Introduction to course 02457

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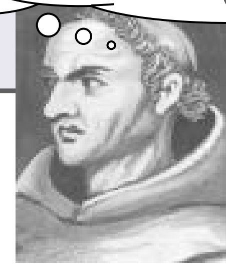
Course 02457   Sep 4, 2017

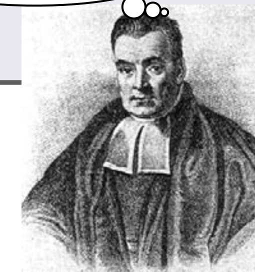
DTU Technical University of Denmark

Outline

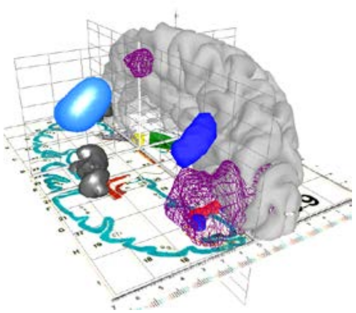
Do not multiply causes!

Multiply likelihood and prior.. $P(a|b) = P(b|a)P(a)/P(b)$

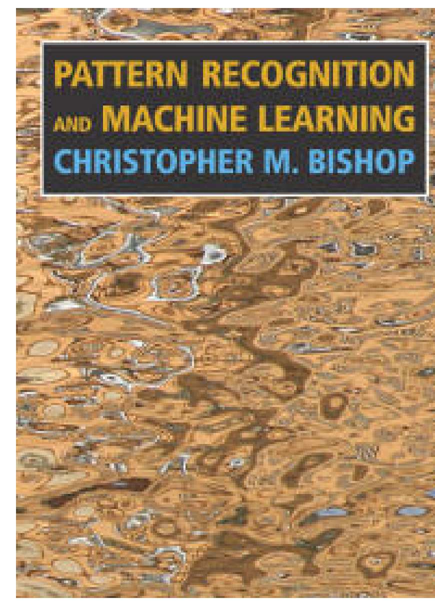




- Introduction
  - What is machine learning?
  - Signal processing in the brain
  - Search engines
- Course overview
  - What kinds of math are necessary?
- Probabilities, densities
  - Bayes theorem
  - Bayes decision theory



Textbook



Cognitive Systems

A Section in DTU Compute
Approx. 50 scientists, students, staff

Digital media

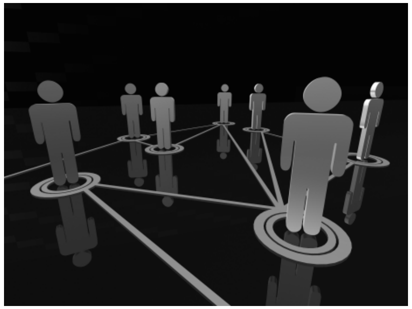
Social computing

Statistical machine learning

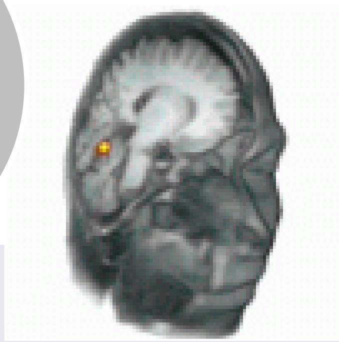
Cognitive Neuroscience

Mobile Systems

Neuro-informatics



Ceci n'est pas un smartphone



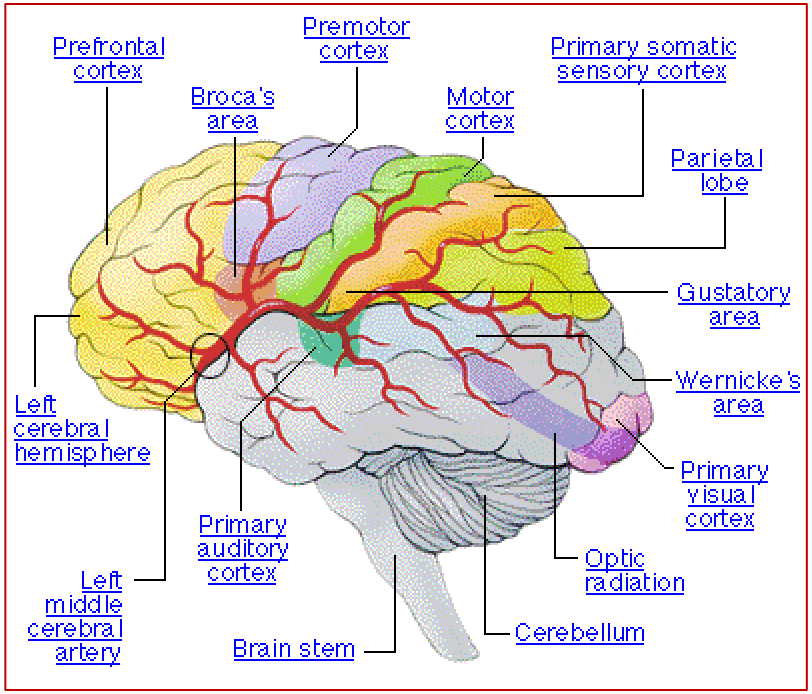
What is machine learning?

Abstraction of generalizable relations from empirical data

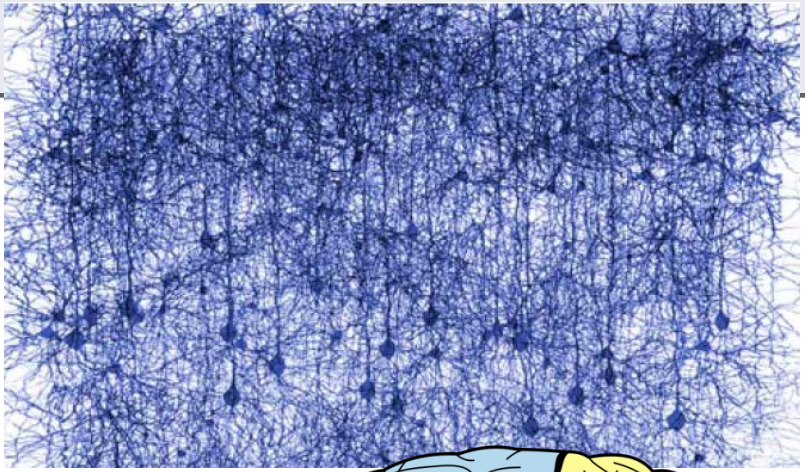
Statistical modeling in domains where we have no or little insight into the physics of the problem

Inspired by / interfacing with the brain – people involved!

Emphasis on
- Prediction
- Scaling
- Robustness
- Interpretability

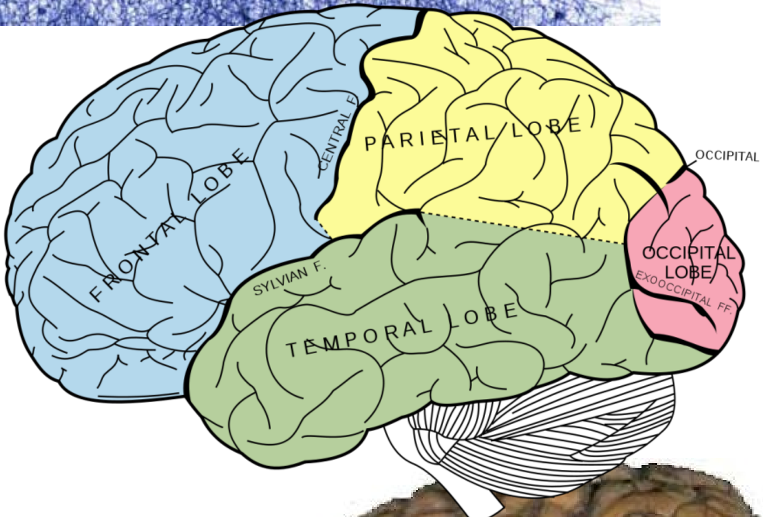


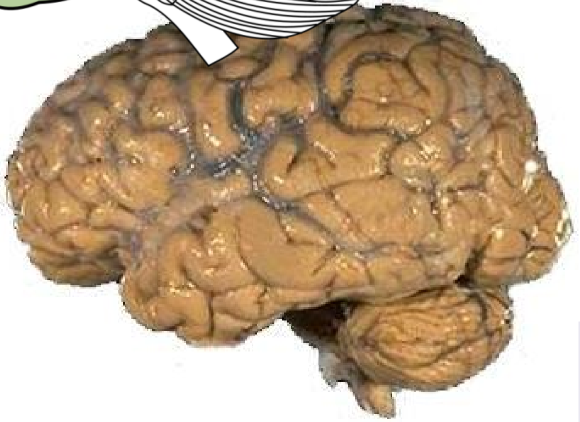
The brain design



Three design principles

i) Division of labor
ii) Neural networks of simple computers
iii) Learning – adaptivity -plasticity





Centers for vision, hearing, smell etc

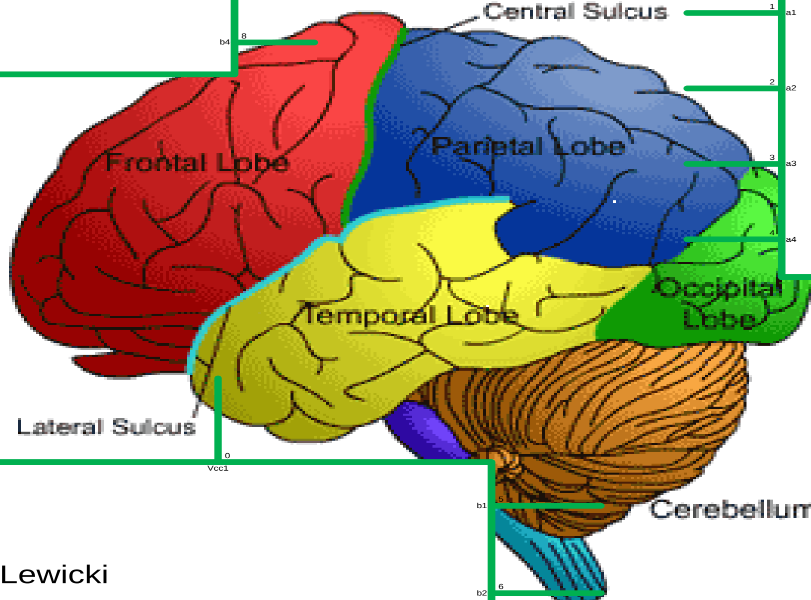
The brain’s ”output”: motor signals

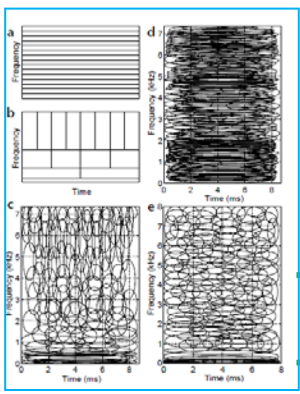
Division of labor: Understanding dependency structures in environment

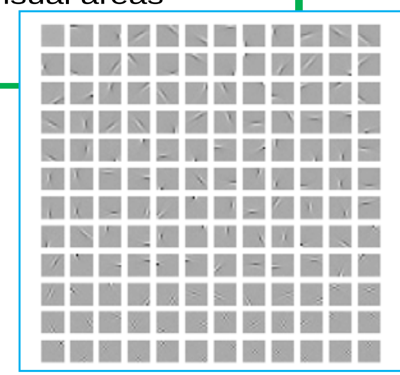
Abstract data???

Barlow, Olshausen, Bell & Sejnowski, Hyvarinen…

Adapted (frozen) ICA in primary visual areas







Lewicki

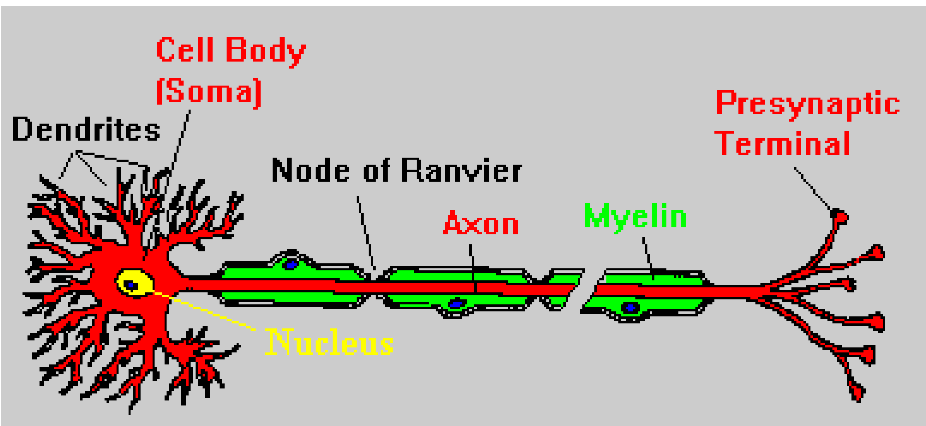
Adapted ICA in primary auditory areas: MEL MFCC's like freq-bandwidth relations

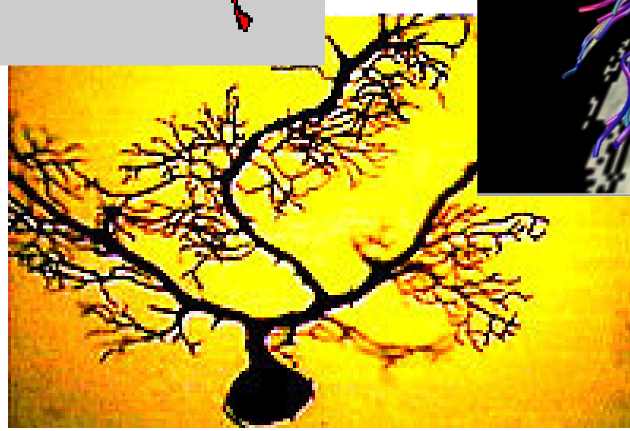
Is the brain optimal?

