Ejercicio 6.3: Obtener una fórmula recurrente para las siguientes integrales:

a) $\int x^n e^{-x}\,dx$ b) $\int_0^{\frac{\pi}{2}} \cos^n(x)dx$

(b) $\int_0^{\pi/2} \cos^n(x)\,dx$

$$\int \cos^n(x)dx = -\operatorname{sen}x\cos^{n-1}x - (n-1)\int \cos^{n-2}x \cdot \underbrace{\operatorname{sen}^2x}_{\operatorname{sen}^2x = 1-\cos^2x}\,dx = -\operatorname{sen}x\cos^{n-1}x - (n-1)\int\left(\cos^{n-2}x - \cos^n x\right)dx =$$

$u = \cos^{n-1}(x) \longrightarrow du = -(n-1)\cos^{n-2}(x)\operatorname{sen}x$

$dv = \cos(x) \longrightarrow v = -\operatorname{sen}x$

$= -\operatorname{sen}x\cos^{n-1}x - (n-1)\int\cos^{n-2}x\,dx + (n-1)\int\cos^n x\,dx \Rightarrow$ Si ponemos $\alpha = \int\cos^n x\,dx$

$(2-n)\alpha = -\operatorname{sen}x\cos^{n-1}x - (n-1)\int\cos^{n-2}x\,dx \Rightarrow \int\cos^n x\,dx = -\operatorname{sen}x\cos^{n-1}x - (n-1)\int\cos^{n-2}x\,dx$

como $\cos\frac{\pi}{2}\cdot\operatorname{sen}\frac{\pi}{2} = 0\cdot 1 = 0 = 1\cdot 0 = \cos 0\operatorname{sen}0$, nos quedaría

$$\boxed{\int_0^{\pi/2}\cos^n x\,dx = -(n-1)\int_0^{\pi/2}\cos^{n-2}x\,dx}$$ FORMULA RECURSIVA