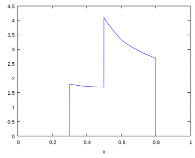
Relación de Ejercicios 1 Javier Gómez López

1 Define la función real g en el intervalo [0,1] como
| 2x - log(x), si 0.3<=x<=0.5
g(x) = | 2/x + |x-0.6|, si 0.5<=x<=0.8
| 0, en otro caso.
Calcula su integral y dibuja su gráfica.

```
--> g( x) : = if x> = 0 . 3 and x< = 0 . 5 then 2· x − log( x) else if x> = 0 . 5 and x< = 0 . 8 then 2 / x + abs( x − 0 . 6) else 0 ;
```

$g(x) := \text{if} x >= 0.3 \text{and} x <= 0.5 \text{then} 2x - \log(x) \text{elseif} x >= 0.5 \text{and} x <= 0.8 \text{then} \frac{2}{x} + \mid x - 0.6 \mid \text{else} 0$

```
--> wxplot2d([ g( x)],[ x, 0, 1]) ;
```




```
--> result1 : integrate( 2· x − log( x), x, 0 . 3, 0 . 5) + integrate( 2 / x + abs( x − 0 . 6), x, 0 . 5, 0 . 8) ;
```

rat: replaced 0.2 by 1/5 = 0.2
rat: replaced 0.3 by 3/10 = 0.3
rat: replaced 0.5 by 1/2 = 0.5
rat: replaced 0.2 by 1/5 = 0.2
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rat: replaced 0.5 by 1/2 = 0.5
rat: replaced 0.2 by 1/5 = 0.2
rat: replaced 0.8 by 4/5 = 0.8
rat: replaced 0.045 by 9/200 = 0.045
rat: replaced 0.125 by 1/8 = 0.125
rat: replaced 0.2 by 1/5 = 0.2
rat: replaced 0.3 by 3/10 = 0.3
rat: replaced 0.5 by 1/2 = 0.5
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rat: replaced 0.5 by 1/2 = 0.5
rat: replaced 0.2 by 1/5 = 0.2
rat: replaced 0.7 by 7/10 = 0.7
rat: replaced −0.5108256237659908 by −21994011/43055810 = −0.5108256237659912
rat: replaced 0.6931471805599453 by 13614799/19642003 = 0.693147180559946
rat: replaced 1.203972804325936 by 24084703/20004358 = 1.203972804325937
rat: replaced 0.5108256237659908 by 21994011/43055810 = 0.5108256237659912
rat: replaced 0.5108256237659908 by 21994011/43055810 = 0.5108256237659912
rat: replaced 0.6931471805599453 by 13614799/19642003 = 0.693147180559946
rat: replaced 1.203972804325936 by 24084703/20004358 = 1.203972804325937
rat: replaced 0.5108256237659908 by 21994011/43055810 = 0.5108256237659912
rat: replaced −0.8465735902799727 by −16628401/19642003 = −0.8465735902799729
rat: replaced −0.6611918412977808 by −26789369/40516787 = −0.6611918412977811
rat: replaced 0.3 by 3/10 = 0.3
rat: replaced 0.5 by 1/2 = 0.5
rat: replaced 0.8 by 4/5 = 0.8
rat: replaced 0.3 by 3/10 = 0.3
rat: replaced −0.6 by −3/5 = −0.6
rat: replaced −0.6 by −3/5 = −0.6
rat: replaced 0.6 by 3/5 = 0.6
rat: replaced 0.3 by 3/10 = 0.3
rat: replaced 0.5 by 1/2 = 0.5
rat: replaced 0.8 by 4/5 = 0.8
rat: replaced 0.3 by 3/10 = 0.3
rat: replaced −0.6 by −3/5 = −0.6
rat: replaced −0.6 by −3/5 = −0.6
rat: replaced −0.004999999999999997 by −1/200 = −0.005
rat: replaced 0.02000000000000001 by 1/50 = 0.02
rat: replaced 0.3 by 3/10 = 0.3
rat: replaced 0.5 by 1/2 = 0.5
rat: replaced 0.8 by 4/5 = 0.8
rat: replaced 0.3 by 3/10 = 0.3
rat: replaced 1.3 by 13/10 = 1.3
rat: replaced 0.3 by 3/10 = 0.3
rat: replaced 0.5 by 1/2 = 0.5
rat: replaced 0.8 by 4/5 = 0.8
rat: replaced 0.3 by 3/10 = 0.3
rat: replaced 0.3 by 3/10 = 0.3
rat: replaced 0.5 by 1/2 = 0.5
rat: replaced 0.8 by 4/5 = 0.8
rat: replaced 0.3 by 3/10 = 0.3
rat: replaced 0.9650072584914712 by 59875457/62046639 = 0.9650072584914712