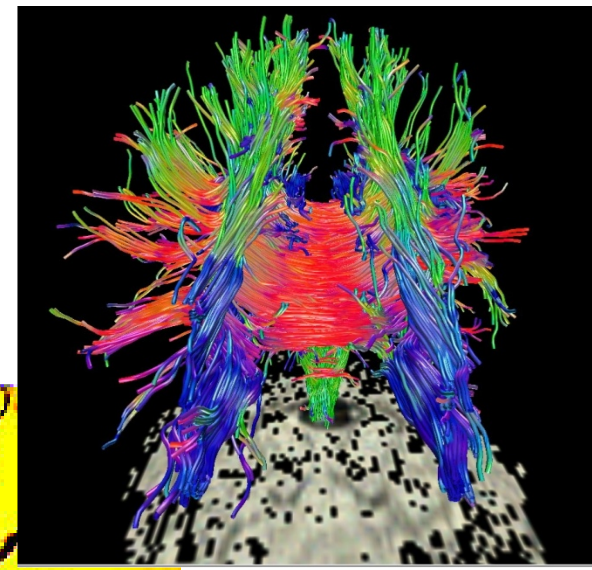
Neural circuitry

Cortex consists of gray matter brain cells / neurons, modeled as simple computers ...1 bit?

and white matter: wires connecting neurons at synapses (+/-)

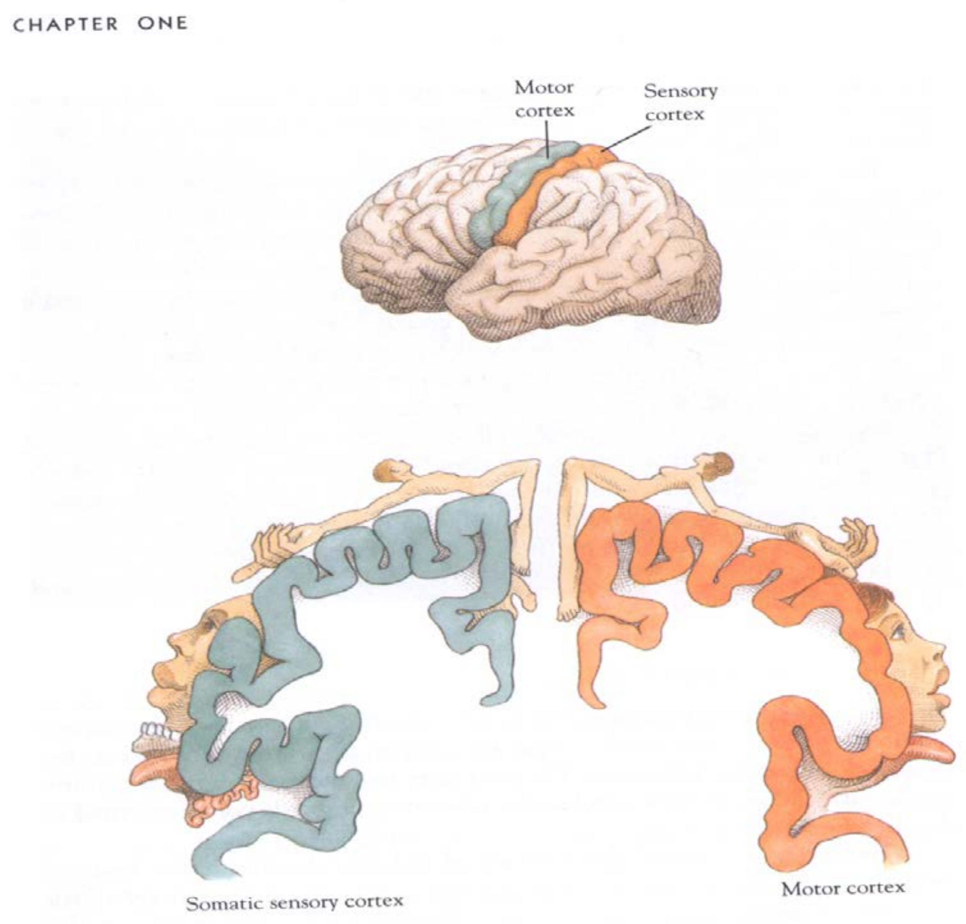






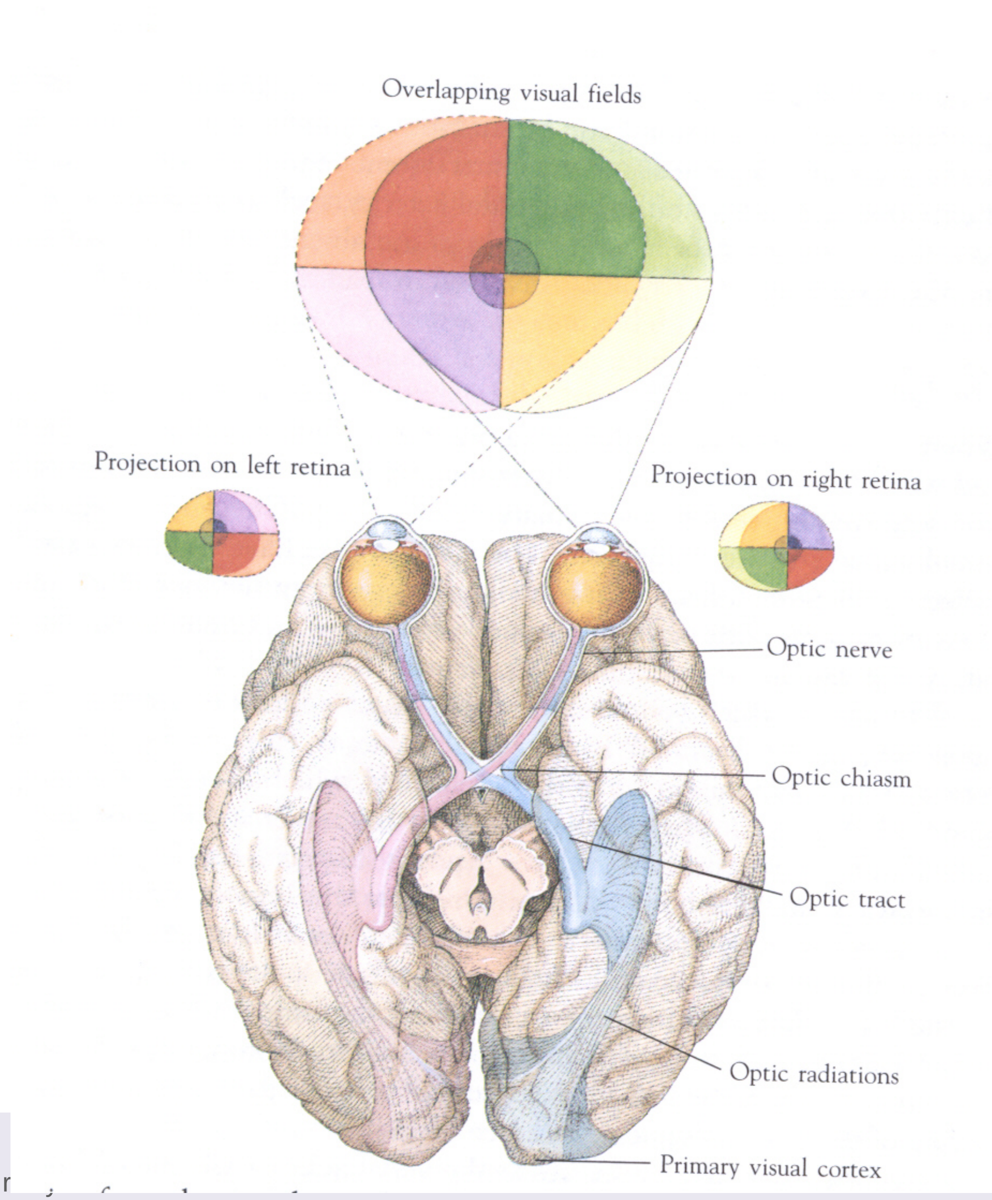
Sensory input - Motor output

- Multiple sensory inputs from vision, ears, skin,...
- Muscles are activated by specific areas in the brain



Vision resides in the back of the human head

The ”GPU” is a massively parallel computer

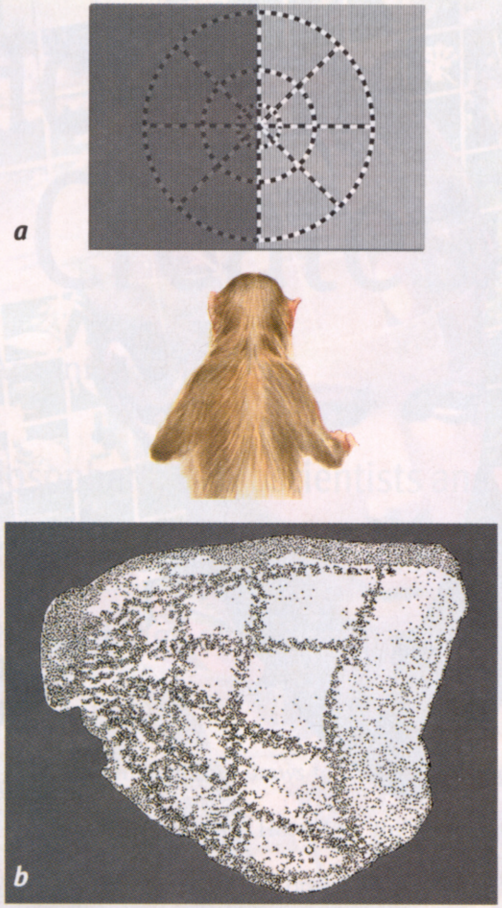


Visual cortex is adapted to natural stimuli

Visual cortex creates multiple maps: contrast, depth, color, shape,... "division of labor"

Visual cortex is plastic

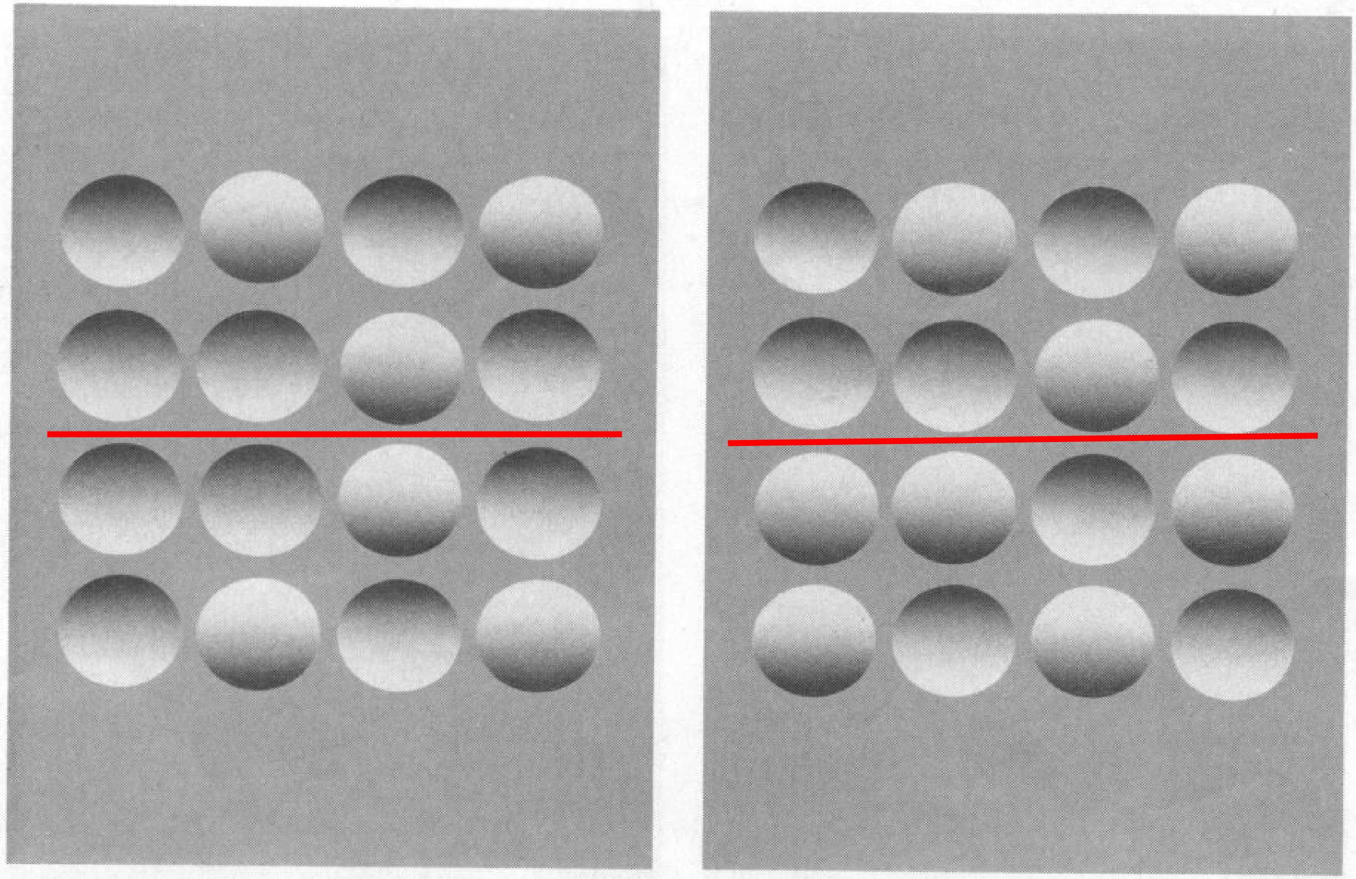
Visual cortex is massively parallel



The brain's business is representing other things. Studies with macaques show a remarkable fidelity between a seen shape (*a*) and the shape of the neural activity pattern (*b*) in one of the layers of the primary visual cortex.

