

6. Demuestre que para todo conjunto de fórmulas $\Gamma \cup \{\alpha, \beta, \gamma\}$:

a) $\text{Con}(\Gamma, \alpha \rightarrow \beta, \alpha \rightarrow (\beta \rightarrow \gamma)) = \text{Con}(\Gamma, \alpha \rightarrow \beta, \alpha \rightarrow \gamma)$

b) $\text{Con}(\Gamma, \alpha \rightarrow (\beta \rightarrow \gamma)) = \text{Con}(\Gamma, (\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \gamma))$

a) Primero vemos que $(\alpha \rightarrow \beta, \alpha \rightarrow (\beta \rightarrow \gamma)) \subseteq (\alpha \rightarrow \beta), (\alpha \rightarrow \gamma)$

$$\alpha \rightarrow (\beta \rightarrow \gamma) = \neg \alpha \vee (\neg \beta \vee \gamma) = (\neg \alpha \vee \neg \beta \vee \gamma) = A^c \cup B$$

$$= (\neg \alpha \vee \neg (\neg \beta \wedge \gamma)) = (\neg \alpha \vee (\neg \neg \beta \wedge \neg \gamma)) =$$

$$= (\neg \alpha \vee (\beta \wedge \gamma)) = (\neg \alpha \vee \beta) \wedge (\neg \alpha \vee \gamma) \stackrel{\downarrow}{=} (\neg \alpha \vee \gamma) = \alpha \rightarrow \gamma$$

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Por tanto, teniendo $(\alpha \rightarrow \beta), (\alpha \rightarrow (\beta \rightarrow \gamma))$, hemos llegado a $(\alpha \rightarrow \beta), (\alpha \rightarrow \gamma)$

$$A^c, (\alpha \rightarrow \beta, \alpha \rightarrow (\beta \rightarrow \gamma)) \subseteq (\alpha \rightarrow \gamma), (\alpha \rightarrow \beta)$$

$$\text{Con}(\Gamma, \alpha \rightarrow \beta, \alpha \rightarrow (\beta \rightarrow \gamma)) = \text{Con}(\Gamma, \alpha \rightarrow \beta, \alpha \rightarrow \gamma)$$

$$b) \text{Con}(\mathcal{I}^*, \alpha \rightarrow (\beta \rightarrow \gamma)) = \text{Con}(\mathcal{I}^*, (\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \gamma))$$

$$\text{veremos que } \alpha \rightarrow (\beta \rightarrow \gamma) = (\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \gamma)$$

$$\begin{aligned} & \neg(\neg\alpha \vee \beta) \vee (\neg\alpha \vee \gamma) = (\neg\neg\alpha \wedge \neg\beta) \vee (\neg\alpha \vee \gamma) \\ & = (\alpha \wedge \neg\beta) \vee (\neg\alpha \vee \gamma) = (\alpha \vee \neg\alpha \vee \gamma) \wedge (\neg\beta \vee \neg\alpha \vee \gamma) = \\ & = \gamma \wedge (\neg\beta \vee \neg\alpha \vee \gamma) = (\neg\beta \vee \neg\alpha \vee \gamma) = (\neg\alpha \vee (\neg\beta \vee \gamma)) = \\ & \alpha \rightarrow (\beta \rightarrow \gamma) = \alpha \rightarrow (\beta \rightarrow \gamma) \end{aligned}$$

Por tanto,

$$(\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \gamma) = \alpha \rightarrow (\beta \rightarrow \gamma)$$

$$\text{Con}(\mathcal{I}^*, \alpha \rightarrow (\beta \rightarrow \gamma)) = \text{Con}(\mathcal{I}^*, (\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \gamma))$$