

The effects of imperfect information on Stern-judging in networks

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Abstract

Indirect reciprocity is one of the most fundamental concepts in the study of population dynamics and cooperation dynamics. We can find the effects and marks of this mechanism all over the planet in a variety of species, including humans. Stern-judging has proven to be one of the most robust assessment rules in indirect reciprocity, generally outperforming others and providing stable rates of cooperation in populations of various sizes when all individuals have access to the reputations of all other members of the population and no information errors occur. In this work, we analyze the effect of imperfect information on the performance of discriminators using stern-judging relative to full cooperators and full defectors, first in a well-mixed population and then in the context of a static random network (maybe scale free? might be too resource consuming). We show that in both of these population scenarios, the norm's effectiveness collapses.

I. Introduction

Reciprocity can be used to describe several behavioural patterns found in nature. The idea of "You scratch my back, I scratch yours" has been present in several species for hundreds of thousands of years. Standard reciprocity as a societal model faces a logical problem [1]. A society as a whole does benefit more from being made up of cooperators than defectors, however, on an individual level, each defector can extract more benefits than each cooperator since helping someone incurs a cost. This thought process would lead to societies full of defectors and no cooperators. This doesn't, however, happen in real life. A variety of biological systems actively use cooperation in their organization [3].

Social norms allow us to craft more accurate models of interactions in populations, especially human ones, that address this collapse in cooperation [2]. These social norms are derived from the idea of indirect reciprocity, which has proven to maintain mutual cooperation in populations [4] [5]. *Indirect reciprocity* makes use of the concept of reputations in individuals which adds a higher level of cognitive capacity to interactions [6]. An individual can, for example, choose to cooperate or not cooperate, based on the reputation of the individual he is interacting with. The way he might use this information on reputation will constitute a specific social norm. The degree of a norm depends on the amount of information used to decide on whether to cooperate or not cooperate, with each level of information constituting a "layer" [7].

Stern-Judging in particular is a second degree social norm that, given an interaction between two individuals, one donor and one recipient, will take into account the reputations of both the donor and recipient as well as the donor's action to calculate the donor's new reputation [7]. Stern-Judging has proven to be the most successful social norm

when it comes to maintaining high levels of cooperation across many population sizes [8], but how does it behave when faced with imperfect information in a way similar to what we would expect to see in real life interactions amongst individuals in a population? If we create a population mix with different types of agents like defectors cooperators and discriminators, what will be the distribution of these agents at the state of equilibrium? What will the average fitness look like? Here we extend on previous work related to imperfect information in social norm models like [9], first in a Well-Mixed population model and then in a random network.

II. Methods

I Evolution Model

Our evolution model, which will be used to evaluate which agents dominate given certain simulation parameters, is grounded in *evolutionary game theory*. This evolution dynamic will be applied to the three types of agents we will be mixing in our populations: cooperators, defectors and discriminators, where discriminators will follow the Stern-Judging social norm. A cooperator can, for example, turn into a defector and vice-versa.

In evolutionary game theory, the success of a strategy is tied to its *fitness*—a measure of how well an individual with that strategy performs relative to others in the population. The higher the fitness, the more likely a strategy is to proliferate. The evolution of strategies in a population follows the *replicator equation*, which describes how the frequency of a given strategy changes over time based on its fitness.

The general form of the *replicator equation* is given by:

$$\dot{x}_i = x_i (f_i(\mathbf{x}) - \phi(\mathbf{x}))$$

Where \dot{x}_i is the rate of change in the frequency of strategy i , x_i is the current frequency of strategy i , $f_i(\mathbf{x})$ is the fitness of individuals using strategy i , and $\phi(\mathbf{x})$ is the average fitness of the population. This equation implies that strategies whose fitness exceeds the population's average fitness will increase in frequency, while strategies with below-average fitness will decrease.

In addition to the natural selection process described by the replicator equation, our model also includes *mutation* based on fitness comparisons where each individual A has a certain probability of imitating individual B based on the difference in their fitness. That probability is given by the following equation:

$$p = \left[1 + e^{-\beta(f_B - f_A)}\right]^{-1}$$

Where p is the probability, f_B is the fitness of individual B and f_A is the fitness of individual A and β will have a value of 0.05.

II Well-Mixed population model, Stern-Judging and imperfect information

A large, well-mixed population of players is considered. From time to time, two players are chosen at random from the population and they engage in a one-shot prisoner's dilemma with the following payoff matrix:

	C	D
C	R, R	S, T
D	T, S	P, P

	C	D
C	0.4, 0.4	-1, 0.6
D	0.6, -1	0.0, 0.0

The values are designed to reflect real-world scenarios where cooperation provides mutual benefits, but defection can be tempting. Each individual has an assessment rule by which to judge the action of the donor. We assume a binary judgment: either the label 'good' or 'bad', represented by 1 and 0, is assigned to the donor. The social norm called stern-judging is considered for discriminator agents while co-operators and defectors always either cooperate or defect.

