Digit Recognition: Nearest Neighbors vs. Perceptron

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Abstract

Optical Character Recognition (OCR) is a complex computer science problem which has several effective machine learning solutions. We implemented the K Nearest Neighbors (KNN) and Kernelized Perceptron algorithms and compared the effectiveness of both methods in classifying a testing data set of 28,000 images. Each image is formatted as an array of 768 pixel values, with each value indicating how dark each pixel was in the image. Both of our algorithms were trained on a training data set of 42,000 images. We used cross validation to tune the K value in KNN and to choose a kernel function and dimension value for Perceptron. Despite its relative simplicity, we found that KNN was more effective than Perceptron in classifying the digits. The nearest neighbors algorithm correctly classified 96.857% of the testing samples on a weighted neighbors algorithm which looked at 4 neighbors. Our best Perceptron implementation correctly classified 95.67% of the testing samples using an exponential kernel with a $\sigma = 5$. Despite its higher accuracy, the runtime of KNN was 8x slower than that of the Perceptron.

1 The Problem

1.1 Image Data

Optical Character Recognition (OCR) is a computational problem where a machine attempts to classify an image of text. We tackled a subset of this problem, attempting to classify the digits 0-9 from data provided by the Kaggle Digit Recognition competition[1].

The Kaggle data provides two groups of data, 42,000 training samples with labels and 28,000 unlabeled testing samples. Each training sample contains a 48 pixel by 48 pixel image containing a handwritten number. The image is represented as an array of 768 pixel values which represent how dark each pixel of the image is. Table 1 is an example of a training sample.

[1] http://www.kaggle.com/c/digit-recognizer

Table 1: Subset of Kaggle Data

Label	Pixel 77	Pixel 78	Pixel 79
8	0	0	0
6	89	208	135

2 K Nearest Neighbors (KNN)

2.1 Overview

K Nearest Neighbors is a relatively simple algorithm which often gets surprisingly good results given its simplicity. Given a testing sample X, the algorithm looks through all the training samples it has previously seen and finds the *K* most similar samples. The algorithm then implements some form of voting mechanism among the *K* samples to classify the testing sample as a digit 0-9. The voting mechanism can either be as straightforward as selecting the majority label, to something more complicated which involves weighting the samples based on some distance metric.

2.2 Implementation Details

The first step in implementing KNN is to find the *K* nearest neighbors. This is accomplished by taking the euclidean distance between the testing sample we are classifying and every training sample in our training data set.

$$Euclidean(TestX, TrainX) = \sum_{i \in TrainX} (TestX - TrainX_i)^2$$

Because the image data is defined as an array of 768 pixels, $TestX - TrainX_i$ is actually doing a 768 dimension comparison between data points.

Once we identified the *K* nearest neighbors to our testing sample, we had to use this information to classify the testing point. We considered two approaches when deciding how to perform this classification.

2.2.1 Unweighted KNN

The first method we implemented to classify our testing sample was a simple majority voting algorithm which selects the majority label from the *K* closest neighbors. For example, if we ran the nearest neighbors algorithm with $K = 5$ and 3 of the neighbors had a label of 6, we would classify our testing sample as a 6.

2.2.2 Weighted KNN

Given the *K* nearest neighbors, the weighted version of the algorithm multiplies each label by a weight which indicates how close it is to our testing sample. These weights help to emphasize points that are closer to our testing sample while reducing the impact further away training points have on classifying our testing sample.

