Energy transfer in ITG/KBM system

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1 model equation

$$\frac{dn}{dt} = a\frac{dn_{eq}}{dr}\nabla_\theta\phi - n_{eq}\nabla_\parallel v + \nabla_\parallel j + \omega_d(n_{eq}\phi - p_e) + D_n\nabla_\perp^2 n \tag{1}$$

$$\frac{d\nabla_\perp^2\phi}{dt} = -aT_{eq}(\frac{1}{n}\frac{dn_{eq}}{dr} + \frac{1}{T_{eq}}\frac{dT_{eq}}{dr})\nabla_\theta\nabla_\perp^2\phi + \frac{1}{n_{eq}}\nabla_\parallel j - \omega_d(T_i + \frac{T_{eq}}{n_{eq}}n + \frac{p_e}{n_{eq}}) + D_U\nabla_\perp^4\phi \tag{2}$$

$$\frac{dv}{dt} = -\nabla_\parallel T_i - 2\frac{T_{eq}}{n_{eq}}\nabla_\parallel n - \beta aT_{eq}(\frac{2}{n_{eq}}\frac{dn_{eq}}{dr} + \frac{1}{T_{eq}}\frac{dT_{eq}}{dr})\nabla_\theta A + D_v\nabla_\perp^2 v \tag{3}$$

$$\beta\frac{\partial A}{\partial t} = -\nabla_\parallel\phi + \frac{T_{eq}}{n_{eq}}\nabla_\parallel n + \beta aT_{eq}\frac{1}{n_{eq}}\frac{dn_{eq}}{dr}\nabla_\theta A + \sqrt{\frac{\pi}{2}\frac{m_e}{m_i}}|\nabla_\parallel|(v - \frac{j}{n_{eq}}) - \eta j \tag{4}$$

$$\frac{dT_i}{dt} = a\frac{\partial T_{eq}}{\partial r}\nabla_\theta\phi - (\Gamma-1)T_{eq}\nabla_\parallel v + T_{eq}\omega_d((\Gamma-1)\phi + (2\Gamma-1)T_i + (\Gamma-1)\frac{T_{eq}}{n_{eq}}n) - (\Gamma-1)\sqrt{\frac{8T_{eq}}{\pi}}|\nabla_\parallel|T_i + D_T\nabla_\perp^2 T_i \tag{5}$$

2 energy balance equation

energy in system

$$\begin{aligned} E_k &= \frac{1}{2} < |\nabla_\perp\phi|^2 > \\ E_m &= \frac{1}{2} < \frac{\beta}{n_{eq}}|\nabla_\perp\psi|^2 > \\ E_n &= \frac{1}{2} < \tau\frac{T_{eq}}{n_{eq}^2}|n|^2 > \\ E_v &= \frac{1}{2} < \frac{\tau}{\tau+1}|v|^2 > \\ E_t &= \frac{1}{2} < \frac{\tau}{(\Gamma-1)(\tau+1)T_{eq}}|T|^2 > \end{aligned} \tag{6}$$

where

$$< f > = \frac{\int f dV}{\int dV} = \frac{\int f \; rdrd\theta d\zeta}{\int \; rdrd\theta d\zeta} \tag{7}$$

total energy

$$E_{total} = E_k + E_m + E_n + E_v + E_t \tag{8}$$

time dependence of energy

$$\frac{d}{dt}E_i = \Gamma_F + \Gamma_Q + \Gamma_C + \Gamma_{LD} + \Gamma_{NL} + \Gamma_S \tag{9}$$

