Relación 6

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Ejercicio 6.1. Calcular las siguientes integrales:

ll) $\int x^3 \sin 3x \, dx$

$$\int x^3 \sin 3x \, dx = \frac{1}{3}\int x^3 3\sin 3x \, dx = \begin{bmatrix} u = x^3 & du = 3x^2 \\ dv = 3\sin 3x & v = -\cos 3x \end{bmatrix} =$$
$$= \frac{1}{3}\left(-x^3\cos 3x - \int -x^2 3\cos 3x \, dx\right) = \begin{bmatrix} u = x^2 & du = 2x \\ dv = 3\cos 3x & v = \sin 3x \end{bmatrix} =$$
$$= \frac{1}{3}\left(-x^3\cos 3x + x^2\sin 3x - \int 2x\sin 3x \, dx\right) =$$
$$= \frac{1}{3}\left(-x^3\cos 3x + x^2\sin 3x + \frac{-2}{3}\int x3\sin 3x \, dx\right) = \begin{bmatrix} u = x & du = 1 \\ dv = 3\sin 3x & v = -\cos 3x \end{bmatrix} =$$
$$= \frac{1}{3}\left(-x^3\cos 3x + x^2\sin 3x + \frac{-2}{3}\left[-x\cos 3x + \int \cos 3x \, dx\right]\right) =$$
$$= \frac{1}{3}\left(-x^3\cos 3x + x^2\sin 3x + \frac{-2}{3}\left[-x\cos 3x + \frac{1}{3}\sin 3x\right]\right) + c =$$
$$= \frac{(9x^2-2)\sin 3x + (6x - 9x^3)\cos 3x}{27} + c$$

m) $\int x \ln 1 + x^2 \, dx$

$$\int x \ln 1 + x^2 \, dx = \begin{bmatrix} u = \ln 1 + x^2 & du = \dfrac{2x}{1+x^2} \\ dv = x & v = \dfrac{x^2}{2} \end{bmatrix} = \frac{x^2}{2}\ln 1 + x^2 - \int \frac{x^2}{2}\frac{2x}{1+x^2} \, dx =$$
$$= \frac{x^2}{2}\ln 1 + x^2 - \int \frac{x^3}{1+x^2} \, dx = \frac{x^2}{2}\ln 1 + x^2 - \int x - \frac{-x}{1+x^2} \, dx =$$
$$= \frac{x^2}{2}\ln 1 + x^2 - \frac{x^2}{2} + \frac{1}{2}\int \frac{2x}{1+x^2} \, dx = \frac{x^2}{2}\ln 1 + x^2 - \frac{x^2}{2} + \frac{1}{2}\ln 1 + x^2 + c =$$
$$= \frac{x^2 \ln 1 + x^2 - x^2 + \ln 1 + x^2}{2} + c$$