o) $\int \frac{x^4-3x^3-3x-2}{x^3-x^2-2x}\,dx$

$$\int \frac{x^4-3x^3-3x-2}{x^3-x^2-2x}\,dx = \int \frac{P(x)}{Q(x)}\,dx$$

Comparamos los grados de ambos polinomios y vemos que $P(x) > Q(x)$ => dividimos

$$\begin{array}{l|l} x^4-3x^3-3x-2 & x^3-x^2-2x \\ \hline -x^4+x^3+2x^2 & x-2 \\ 0\ -2x^3+2x^2-3x-2 & \\ \quad +2x^3-2x^2-4x & \\ \qquad -7x-2 & \end{array}$$

$$\Rightarrow \frac{P(x)}{Q(x)} = (x-2) - \frac{(7x+2)}{x^3-x^2-2x}$$

$$\int x-2-\frac{P'(x)}{Q'(x)}\,dx = \frac{x^2}{2}-2x-\int \frac{P'(x)}{Q'(x)}\,dx$$ => comparamos los grados: $P'(x) < Q'(x)$ => Factorizamos $Q'(x)$

$$Q'(x) = x(x^2-x-2) = 0 \Rightarrow \begin{cases} x=0 \\ x=2 \\ x=-1 \end{cases}$$

$$\frac{P'(x)}{Q'(x)} = \frac{7x+2}{x(x+1)(x-2)}$$

$$\int \frac{P'(x)}{Q'(x)} = A\int\frac{dx}{x} + B\int\frac{dx}{(x-2)} + C\int\frac{dx}{(x+1)}$$

$$\frac{7x+2}{x(x+1)(x-2)} = \frac{A}{x}+\frac{B}{(x-2)}+\frac{C}{(x+1)}$$

$$7x+2 = (x-2)(x+1)A+(x)(x+1)B+x(x-2)C$$

si $x=0$: $2=(-2)(1)A \Rightarrow 2=-2A \Rightarrow A=-1$

si $x=-1$: $-5=(-1)(-3)C \Rightarrow -5=3C \Rightarrow C=-\frac{5}{3}$

si $x=2$: $16=(2)(3)B \Rightarrow B=\frac{16}{6}=\frac{8}{3}$

$$\int\frac{P'(x)}{Q'(x)}\,dx = -\int\frac{dx}{x}-\frac{5}{3}\int\frac{dx}{x-2}+\frac{8}{3}\int\frac{dx}{x+1} = -\ln|x|-\frac{5}{3}\ln|x-2|+\frac{8}{3}\ln|x+1|$$

sol: $\frac{x^2}{2}-2x+\ln|x|+\ln|x-2|-\ln|x+1|$

$\Rightarrow \frac{x^2}{2}-2x+\ln|x(x-2)|-\ln|x+1|$

$\Rightarrow \frac{x^2}{2}-2x+\ln\left|\frac{x(x-2)}{x+1}\right| = \frac{1}{2}x^2-2x+\ln\left|\frac{x(x-2)}{x+1}\right|+k$

p) $\int \frac{x+1}{x^2-3x+3}\,dx$

$$\int \frac{x+1}{x^2-3x+3}\,dx = \int \frac{P(x)}{Q(x)}\,dx$$

comparamos los grados y vemos que $P(x) < Q(x)$

$$\int \frac{x+1}{x^2-3x+3}\,dx = \frac{1}{2}\int \frac{2x-3+3+2}{x^2-3x+3}\,dx = \frac{1}{2}\int \frac{2x-3}{x^2-3x+3}\,dx + \frac{1}{2}\int \frac{5}{x^2-3x+3}\,dx = \frac{1}{2}\ln|x^2-3x+3| + \frac{1}{2}\int \frac{5}{x^2-3x+3}\,dx$$

$$\Rightarrow \frac{1}{2}\int \frac{5}{x^2-3x+\frac{9}{4}+\frac{3}{4}}\,dx = \frac{5}{2}\int \frac{dx}{\underbrace{(x-\frac{3}{2})^2}_{\text{sustituimos por } t^2}+\frac{3}{4}} = \frac{5}{2}\int \frac{dt}{t^2+\frac{3}{4}} = \frac{5}{2}\cdot\frac{2}{\sqrt{3}}\arctan\left(\frac{2t}{\sqrt{3}}\right) = \frac{5}{\sqrt{3}}\arctan\left(\frac{2(x-\frac{3}{2})}{\sqrt{3}}\right)$$

$$= \frac{5}{\sqrt{3}}\arctan\left(\frac{2x-3}{\sqrt{3}}\right)$$

sol: $\frac{1}{2}\ln|x^2-3x+3| + \frac{5}{\sqrt{3}}\arctan\left(\frac{2x-3}{\sqrt{3}}\right)$