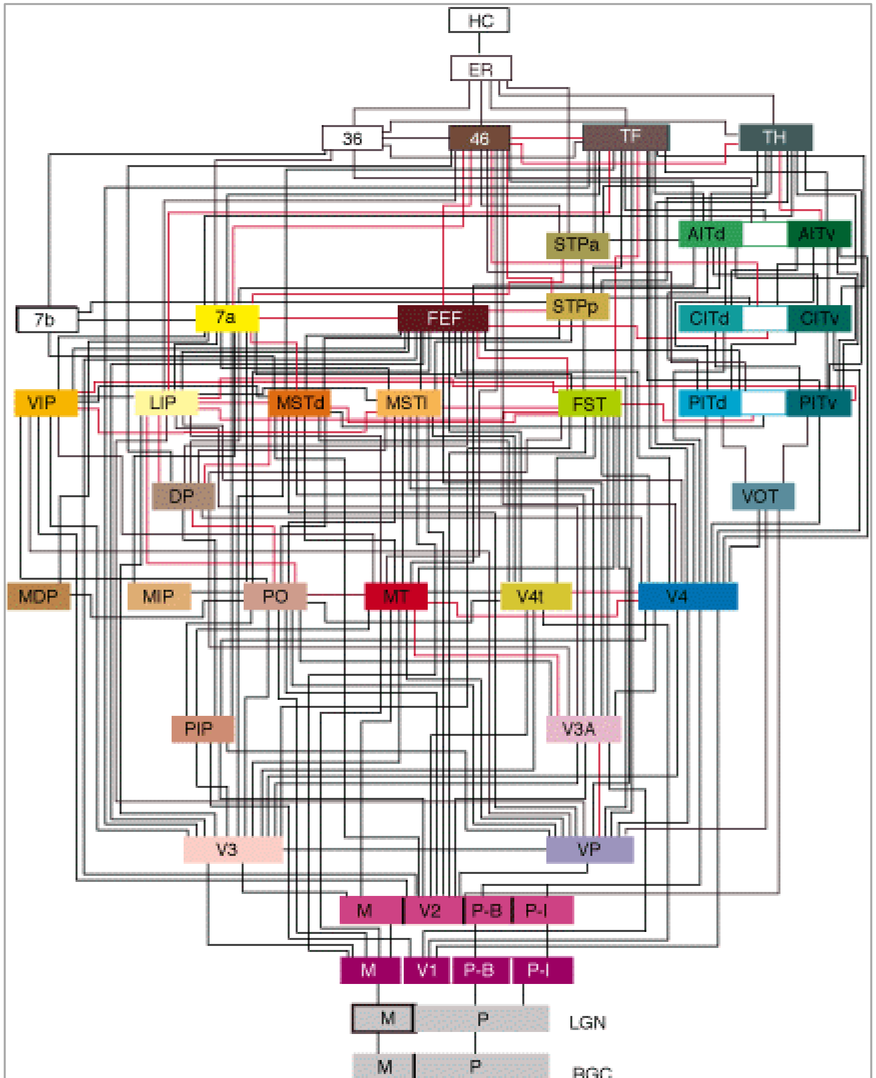
Vision software

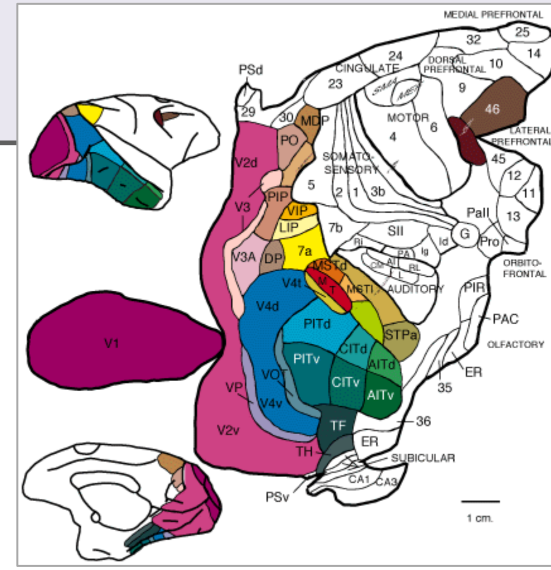
The human brain's context setting determines visual interpretation



SYMMETRY PERCEPTION occurs after the brain extracts shape from shading. In the left-hand array spheres and cavities seem arranged symmetrically about a horizontal axis. But in two dimensions it is the right-hand array that is truly symmetrical.

Visual system hierarchy (Van Essen)





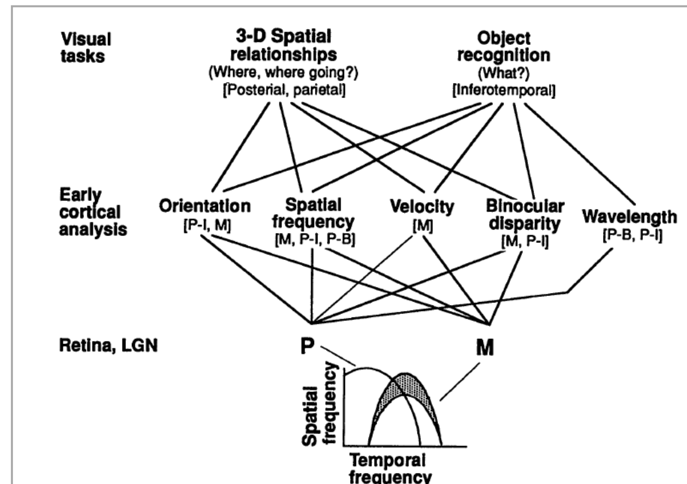
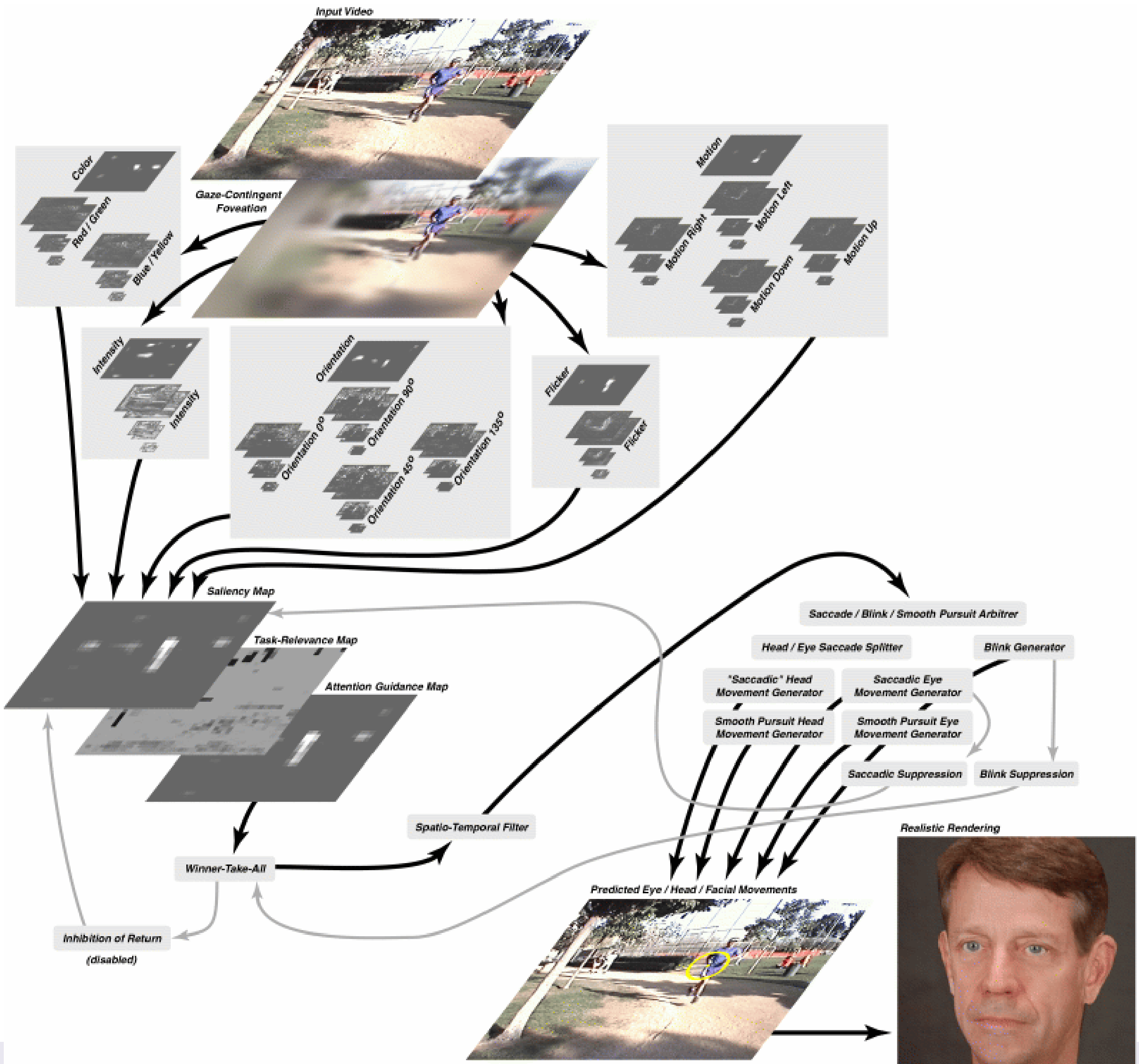


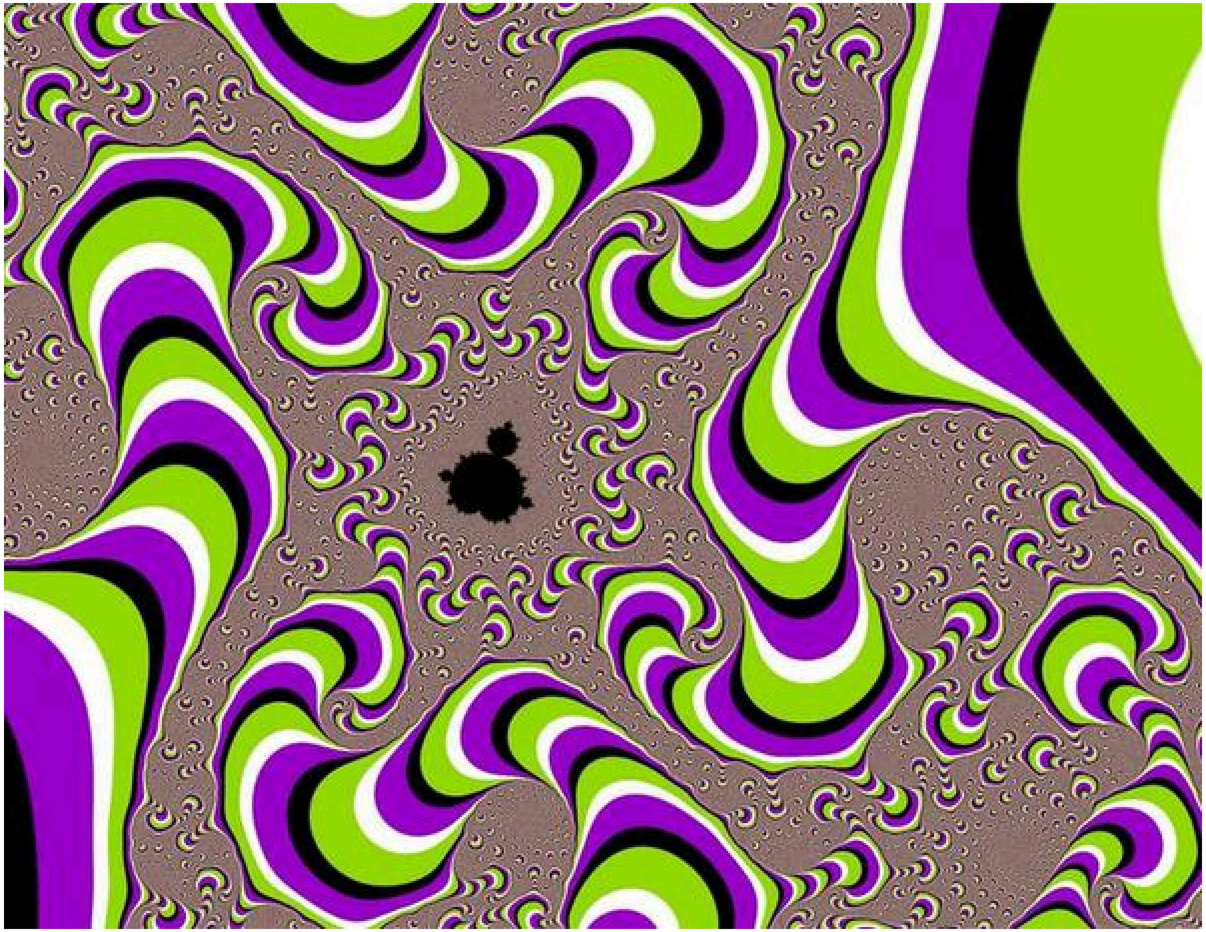
Fig. 3. Convergence and divergence in visual processing. Arrows represent major lines of information flow from subcortical P and M streams (bottom) to the selectivities represented among neurons at early stages of cortical analysis (middle) and from there to two general tasks of vision (top level). The hatched portion of the M cell curve represents their nonlinear component of processing. The processing streams associated with each property in the middle row are assigned on the basis of a high incidence of selectivity recorded physiologically (*6*, *7*).

