\Phi_k(t = 0)} = \frac{\epsilon^{cl}}{\epsilon^{cl} + \epsilon^{nc}} \tag{23}$$

Using the Hinton formula, we have,

$$\frac{\Phi_k(t = \infty)}{\Phi_k(t = 0)} = \frac{1}{1 + 1.6q^2/\sqrt{\epsilon}} \tag{24}$$

Geodesic acoustic mode

- first prediction by [Winsor 1968]
- Fully forgotten between 1968 and 1996
- dispersion relation, frequency, radial structure and propagation
- close relation to alfven eigen mode
- experimental observations of GAM, H1-heliac,[Shats PRL 2002]; DIII- D, [McKee PoP 2003]

Geodesic acoustic mode

The starting equations are the follows:

$$
\begin{aligned}
&\rho_0 \frac{\partial \boldsymbol{v}}{\partial t} = \frac{1}{c}\boldsymbol{J} \times \boldsymbol{B} - \nabla p \\
&\frac{\partial \rho}{\partial t} + \nabla \cdot \rho_0 \boldsymbol{v} = 0 \\
&\nabla \phi = \frac{1}{c}\boldsymbol{b} \times \boldsymbol{B} \\
&\nabla \cdot \boldsymbol{J} = 0 \\
&\rho_0^{-\gamma} - \gamma p_0 \rho_0^{-\gamma-1}\frac{\partial \rho}{\partial t} + \boldsymbol{v} \cdot \nabla (p_0 \rho_0^{-\gamma}) = 0
\end{aligned}
\tag{25}
$$

Geodesic acoustic mode

finally, we can deduce the dispersion relation:

$$\omega^2 \int |\rho|^2 \mathcal{J}\, dS =$$

$$\frac{\gamma p_0}{\rho_0}\left(\left|\int \rho_0 \frac{\boldsymbol{B} \times \nabla\psi \cdot \nabla B^2}{B^4} \mathcal{J}\, dS\right|^2 \Big/ \int \frac{|\nabla\psi|^2}{B^2} \mathcal{J}\, dS + \int \frac{|\boldsymbol{B} \cdot \nabla\rho|^2}{B^2} \mathcal{J}\, dS\right) \tag{26}$$

The first term is due to motion in the $\boldsymbol{B} \times \nabla\psi$direction.It is associated with geodesic curvature, i.e., the surface component of the magnetic field line curvature. And the second **ordinary sound propagation propagating along the field lines**.

Geodesic acoustic mode

In the limit of circular cross section with large aspect ratio($r \ll R$), the dispersion relation becomes,

$$\omega^2 = \frac{2\gamma p_0}{\rho_0 R^2}[1 + 1/(2q^2)] = 2\frac{C_s^2}{R^2}(1 + 1/(2q^2)) \tag{27}$$

with the defination of sound wave velocity in neutral gas $C_s = (\gamma p_0/\rho_0)^{1/2}$.

question: relation between RHF and GAM