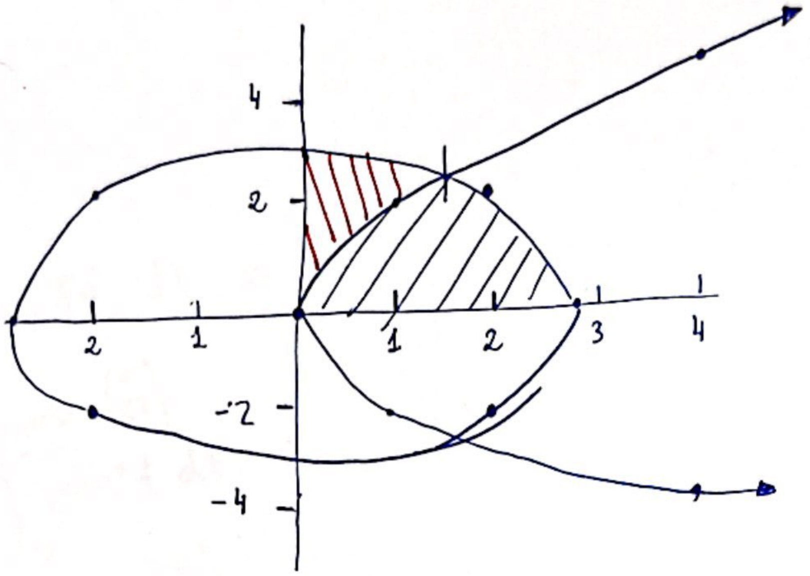
$\boxed{7.12}$



· Punto de corte:

$$x^2 + 4x - 8 = 0 \iff x = \begin{cases} -2+2\sqrt{3} > 0 \\ -2-2\sqrt{3} < 0 \end{cases}$$

· Calcularemos el área de la parte positiva, ya que por simetría será la mitad del área deseada.

· Pongamos $\boxed{\alpha = -2+2\sqrt{3}}$ y calculemos el área roja:

$$S = \int_0^\alpha \left(\sqrt{8-x^2} - 2\sqrt{x}\right)dx = \int_0^\alpha \sqrt{8-x^2}\,dx - \int_0^\alpha 2\sqrt{x}\,dx =$$

$$= \int_0^\alpha \sqrt{8-x^2}\,dx - \underbrace{\cancel{\tfrac{2}{3}}}_{\frac{4}{3}}\left[x^{3/2}\right]_0^\alpha = \int_0^\alpha \sqrt{8-x^2}\,dx - \frac{4}{3}\alpha^{3/2}$$

$$\int_0^{a} \sqrt{8-x^2}\,dx = \int_0^{\operatorname{arcsen}\left(\frac{a}{\sqrt{8}}\right)} \sqrt{8(1-\operatorname{sen}^2 t)}\,\cos t\cdot\sqrt{8}\cdot dt =$$

$$\boxed{\begin{aligned} x &= \sqrt{8}\,\operatorname{sen} t \\ dx &= \sqrt{8}\cos t\,dt \end{aligned}}$$

$$= 8\int_0^{\operatorname{arcsen}\left(\frac{a}{\sqrt{8}}\right)} \cos^2 t\,dt =$$

$$= 8\int_0^{\operatorname{arcsen}\left(\frac{a}{\sqrt{8}}\right)} \frac{1+\cos 2t}{2}\,dt =$$

$$= 4\int_0^{\operatorname{arcsen}\left(\frac{a}{\sqrt{8}}\right)} (1+\cos 2t)\,dt = \cancel{4\left[t+\frac{\operatorname{sen} 2t}{2}\right]}$$

$$= 4\left[\int_0^{\operatorname{arcsen}\left(\frac{a}{\sqrt{8}}\right)} dt + \int_0^{\operatorname{arcsen}\left(\frac{a}{\sqrt{8}}\right)} 2\cos 2t\,dt\cdot\frac{1}{2}\right] =$$

$$= 4\left[\operatorname{arcsen}\left(\frac{a}{\sqrt{8}}\right) + \frac{1}{2}\operatorname{sen}\left(2\operatorname{arcsen}\left(\frac{a}{\sqrt{8}}\right)\right)\right] =$$

$$= 4\operatorname{arcsen}\left(\frac{a}{\sqrt{8}}\right) + 2\operatorname{sen}\left(2\operatorname{arcsen}\left(\frac{a}{\sqrt{8}}\right)\right)$$

Por tanto:

$$S = 4\,\text{arcsen}\left(\frac{\alpha}{\sqrt{8}}\right) + 2\,\text{sen}\left(2\,\text{arcsen}\left(\frac{\alpha}{\sqrt{8}}\right)\right) - \frac{4}{3}\alpha^{3/2}$$

El área azul será: $4\pi - 2S$

(Área del círculo $= 8\pi$)