Felleman, D.J., and D.C. Van Essen. "Distributed hierarchical processing in the primate cerebral cortex." *Cerebral cortex* 1, no. 1 (1991): 1-47.

Itti, Dhavale & Pighin, Proc. SPIE, 2003



Damn you visual cortex! Other domains.. Cognitive biases



the vision

Computeren bliver så småt en del af os

Fremtidens computere og mobiltelefoner behøver ikke at ligge i tasker eller bukselommer. Om få år kan de sidde på armen, næsen og andre steder på kroppen eller ligefrem i tøjet, vurderer eksperter.

a vision for the effective ~~market~~ society:

The information you need when you need it

Support productivity and well-being
Respect democratic living and privacy

~jaguar - Google-søgning - Windows Internet Explorer

http://www.google.dk/#hl=da&rlz=1R2GFRE_daDK376&q=~jaguar&aq=f&aqi=g10&aql=&oq=&gs_rfai=&fp=511881e210c1f1b1

Google ~jaguar Søg Del Sidewiki Bogmærker Kontroller Oversæt Lars.K... Convert Select

Favorites Web Slice Gallery

~jaguar - Google-søgning

Page Safety Tools

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Google

~jaguar Søg

Ca. 725.000.000 resultater (0,41 sekunder) Avanceret søgning

Alle
Billeder
Mere

Nettet
Sider på dansk
Sider fra Danmark

Ethvert tidsinterval
Seneste 4 dage

Alle resultater
Stjernevisning
Websteder med billeder

Flere værktøjer

Jaguar.dk
Salg af nye og brugte biler. Værksted, reservedele og tilbehør. Sitet indeholder oplysning om modeller, billeder og nyheder. Virtuelt showroom med butik og **...**
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TIGER - Det er for vildt!
Tiger har altid noget du kan bruge - hele året rundt. Masser af sjove, nyttige ting inden for bolig, pleje, legetøj, dvd'er, kontor, krydderier og meget **...**
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jaguars.com - 2010 Season Tickets On Sale Now! - [Oversæt denne side]
The official team site with scores, news items, game schedule, and roster.
2010 Draft - Tickets - Message Board - TEAM
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Billeder af **~jaguar** - Rapporter billeder

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Done Internet | Protected Mode: Off 100%

DA 16:08 18-05-2010

Google getting smarter - semantics

jaguar.dk - Jaguar i Danmark
Annonce www.jaguar.dk/
officiel importør af **Jaguar** - se alle modeller her!

Om Jaguar
Bliv klogere på Jaguar &
vores spændende historie.

Find forhandler
Find nærmeste forhandlere
og værksteder her.

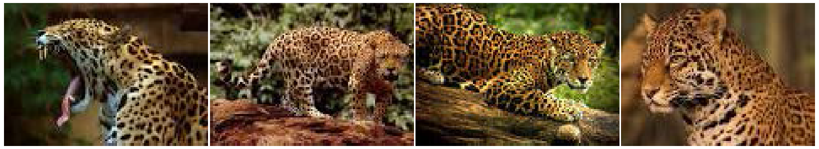
Jaguar XF Sportbrake
Se den flotte Jaguar XF Sportbrake.
- Spændende og funktionelt design.

Jaguar Tilbehør
Få endnu mere ud af din Jaguar.
Giv Jaguaren et personligt præg.

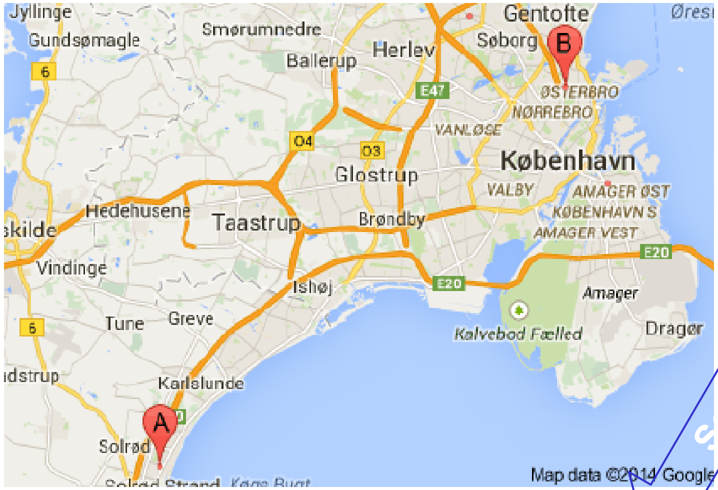
Forside | Jaguar - Jaguar
www.**jaguar**.dk/
Velkommen til **Jaguars** officielle hjemmeside i Danmark.

Jaguar
www.**jaguar**.com/ Oversæt denne side
Official worldwide web site of **Jaguar** Cars. Directs users to pages tailored to country-specific markets and model-specific websites.

Billeder af jaguar — Rapportér billeder



Flere billeder af jaguar



Kort over jaguar

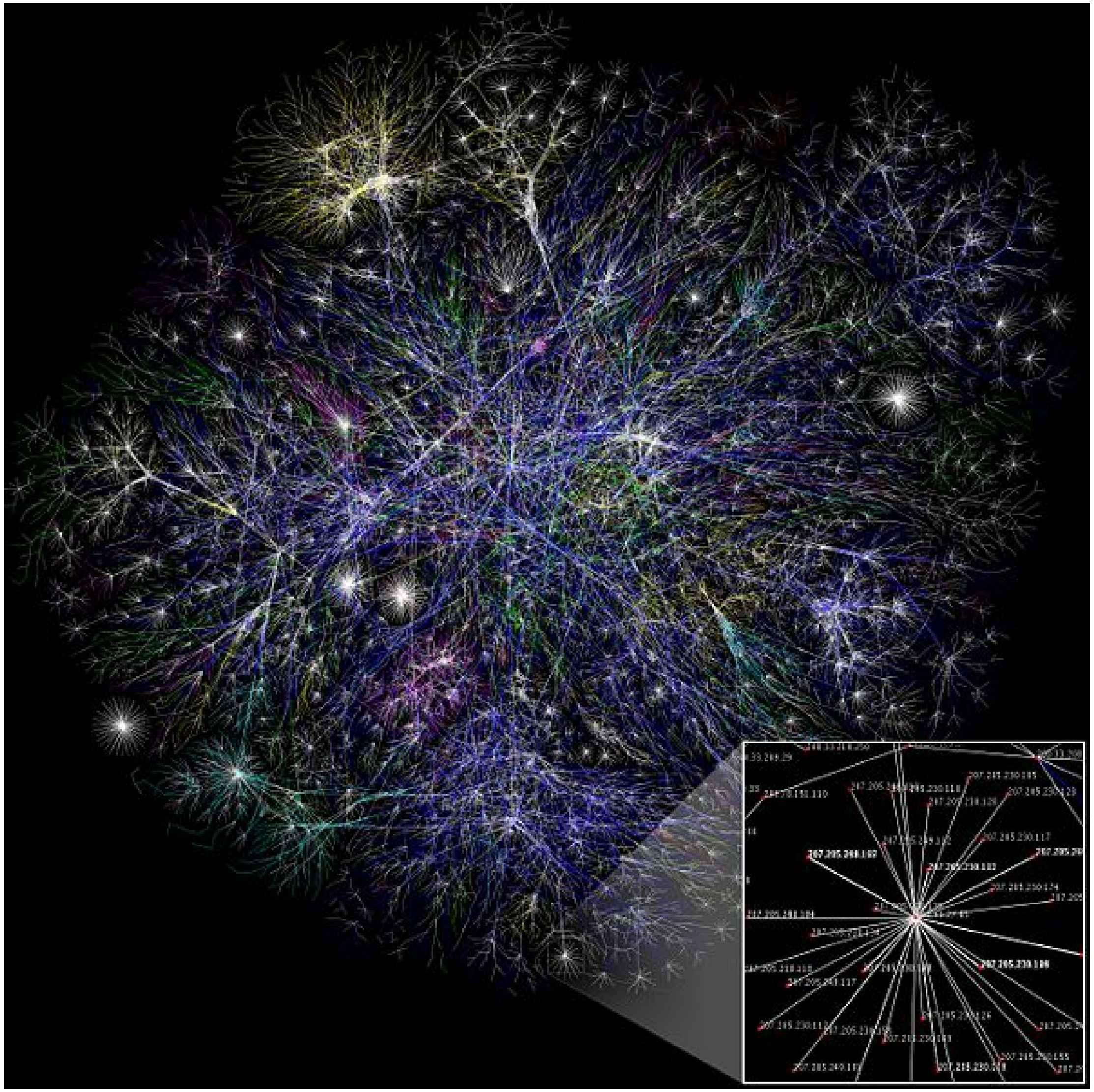
Se resultater om

Jaguar Cars
Jaguar Cars Limited er et britisk luksusbilmærke ejet af indiske Tata Motors med hovedkvarter i Browns ...

Jaguar
Jaguaren er et af de store kattedyr i kattefamilien. Den lever i Syd og Centralamerika og er tæt ...

… numerous specialized big data indexes for specialized search

Wikipedia (facts)
Facebook (friends)
Youtube (video)
Spotify (music)
Dating.dk (dates)
Twitter (news)



Search engine modeling

A search engine is a database of documents, an Index, and a retrieval function *Query -> Ranked retrieval*

Purpose

Return relevance ranked information in response to a *query*

Search related machine learning research issues

What should be indexed? ....what are the ”words”?
Query expansion –query is a document (including context?)
Retrieval of documents and context information

Ranking of retrieval set

Relevance feedback, user modeling

Organization of multi-dimensional responses

list, tree, groups etc

Maintenance issues:

Cleaning, updating, forecasting etc.

