Javier Gómez López

1.

$a \in \mathbb{R}$

$$\begin{aligned} ax + z + t &= 0 \\ ax + y + 2z &= a \\ y + z + at &= a \end{aligned}$$

Sea $A = \begin{pmatrix} a & 0 & 1 & 1 \\ a & 1 & 2 & 0 \\ 0 & 1 & 1 & a \end{pmatrix}$ $A^* = \begin{pmatrix} a & 0 & 1 & 1 & 0 \\ a & 1 & 2 & 0 & a \\ 0 & 1 & 1 & a & a \end{pmatrix}$

$rg(A) \leq 3$

$\begin{vmatrix} 0 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 1 & a \end{vmatrix} = 1 - 2 - a = -a - 1$ esto es 0 si $a = -1$

Si $a = -1$ Discutimos

$A = \begin{pmatrix} -1 & 0 & 1 & 1 \\ -1 & 1 & 2 & 0 \\ 0 & 1 & 1 & -1 \end{pmatrix}$ $\begin{vmatrix} 0 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 1 & -1 \end{vmatrix} = 0$ $\begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix} = -1 \neq 0 \Rightarrow$

$rg(A) \geq 2$
$rg(A) \leq 3$

$\begin{vmatrix} -1 & 0 & 1 \\ -1 & 1 & 2 \\ 0 & 1 & 1 \end{vmatrix} = -1 - 1 + 2 = 0$ $rg(A) = 2$

$A^* = \begin{pmatrix} -1 & 0 & 1 & 1 & 0 \\ -1 & 1 & 2 & 0 & -1 \\ 0 & 1 & 1 & -1 & -1 \end{pmatrix}$ Puesto que $\begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix} = -1 \neq 0$ $rg(A^*) \geq 2$

$rg(A^*) \leq 3$

$\begin{vmatrix} 0 & 1 & 0 \\ 1 & 2 & -1 \\ 1 & 1 & -1 \end{vmatrix} = -1 + 1 = 0$ $rg(A^*) = 2$

$rg(A) = rg(A^*) = 2 \neq 4 =$ nº incógnitas $\xRightarrow{R.F} S.C.I$

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Resolvemos para $a=-1$

$$\left.\begin{array}{l} -x + z + t = 0 \\ -x + y + 2z = -1 \\ y + z - t = -1 \end{array}\right\} \quad \text{Sea } z=\alpha \text{ y } t=\beta$$

$$\left.\begin{array}{l} -x + \alpha + \beta = 0 \\ -x + y + 2\alpha = -1 \\ y + \alpha - \beta = -1 \end{array}\right\} \Longrightarrow \begin{array}{l} x = \alpha + \beta \\ y = -1 + \beta - \alpha \\ z = \alpha \\ t = \beta \end{array} \qquad \alpha, \beta \in \mathbb{R}$$

Discutimos para $a \neq -1$

Para este caso, $|A| \neq 0 \Longrightarrow rg(A) = rg(A^*) = 3 \neq 4 = n^{\circ}$ incógnitas

Por Rouché-Frobenius S.C.I

$$\left.\begin{array}{l} ax + z + t = 0 \\ ax + y + 2z = a \\ y + z + at = a \end{array}\right\} \quad a \neq -1$$

$$x = -\frac{z+t}{a} \quad \text{si } a \neq 0$$

$$\left.\begin{array}{l} a \cdot \left(-\frac{z+t}{a}\right) + y + 2z = a \\ y + z + at = a \end{array}\right\} \Longrightarrow \left.\begin{array}{l} z - t + y = a \\ y + z + at = a \end{array}\right\}$$

$$z = a + t - y$$

$$y + a + t - y + at = a \Longrightarrow a + t + at = a \Longrightarrow t + at = 0$$

$$t(a+1) = 0$$

$$t = 0$$

$$z = a - y \qquad x = \frac{-a+y-t}{a} = \frac{y-a}{a}$$

<u>Sol</u> Si $y = \mu$ $\quad x = \frac{\mu - a}{a}, y = \mu, z = a - \mu \quad t = 0 \quad a \neq 0$

$$(x, y, z, t) = \left(\frac{\mu - a}{a}, \mu, a - \mu, 0\right) \quad \mu, a \in \mathbb{R}, \ a \neq 0$$

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2.

