⑥ Estudiar la convergencia y, cuando la haya, calcular el valor de las siguientes integrales:

k) $\displaystyle\int_{-\infty}^{+\infty} \frac{x^2-x+2}{x^4+10x^2+9}\,dx.$

$$\lim_{t\to+\infty} \int_{-t}^{t} \frac{x^2-x+2}{x^4+10x^2+9}\,dx = \lim_{t\to+\infty}\left(\int_{-t}^{t}\frac{x^2}{(x^2+1)(x^2+9)}dx - \int_{-t}^{t}\frac{x}{(x^2+1)(x^2+9)}dx + 2\int_{-t}^{t}\frac{1}{(x^2+1)(x^2+9)}dx\right) =$$

$$= \lim_{t\to+\infty}\left(-\cancel{\int_{-t}^{t}\frac{1}{8(x^2+1)}dx} + \int_{-t}^{t}\frac{9}{8(x^2+9)}dx - \int_{-t}^{t}\frac{x}{8(x^2+1)}dx + \int_{-t}^{t}\frac{x}{8(x^2+9)}dx + \cancel{2}\cdot\int_{-t}^{t}\frac{1}{8(x^2+1)}dx - \right.$$

$$\left. - 2\cdot\int_{-t}^{t}\frac{1}{8(x^2+9)}dx\right) = \lim_{t\to+\infty}\left[\frac{3}{8}\operatorname{arctg}\left(\frac{x}{3}\right) - \frac{1}{16}\ln(x^2+1) + \frac{1}{16}\ln(x^2+9) + \frac{1}{8}\operatorname{arctg}(x) - \frac{1}{12}\operatorname{arctg}\left(\frac{x}{3}\right)\right]_{-t}^{t} =$$

$$= \lim_{t\to+\infty}\left[\frac{7}{24}\operatorname{arctg}\left(\frac{x}{3}\right) + \frac{1}{8}\operatorname{arctg}(x) + \frac{1}{16}\left(\ln(x^2+9) - \ln(x^2+1)\right)\right]_{-t}^{t} =$$

$$= \lim_{t\to+\infty}\left(\frac{7}{24}\operatorname{arctg}\left(\frac{t}{3}\right) + \frac{1}{8}\operatorname{arctg}(t) + \cancel{\frac{1}{16}\left(\ln(t^2+9) - \ln(t^2+1)\right)} - \frac{7}{24}\operatorname{arctg}\left(\frac{-t}{3}\right) - \frac{1}{8}\operatorname{arctg}(-t) - \right.$$

$$\left. - \cancel{\frac{1}{16}\left(\ln((-t)^2+9) - \ln((-t)^2+1)\right)}\right) = \lim_{t\to+\infty}\left(\frac{7}{24}\operatorname{arctg}\left(\frac{t}{3}\right) - \frac{7}{24}\operatorname{arctg}\left(\frac{-t}{3}\right) + \right.$$

$$\left. + \frac{1}{8}\operatorname{arctg}(t) - \frac{1}{8}\operatorname{arctg}(-t)\right) = \frac{7}{24}\left(\lim_{t\to+\infty}\left(\operatorname{arctg}\left(\frac{t}{3}\right)\right) - \lim_{t\to+\infty}\left(\operatorname{arctg}\left(\frac{-t}{3}\right)\right)\right) +$$

$$+ \frac{1}{8}\left(\lim_{t\to+\infty}\left(\operatorname{arctg}(t)\right) - \lim_{t\to+\infty}\left(\operatorname{arctg}(-t)\right)\right) = \frac{7}{24}\left(\frac{\pi}{2} + \frac{\pi}{2}\right) + \frac{1}{8}\left(\frac{\pi}{2} + \frac{\pi}{2}\right) = \frac{5}{12}\pi.$$

l) $\int_0^{+\infty} \frac{\ln(x)}{x^2}\,dx.$

$$\lim_{t\to+\infty} \int_0^t \frac{\ln(x)}{x^2}\,dx = \begin{bmatrix} u=\ln(x) \ ; du=\frac{1}{x} \\ dv=\frac{1}{x^2} \ ; v=-\frac{1}{x} \end{bmatrix} = \lim_{t\to+\infty}\left(-\frac{\ln(x)}{x} - \int -\frac{1}{x^2}\,dx\right) =$$

$$= \lim_{t\to+\infty}\left[\frac{-\ln(x)-1}{x}\right]_0^t \Rightarrow \text{divergente.}$$