radial transport items

$$
\begin{aligned}
\Gamma_{Q,n} &=< \tau a\frac{T_{eq}}{n_{eq}^2}n\nabla_\theta\phi > + < \beta\tau\frac{T_{eq}}{n_{eq}^2}\frac{\partial j_0}{\partial r}n\nabla_\theta A > \\
\Gamma_{Q,v} &= - < \frac{\tau}{\tau+1}\beta a T_{eq}(\frac{2}{n_{eq}}\frac{dn_{eq}}{dr} + \frac{1}{T_{eq}}\frac{dT_{eq}}{dr})v\nabla_\theta A > \\
\Gamma_{Q,t} &=< \frac{\tau}{(\Gamma-1)(\tau+1)T_{eq}}a\frac{\partial T_{eq}}{\partial r}T\nabla_\theta\phi > \\
\Gamma_{Q,k} &=< -aT_{eq}(\frac{1}{n}\frac{dn_{eq}}{dr} + \frac{1}{T_{eq}}\frac{dT_{eq}}{dr})\phi\nabla_\theta\nabla_\perp^2\phi > + < \frac{\beta}{n_{eq}}\frac{\partial j_0}{\partial r}\phi\nabla_\theta A > \\
\Gamma_{Q,m} &=< \frac{\beta}{n_{eq}}aT_{eq}\frac{1}{n_{eq}}\frac{dn_{eq}}{dr}j\nabla_\theta A >
\end{aligned}
\tag{10}
$$

parallel transport items

$$\Gamma_{F,k} =<> \tag{11}$$

curvature items

$$\Gamma_{C,k} =<> \tag{12}$$

landau damping items

$$
\begin{aligned}
D_m &=< \frac{1}{n_{eq}}\sqrt{\frac{\pi\tau}{2}\frac{m_e}{m_i}}j\nabla_\parallel|(v - \frac{j}{n_{eq}}) > \\
D_t &= - < \frac{\tau}{(\tau+1)T_{eq}}\sqrt{\frac{8T_{eq}}{\pi}}T|\nabla_\parallel|T >
\end{aligned}
\tag{13}
$$

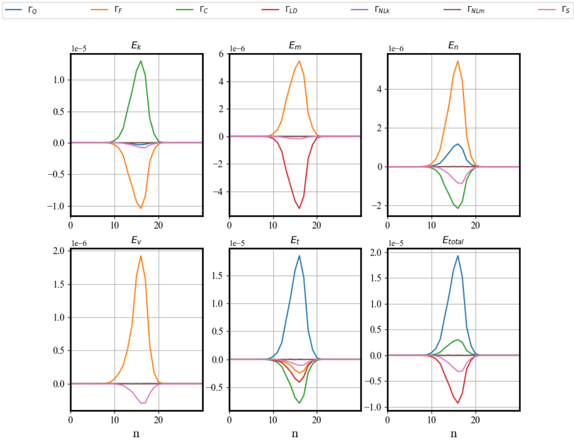
diffusion items

$$
\begin{aligned}
\Gamma_{S,k} &= -D_U < |\nabla_\perp^2\phi|^2 > \\
\Gamma_{S,m} &= -\frac{1}{n_{eq}}D_m < |j|^2 > \\
\Gamma_{S,n} &= \tau\frac{T_{eq}}{n_{eq}^2}D_n < |\nabla_\perp n|^2 > \\
\Gamma_{S,v} &= -\frac{\tau}{\tau+1}D_v < |\nabla_\perp v|^2 > \\
\Gamma_{S,t} &= -\frac{\tau}{(\Gamma-1)(\tau+1)T_{eq}}D_t < |\nabla_\perp T|^2 >
\end{aligned}
\tag{14}
$$

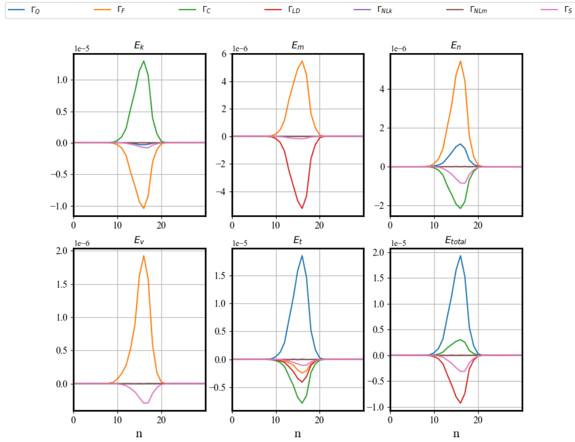
time dependence of energy

$$\frac{\partial}{\partial t}E_{total} = \Sigma_i[\frac{1}{2}(\Gamma_{Q,i} + \Gamma_{C,i} + \Gamma_{LD,i}) + \Gamma_{S,i}] \tag{15}$$