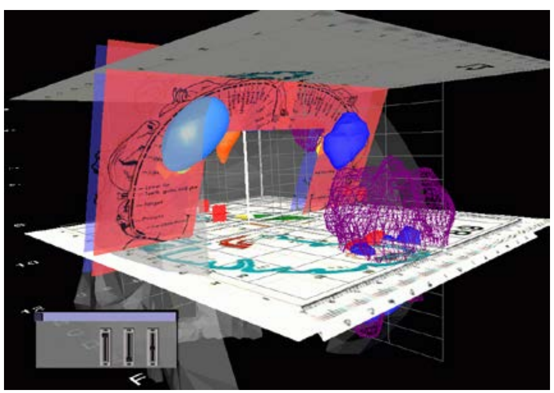
Search Apps

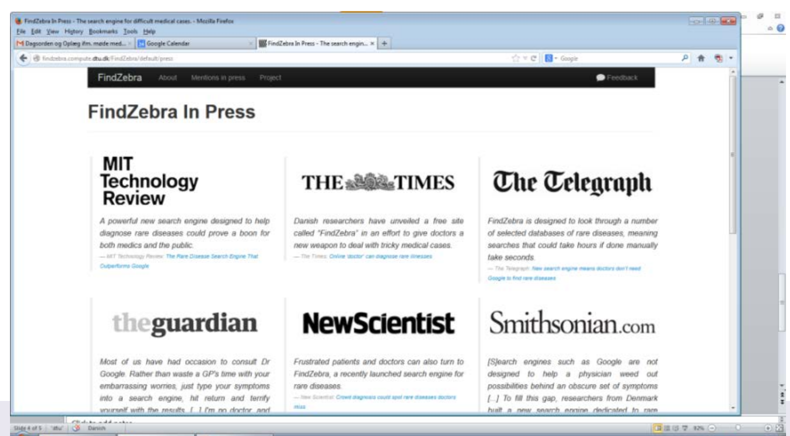
audio engines

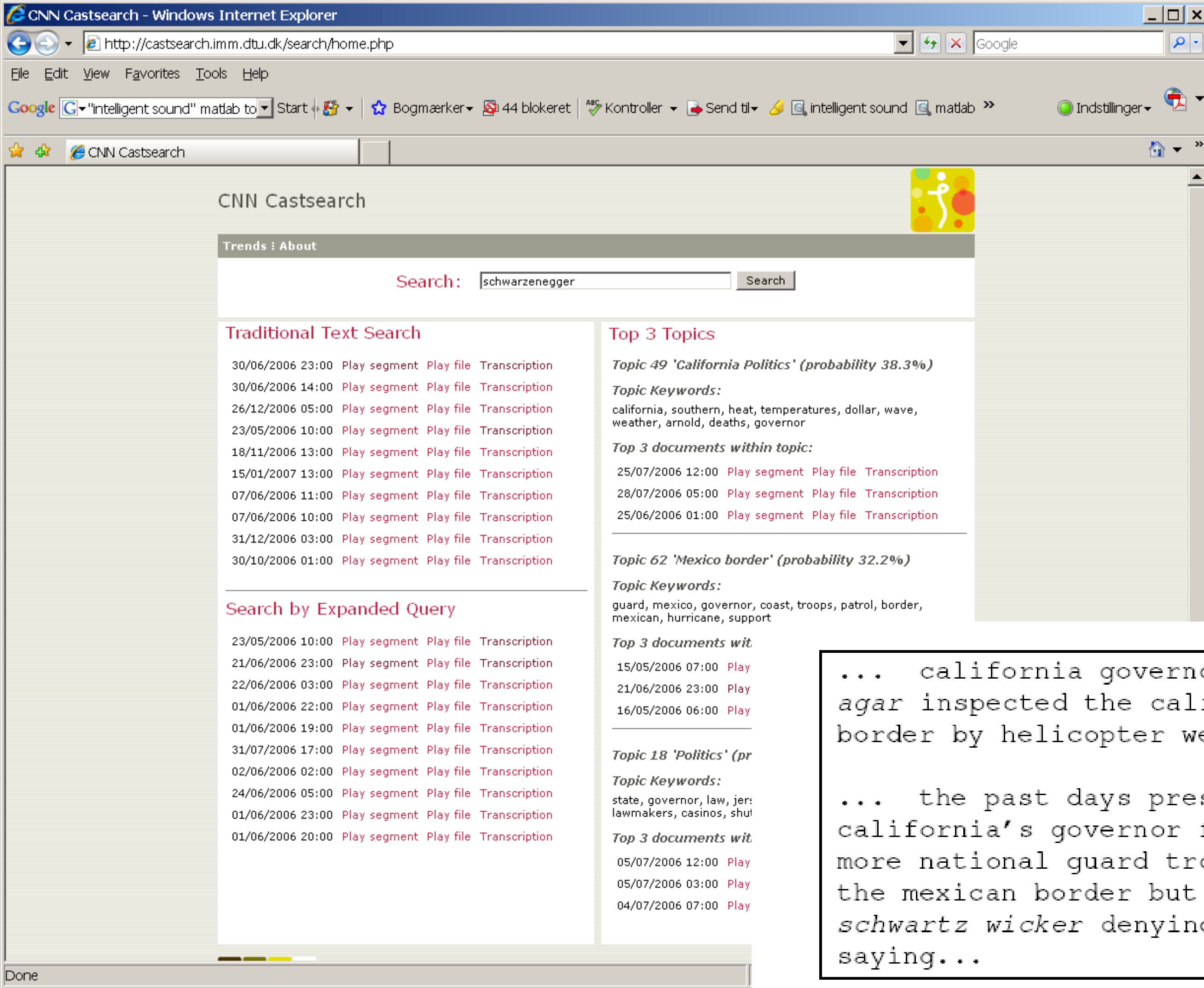
MIRocket (2006)
CastSearch (2007)
Muzeeker(2009)
CoSound (2014)

medical search engines

Brede Search neuroinformatics (2005)
FindZebra, diagnostic queries (2013)







... california governor arnold's *fortson agar* inspected the california mexico border by helicopter wednesday to see ...

... the past days president bush asking california's governor for fifteen hundred more national guard troops to help patrol the mexican border but governor *orville schwartz wicker* denying the request saying...

Fig. 2. Two examples of the retrieved text for a query on 'schwarze-negger'.

castsearch.imm.dtu.dk

CASTSEARCH - CONTEXT BASED SPEECH DOCUMENT RETRIEVAL

Lasse Lohilahti Mølgaard, Kasper Winther Jørgensen, and Lars Kai Hansen

Informatics and Mathematical Modelling
Technical University of Denmark Richard Petersens Plads
Building 321, DK-2800 Kongens Lyngby, Denmark

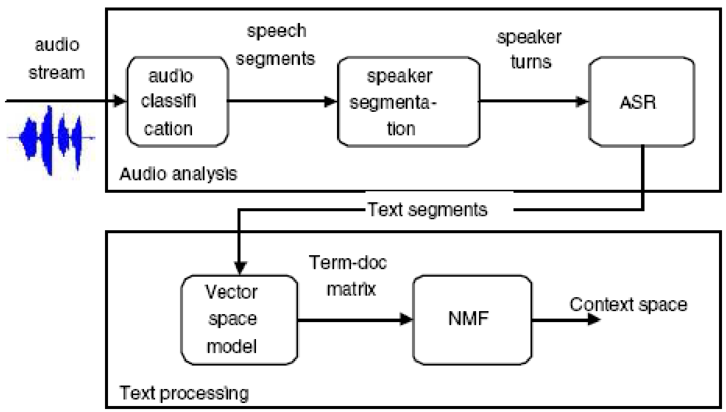
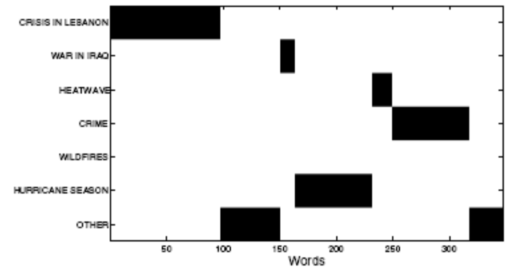
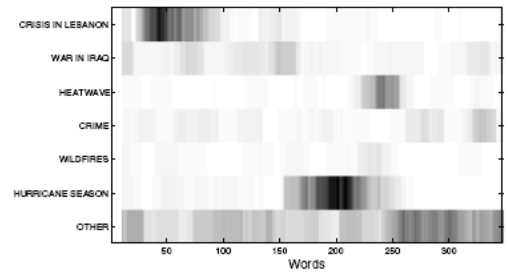


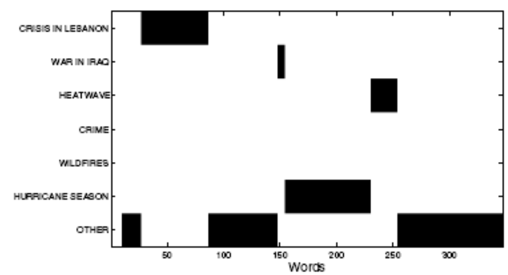
Fig. 1. The system setup. The audio stream is first processed using audio segmentation. Segments are then using an automatic speech recognition (ASR) system to produce text segments. The text is then processed using a vector representation of text and apply non-negative matrix factorization (NMF) to find a topic space.



(a) Manual segmentation.



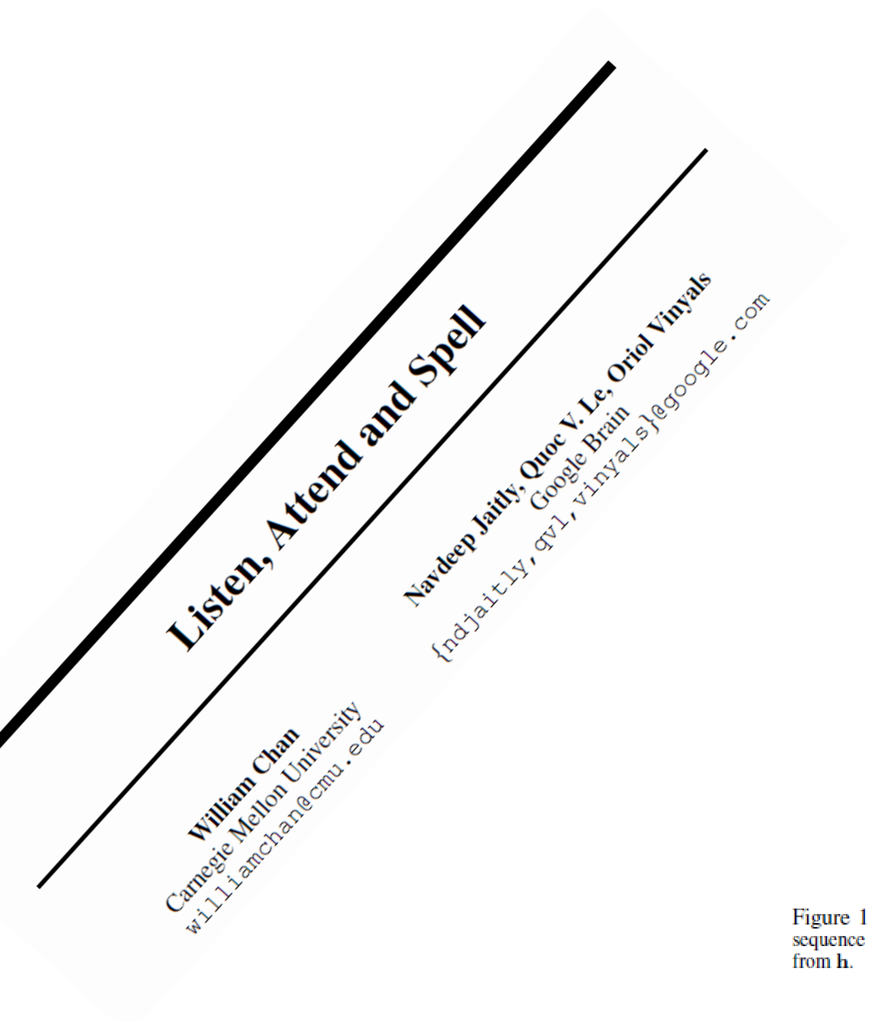
(b) $p(k|d^*)$ for each context. Black means high probability.



(c) The segmentation based on $p(k|d^*)$.

Fig. 3. Figure 3(a) shows the manual segmentation of the news show into 7 classes. Figure 3(b) shows the distribution $p(k|d^*)$ used to do the actual segmentation shown in figure 3(c). The NMF-segmentation is in general consistent with the manual segmentation. Though, the segment that is manually segmented as 'crime' is labeled 'other' by the NMF-segmentation

From pipelines to deep networks


Listen, Attend and Spell

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Navdeep Jaitly, Quoc V. Le, Oriol Vinyals
Google Brain
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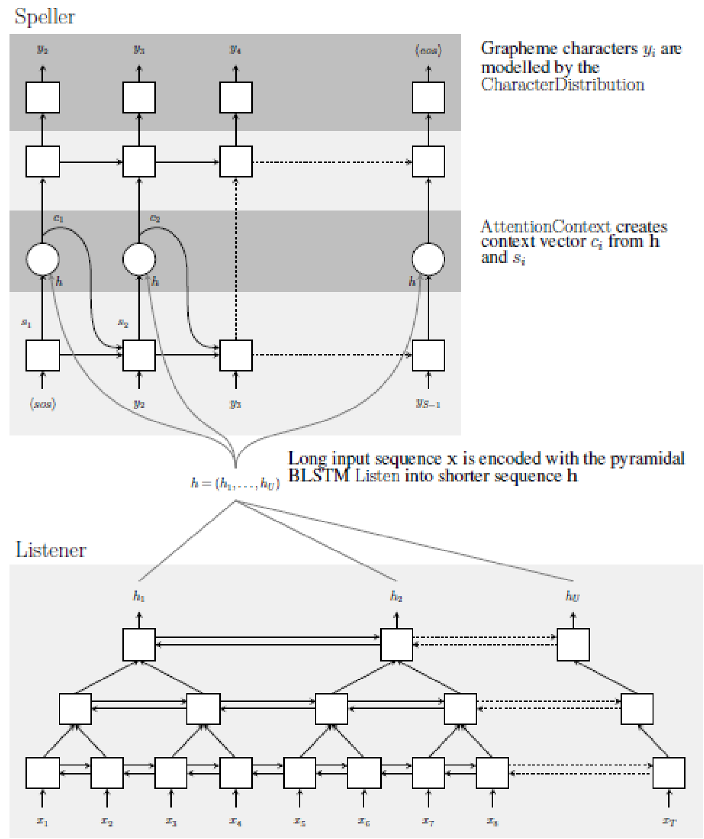


Figure 1: Listen, Attend and Spell (LAS) model: the listener is a pyramidal BLSTM encoding our input sequence $\mathbf{x}$ into high level features $\mathbf{h}$, the speller is an attention-based decoder generating the $\mathbf{y}$ characters from $\mathbf{h}$.

Machine learning: strategy for modeling objects in complex vector space representations (x)

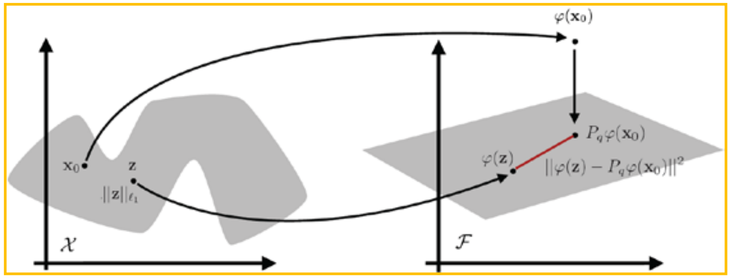
Measurements represented in vector spaces –
a point in a high-dimensional ”feature space”

Simplest learning idea: object similarity ~ spatial proximity
–similarity measures / distance metrics become important,

More realistic: proximity in complex manifolds representing the physical constraints
- ”long range” dependency

May need complex statistical models and lots of data (deep networks / big data)
to discover such representations from data

...e.g. ”objects” in image space occupy highly complex manifolds determined by
(approximate) invariances



Unsupervised learning in vector spaces

Identification of structure

- Preprocessing/ dim redux
- Missing data
- De-noising
- Novelty / outlier detection

Methods

- Clustering
- Signal separation linear/non-linear

$$p(\mathbf{x}|\theta)$$

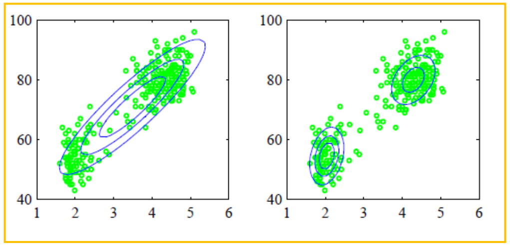
Measurement **Parameters**

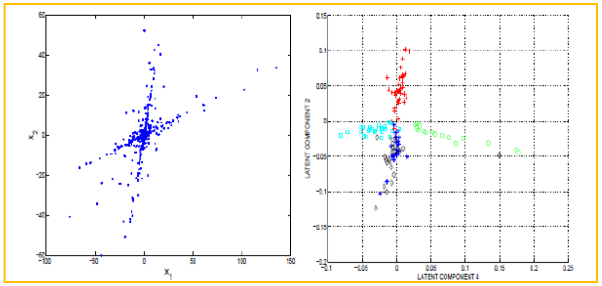
Analysis by synthesis: generative models

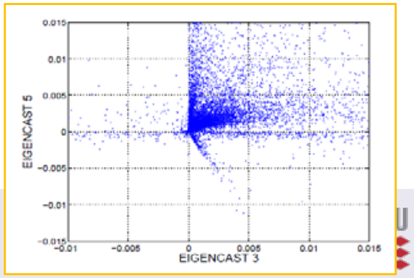
Cautionary note:

Careful distinction between **prior** (say clustering assumption) and **posterior** (how well does the assumption hold...)

