

18.335 Problem Set 2

Due Fri. Mar 12, 2021, at 3pm via Stellar.

Problem 1: Stability

- (a) Trefethen, exercise 15.1. You will need to review the definition of “stability,” of which backwards stability is a (common) special case, in that chapter. [In parts (e) and (f), assume that $\frac{1}{k!}$ can be computed to $O(\epsilon_{\text{machine}})$ and concentrate on the accumulation of errors in the summations.]

Note: in part 15.1(c), the online version of Trefethen has a misprint: it should be $\tilde{f}(x) = x \odot x$ as an algorithm for $f(x) = 1$, **not** “ $\tilde{f}(x) = x$.”

- (b) Trefethen, exercise 16.1.

Hint: for $f(A) = QA$ where Q is unitary, it might be convenient to show that backwards stability of $\tilde{f}(A)$ **in this case only** is equivalent to proving $\|\tilde{f}(A) - f(A)\| = \|f(A)\|O(\epsilon_{\text{machine}})$ if you choose $\|\cdot\|$ to be the L_2 induced norm or the Frobenius norm. i.e. for multiplying by a *unitary* matrix, backwards stability is equivalent to the *forwards* error being small, which might be easier to analyze. (Essentially, this happens because unitary matrices have condition number $\kappa(Q) = 1$.)

Problem 2: Norms

- (a) Derive Trefethen eq. (3.10) (for which Trefethen only writes “by much the same argument”). Find the code that computes the induced $\|A\|_\infty$ norm in Julia, the `opnorm(A, Inf)` function, on github.com/JuliaLang/julia in `stdlib/LinearAlgebra/src/generic.jl` and satisfy yourself that it is equivalent to (3.10).
- (b) Trefethen, problem 3.4. Check your result for a random 10×7 matrix A in Julia, constructed by `A=randn(10,7)` with the induced $p = 2$ norm as computed by `opnorm(A)` in Julia.

Problem 3: SVD and low-rank approximations

- (a) Trefethen, problem 4.5.
- (b) Trefethen, problem 5.2.
- (c) Trefethen, problem 5.4.

Problem 4: Least squares

Trefethen, problem 11.2. Note that the $\Gamma(x)$ function is provided as `gamma(x)` by the `SpecialFunctions` package in Julia (execute `] add SpecialFunctions` to install this package).