$$\frac{16176303347167115247131 41}{1234465739174192139816975}$$

```
--> float( result1) ;
```

1.310389007473663

Primero calculamos la integral definida (lo hacemos a trozos) y posteriormente lo mostramos el resultado decimal por pantalla

2 Halla el radio espectral de la matriz de orden 4x4 cuyo coeficiente (i,j) es |2i-4j|.

Primero generamos la matriz

```
--> matrix2 : genmatrix( lambda([ i, j], abs( 2· i − 4· j)), 4, 4) ;
```

$$\begin{pmatrix} 2 & 6 & 10 & 14 \\ 0 & 4 & 8 & 12 \\ 2 & 2 & 6 & 10 \\ 4 & 0 & 4 & 8 \end{pmatrix}$$

Después buscamos sus valores propios, los ponemos en valor absoluto y calculamos su máximo

```
--> eigenvalues( matrix2) ;
```

$$[[\frac{376\left(\frac{\sqrt{3}\%i}{2}+\frac{-1}{2}\right)}{9\left(\frac{16\sqrt{6409}}{3^{\frac{3}{2}}}+\frac{9872}{27}\right)^{\frac{1}{3}}}+\left(\frac{16\sqrt{6409}}{3^{\frac{3}{2}}}+\frac{9872}{27}\right)^{\frac{1}{3}}\left(\frac{-1}{2}-\frac{\sqrt{3}\%i}{2}\right)+\frac{20}{3}, \left(\frac{16\sqrt{6409}}{3^{\frac{3}{2}}}+\frac{9872}{27}\right)^{\frac{1}{3}}\left(\frac{\sqrt{3}\%i}{2}+\frac{-1}{2}\right)+\frac{376\left(\frac{-1}{2}-\frac{\sqrt{3}\%i}{2}\right)}{9\left(\frac{16\sqrt{6409}}{3^{\frac{3}{2}}}+\frac{9872}{27}\right)^{\frac{1}{3}}}+\frac{20}{3}, \left(\frac{16\sqrt{6409}}{3^{\frac{3}{2}}}+\frac{9872}{27}\right)^{\frac{1}{3}}+\frac{376}{9\left(\frac{16\sqrt{6409}}{3^{\frac{3}{2}}}+\frac{9872}{27}\right)^{\frac{1}{3}}}+\frac{20}{3}, 0], [1, 1, 1, 1]]$$

```
--> a : abs( eigenvalues( matrix2)[ 1]) ;
```

