18.335 Problem Set 2

Due Fri. Mar 4, 2022, at 12pm. Submit via Gradescope (available from Canvas navigation bar).

Problem 1: Stability

(a) Trefethen, exercise 15.1. You will need to review the definition of "stability," of which backwards stability is a (common) special case, in that chapter. [In parts (e) and (f), assume that $\frac{1}{k!}$ can be computed to $O(\epsilon_{\text{machine}})$ and concentrate on the accumulation of errors in the summations.]

Note: in part 15.1(c), the online version of Trefethen has a misprint: it should be $\tilde{f}(x) = x \oslash x$ as an algorithm for $f(x) = 1$, **not** "$\tilde{f}(x) = x$."

(b) Trefethen, exercise 16.1.

Hint: for $f(A) = QA$ where Q is unitary, it might be convenient to show that backwards stability of $\tilde{f}(A)$ **in this case only** is equivalent to proving $\|\tilde{f}(A) - f(A)\| = \|f(A)\|O(\epsilon_{\text{machine}})$ if you choose $\|\cdot\|$ to be the L_2 induced norm or the Frobenius norm. i.e. for multiplying by a *unitary* matrix, backwards stability is equivalent to the *forwards* error being small, which might be easier to analyze. (Essentially, this happens because unitary matrices have condition number $\kappa(Q) = 1$.) Once you have proved stability for multiplying by *one* Q, you can prove backwards stability for multiplying by *many* Q's using induction, for example.

Problem 2: Norms

(a) Derive Trefethen eq. (3.10) (for which Trefethen only writes "by much the same argument"). Find the code that computes the induced $\|A\|_\infty$ norm in Julia, the `opnorm(A, Inf)` function, on `github.com/JuliaLang/julia` in stdlib/LinearAlgebra/src/generic.jl and satisfy yourself that it is equivalent to (3.10).

(b) Trefethen, problem 3.3. Check that your proposed vectors and matrices achieve equality with `norm(x)` and/or `opnorm(A)` in Julia.

Problem 3: SVD and low-rank approximations

(a) Trefethen, problem 5.2.

(b) Trefethen, problem 5.4.

Problem 4: Gaussian elimination

(a) Trefethen, problem 20.2

(b) Trefethen, problem 21.4

(c) Trefethen, problem 23.3. Set up each experiment in Julia and compare timings with the results from the 1991 workstation included in the text.