

1.

$$\varphi(122) = \varphi(2) \cdot \varphi(61) = 60$$

$$\varphi(p^n) = (p-1) \cdot p^{n-1}$$

$$\begin{array}{r} 122 \overline{) 2} \\ 61 \overline{) 61} \\ 1 \end{array}$$

$$\boxed{\varphi(122) = 60}$$

$$19^{-1234567}$$

mod 122

Usamos Teorema de Euler

$$\text{si } (n, a) = 1 \Rightarrow a^{\varphi(n)} \equiv 1 \pmod{n}$$

Itemos comprobar que $(19, 122) = 1$.

Calculamos $(19, 122)$

$$\begin{array}{r} 122 \overline{) 19} \\ 8 \overline{) 6} \\ 2 \overline{) 2} \\ 2 \overline{) 2} \\ 0 \end{array}$$

$$\begin{array}{r} 19 \overline{) 8} \\ 3 \overline{) 2} \\ 1 \overline{) 1} \\ 1 \end{array}$$

$$(19, 122) = 1$$

Ahora calculamos $-1234567 \pmod{\varphi(122) = 60}$

$$\begin{array}{r} -1234567 \\ 120 \overline{) 60} \\ -345 \\ +300 \\ -456 \\ +480 \\ -767 \end{array}$$

$$\begin{array}{r} -1234567 \overline{) 60} \\ 120 \overline{) -20(-5) \cdot (-7) \cdot (-8)} \\ -345 \\ +300 \\ -456 \\ +480 \\ -767 \\ 420 \\ 113 \end{array}$$

$$-1234567 : 60 \approx 20578$$

Ahora calculamos $113 \pmod{60}$

$$\begin{array}{r} 113 \overline{) 60} \\ 53 \end{array}$$

$$113 \pmod{60} = 53 \pmod{60}$$

$$19^{53} \pmod{122} = (19^5)^{10} \cdot 19^3$$

$$19^5 \pmod{60}$$

$$19^5 \pmod{60} = (19^6)^6 \cdot 19^6$$

$$19^6 \pmod{60} = 1$$

$$19^5 \pmod{60} = (1)^6 \cdot 1 = 1^9 = 1 \pmod{122}$$

$$\text{Sol } 19^{-1234567}$$

$$\equiv 1 \pmod{122}$$