$$[\sqrt{\left(\frac{188}{3^{\frac{3}{2}}\left(\frac{16\sqrt{6409}}{3^{\frac{3}{2}}}+\frac{9872}{27}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\left(\frac{16\sqrt{6409}}{3^{\frac{3}{2}}}+\frac{9872}{27}\right)^{\frac{1}{3}}}{2}\right)^2+\left(-\frac{\left(\frac{16\sqrt{6409}}{3^{\frac{3}{2}}}+\frac{9872}{27}\right)^{\frac{1}{3}}}{2}-\frac{188}{9\left(\frac{16\sqrt{6409}}{3^{\frac{3}{2}}}+\frac{9872}{27}\right)^{\frac{1}{3}}}+\frac{20}{3}\right)^2}, \sqrt{\left(\frac{\sqrt{3}\left(\frac{16\sqrt{6409}}{3^{\frac{3}{2}}}+\frac{9872}{27}\right)^{\frac{1}{3}}}{2}-\frac{188}{3^{\frac{3}{2}}\left(\frac{16\sqrt{6409}}{3^{\frac{3}{2}}}+\frac{9872}{27}\right)^{\frac{1}{3}}}\right)^2+\left(-\frac{\left(\frac{16\sqrt{6409}}{3^{\frac{3}{2}}}+\frac{9872}{27}\right)^{\frac{1}{3}}}{2}-\frac{188}{9\left(\frac{16\sqrt{6409}}{3^{\frac{3}{2}}}+\frac{9872}{27}\right)^{\frac{1}{3}}}+\frac{20}{3}\right)^2}, \left(\frac{16\sqrt{6409}}{3^{\frac{3}{2}}}+\frac{9872}{27}\right)^{\frac{1}{3}}+\frac{376}{9\left(\frac{16\sqrt{6409}}{3^{\frac{3}{2}}}+\frac{9872}{27}\right)^{\frac{1}{3}}}+\frac{20}{3}, 0]$$

```
--> result2 : apply( max, a) ;
```

$$\left(\frac{16\sqrt{6409}}{3^{\frac{3}{2}}}+\frac{9872}{27}\right)^{\frac{1}{3}}+\frac{376}{9\left(\frac{16\sqrt{6409}}{3^{\frac{3}{2}}}+\frac{9872}{27}\right)^{\frac{1}{3}}}+\frac{20}{3}$$

```
--> float( result2) ;
```

20.07783624796722

3 Calcula la suma de los cubos de los 23 primeros números naturales mediante un adecuado bucle. Comprueba que el resultado obtenido es correcto mediante el comando apply.

```
--> result3 : 0 ;
```

0

```
--> for i : 1 thru 23 do result3 : result3 + i ^ 3 ;
```

done

```
--> result3 ;
```

76176

Ahora lo haremos con apply

```
--> b : makelist( i ^ 3, i, 1, 23) ;
```

[1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, 1331, 1728, 2197, 2744, 3375, 4096, 4913, 5832, 6859, 8000, 9261, 10648, 12167]

```
--> apply( "+", b) ;
```

76176

Vemos que el resultado es el mismo

4 Halla el producto de los inversos de los números naturales comprendidos entre 6 y 19 haciendo uso de un bucle, y comprueba la validez de tu respuesta con la orden apply.

```
--> result4 : 1 ;
```

1

```
--> for i : 6 thru 19 do result4 : result4· 1 / i ;
```

done

```
--> result4 ;
```

$$\frac{1}{1013709170073600}$$

```
--> float( result4) ;
```

$9.86476229594919710^{-16}$

Ahora lo haremos con un apply

```
--> c : makelist( 1 / i, i, 6, 19) ;
```

$$[\frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \frac{1}{10}, \frac{1}{11}, \frac{1}{12}, \frac{1}{13}, \frac{1}{14}, \frac{1}{15}, \frac{1}{16}, \frac{1}{17}, \frac{1}{18}, \frac{1}{19}]$$

```
--> d : apply( "·", c) ;
```

$$\frac{1}{1013709170073600}$$

```
--> float( d) ;
```

$9.86476229594919710^{-16}$

Vemos que el resultado coincide

5 Determinar el termino 43º de la sucesión de Fibonnacci de forma recurrente y a partir de la expresión explícita de dicho término.

Primero lo haremos de manera recurrente

```
--> fibo : makelist( 1, i, 1, 43) ;
```

[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]

```
--> for i : 3 thru 43 do
        fibo[ i] : fibo[ i − 1] + fibo[ i − 2] ;
```

done

```
--> fibo[ 43] ;
```

433494437

Ahora lo haremos con la expresión explícita que aparece en las transparencias

```
--> φ :( 1 + sqrt( 5)) / 2 ;
```

$$\frac{\sqrt{5}+1}{2}$$

```
--> fibo_43 :( 1 / sqrt( 5))·( φ ^ 43 −(( 1 − sqrt( 5)) / 2) ^ 43) ;
```

$$\frac{\frac{(\sqrt{5}+1)^{43}}{8796093022208}-\frac{(1-\sqrt{5})^{43}}{8796093022208}}{\sqrt{5}}$$

```
--> float( fibo_43) ;
```

4.3349443710^{8}

Observamos que la variación es nula

6 Calcula, aplicando el comando float, el valor de √(5+10^-n)-√5, para n=1,...,20. Repite los cálculos, reescribiendo la expresión √(5+10^-n)-√5 como 10^(-n)/(√(5+10^-n)+√5). Interpreta los resultados.

