1) Let $X$ be a discrete random variable with the probability mass function given in table. Define a new random variable $Y = (X+1)^2$. The range of $Y$ is

| $x$ | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $P(X = x)$ | 0.1 | 0.2 | 0.2 | 0.3 | 0.2 |

$$Y = (X + 1)^2$$

At $X = -2$ $\quad\Big|\quad$ At $X = -1$ $\quad\Big|\quad$ At $X = 1$ $\quad\Big|\quad$ At $X = 2$
$\Rightarrow Y = 1$ $\qquad\qquad Y = 0$ $\qquad\qquad Y = 4$ $\qquad\qquad Y = 9$

$$\Rightarrow \text{Range of } Y = \{0, 1, 4, 9\}$$

2) $X \sim$ Geometric(0.8). Define a function $f(x)$ as

$$x = \begin{cases} x^2 & \text{for } x < 5 \\ 25 & \text{for } x \geq 5 \end{cases}$$

Find the range of $f(X)$.

Note: The number of trials is a stepwise function, i.e.,
$$X \sim [1, \infty)$$

Range of $f(X) = \{1, 4, 9, 16, 25\}$

3) Suppose $X \sim$ Uniform($\{-3, -2, -1, 0, 1, 2, 3\}$) and $f(x) = x^2$. Find $P(f(X) = 4)$.

$f(X) = 4$ when,
$\quad X = -2 \quad$ or $\quad X = 2$
$\Rightarrow P(f(X) = 4) = P(f(X = -2)) + P(f(X = 2))$
$\qquad\qquad\qquad\Rightarrow \quad 1/7 \quad + \quad 1/7$
$\qquad\qquad\qquad\Rightarrow \quad 2/7$