$P^3(\mathbb{R})$ $\quad U = L(\{1-x, 1+x^2\})$ $\quad W = \{p(x) \in P^3(\mathbb{R}) : p(1)=0, p''(1)=0\}$

• Calcular $U+W$ y $U \cap W$. Argumentar si la suma es directa

$p(x) \in \mathbb{R}_3[x] = ax^3 + bx^2 + cx + d$

$B_U = \{(0,0,-1,1), (0,1,0,1)\}$ Al no ser proporcionales son Li y esto base de U.

$p'(x) = 3ax^2 + 2bx + c$ $\qquad a+b+c+d=0$
$p''(x) = 6ax + 2b$ $\qquad 6a + 2b = 0$

$a = \alpha \quad b = \frac{-6a}{2} = -3\alpha \quad c = \lambda \quad d = -\lambda + 3\alpha - \alpha = -\lambda + 2\alpha$

$B_W = \{(1,-3,0,2), (0,0,1,-1)\}$ No son proporcionales ⟹ son Li y esto es base de W.

• Base de $U+W$

$$\begin{pmatrix} 1 & -3 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix} \Rightarrow \text{Tiene 3 pivotes} \quad B_{U+W} = \{(0,0,-1,1), (0,1,0,1), (1,-3,0,2)\}$$

$B_{W+U} = \{(-x+1), (x^2+1), (x^3-3x^2+2)\}$

Fórmula de las dimensión

$$\underset{3}{\dim(U+W)} = \underset{2}{\dim(U)} + \underset{2}{\dim(W)} - \underset{1}{\dim(U \cap W)}$$

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Calculamos ecuaciones cartesianas de U

$B_U = \{(0,0,-1,1),(0,1,0,1)\}$

$(a,b,c,d) = \alpha\cdot(0,0,-1,1) + \beta\cdot(0,1,0,1)$

$a = 0$
$b = \beta$
$c = -\alpha$
$d = \alpha + \beta$

$d + c - b = 0$

$U \cap W = \{p(x) \in U \wedge p(x) \in W\}$:
$$\left.\begin{aligned} a+b+c+d&=0\\ 6a+2b&=0\\ a&=0\\ -b+c+d&=0 \end{aligned}\right\}$$

$$\begin{pmatrix} 1 & 1 & 1 & 1\\ 6 & 2 & 0 & 0\\ 1 & 0 & 0 & 0\\ 0 & -1 & 1 & 1 \end{pmatrix} \xRightarrow[F_3=F_3-F_1]{F_2=F_2-6F_1} \begin{pmatrix} 1 & 1 & 1 & 1\\ 0 & -4 & -6 & -6\\ 0 & -1 & -1 & -1\\ 0 & -1 & 1 & 1 \end{pmatrix} \Rightarrow \text{(Regla de Chío)}$$

$$1\cdot\begin{vmatrix} -4 & -6 & -6\\ -1 & -1 & -1\\ -1 & 1 & 1 \end{vmatrix} = 1\cdot 0 = 0$$ Entonces son L.D

$$\begin{vmatrix} 1 & 1 & 1\\ 6 & 2 & 0\\ 1 & 0 & 0 \end{vmatrix} = 2 \neq 0$$

$U \cap W = \{p(x) \in U \wedge p(x) \in W$:
$$\left.\begin{aligned} a+b+c+d&=0\\ 6a+2b&=0\\ a&=0 \end{aligned}\right\}$$

$a = 0$
$b = 0$
$c = \alpha$
$d = -\alpha$

$B_{U\cap W} = \{(0,0,1,-1)\}$

$B_{U\cap W} = \{(x-1)\}$

Por lo tanto la intersección $\neq \{0\}$ y la suma no es directa.

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• Encontrar una base de $P^3(\mathbb{R})/U \cap W$

$\dim(P^3[x]/U \cap W) = 4 - 1 = 3$

$B_{U\cap W} = \{(0,0,1,-1)\}$ Ampliamos a $P^3[x]$

$$\begin{pmatrix} 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad \begin{vmatrix} 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{vmatrix} \neq 0 \Rightarrow L.i$$

Luego una base $P^3[x]/U \cap W =$

$\{x^3 + U \cap W,\ x^2 + U \cap W,\ x + U \cap W\}$

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