3 turbulence

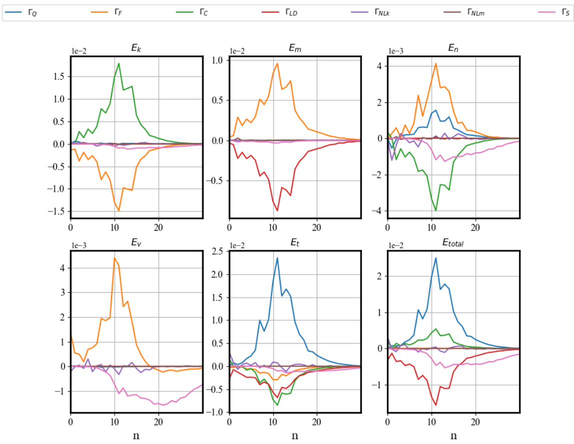


(a)



(b)

Figure 1: (a),(b)



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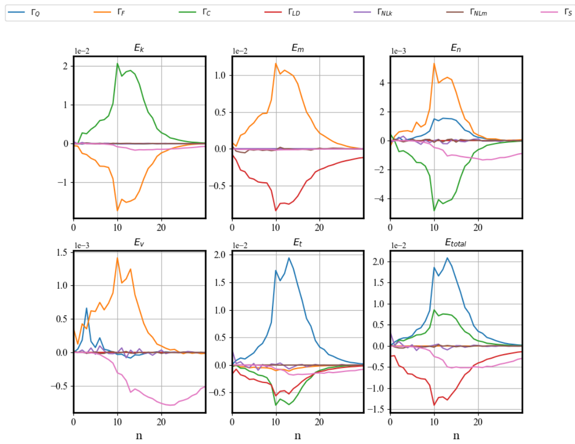
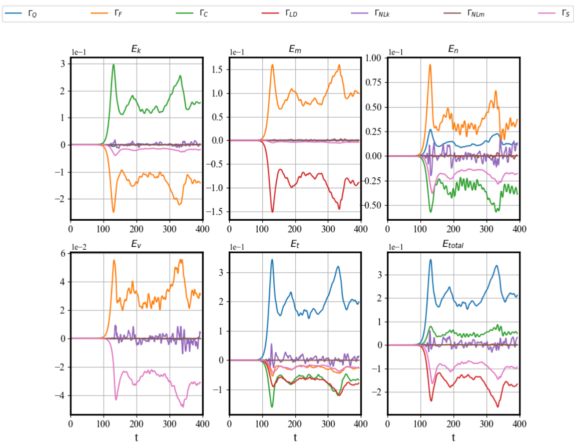
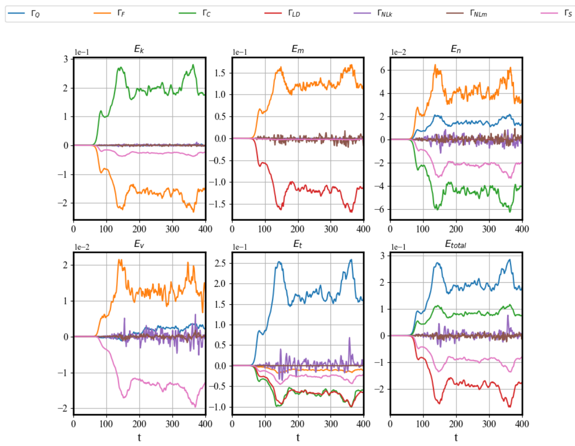


(b)

Figure 2: (a),(b)