Calculamos con la primera expresión

```
--> e : makelist(( sqrt( 5 + 10 ^( − n)) − sqrt( 5)), n, 1, 20) ;
```

$$[\frac{\sqrt{51}}{\sqrt{10}}-\sqrt{5}, \frac{\sqrt{501}}{10}-\sqrt{5}, \frac{\sqrt{5001}}{10^{\frac{3}{2}}}-\sqrt{5}, \frac{\sqrt{50001}}{100}-\sqrt{5}, \frac{\sqrt{500001}}{10^{\frac{5}{2}}}-\sqrt{5}, \frac{\sqrt{5000001}}{1000}-\sqrt{5}, \frac{\sqrt{50000001}}{10^{\frac{7}{2}}}-\sqrt{5}, \frac{\sqrt{500000001}}{10000}-\sqrt{5}, \frac{\sqrt{5000000001}}{10^{\frac{9}{2}}}-\sqrt{5}, \frac{\sqrt{50000000001}}{100000}-\sqrt{5}, \frac{\sqrt{500000000001}}{10^{\frac{11}{2}}}-\sqrt{5}, \frac{\sqrt{5000000000001}}{1000000}-\sqrt{5}, \frac{\sqrt{50000000000001}}{10^{\frac{13}{2}}}-\sqrt{5}, \frac{\sqrt{500000000000001}}{10000000}-\sqrt{5}, \frac{\sqrt{5000000000000001}}{10^{\frac{15}{2}}}-\sqrt{5}, \frac{\sqrt{50000000000000001}}{100000000}-\sqrt{5}, \frac{\sqrt{500000000000000001}}{10^{\frac{17}{2}}}-\sqrt{5}, \frac{\sqrt{5000000000000000001}}{1000000000}-\sqrt{5}, \frac{\sqrt{50000000000000000001}}{10^{\frac{19}{2}}}-\sqrt{5}, \frac{\sqrt{500000000000000000001}}{10000000000}-\sqrt{5}]$$

```
--> float( e) ;
```

$[0.02224998062745342, 0.00223495106014937, 2.23595618527028210^{-4}, 2.23605679727789610^{-5}, 2.2360668558340991 0^{-6}, 2.23606786864394210^{-7}, 2.23606781979412910^{-8}, 2.23606821947441810^{-9}, 2.23604690319234510^{-10}, 2.23607798943703510^{-11}, 2.23376872554581510^{-12}, 2.23376872554581510^{-13}, 1.90958360235526910^{-14}, 2.22044604925031310^{-15}, -3.10862446895043810^{-15}, 0.0, -3.99680288865056310^{-15}, 0.0, -4.44089209850062610^{-15}, 0.0]$

```
--> f : makelist(( 10 ^( − n)) /( sqrt( 5 + 10 ^( − n)) + sqrt( 5)), n, 1, 20) ;
```