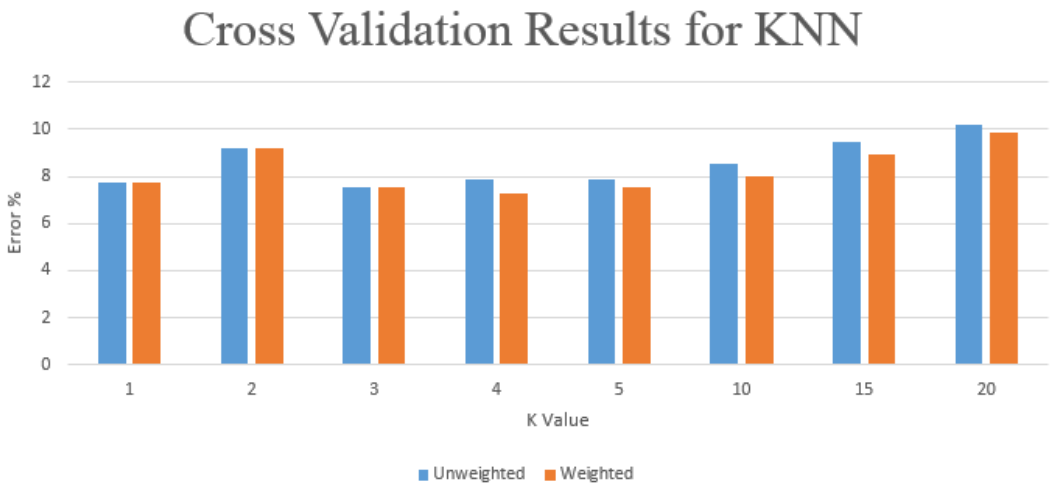
2.3 Cross Validation

Our implementation of KNN has a number of parameters which require tuning. In particular, we need to decide whether or not to use the weighted version of our KNN algorithm and how many neighbors to consider when implementing our algorithm.

To do this, we implemented K folds cross validation using 5,000 training samples. The reduced training set allowed us to run our algorithm using various configurations to see which configuration was most accurate.

Table 2: KNN Cross Validation Error Rates

K	Unweighted KNN Error	Weighted KNN Error
1	.077	.077
2	.092	.092
3	.075	.075
4	.078	.073
5	.078	.075
10	.085	.079
15	.095	.089
20	.102	.098



Our cross validation results suggest that running weighted KNN with $K = 4$ yields the most successful classification rate. One caveat to this is that our cross validation code ran using 5,000 training samples instead of the 42,000 samples in our full training set. It is possible that data we used to cross validate is no representative of the full data set.

2.4 Results

Our KNN implementation was extremely successful, correctly classifying 96.857% of the testing samples. One downside to it however is the runtime. Training is near instantaneous since the algorithm simply loads the data into an array. To classify a sample the algorithm needs to compute the euclidean distance between the testing sample and every training sample. In the case of the full data set this is 42,000 samples, each of which has 768 dimensions. As a result, each testing sample takes approximately. Altogether it takes close to 8 hours to classify all 28,000 testing samples.

3 Kernelized Perceptron

3.1 Overview

The perceptron algorithm attempts to separate a training data set into two classes and attempts to classify future testing samples as belonging to one of these two classes. It does this by storing a list of all the mistakes it has made previously and using this knowledge to adjust future predictions in the hopes of improving classification accuracy.

In its most basic implementation, the perceptron algorithm divides the data linearly. However, the data is often not linearly separable. To address this issue we use a kernel function to create a non-linear decision boundary. This allows us to have a more complex decision boundary which potentially fits the data better.

3.2 Implementation Details

Before we can begin classifying test points, we need to train the perceptron algorithm using our training data set. The training process attempts to classify each training point sequentially. If the perceptron correctly classifies the point nothing changes and the training process moves on to the next training sample. If the training sample is incorrectly classified, however, the mistake is added to an array of mistakes which is taken into consideration for future classifications.

One problem that needs to be addressed is that a standard perceptron is a binary classifier meaning it selects between one of two classes. We resolved this issue by training $\binom{10}{2=45}$ classifiers. Each classifier is responsible for classifying between two digits. For example, we would train a perceptron to classify between 0 and 1, and a second to classify between 0 and 2, and so on. When we classify we vote between all of our trained classifiers and output the majority element.

The actual classification is performed by:

$$classify(X) = \sum_{i \in Mistakes} Y_i K(X_i, X)$$

The kernel function is represented by $K(X_i, X)$. There are a number of choices for this kernel function.