ICA: components are independent in prior but not necessarily in the posterior







Supervised Learning in vector spaces

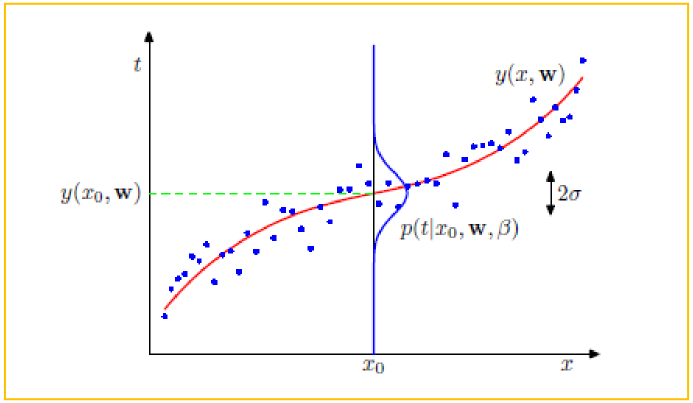
Issues

- Selection of model family
- **How to incorporate unlabeled samples?**
- Discriminative vs generative learning
- A priori knowledge constraints/probabilistic Learning (ML, MAP, Bayes)
- Outlier detection
  - Erroneous label
  - Unknown label
- Model structure optimization
- Performance issues
  - Generalization
  - Confidence
  - Consensus methods
  - Reproducibility
- Visualization/interpretation
  - Importance analysis
  - Why should I trust you?

$$p(t|\mathbf{x},\theta)$$

Parameters

Labels **Measurement**



TOWARDS COGNITIVE COMPONENT ANALYSIS

Lars Kai Hansen, Peter Ahrendt, and Jan Larsen

Intelligent Signal Processing,
Informatics and Mathematical Modelling,
Technical University of Denmark B321,
DK-2800 Kgs. Lyngby, Denmark

Unsupervised learning

The DNA of music?
Speech: Phonemes, gender, identity

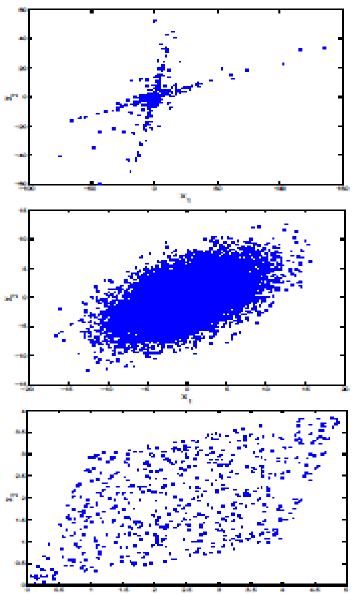


Figure 1. Prototypical feature distributions produced by a linear mixture, based on sparse (top), normal (middle), or dense source signals (bottom), respectively. The characteristic of the sparse signal is that it consists of relatively few large magnitude samples on a background of small signals.

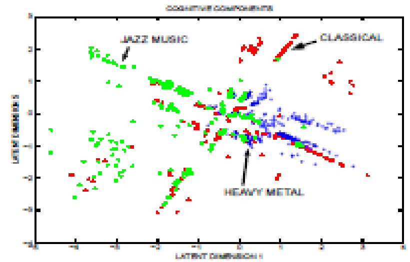


Figure 4. We represent three music tunes (with labels: heavy metal, jazz, classical) by their spectral content in overlapping small time frames (w = 30msec, with an overlap of 10msec, see [22], for details). To make the visualization relatively independent of 'pitch', we use the so-called mel-cepstral representation (MFCC, $K = 13$ coefficients pr. frame). To reduce noise in the visualization we have 'sparsified' the amplitudes. This was achieved simple by keeping coefficients that belonged to the upper 5% magnitude fractile. The total number of frames in the analysis was $F = 10^5$. Latent semantic analysis provided unsupervised subspaces with maximal variance for a given dimension. We show the scatter plot of the data on a 2D subspace within an original 5D PCA. For interpretation we have coded the data points with signatures of the three genres involved: classical ($*$), heavy metal (diamond), jazz ($+$). The ICA ray-structure is striking, however, note that the situation is not one-to-one (ray to genre) as in the small text databases. A component (ray) quantifies a characteristic musical 'theme' at the temporal level of a frame (30msec), i.e., an entity similar to the 'phoneme' in speech.

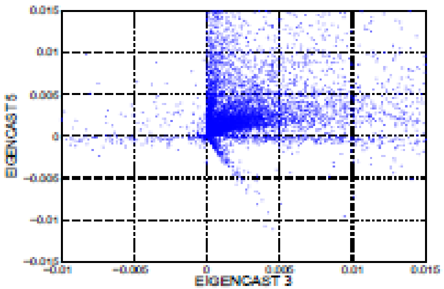


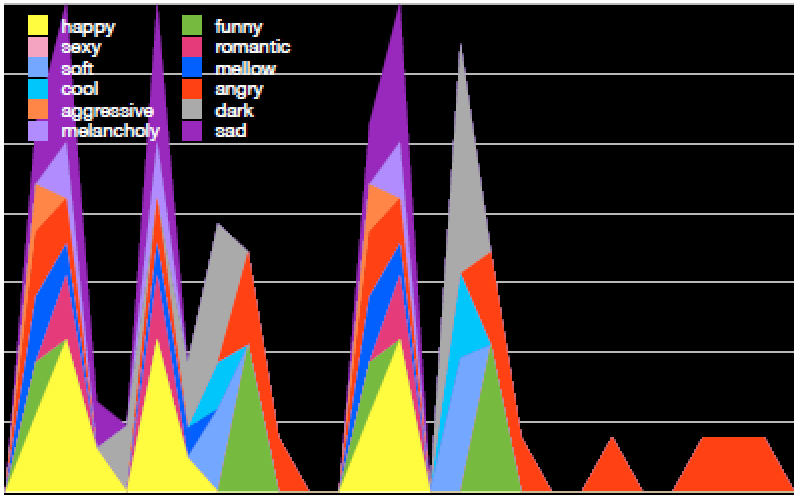
Figure 3. The so-called actor network quantifies the collaborative pattern of 382.000 actors participating in almost 128.000 movies. For visualization we have projected the data onto principal components (LSI) of the actor-actor co-variance matrix. The eigenvectors of this matrix are called 'eigencasts' and they represent characteristic communities of actors that tend to co-appear in movies. The network is extremely sparse, so the most prominent variance components are related to near-disjunct sub-communities of actors with many common movies. However, a close up of the coupling between two latent semantic components (the region $\sim (0,0)$) reveals the ubiquitous signature of a sparse linear mixture: A pronounced 'ray' structure emanating from (0,0). We speculate that the cognitive machinery developed for handling of independent events can also be used to locate independent sub-communities, hence, navigate complex social networks, a hallmark of successful cognition.

Supervised learning of sentiment

A step towards understanding subjectivity, opinion

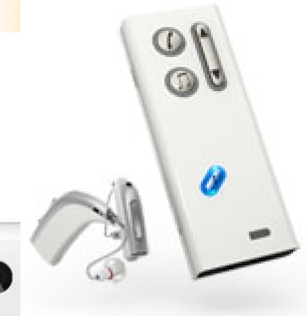
Important to many services
- Recommender (Amazon reviews)
- Information navigation, e.g. navigating music

Oasis ”Wonderwall”
Emotional content in the song lyrics through time



M.K. Petersen: Modeling media as latent semantics based on cognitive components (Ph.D. Thesis, DTU,2010)

- International
  - Google, Yahoo!, Microsoft
  - Recommender services: Amazon, NetFlix, Spotify
- DK
  - Hearing aids: Oticon, Widex, GN
  - Communication, digital media: Adform,
  - Monitoring: MAN B&W Diesel, Bruel & Kjær



Learning objectives

A student who has met the objectives of the course will:

Be able to discuss fundamental concepts in machine learning: Generalizability, likelihood functions, Bayesian modeling, and the bias-variance trade-off.

Be able to discuss fundamental concepts of signal detection: Bayesian decision theory, the role of posterior probabilities and loss functions.

Establish likelihood functions for signal modeling and detection.

Design adaptive linear and non-linear signal modeling and detection systems

Use cross validation to obtain un-biased performance estimates.

Evaluate adaptive linear and non-linear signal modeling and detection systems

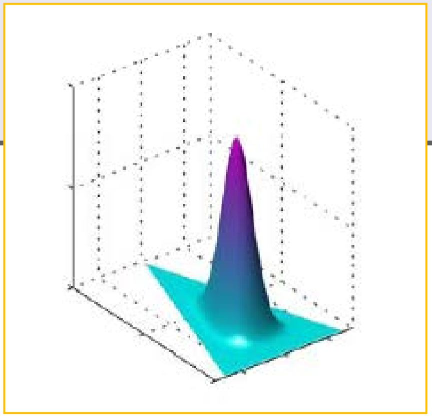
Explain implementations of adaptive systems and evaluation methods in Matlab.

Be familiar with applications of machine learning in audio signal detection.

Be familiar with applications of machine learning in bio-medical data.

Give a verbal presentation of results obtained in the course's Matlab exercises

What kinds of math are necessary?



Vectors and matrices

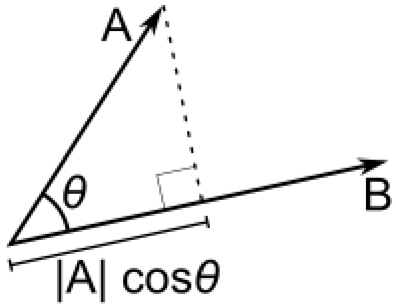
Eigenvectors and eigenvalues

Trace and determinant

Sequences and summation

Taylor and Fourier series

Functions on vector spaces and their derivatives (gradient and Hessian)



Integrals in one and “many” dimensions

Optimization theory:

Gradient descent,

Newton methods

slice

$$\begin{bmatrix} 2 & 1 & 2 \\ 3 & 2 & 3 \\ 4 & 1 & 1 \end{bmatrix} \mathbf{x} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2*1 + 1*0 + 2*1 \\ \\ \end{bmatrix}$$

Probabilities

We define the probability of an event as the limit of a frequency in a sampling process.

Flip a coin, count the number of heads (N_{head})

$$P(\text{head}) = \lim_{N \to \infty} \frac{N_{\text{head}}}{N}$$

Joint probabilities

head

Probabilities are limits of relative # counts (frequencies)

Event characterized by two "values" (x,y)

We observe pairs (x,y)

Examples

- Signal level vs bit (0,1)
- Foot length vs shoe size
- Image of surface vs quality of object

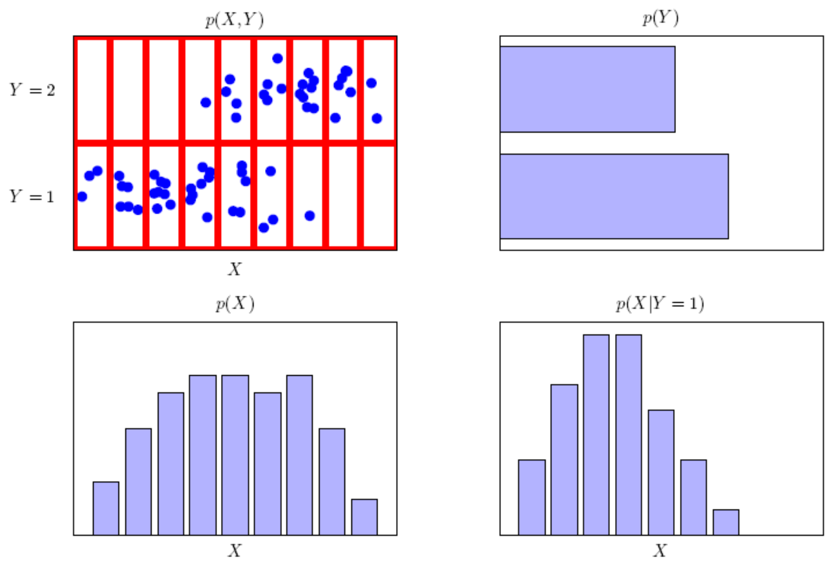
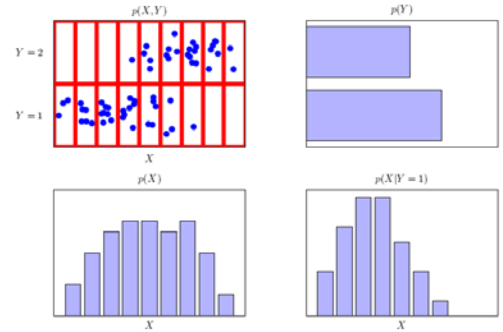


Figure 1.11 An illustration of a distribution over two variables, X, which takes 9 possible values, and Y, which takes two possible values. The top left figure shows a sample of 60 points drawn from a joint probability distribution over these variables. The remaining figures show histogram estimates of the marginal distributions $p(X)$ and $p(Y)$, as well as the conditional distribution $p(X|Y = 1)$ corresponding to the bottom row in the top left figure.