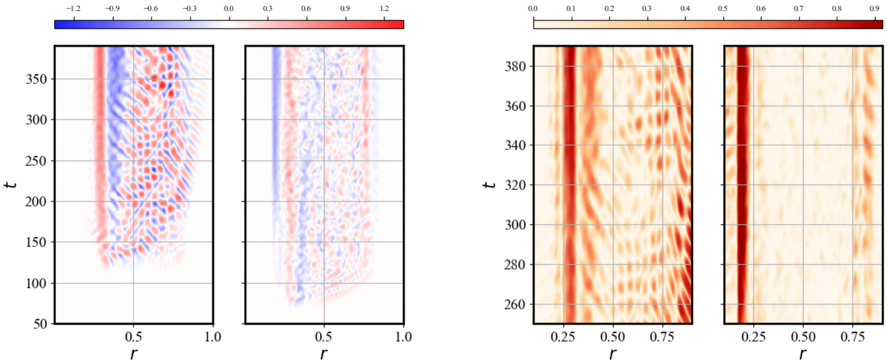
(a)



(b)

Figure 3: (a),(b)

4 zonal flow character



(a) (b)

Figure 4: (a)zonal flow as a function of radius and time,(b)$E_{k,ZF}/E_{k,all}$as a function of radius and time

5 energy drive of $E_{k,m=0,n=0}$ and $E_{p,m=1,n=0}$

for zonal flow energy:

$$\frac{\partial}{\partial t}E_k = \Gamma_{NLk,k} + \Gamma_{NLm,k} + \Gamma_{C,k} \tag{16}$$

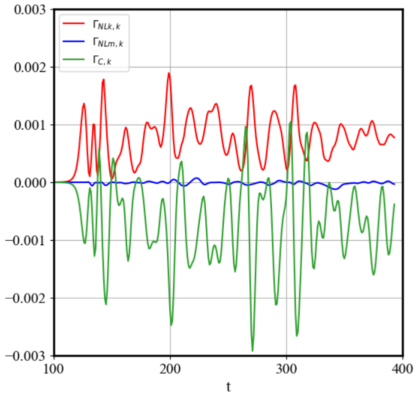
for pressure:

$$p = p_i + p_e = n_0 T + (1+\tau)T_0 n \tag{17}$$

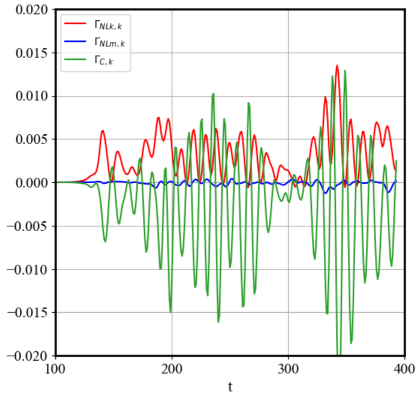
$$\frac{\partial}{\partial t}E_p = \Gamma_{Q,p} + \Gamma_{F,p} + \Gamma_{C,p} + \Gamma_{LD,p} + \Gamma_{NL,p} + \Gamma_{S,p} \tag{18}$$

$$\begin{aligned}
&\Gamma_{Q,p} =< a[(1+\tau)T_{eq}\frac{\partial n_e q}{\partial r} + n_{eq}\frac{\partial T_{eq}}{\partial r}]p\nabla_\theta\phi > + < (1+\tau)\beta T_{eq}\frac{\partial j_0}{\partial r}p\nabla_\theta A >\\
&\Gamma_{F,p} = - < (\Gamma+\tau)n_{eq}T_{eq}p\nabla_\parallel v > + < (1+\tau)T_{eq}p\nabla_\parallel j >\\
&\Gamma_{LD,p} = - < (\Gamma-1)\sqrt{\frac{8T_{eq}}{\pi}}n_{eq}p|\nabla_\parallel|T >\\
&\Gamma_{C,p} =< p\omega_d((\Gamma+\tau)n_{eq}T_{eq}\phi) > + < p\omega_d((2\Gamma-1)n_{eq}T_{eq}T) > + < p\omega_d(((\Gamma-1)-\tau(\tau+1))T_{eq}^2 n) >\\
&\Gamma_{NL,p} = - < p((1+\tau)T_{eq}[\phi,n] + n_{eq}[\phi,T]) > + < p(\Gamma+\tau)n_{eq}T_{eq}\beta[A,v] > - < p(1+\tau)T_{eq}\beta[A,j] >\\
&\Gamma_{S,p} =< p[(1+\tau)T_{eq}D_n\nabla_\perp^2 n + n_{eq}D_T\nabla_\perp^2 T] >
\end{aligned} \tag{19}$$