3.3 Cross Validation for Kernel Selection

We implemented two kernel functions to use with our perceptron algorithm. One is a polynomial kernel which takes the form:

$$Polynomial(x, y, d) = (dot(x, y) + 1)^d$$

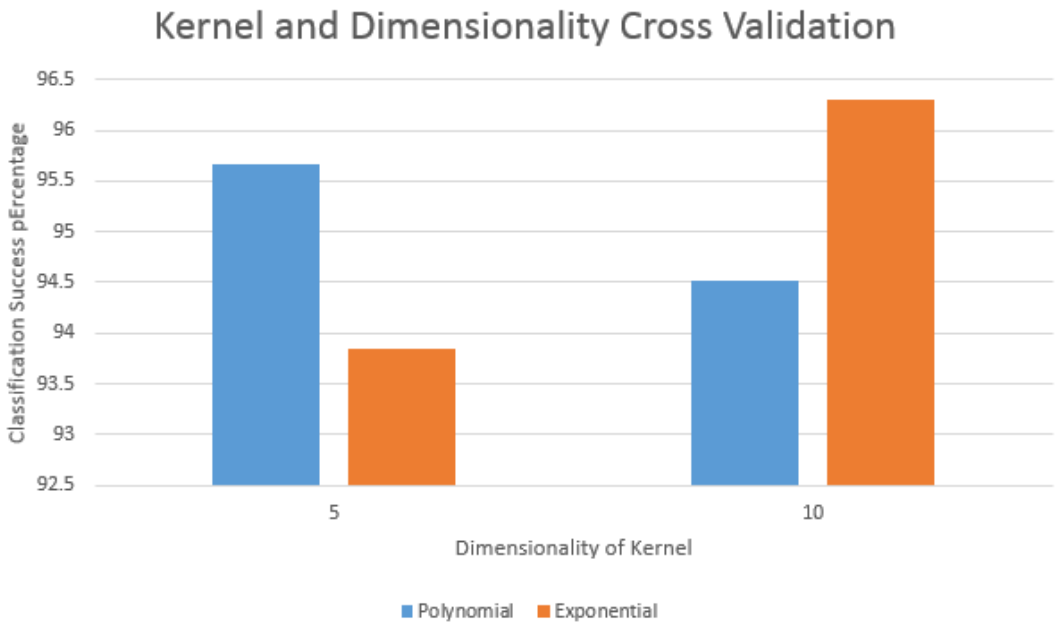
The second is an exponential kernel:

$$Exponential(x, y, \sigma) = exp(-\frac{||x-y||}{2\sigma^2})$$

Both of these kernels have a parameter which needs to be tuned, d in the case of the polynomial kernel and σ in the case of the exponential kernel. We used cross validation to find out which kernel gave us the best result as well as which value of d and σ was optimal.

Table 3: Perceptron Cross Validation Success Rates

Kernel	$d, \sigma = 5$	$d, \sigma = 10$
Polynomial	95.67%	94.52%
Exponential	93.84%	96.3%



3.4 Results

Our cross validation indicates that the exponential kernel with a σ of 10 leads to the highest success rate of 96.3%. An interesting observation is that our polynomial kernel actually became less successful when it's dimension was increased whereas the exponential kernel got better. We suspect that this is because the polynomial kernel began to overfit the training data.

Finally, the runtime of the perceptron algorithm is a little over an hour. It takes approximately 45 minutes to train and about 20 minutes to test the data. The faster runtime allowed us to cross validate on the full data set.

4 Conclusion

Despite its seeming simplicity, we found that KNN was extremely successful when classifying data points in the Kaggle set.

Algorithm	Percent Success	runtime
Nearest Neighbors, $k = 4$	96.857%	$\sim$8 hours
Perceptron	96.300%	$\sim$1 hour

Table 4 summarizes the accuracy and runtimes of weighted KNN with $K = 4$ and perceptron with the polynomial kernel and $d = 10$. KNN is clearly more accurate, however because perceptron is 8x faster and almost as accurate, there could be many cases where it is a better choice.

In addition to being faster, perceptron likely has a lower memory footprint. While KNN must store every sample in the training set, perceptron only stores mistakes which will be a subset of the entire training set.

5 Future Work

There are three specific expansions to our work we would have liked to explore had we had time.

- Expanded cross validation for KNN using the full training set. This would help validate that the weighted voting algorithm with $K = 4$ leads to the most accurate.
- More kernels and a greater assortment of σ and d values to see if we can increase the effectiveness of the perceptron algorithm.
- An SVM implementation which regularizes the data to avoid overfitting. If the perceptron implementation overfits the data, this would be a potential way to increase the accuracy.