$$[\frac{1}{10\left(\frac{\sqrt{51}}{\sqrt{10}}+\sqrt{5}\right)}, \frac{1}{100\left(\frac{\sqrt{501}}{10}+\sqrt{5}\right)}, \frac{1}{1000\left(\frac{\sqrt{5001}}{10^{\frac{3}{2}}}+\sqrt{5}\right)}, \frac{1}{10000\left(\frac{\sqrt{50001}}{100}+\sqrt{5}\right)}, \frac{1}{100000\left(\frac{\sqrt{500001}}{10^{\frac{5}{2}}}+\sqrt{5}\right)}, \frac{1}{1000000\left(\frac{\sqrt{5000001}}{1000}+\sqrt{5}\right)}, \frac{1}{10000000\left(\frac{\sqrt{50000001}}{10^{\frac{7}{2}}}+\sqrt{5}\right)}, \frac{1}{100000000\left(\frac{\sqrt{500000001}}{10000}+\sqrt{5}\right)}, \frac{1}{1000000000\left(\frac{\sqrt{5000000001}}{10^{\frac{9}{2}}}+\sqrt{5}\right)}, \frac{1}{10000000000\left(\frac{\sqrt{50000000001}}{100000}+\sqrt{5}\right)}, \frac{1}{100000000000\left(\frac{\sqrt{500000000001}}{10^{\frac{11}{2}}}+\sqrt{5}\right)}, \frac{1}{1000000000000\left(\frac{\sqrt{5000000000001}}{1000000}+\sqrt{5}\right)}, \frac{1}{10000000000000\left(\frac{\sqrt{50000000000001}}{10^{\frac{13}{2}}}+\sqrt{5}\right)}, \frac{1}{100000000000000\left(\frac{\sqrt{500000000000001}}{10000000}+\sqrt{5}\right)}, \frac{1}{1000000000000000\left(\frac{\sqrt{5000000000000001}}{10^{\frac{15}{2}}}+\sqrt{5}\right)}, \frac{1}{10000000000000000\left(\frac{\sqrt{50000000000000001}}{100000000}+\sqrt{5}\right)}, \frac{1}{100000000000000000\left(\frac{\sqrt{500000000000000001}}{10^{\frac{17}{2}}}+\sqrt{5}\right)}, \frac{}{1000000000000000000(\sqrt{}}$$

```
--> float( f) ;
```

$[0.02224998062745328, 0.002234951060149439, 2.23595618527985710^{-4}, 2.23605679727170310^{-5}, 2.23606685946691910^{-6}, 2.23606786569640110^{-7}, 2.2360679663194510^{-8}, 2.23606797638175510^{-9}, 2.23606797738798710^{-10}, 2.23606797748860910^{-11}, 2.23606797749867210^{-12}, 2.23606797749967710^{-13}, 2.2360679774997810^{-14}, 2.23606797749978810^{-15}, 2.23606797749979110^{-16}, 2.23606797749978910^{-17}, 2.23606797749979110^{-18}, 2.2360679774997910^{-19}, 2.23606797749979210^{-20}, 2.23606797749978910^{-21}]$

Podemos observar, que, aunque la expresión es la misma, los resultados no son nada iguales. En el primer array, a medida q n es mayor, se produce un error de cancelación porque estamos restando números positivos, pero en el segundo no se produce, pues tenemos cocientes y sumas. En el primero podemos observar que en ciertas ocasiones obtenemos 0, lo cual es imposible.

7 Programa (no necesariamente módulo) el calculo de la norma 1 de una matriz cuadrada cualquiera.

```
--> norm1( m) : = block( b : transpose( m), apply( max, makelist( apply( "+", abs( b[ i])), i, 1, matrix_size( m)[ 1]))) ;
```

$$\text{norm1}(m) := \text{block}\left(b: \text{transpose}(m), \text{apply}\left(\max, \text{makelist}\left(\text{apply}\left(+, \mid b_i \mid\right), i, 1, (\text{matrix\_size}(m))_1\right)\right)\right)$$

Lo probaremos ahora:

```
--> I : matrix(
     [ 1, 0, 4],
     [ 0, 2, 0],
     [ 0, 0, 3]
     ) ;
```

$$\begin{pmatrix}1 & 0 & 4\\0 & 2 & 0\\0 & 0 & 3\end{pmatrix}$$

```
--> norm1( I) ;
```

$$7$$

Observamos que efectivamente nos da el resultado correcto

8 Diseña un programa que, a partir de una matriz cuadrada, genere como salida su condicionamiento (norma infinito) si es regular, o el mensaje "la matriz no es regular" en caso contrario.

