Let $Y = XZ, X \sim \text{Uniform}\{-1, 0, 1\}$ and $Z \sim \text{Normal}(1, 2)$, where $X$ and $Z$ are independent.

Q5) Find $f_Y(1)$

$\Rightarrow f_Y(Y) = \text{Normal}(-1,2) + \text{Normal}(0,0) + \text{Normal}(1,2)$

$$\Rightarrow f_Y(y) = \left[\frac{1}{\sqrt{2\pi}\sqrt{2}} \cdot e^{-\frac{1}{2}\left(\frac{y+1}{2}\right)^2} + \frac{1}{\sqrt{2\pi}\sqrt{2}} \cdot e^{-\frac{1}{2}\left(\frac{y}{2}\right)^2} + \frac{1}{\sqrt{2\pi}\sqrt{2}} \cdot e^{-\frac{1}{2}\left(\frac{y-1}{2}\right)^2}\right] \times \frac{1}{3}$$

$$\Rightarrow f_Y(1) = \frac{1}{3\sqrt{4\pi}} \left[ e^{-1} + 0 + 1 \right]$$

5) Find $f_Y(1)$.

- ○ $\frac{1}{3\sqrt{4\pi}}(2 + e^{-1})$
- ○ $\frac{1}{3\sqrt{4\pi}}(1 + e^{\frac{1}{2}})$
- ○ $\frac{1}{3\sqrt{4\pi}}(2 + e^{-\frac{1}{2}})$
- ○ $\frac{1}{3\sqrt{4\pi}}(1 + e^{-1})$ ✓

6) Choose the correct option(s) from the following:

- ☐ $f_{X|Y=1}(0) = 0$ a) ✓
- ☐ $f_{X|Y=1}(-1) = \frac{e^{-1}}{1+e^{-1}}$ b) ✓
- ☐ $f_{X|Y=1}(1) = \frac{1}{1+e^{-1}}$ c)
- ☐ $f_{X|Y=1}(1) = \frac{1}{1+e^{\frac{1}{2}}}$ d)

a)

$$f_{X|Y=1}(0) = \frac{f_{Y=1|X=0} \cdot f_{X=0}}{f_{Y=1}}$$

$$f_X(0) = \frac{1}{3}$$

$$f_Y(1) = \frac{1}{3\sqrt{4\pi}} \left[ 1 + e^{-1} \right]$$

$$f_{Y=1|X=0} = \underbrace{\text{Normal}(0,0)}_{0}$$

$$\therefore f_{X|Y=1}(0) = \frac{0 \times 1/3}{\frac{1}{3\sqrt{4}}\left[1 + e^{-1}\right]} = 0$$

b) $f_{X|Y=1}(-1) = \dfrac{f_{Y=1|X=-1} \cdot f_X(-1)}{f_Y(1)}$

$f_{Y=1|X=-1} = \text{Normal}(-1, 2)$

$$\Rightarrow \frac{1}{\sqrt{4\pi}}\left(e^{-\frac{1}{4}(1+1)^2}\right)$$

$$\Rightarrow \frac{1}{\sqrt{4\pi}}\left(e^{-1}\right)$$

$f_X(-1) = \dfrac{1}{3}$

$$f_{Y=1|X=-1} = \frac{\cancel{\frac{1}{\sqrt{4\pi}}}\left(e^{-1}\right) \cdot \frac{1}{3}}{\cancel{\frac{1}{3\sqrt{4\pi}}}\left[e^{-1} + 1\right]}$$

$$\Rightarrow \frac{\cancel{\frac{1}{3}} \cdot e^{-1}}{\cancel{\frac{1}{3}}\left[e^{-1} + 1\right]} \Rightarrow \frac{e^{-1}}{e^{-1} + 1}$$