EJERCICIO 6.6

a) $\int_a^{+\infty} x^n \, dx \quad (a>0)$

Vamos a diferenciar tres casos: $n>-1$, $n<-1$, $n=-1$.

$$F(b) = \int_a^{b} x^n \, dx$$

Si $n \neq -1$:

$$F(b) = \int_a^b x^n = \left[\frac{x^{n+1}}{n+1}\right]_a^b = \frac{b^{n+1}}{n+1} - \frac{a^{n+1}}{n+1}$$

$\boxed{n<-1}$

$$\lim_{b\to\infty} F(b) = \cancel{\frac{\frac{1}{\infty}}{n+1}} - \frac{a^{n+1}}{n+1}$$

0

$$\lim_{b\to\infty} F(b) = \frac{-a^{n+1}}{n+1}$$

↳ La integral converge

$\boxed{n>-1}$

$$\lim_{b\to\infty} F(b) = \infty - \frac{a^{n+1}}{n+1} = \infty$$

↳ La integral diverge.

Si $n=-1$:

$$F(b) = \int_a^b x^{-1} = [\ln x]_a^b = \ln b - \ln a.$$

$\lim_{b\to\infty} \ln b - \ln a = \infty \to$ La integral diverge.

b) $\int_{-\infty}^{0} e^x \, dx \to \int_a^0 e^x \, dx$

$$F(a) = \int_a^0 e^x \, dx = [e^x]_a^0 = e^0 - e^a = 1 - e^a.$$

$\lim_{a\to-\infty} F(a) = \lim_{a\to-\infty} 1 - e^a = 1 - \cancel{\frac{1}{e^{\infty}}} = 1. \to$ La integral converge.

0

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