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Conditional probability

- Bayes' theorem

$$p(x,y) = \lim_{N\to\infty} \frac{N_{\mathrm{x,y}}}{N} = \lim_{N\to\infty} \boxed{\frac{N_{\mathrm{x,y}}}{N_x}} \frac{N_{\mathrm{x}}}{N} = p(y\,|\,x)\,p(x)$$

$$p(x,y) = \lim_{N\to\infty} \frac{N_{\mathrm{x,y}}}{N} = \lim_{N\to\infty} \frac{N_{\mathrm{x,y}}}{N_y} \frac{N_{\mathrm{y}}}{N} = p(x\,|\,y)\,p(y)$$

Probability density functions

$$P(x \in [a,b]) = \int_a^b p(x)dx$$

- Events with finite probability are intervals

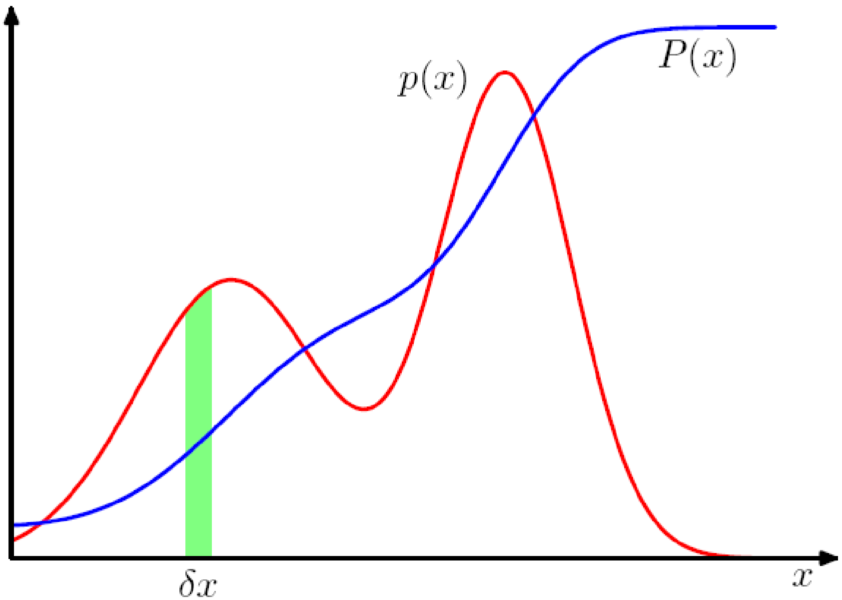


Figure 1.12 The concept of probability for discrete variables can be extended to that of a probability density $p(x)$ over a continuous variable x and is such that the probability of x lying in the interval $(x, x + \delta x)$ is given by $p(x)\delta x$ for $\delta x \to 0$. The probability density can be expressed as the derivative of a cumulative distribution function $P(x)$.

Expectations computed from the pdf

$$\mathcal{E}(f(x)) = \int_{\text{Domain of x}} f(x)p(x)dx$$

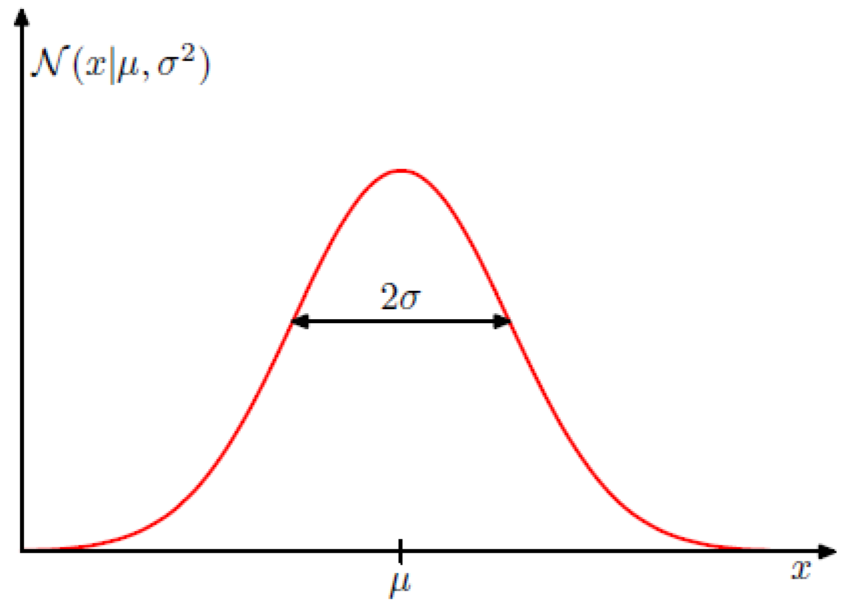
$$P(x \in \text{Domain of x}) = \int_{\text{Domain of x}} p(x)dx = 1$$

$$\mathcal{E}(x) \equiv \mu = \int_{\text{Domain of x}} xp(x)dx$$

The typical "spread" of the data

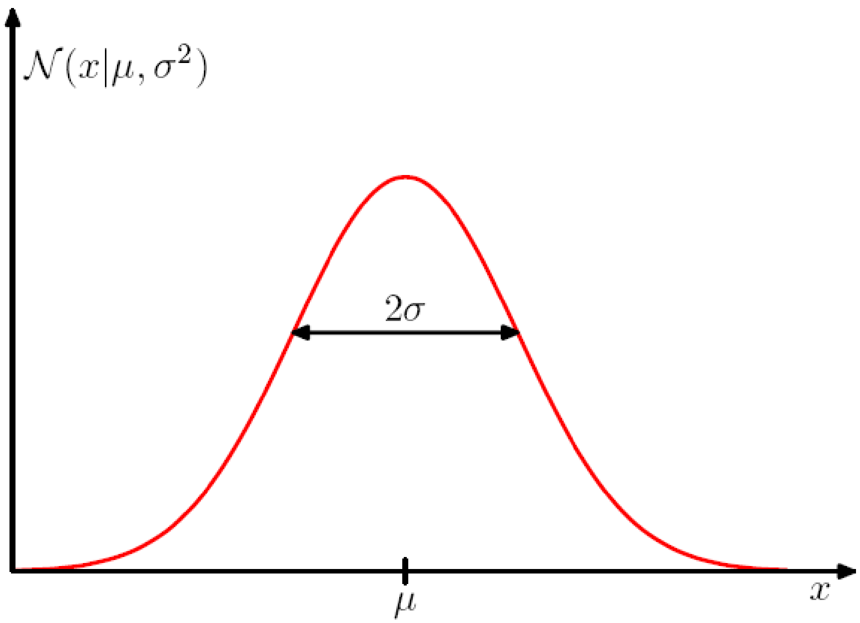
$$\sigma = \sqrt{\int_{\text{Domain of x}} (x-\mu)^2 p(x) dx}$$

Figure 1.13 Plot of the univariate Gaussian showing the mean μ and the standard deviation σ.



The normal distribution as a signal model

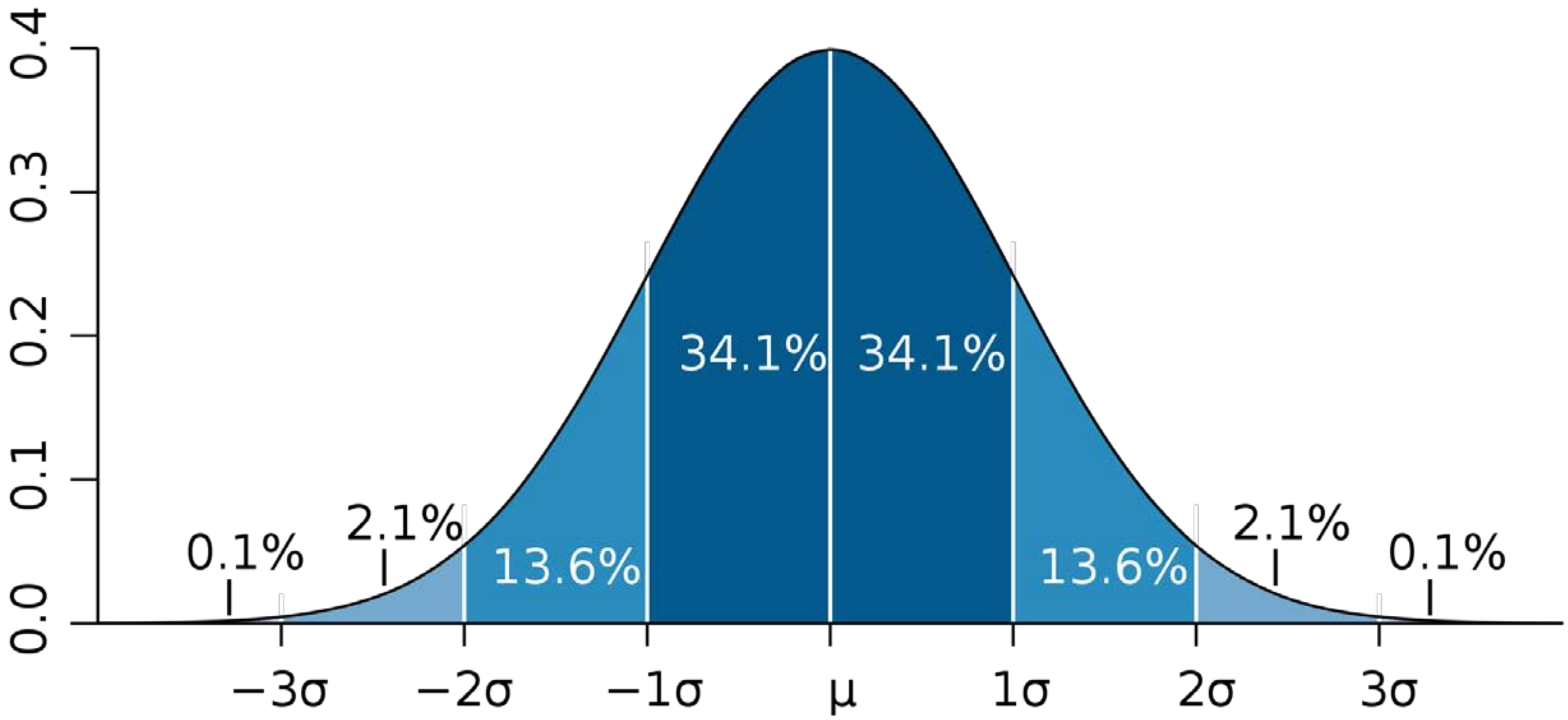
Figure 1.13 Plot of the univariate Gaussian showing the mean μ and the standard deviation σ.



If events can be described as a "mean effect" with additive noise

$$p(x|\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2\sigma^2}(x-\mu)^2\right)$$

Probabilities under the normal pdf curve



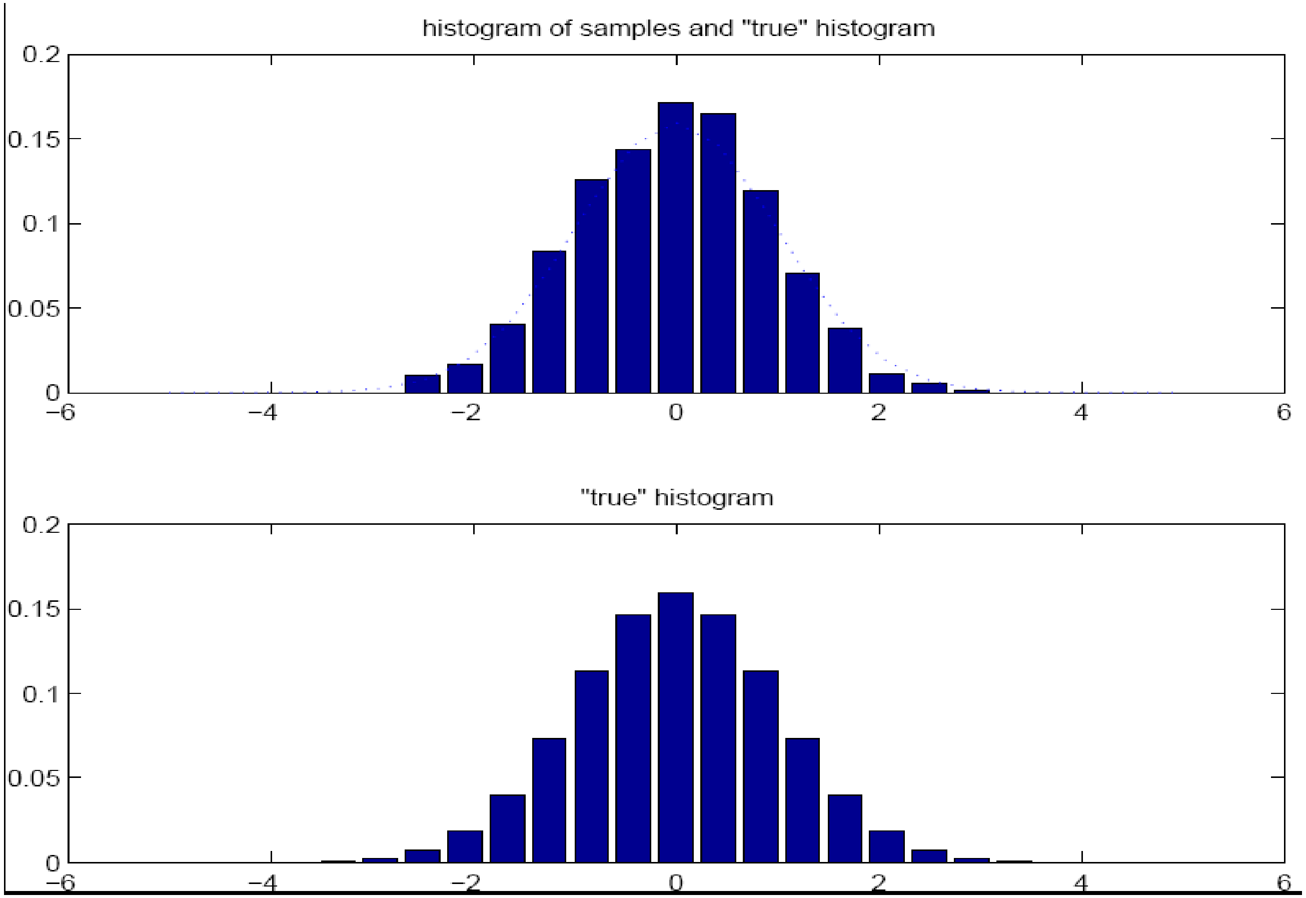
Parameters in the normal distribution

$$p(x|\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2\sigma^2}(x-\mu)^2\right)$$

$$\mu = \int_{-\infty}^{\infty} x \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2\sigma^2}(x-\mu)^2\right) dx$$