5.1 case of $\beta = 0.1\%$

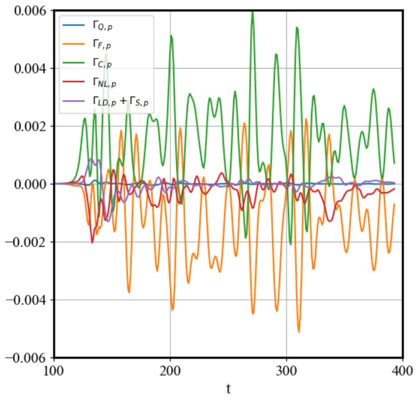


(a)

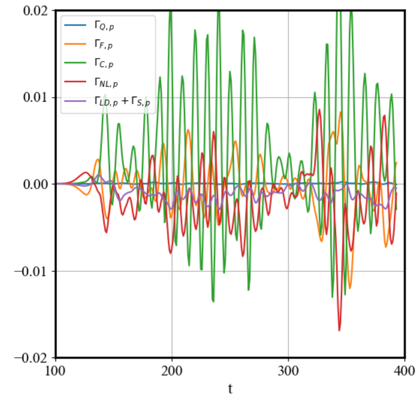


(b)

Figure 5: (a),(b)



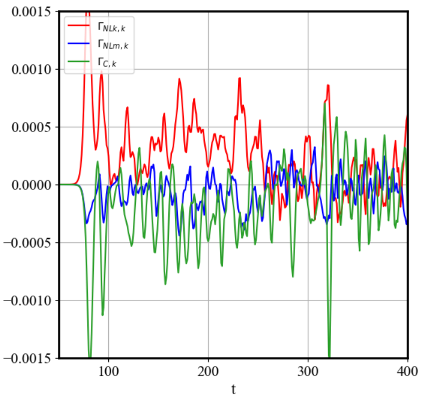
(a)



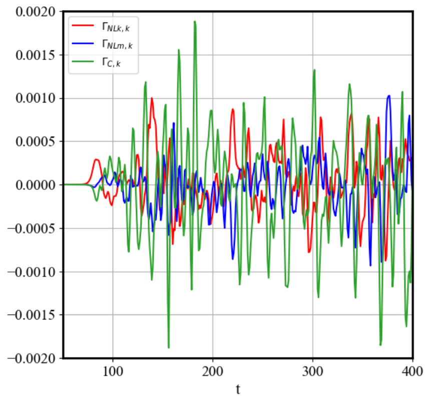
(b)

Figure 6: (a),(b)

5.2 case of $\beta = 1.0\%$

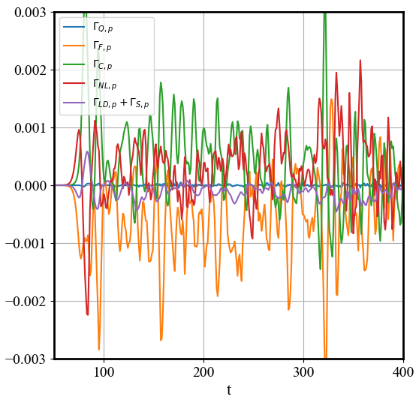


(a)

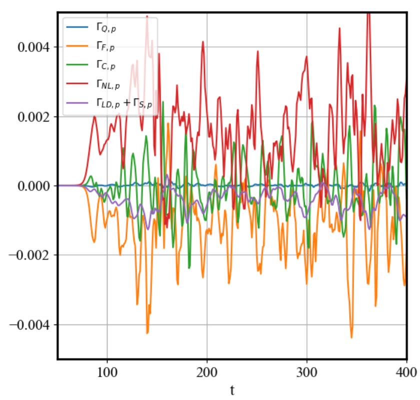


(b)

Figure 7: (a),(b)



(a)



(b)

Figure 8: (a),(b)