# 18.335 Problem Set 4

Due Friday, April 9, 2021.

## Problem 1: Generalized eigenproblems

Consider the "generalized" eigenvalue problem $Ax = \lambda Bx$ where $A$ and $B$ are Hermitian and $B$ is positive-definite. You could turn this into an ordinary eigenvalue problem $B^{-1}Ax = \lambda x$, but then you disguise the Hermitian nature of the underlying problem.

(a) Show that the eigenvalue solutios $\lambda$ of $Ax = \lambda Bx$ must be real (with $x \neq 0$ as usual) and that eigenvectors $x_1$ and $x_2$ for distinct eigenvalues $\lambda_1 \neq \lambda_2$ must satisfy a modified orthogonality relationship $x_1^* B x_2 = 0$.

(b) A Hermitian matrix $A$ can be diagonalized as $A = Q\Lambda Q^*$; write down an analogous diagonalization formula arising from the eigenvectors $X$ (suitably normalized) and eigenvalues of the generalized problem, in terms of $\{A, B, X, \Lambda\}$. (Your formula should contain no matrix inverses, only adjoints like $X^*$.)

(c) Using the Cholesky factorization of $B$, show that you can derive an ordinary Hermitian eigenvalue problem $Hy = \lambda y$ where $H = H^*$ is Hermitian, the eigenvalues $\lambda$ are the same as those of $Ax = \lambda Bx$, and there is a simple relationship between the corresponding eigenvectors $x$ and $y$.

## Problem 2: Shifted-inverse iteration

Trefethen, problem 27.5.

## Problem 3: Arnoldi

Trefethen, problem 33.2.