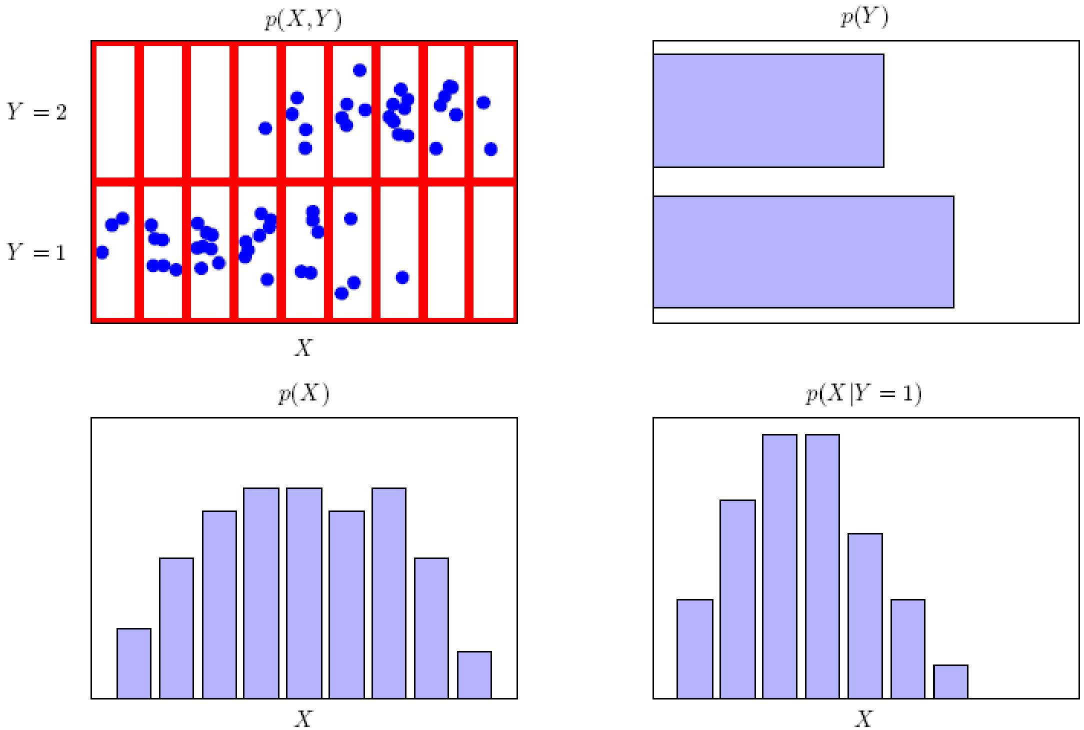
$$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2\sigma^2}(x-\mu)^2\right) dx$$

Models vs data



$$\mu = \int_{-\infty}^{\infty} x \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2\sigma^2}(x-\mu)^2\right) dx \qquad \mu_{\mathrm{ML}} = \frac{1}{N}\sum_{n=1}^{N} x_n$$

Signal detection problem



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$$P(\mathcal{C}_k, \mathbf{x}) = p(\mathbf{x}|\mathcal{C}_k)P(\mathcal{C}_k)$$
$$P(\mathcal{C}_k, \mathbf{x}) = P(\mathcal{C}_k|\mathbf{x})p(\mathbf{x})$$

$$P(\mathcal{C}_k|\mathbf{x}) = \frac{p(\mathbf{x}|\mathcal{C}_k)P(\mathcal{C}_k)}{p(\mathbf{x})}$$
$$p(\mathbf{x}|\mathcal{C}_k) = \frac{P(\mathcal{C}_k|X^l)p(\mathbf{x})}{P(\mathcal{C}_k)}$$

$$\sum_{k=1}^{c} P(\mathcal{C}_k|\mathbf{x}) = 1$$

$$\sum_{k=1}^{c} p(\mathbf{x}|\mathcal{C}_k)P(\mathcal{C}_k) = p(\mathbf{x})$$

Signal detection: Bayes decision theory

- A signal detection system (or pattern classifier) provides a rule for assigning a measurement to a given signal category (class)
- Hence, a classifier divides measurement space (feature space) into disjoint regions $\mathcal{R}_1, \mathcal{R}_2, ..., \mathcal{R}_c$, such that measurements that fall into region $\mathcal{R}_k$ are assigned with class $\mathcal{C}_k$.
- Boundaries between regions are denoted decision surfaces or decision boundaries

Signal Detection: Bayes decision theory

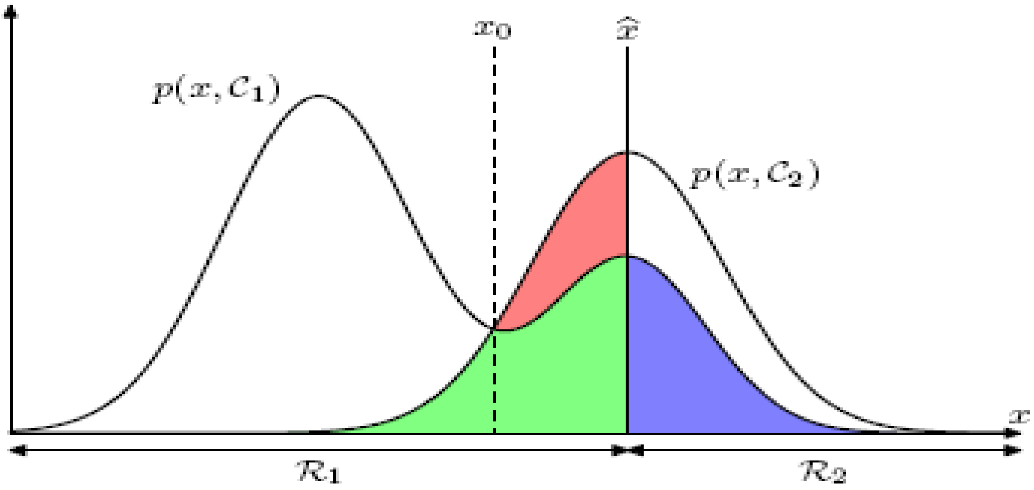

Figure 2: Schematic plot of the densities for a measured signal drawn from either of two populations $\mathcal{C}_1, \mathcal{C}_2$

$$
\begin{aligned}
P(\text{error}) &= P(x \in \mathcal{R}_2, \mathcal{C}_1) + P(x \in \mathcal{R}_1, \mathcal{C}_2) \\
&= P(x \in \mathcal{R}_2 | \mathcal{C}_1) P(\mathcal{C}_1) + P(x \in \mathcal{R}_1 | \mathcal{C}_2) P(\mathcal{C}_2) \\
&= \left(\int_{\mathcal{R}_2} p(x|\mathcal{C}_1) dx \right) P(\mathcal{C}_1) + \left(\int_{\mathcal{R}_1} p(x|\mathcal{C}_2) dx \right) P(\mathcal{C}_2)
\end{aligned}
$$

- The probability of error is minimized if we assign points to $\mathcal{R}_1$, whenever $p(x|\mathcal{C}_1)P(\mathcal{C}_1) > p(x|\mathcal{C}_2)P(\mathcal{C}_2)$

Signal detection: Use posterior for decision

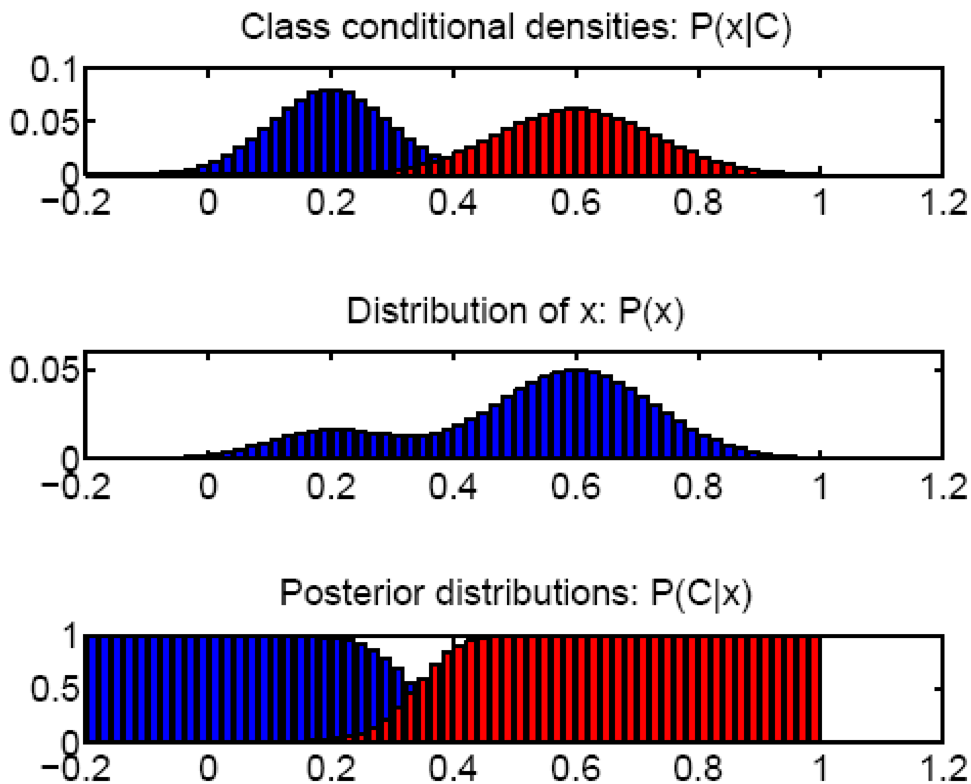

Figure 1: Schematic plot of the histograms for a measured signal drawn from either of t
populations $\mathcal{C}_1, \mathcal{C}_2$, density of x, and the corresponding posteriors $P(\mathcal{C}|X)$'s

$$P(\mathcal{C}_k, \mathbf{x}) = p(\mathbf{x}|\mathcal{C}_k)P(\mathcal{C}_k)$$

$$P(\mathcal{C}_k, \mathbf{x}) = P(\mathcal{C}_k|\mathbf{x})p(\mathbf{x})$$

$$P(\mathcal{C}_k|\mathbf{x}) = \frac{p(\mathbf{x}|\mathcal{C}_k)P(\mathcal{C}_k)}{p(\mathbf{x})}$$

$$p(\mathbf{x}|\mathcal{C}_k) = \frac{P(\mathcal{C}_k|X^l)p(\mathbf{x})}{P(\mathcal{C}_k)}$$

$$\sum_{k=1}^{c} P(\mathcal{C}_k|\mathbf{x}) = 1$$

$$\sum_{k=1}^{c} p(\mathbf{x}|\mathcal{C}_k)P(\mathcal{C}_k) = p(\mathbf{x})$$

Classification with asymmetric loss

$$
\begin{aligned}
R_k &= \sum_{j=1}^{c} L_{k,j} \int_{\mathcal{R}_j} p(\mathbf{x}|\mathcal{C}_k) d\mathbf{x} \\
R &= \sum_{k=1}^{c} R_k P(\mathcal{C}_k) \\
&= \sum_{j=1}^{c} \int_{\mathcal{R}_j} \sum_{k=1}^{c} L_{k,j} p(\mathbf{x}|\mathcal{C}_k) P(\mathcal{C}_k) d\mathbf{x}
\end{aligned}
$$

$$
\boxed{\sum_{k=1}^{c} L_{k,j} p(\mathbf{x}|\mathcal{C}_k) P(\mathcal{C}_k) < \sum_{k=1}^{c} L_{k,i} p(\mathbf{x}|\mathcal{C}_k) P(\mathcal{C}_k)}
$$