Stern-judging views those as good who, in their previous game, gave help to a good recipient or refused help to a bad recipient [7]. Discriminators using Stern-Judging will make use a reputation matrix to keep track of which individuals they consider 'good' or 'bad' and update said Matrix based on observations of actions. We assume that an assessment error occurs with a small probability.

When it comes to implementing the idea of imperfect information in this model, we have followed studies on the

Three Degrees of Influence [10], where information has a certain probability of reaching individuals depending on their distance to the action. We have used the value of 61%, used in [10], as the experimental value of "influence" for first-degree neighbours. Only this value is needed since in a well mixed population there are only first-degree neighbours. In the case of networks, the percentages for second and third-degree neighbours will be obtained from the same source. It should be noted that only discriminators make use of reputation to guide their decisions.

III Reputation matrix for Well-Mixed Population

Let us the notation r_{ij} to represent the reputation of player j in the eyes of player i . The values $r_{ij} = 1$ and $r_{ij} = 0$ correspond to the situations where i thinks that j is good and bad, respectively. The matrix (r_{ij}) is called the *reputation matrix*.

The update rule is described as follows: let u , v , and W denote two random individuals that interact, with u deciding on an action towards v , and the set of observers, which will be, on average 61% of the whole population. The number of observers is qN , where N is the total number of individuals in the population. For the analytical calculation, the population is assumed to be infinite.

After the game, the updated reputation matrix for discriminators is given by:

$$r'_{ij} = \begin{cases} r_{ij} & \text{if } i \notin W \text{ or } j \neq u \\ f(i)(a_u, r_{iv}) & \text{if } i \in W \text{ and } j = u \end{cases}$$

where a_u is the action of u toward v in the game, and r_{iv} is the current reputation of v in the reputation matrix.

Using the abbreviations C for "help" and D for "refuse", the function $f(i)(a, b) \in \{0, 1\}$ with $a \in \{C, D\}$ and $b \in \{0, 1\}$ is the assessment, from the viewpoint of i , of the action of a random node towards the recipient. For *stern-judging*, the function $f(i)$ is defined as:

$$f(i)(C, 1) = f(i)(D, 0) = 1$$

$$f(i)(C, 0) = f(i)(D, 1) = 0$$

and

$$f(i)(C, 1) = f(i)(D, 0) = 0$$

$$f(i)(C, 0) = f(i)(D, 1) = 1$$

Since the action a_u is probabilistically determined by r_{uv} , the new assessment r'_{wu} in the eyes of observer $w \in W$ probabilistically depends on the assessments of recipient v both in the eyes of u (r_{uv}) and w (r_{wv}) before the game. That is to say, the updated image matrix is probabilistically determined by the old image matrix.

IV Network Model

V Evaluation Criteria

For the purposes of evaluating our simulation results, we shall be using four criteria: Average fitness of the population at the end of a simulation, the percentage of cooperative interactions per time-step in the simulation, the overall population mix in comparison the starting mix, overall imperfection of information at the end of the simulation, taking a sample of 5% of the population for that calculation.

III. Results and Discussion

IV. Conclusion

V. Future work

In this work we built models to access the robustness of Stern-Judging under the strain of imperfect information in a variety of different contexts. We tested Stern-Judging discriminators with imperfect information against standard cooperators and standard defectors in a variety of different mixes. We also tested two different settings: well-mixed population and a random network. Other approaches could have been taken, however. For example, other network topologies like Scale-Free networks [13] [14] or a Watts-Strogatz network [12] could have been tested and compared to the results we obtained for random networks. Dynamic networks could be studied in this context as well.

Other social norms could have been analyzed and compared as well, like the work presented in [11]. Likewise, there are many possible approaches to imperfect information and many other models could have been used. In [11], imperfect information is represented by a cost associated with acquiring information about interactions. In [9], the sample of observers is chosen randomly and its number is random as well, while ours is fixed and their imperfect information comes in the form of an assessment error where an individual i has a certain chance of incorrectly reading an action and making the opposite judgment. Our model considers only the possibility of information not reaching certain individuals. Adding an assessment error could yield slightly different trends.

For networks, further degrees of neighbourhood could have been considered and the probabilities of information propagation could have been changed to investigate their impact. The values used in our model, associated with the Prisoner's Dilemma, could be varied to resemble a Stag-Hunt game [17] or a Snowdrift game [16] for example.

We treated reputation as a binary variable, with individuals being perceived as either "good" (1) or "bad" (0). This was done to facilitate computation. However, reputation could be a discreet variable with a limited number of possible values ranging between 0 and 1 or even a continuous variable between 0 and 1. What trends could be derived

from making such a change to nature of reputation?

Imperfect information models, because of their versatility, can be studied in many different ways, using many different criteria with many different purposes and therefore, there is much that can still be explored.

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A Appendix