Primero insertamos de los apuntes la norma infinito

```
--> normmax( b) : = block( apply( max, makelist( apply( "+", abs( b[ i])), i, 1,
    matrix_size( b)[ 1]))) ;
```

$$\mathrm{normmax}(b) := \mathrm{block}\left(\mathrm{apply}\left(\max, \mathrm{makelist}\left(\mathrm{apply}\left(+, |b_i|\right), i, 1, (\mathrm{matrix\_size}(b))_1\right)\right)\right)$$

```
--> cond( m) : = block(if determinant( m) = 0 then print( "La matriz no es regular") else condicionamiento : normmax( m)· normmax( invert( m))) ;
```

$$\mathrm{cond}(m) := \mathrm{block}\,(\mathrm{ifdeterminant}\,(m){=}0\mathrm{thenprint}\,(\text{ La matriz no es regular })\mathrm{elsecondicionamiento}{:}\mathrm{normmax}\,(m)\mathrm{normmax}\,(\mathrm{invert}\,(m)))$$

```
--> h : matrix(
    [ 34, 0, 0],
    [ 0, 3, 3],
    [ 5, 0, 7]
    ) ;
```

$$\begin{pmatrix}34 & 0 & 0\\0 & 3 & 3\\5 & 0 & 7\end{pmatrix}$$

```
--> cond( h) ;
```

$$\frac{355}{21}$$

```
--> float( cond( h)) ;
```

$$16.9047619047619$$

```
--> null_8 : matrix(
    [ 1, 2, 3],
    [ 4, 5, 6],
    [ 7, 8, 9]
    ) ;
```

$$\begin{pmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 9\end{pmatrix}$$

```
--> cond( null_8) ;
```

La matriz no es regular
La matriz no es regular

Comprobamos que funciona

9 Calcula la norma euclidea de la matriz 2x4 cuyo coeficiente (i,j) sea i/(i+j+1).

Generamos la matriz

```
--> matrix_9 : genmatrix( lambda([ i, j], i /( i + j + 1)), 2, 4) ;
```

$$\begin{pmatrix}\frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6}\\ \frac{1}{2} & \frac{2}{5} & \frac{1}{3} & \frac{2}{7}\end{pmatrix}$$

La norma euclidea se puede definir como la raíz cuadrada positiva de el radio espectral de la matriz generada por la matriz A por su traspuesta

Primero calculamos la matriz por su traspuesta

```
--> matrix_9_t : transpose( matrix_9) ;
```

$$\begin{pmatrix}\frac{1}{3} & \frac{1}{2}\\ \frac{1}{4} & \frac{2}{5}\\ \frac{1}{5} & \frac{1}{3}\\ \frac{1}{6} & \frac{2}{7}\end{pmatrix}$$

```
--> matriz_9_g : matrix_9_t. matrix_9 ;
```

$$\begin{pmatrix}\frac{13}{36} & \frac{17}{60} & \frac{7}{30} & \frac{25}{126}\\ \frac{17}{60} & \frac{89}{400} & \frac{11}{60} & \frac{131}{840}\\ \frac{7}{30} & \frac{11}{60} & \frac{34}{225} & \frac{9}{70}\\ \frac{25}{126} & \frac{131}{840} & \frac{9}{70} & \frac{193}{1764}\end{pmatrix}$$

Ahora su radio espectral

```
--> eigenvalues( matriz_9_g) ;
```

$$\left[\left[-\frac{\sqrt{22126530049}-148905}{352800}, \frac{\sqrt{22126530049}+148905}{352800}, 0\right], [1, 1, 2]\right]$$

```
--> n : abs( eigenvalues( matriz_9_g)[ 1]) ;
```

$$\left[-\frac{\sqrt{22126530049}-148905}{352800}, \frac{\sqrt{22126530049}+148905}{352800}, 0\right]$$

```
--> radio_es_9 : apply( max, n) ;
```

$$\frac{\sqrt{22126530049}+148905}{352800}$$

```
--> result_9 : sqrt( radio_es_9) ;
```

$$\frac{\sqrt{\sqrt{22126530049}+148905}}{1052^{\frac{5}{2}}}$$

```
--> float( result_9) ;
```

$$0.9185276269534124$$

Created with wxMaxima.